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New argument in favor of asymptotic SU(3): $\frac{3}{2}^+$-decuplet mass spacing and decays

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We argue that the recent indication from high-statistics experiments that the $\frac{3}{2}^+$ decuplet satisfies equal mass-squared spacing favors a nonperturbative point of view towards broken SU(3), i.e., an algebraic approach based on the hypothesis of asymptotic SU(3) in which, apart from a mixing effect, the Gell-Mann–Okubo mass formula is exact. A further test in the strong decays of hadrons involving pseudoscalar-meson emission is also discussed. We also add a critical remark about the SU(3) formula traditionally used in the SU(3) test of strong decays of hadrons involving pseudoscalar-meson emission.

The Gell-Mann–Okubo (GMO) mass formula was originally derived as a perturbation-theoretic formula but was found to fit the mass splittings of $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons well. However, it was then pointed out, in particular by Sakurai, that the inclusion of large singlet-octet mixing is necessary for boson mass splittings. For the past decade, the GMO formula including mixing has been treated as if it were exact, and general practice (except for a few attempts) has been to use a *linear* mass formula for baryons and a *mass-squared* one for bosons. We wish to discuss the following two questions: (i) Why does the first-order formula work so well? (ii) In what form (mass or mass squared) should the SU(3) mass formula be valid?

We now remark on the rather striking recent experimental development regarding the masses of the $\frac{3}{2}^+$ decuplet.

The recently improved values of Ξ^{*-} and Ω^- masses are 1535.0 ± 0.6 MeV and 1672.5 ± 0.5 MeV, respectively.[1] The mass value of Y_1^{*-} of the recent high-statistics experiment of Borenstein *et al.*[2] is 1383 ± 2 MeV (where the error includes systematic effects), so that (we use $m_{\Omega^-} \equiv \Omega^-$, etc.)

$$\begin{aligned} \Omega^- - \Xi^{*-} &= 137.5 \pm 0.8 \text{ MeV}, \\ \Xi^{*-} - Y_1^{*-} &= 152 \pm 2 \text{ MeV}. \end{aligned} \tag{1}$$

Therefore, equal *mass* spacing is appreciably violated. However, Borenstein *et al.* also pointed out[2] that the *mass-squared* spacings are given by

$$\begin{aligned} (\Omega^-)^2 - (\Xi^{*-})^2 &= 0.443 \pm 0.006 \text{ GeV}^2, \\ (\Xi^{*-})^2 - (Y_1^{*-})^2 &= 0.441 \pm 0.003 \text{ GeV}^2, \end{aligned} \tag{2}$$

and these are *equal* within errors.

Another slightly earlier high-statistics experiment of Baltay *et al.*[3] also pointed out a sizable violation of equal *mass* spacing for $\Omega^- - \Xi^{*-}$ and $\Xi^{*-} - Y_1^{*-}$. The remaining member of the decuplet, Δ^-, has a broader width ($\simeq 100$ MeV), and the central value of its mass is harder to determine. However, the extension of equal *mass-squared* spacing to $(Y_1^{*-})^2 - (\Delta^-)^2$ gives[2] $\Delta^- = 1213 \pm 2.5$ MeV and is close to the "pole" value[4] of Δ. The $\frac{1}{2}^+$ octet is known to satisfy also a *mass-squared* formula $(\Sigma^0)^2 + 3(\Lambda^0)^2 = 2[(n^0)^2 + (\Xi^0)^2]$ well.

In this note we wish to point out that if the $\frac{3}{2}^+$ decuplet indeed satisfies the mass-squared formula well, along with the $\frac{1}{2}^+$ octet, as the recent experiments indicate, it provides a favorable argument for a *nonperturbative* approach to broken SU(3) symmetry.

We call the reader's attention to the following: (i) A nonperturbative derivation of the SU(3) mass formula is possible within the framework of asymptotic SU(3) proposed some time back by Matsuda and Oneda[5] and reformulated by Oneda, Umezawa, and Matsuda.[6] (ii) The derived SU(3) mass formula takes the mass-squared form, including various types of SU(3) mixings, irrespective of spins of hadrons.[7]

We also discuss SU(3) in the rates of $\frac{3}{2}^+ \to \frac{1}{2}^+ + \pi$ decays and add a critical remark about the traditional treatment of SU(3) in the strong decays of hadrons involving pseudoscalar-meson emission.

