Triple-Regge coupling and the multiperipheral or Mueller-Regge model*

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In the pole approximation the Mueller-Regge and multiperipheral models have been shown to be equivalent. The couplings in the two forms are related by a transformation by a particular orthogonal matrix. This same matrix and the above couplings also yield the triple-Regge coupling constants.

Recently it has been realized by a number of workers that the multiperipheral model (MPM) and the Mueller-Regge model (MRM) are equivalent.[1,2] This is demonstrated in the pole approximation most economically in Pinsky, Snider, and Thomas[2] (PST) whose notation we will use. They found that there is an orthogonal transformation between the Mueller-Regge (central region) couplings and the multiperipheral couplings. This orthogonal matrix also relates the inclusive poles, the exclusive poles, and the coupling constants.[3]

Here we will show that the triple-Regge couplings[4] are also related to the multiperipheral couplings by this orthogonal matrix. This relation will be another constraint for anyone trying to construct a realistic Mueller-Regge or multiperipheral model.

We are working in the approximation of a two-component model,[5] where the Pomeron is not included in the set of exclusive trajectories (those used in the production amplitudes), but is included among the inclusive trajectories. Therefore we will be able to find expressions for g_{iiP} and g_{iij}, $i \neq P \neq j$, but not for g_{PPi} or g_{PPP}. The papers by Abarbanel, Chew, Goldberger, and Saunders[6] are the analogous investigation for g_{PPP}.

Although if one started with a three-dimensional MPM he would find an expression for $g_{iij}(t,t,0)$, we will consider only the one-dimensional model and obtain an expression for a t integral involving this triple-Regge coupling constant.

Before proceeding we review the notation of PST[2] for an N-channel description of nature. We work with the Mellin-transformed cross sections since everything is diagonal in J:

$$s\sigma^n(s)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dJ\left(\frac{s}{s_0}\right)^J Q_n(J). \tag{1}$$

Then in PST

$$Q_n(J)=D^T F(J)[GF(J)]^{n-2}D, \tag{2}$$

where G and F are $N\times N$ matrices and D is an N-dimensional vector. (We assume just one kind of incident particle to reduce notation.) $F(J)$ is diagonal and has the form

$$F(J)=\begin{bmatrix} \frac{1}{J-l_1} & & \\ & \frac{1}{J-l_2} & \\ & & \ddots \end{bmatrix} \tag{3}$$

where $l_1=2\alpha_i-1$ are the exclusive poles. G is the matrix of "coupling constants." The analogous Laplace transform of the fully integrated n-particle inclusive cross section from the MRM is called $P_n(J)$. (We exclude a single particle from each fragmentation region for ease of counting so it is like $pp\to n\pi$'s + anything integrated.) It is

$$P_n(J)=\Delta^T\Phi(J)[\Gamma\Phi(J)]^n\Delta, \tag{4}$$

where Γ, Φ, and Δ are analogous to G, F, and D in every way. PST show that there exists an orthogonal $N\times N$ matrix, S, found by "solving" either model, which relates the two models by

$$\Gamma=S^TGS,\quad \Delta=S^TD,\quad \text{and } \Phi=S^T(F^{-1}-G)^{-1}S; \tag{5}$$

furthermore, S is J-independent. This establishes the notation; now we want to look in the triple-Regge region.

The triple-Regge cross section is given by

$$\frac{d\sigma}{dM^2dt}=\frac{s_0}{16\pi s^2}\sum_{i\nu}\beta_i{}^2(t)\beta_\nu(0)\,|\,\xi_i(t)\,|^2 g_{ii\nu}(t) \times\left(\frac{s}{M^2}\right)^{2\alpha_i(t)}\left(\frac{M^2}{s_0}\right)^{\alpha_\nu(0)}. \tag{6}$$