Physical (i.e., incoming) hadrons are denoted by $B_{\alpha,s}$, where α denotes the physical SU(3) members such as Δ_s, Y_s^*, Ξ_s^*, Ω_s, etc., and s stands for J^{PC} and other quantum numbers. Without taking the usual perturbation-theoretic point of view, we suspect[5,6] that SU(3) is broken in such a way that linearity of the SU(3) transformation is still maintained in some asymptotic limit.

We thus demand[5,6] that the annihilation (or creation) operator $a_{\alpha,s}(\vec{k},\lambda)$ of $B_{\alpha,s}$ (λ denotes helicity) transforms still linearly in broken SU(3) but only in the limit $\vec{k}\to\infty$.

We write,[8] with the SU(3) generator V_i,

$$[V_i, a_{\alpha,s}(\vec{k},\lambda)] = i\sum_{\beta,t} u_{i\alpha\beta,st}\, a_{\beta,t}(\vec{k},\lambda) + \delta u_{i\alpha}\,, \qquad (3)$$

picking up all possible terms linear in $a_{\beta,t}(\vec{k},\lambda)$. Here the subscript t includes not only s but also other hadrons with the *same* J^{PC} or J^P as s, in anticipation of the possible occurrence of B_s-B_t mixing. The remainder is denoted by $\delta u_{i\alpha}$. We then demand that $\delta u_{i\alpha}\to 0$ as $\vec{k}\to\infty$.

Therefore, $a_{\alpha,s}(\vec{k},\lambda)$ can be *linearly* related (but only at $\vec{k}\to\infty$) to the (hypothetical) exact SU(3) representation operator $a_{j,t}(\vec{k},\lambda)$, i.e., $a_{\alpha,s}(\vec{k},\lambda) = \sum_{j,t} C_{\alpha j,st}\, a_{j,t}(\vec{k},\lambda)$, as $\vec{k}\to\infty$. (t is defined above.) $C_{\alpha j,st}$ involves SU(3) mixing parameters.[5,6]

The imposition of the SU(3) charge commutation relation (CR), $[V_i, V_j] = if_{ijk}V_k$, valid in *broken* SU(3), then enables us to fix the asymptotic values of $u_{i\alpha\beta,st}$ in Eq. (3) in the process of realizing the CR in the limit $\vec{k}\to\infty$. The net results are that whereas diagonal matrix elements of the SU(3) charge V_K ($V_{K^0} = V_6 + iV_7$, etc.), such as

$$\langle B_{\alpha,s}(\vec{k},\lambda)|V_K|B_{\beta,t}(\vec{k},\lambda)\rangle,$$

will take the SU(3) plus mixing values (prescribed by $C_{\alpha j,st}$) in the limit $\vec{k}\to\infty$ according to Eq. (3), all nondiagonal matrix elements vanish as $\vec{k}\to\infty$, i.e., for example,

$$\langle B_{\alpha,s}(\vec{k},\lambda)|V_K|B_{\beta,u}(\vec{k},\lambda')\rangle = 0,\ \vec{k}\to\infty \text{ and } s\neq u. \qquad (4)$$

In exact SU(3) Eq. (4), of course, holds for any $\vec{k}$.

If the SU(3)-breaking interaction belongs to an $I = Y = 0$ octet, the following exotic CR's involving $\dot{V}_K = (d/dt)V_K$ are satisfied[9]: $[\dot{V}_{K^0}, V_{K^0}] = 0$, etc. We now discover the SU(3) mass formula as a constraint in the process of realizing the CR in our asymptotic limit. Insert the CR between the states $\langle Y_1^{*-}(\vec{k},\lambda)|$ and $|\Omega^-(\vec{k},\lambda)\rangle$ with $\vec{k}\to\infty$. We obtain, extracting the diagonal contribution [neglecting possible SU(3) mixing and keeping SU(2) symmetry] out of the infinite sum over intermediate states n,

$$[(E_Y - E_{\Xi^*}) - (E_{\Xi^*} - E_\Omega)]\langle Y_1^-|V_{K^0}|\Xi^{*-}\rangle\langle \Xi^{*-}|V_{K^0}|\Omega^-(\vec{k})\rangle + \sum_n [(E_Y - E_n) - (E_n - E_\Omega)]\langle Y_1^-|V_{K^0}|n\rangle\langle n|V_{K^0}|\Omega^-(\vec{k})\rangle = 0\,,$$

where $E_\Omega = (\Omega^2 + \vec{k}^2)^{1/2}$, etc. Consider a single-particle (stable and resonance) approximation for the state n. As $\vec{k}\to\infty$ the diagonal term, $\langle Y_1^*|V_K|\Xi^*\rangle\langle\Xi^*|V_K|\Omega\rangle$, approaches a finite SU(3) value, whereas the nondiagonal terms, $\langle Y_1^*|V_K|n\rangle \times\langle n|V_K|\Omega\rangle$, vanish due to Eq. (4). Therefore, our sum rule (obtained in order $1/|\vec{k}|$) yields mass-squared equal spacing $(\Xi^*)^2 - (Y_1^{*-})^2 = (\Omega^-)^2 - (\Xi^{*-})^2$. Even without the single-particle approximation, a similar argument may be made, if $\delta u_{i\alpha}$ in Eq. (3), which governs the asymptotic behavior of the nondiagonal matrix element of V_K, vanishes sufficiently fast as $\vec{k}\to\infty$. This is, of course, a dynamical possibility. Repeating the same argument, we obtain

$$(\Omega_s^-)^2 - (\Xi_s^{*-})^2 = (\Xi_s^{*-})^2 - (Y_s^-)^2 = (Y_s^-)^2 - (\Delta_s^-)^2,$$

and

$$\Sigma_s^{\,2} + 3\Lambda_s^{\,2} = 2(n_s^{\,2} + \Xi_s^{\,2})$$

for the decuplet and octet baryons and

$$\sin^2\theta_s = (3\eta_s^{\,2} - 4K_s^{\,2} + \pi_s^{\,2})3^{-1}(\eta_s^{\,2} - \eta_s'^{2})^{-1}$$

for the boson nonet $(\pi_s, K_s, \eta_s, \eta_s')$ with η_s-η_s' mixing angle θ_s. Note that in deriving these mass-squared SU(3) mass formulas no perturbation-theoretic argument has been involved. Therefore, the formulas are *exact*, apart from the effect of possible further mixing. We have considered only the most important singlet-octet mixing of the boson nonet, which probably belongs to the *same* orbital excitation as the simple quark model. The large values of such mixing angles are not surprising, since we do not use any perturbation-theoretic argument. Other types of boson mixing will be less important.[10] For the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ *ground*-state baryons, SU(3) mixing is possible *only* with the neighboring *radially* excited states. This type of mixing among different levels of excitation is expected to be small and the order of magnitudes of the mixing parameters is denoted by ϵ. [We note, however, that the SU(3) mixing among the baryons belonging to the *same* excitation can be very large.] From its derivation, our SU(3) mass formula always involves only the *squares* of mixing parameters. Thus, the effect of mixing on our $\frac{1}{2}^+$ and $\frac{3}{2}^+$ mass formulas is only of order ϵ^2. We estimate[11] $\epsilon \simeq 0.1$, since the deviation from our mass-squared mass formula is only around 1.5% for the $\frac{1}{2}^+$ octet. It will then be natural to expect a similar accuracy also for our mass-squared $\frac{3}{2}^+$ mass formula. Therefore, the recent rather drastic change in the masses of $\frac{3}{2}^+$ decuplet which now favors equal mass-squared spacing lends a support for our nonperturbative derivation of SU(3) mass formula.