We have neglected g_{ijk}, $i\neq j$ terms; they could be included if the ij channel were retained in the original multiperipheral model. To correspond with the previous notation i should vary over the N MPM channels and ν over the N MRM channels. The elements of the vector Δ are just the β's above evaluated at $t=0$, i.e., $\delta_\nu=\beta_\nu(0)$. To compare with a one-dimensional model we must integrate over t. We approximate the integration range by $-\infty<t<0$ and neglect the t dependence of $\alpha_i(t)$. This yields

$$\frac{d\sigma}{dM^2}=\frac{1}{s^2}\sum_{i\nu} R_{i\nu}\delta_\nu\left(\frac{s}{M^2}\right)^{2\alpha_i}\left(\frac{M^2}{s_0}\right)^{\alpha_\nu}, \tag{7}$$

where

$$R_{i\nu}=\frac{s_0}{16\pi}\int_{-\infty}^{0} dt\,\beta_i{}^2(t)\,|\xi_i(t)|^2 g_{ii\nu}(t). \tag{8}$$

We make a change of variables by defining Y and y by

$$Y=\ln\left(\frac{s}{s_0}\right) \text{ and } y=\ln\left(\frac{M^2}{s_0}\right) \tag{9}$$

to obtain

$$M^2\left|\frac{d\sigma}{dM^2}=\frac{d\sigma}{dy}=\frac{e^{-Y}}{s_0}\sum_{i\nu} R_{i\nu}\delta_\nu e^{(Y-y)(2\alpha_i-1)}\,e^{y\alpha_\nu}.\right. \tag{10}$$

We introduce the double Laplace transform

$$f(j,J)=\int_0^\infty dy\, e^{-jy}\int_0^\infty dY\, e^{-J(Y-y)}\theta(Y-y) \times s_0 e^Y \frac{d\sigma}{dy}(y,Y). \tag{11}$$

Notice that f is defined in such a way that $f(J,J)$ is just the usual Laplace transform of the *integrated* triple-Regge cross section. Then

$$f(j,J)=\sum_{i,\nu} R_{i\nu}\,\delta_\nu\frac{1}{J-(2\alpha_i-1)}\,\frac{1}{j-\alpha_\nu}. \tag{12}$$

We note that j is just the Mellin conjugate variable of the missing mass squared, M^2, or the Laplace conjugate of the corresponding rapidity interval, $\ln(M^2/s_0)$, while J is the Laplace conjugate of the rapidity gap between the leading particle and the missing mass, $\ln(s/M^2)$.

Since the missing mass can contain any number of particles (greater than 1—elastic scattering is excluded) $f(j,J)$ in the notation of PST is

$$\begin{aligned} f(j,J)&=\sum_{n=1}^{\infty} D^T F(J)[GF(j)]^n D \\ &=D^T F(J) G[F(j)^{-1}-G]^{-1} D \\ &=D^T F(J) GS\Phi(j)\Delta. \end{aligned} \tag{13}$$

These two expressions agree if $R_{i\nu}=d_i(GS)_{i\nu}=d_i(S^T\Gamma)_{i\nu}$. Thus we have

$$\begin{aligned}\frac{s_0}{16\pi}\int_{-\infty}^{0} dt\,\beta_i{}^2(t)\,|\xi_i(t)|^2 g_{ii\nu}(t)&=d_i(GS)_{i\nu} \\ &=d_i(S^T\Gamma)_{i\nu},\end{aligned} \tag{14}$$

which is the desired result.

If we further assume $g_{ii\nu}(t)$ has the same t dependence as $\beta_i{}^2(t)$, then by comparing the expressions for the elastic cross section in both forms we get

$$\begin{aligned}\frac{g_{ii\nu}(0)d_i{}^2}{\delta_i{}^2}&=\frac{g_{ii\nu}(0)}{\beta_i{}^2(0)}\left(\frac{1}{16\pi s_0}\right)\int_{-\infty}^{0} dt\,\beta_i{}^4(t)\,|\xi_i(t)|^2 \\ &=\frac{d_i}{s_0{}^2}(GS)_{i\nu}.\end{aligned} \tag{15}$$

Hence

$$g_{ii\nu}(0)=(GS)_{i\nu}\frac{\delta_i{}^2}{d_i s_0{}^2}. \tag{16}$$

Here we have assumed that at least for some trajectories the bootstrap problem is solved, so that among the inclusive (output) trajectories is one corresponding to ith exclusive (input) singularity; δ_i is the external coupling for that trajectory.

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[3]Trying to satisfy this relation while setting the exclusive pole positions consistent with the inclusive pole positions is, of course, the old multiperipheral bootstrap problem, for which a good solution has not yet been found.

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