A further (less precise but useful) test of asymptotic SU(3) can be obtained by testing, for example, the broken SU(3) sum rules based on the CR, $[V_i, A_j] = if_{ijk}A_k$, and asymptotic SU(3) (but now allowing for the effect of mixing at least of order ϵ). We discuss $B \to B' + P$ decays including $\frac{3}{2}^+ \to \frac{1}{2}^+ + \pi$ decays. Our result challenges the usual perturbation-theoretic belief that coupling constants are not sensitive to broken SU(3). As an illustration we discuss the decay $B_\alpha(J = l + \frac{1}{2},\ P = \pm 1) \to B'_\beta(\frac{1}{2}^+) + P_\gamma(0^-)$. Our procedure can also be extended to scattering.[12] The physical coupling constant can be defined by

$$\langle B'_\beta(\vec{k}')|J_{P_\gamma}(0)|B_\alpha(\vec{k})\rangle = ig_{B'BP}[B'B/E(B')E(B)]^{1/2}\bar{u}(\vec{k}')[1 \text{ or } \gamma_5]u_{\mu_1\cdots\mu_l}(\vec{k})q_{\mu_1}\cdots q_{\mu_l}\,.$$

Here $q = k' - k$ and $u_{\mu_1\cdots\mu_l}(\vec{k})$ with $\mu_i = 1, 2, 3, 4$ is the Rarita-Schwinger spinor of B_α. $J_{P_\gamma}(x)$ is the source function of the pseudoscalar meson P_γ. The rate of $B_\alpha \to B'_\beta + P_\gamma$ is then given by

$$\Gamma(B_\alpha \to B'_\beta + P_\gamma) = \left(\frac{g_{B'BP}{}^2}{4\pi}\right)\frac{2^l(l!)^2}{(2l+1)!}\left[\frac{(B \pm B')^2 - P_\gamma{}^2}{2B^2}\right]p^{2l+1}\,. \quad (5)$$

p is the c.m. momentum of P_γ, and $\pm$ signs must be chosen depending on the parity $P = \pm 1$. Define $G_{B'_\beta B_\alpha P_\gamma} \equiv \langle B'_\beta(\vec{k})|A_\gamma|B_\alpha(\vec{k})\rangle$, $\vec{k} \to \infty$. A_γ is the axial charge with SU(3) index γ. $g_{B'BP}$ is now related to $G_{B'BP}$, because of partial conservation of axial-vector current (PCAC), in the soft-meson limit[7]

$$g_{B'_\beta B_\alpha P_\gamma}(P_\gamma{}^2 = 0) = G_{B'_\beta B_\alpha P_\gamma}\left(\frac{(2l+1)!!}{(l+1)!}\right)^{1/2} \times (f_{P_\gamma}{}^{-1})\left(\frac{2B}{B \pm B'}\right)\left(\frac{2B}{B^2 - B'^2}\right)^{l-1}. \quad (6)$$

(f_{P_γ} stands for f_π, f_K, and f_{η}.) Now the realization of the CR, $[V_i, A_j] = if_{ijk}A_k$, in our asymptotic limit using asymptotic SU(3) yields[5] that $G_{B'_\beta B_\alpha P_\gamma}$ (but not the couplings $g_{B'BP}$) can be parameterized in terms of the usual *exact* SU(3) *plus mixing* prescription. Equation (6), therefore, explicitly demonstrates the effect of mass splitting and mixing on the coupling constants (defined in the *soft*-meson limit $P_\gamma{}^2 = 0$). Inserting Eq. (6) into Eq. (5), we obtain broken SU(3) relation for decay rates. We can actually write this relation (in the soft-meson limit) in a remarkably simple form by noting $p = (B^2 - B'^2)/2B$,

$$\Gamma(B_\alpha \to B'_\beta + P_\gamma) = [\text{“SU(3) factor”}]f_{P_\gamma}{}^{-2}p^3\,. \quad (7)$$

Equation (7) is actually valid for any values of J^P of B_α and B'_β, i.e., even when two or more partial waves are involved. The effect of broken SU(3) is now exhibited [apart from mixing appearing in the “SU(3) factor” of Eq. (7)] as the appearance of *universal* effective p-wave barrier, irrespective of the nature of partial waves involved. We have recently shown[12] that a similar effective barrier effect appears in the SU(3) comparison of scattering involving pseudoscalar mesons and resolves the outstanding discrepancies between the naive prediction of SU(3) and experiments. Equation (7) is considerably different from the formulas traditionally used in SU(3) analysis. For example, the formula used by Samios *et al.*[13] is

$$\Gamma(B_\alpha \to B'_\beta + P_\gamma) = [\text{“SU(3) factor”}]\,\frac{p^{2l+1}}{B_\alpha}\,. \quad (8)$$

We believe, however, that Eq. (7) has a firmer theoretical basis. We have neither applied exact SU(3) argument nor introduced a particular barrier effect. When more than one partial wave are involved, our Eq. (5) contains more terms corresponding to each partial wave. Nevertheless, our Eq. (7) remains unchanged (in the *soft*-meson limit).

In our theoretical framework, Eqs. (6) and (7) are exact in the soft-meson limit and narrow-width approximation for resonances used in derivation. Since the extrapolation from soft to physical meson is complicated, we may tentatively assume that Eq. (6) or Eq. (7) is valid also for physical processes. This will be a good approximation if $B - B' \gg P$. If $B - B' \gtrsim P$, some allowance has to be made. The effect may appear, for example, as a shift of the value of p appearing in Eq. (7) to some effective value. We may find experimentally some clue to the correct extrapolation.

Table I shows a comparison of Eq. (7) with experiments for the $\frac{3}{2}^+ \to \frac{1}{2}^+ + \pi$ decays. We have used the value of p^3 averaged over the mass distribution

TABLE I. Width prediction from our p^3 formula for the $J^P = \frac{3}{2}^+$ decuplet with input $\Gamma(\Sigma^+ \to \Lambda\pi^+) = 30$ MeV. Values of p^3 averaged over the mass distribution of the decuplet have been used. Experimental data are taken from Ref. 2.

Decay mode	Prediction (MeV)	Experiment (MeV)
$\Delta^{++}(1211) \to p\pi^+$	98	99 ± 3.6
$\Sigma^+(1381) \to (\Sigma\pi)^+$	6.1	5.8 ± 1.5
$\Xi^{*0}(1531) \to (\Xi\pi)^0$	10.6	9.1 ± 0.5

of the decuplet as has been used in Ref. 2. The experimental width of Δ is chosen (as in Ref. 2) to be "pole" value[4] of the Δ width, encouraged by the fact that our mass-squared formula predicts a Δ mass close to the pole value of Δ. Our prediction with our input $\Gamma(Y_1^* \to \Lambda\pi) \simeq 30$ MeV achieves an agreement with experiments similar to the one [based on Eq. (8)] claimed in Ref. 2. We should also keep in mind that there could further be an effect of mixing of order ϵ neglected. Since the decays under consideration involve only the $l = 1$ wave, there is not much difference between our Eq. (7) and the conventional formula, Eq. (8), except for the factor $1/B_\alpha$. To distinguish between the p^3 and p^{2l+1} behaviors, we need to study reactions with $l \neq 1$. It is also desirable to choose the decays with $B_\alpha - B'_\beta \gg P_\gamma$ to minimize the effect of extrapolation. An indication that our broken-SU(3) coupling-constant relation, Eq. (6), works comes from the reaction $\frac{1}{2}^- \to \frac{1}{2}^+ + 0^-$. Equation (6) gives[7] for the ratio of the $Y(1405)$ couplings $(g_{YpK}/g_{Y\Sigma\pi}) \simeq (Y-p)/(Y-\Sigma)$, with $f_\pi = f_K$. Experimentalists[1] now add a mass factor $(B_\alpha - B'_\beta)$, which is exactly the factor we obtain from Eq. (6), to the analysis of $\frac{1}{2}^- \to \frac{1}{2}^+ + 0^-$. We urge experimentalists to use Eq. (6) for the vertex [or Eq. (7) for the rate] for other reactions as well.

In conclusion, our argument presented here seems to suggest that our nonperturbative algebraic approach towards broken SU(3) is promising and provides a more precise way[5] to approach hadron spectroscopy.

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