Duality constraints on the constituent-current quark picture for the decay of the B meson

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We show how duality constrains the constituent-current quark picture to give, for the $B \to \omega + \pi$ decay, a helicity amplitude ratio in good agreement with experimental data. We also give the predictions for the $\underline{35}(L=1) \to \underline{35}(L=0)+\pi$ decays.

The $B(1235) \to \omega + \pi$ decay has been analyzed extensively both theoretically and experimentally.

Most of the experiments[1] seem to favor $J^P = 1^+$ as the spin-parity assignment for the B meson, while the ratio of the two helicity amplitudes is yet subject to deviation from one experiment to another. Nevertheless, it seems established that the zero-helicity decay amplitude, although smaller than the helicity-one decay amplitude, is not zero. If one normalizes helicity amplitudes F_λ such that

$$|F_0|^2 + 2|F_1|^2 = 1\,, \tag{1}$$

most experiments are in agreement[1] with $|F_0|^2$ less than $\frac{1}{3}$, implying a nonzero D-wave contribution.

An average value for $|F_0|^2$—with normalization as given in (1)—is

$$|F_0|^2 = 0.13 \pm 0.05\,. \tag{2}$$

From a theoretical point of view, the $B \to \omega + \pi$ decay has led one to disregard $SU(6)_W \times O(2)_{L_z}$ as a good vertex symmetry since it predicts a pure longitudinal decay of the B.[3] Many phenomenological models have been built avoiding this problem. Among them, the $^3P_0$ model,[4] based on a quark-pair-creation approach,[5] has been able to fit $|F_0|^2 = 0.08$ (see Ref. 2), taking into account kinematic barrier factors.

More recently, a fit[2] using the parametrization of the constituent quark picture[7,8] based on the work of Melosh,[6] together with partial conservation of axial-vector current (PCAC), gave $|F_0|^2 = 0.04$. In fact, in the case of pionic decays, these two models have been shown[8] to be equivalent up to kinematic factors.

What we want to show is how duality constrains these two models to give, independently of any experimental input, a value for $|F_0|^2$ in full agreement with (2), namely $|F_0|^2 = 0.11$.

The decay of B into $\omega\pi$ has already been analyzed[17] by saturating finite-energy sum rules over an extended t region with explicitly known resonances only. Here, we assume from the start exchange-degenerate sets of Regge trajectories of both natural parities and concentrate on the behavior of amplitudes near $t = -0.6$ GeV^2.

We first recall[9] how duality, via the $\pi + \rho \to \pi + \rho$ charge-exchange (CEX) process, constrains the $A_1 \to \rho\pi$ decay, which is then related to the $B \to \omega + \pi$ decay by one of the two models mentioned above.

One knows that the duality hypothesis, together with the absence of exotic resonances in the $\pi\pi$ channel, predicts the exchange degeneracy of the ρ and f trajectories.[10] This in turn leads to a nonsense-choosing mechanism for the ρ trajectory and implies a zero in the t-channel amplitudes at a t value for which $\alpha_\rho(t) = 0$, i.e., $t \approx -0.6$ GeV^2 for processes where only the ρ can be exchanged (CEX).

A question of consistency arises from this requirement, namely, can we saturate the s-channel amplitudes by resonances in such a way that, by crossing to the t channel, we recover this fixed-t structure?

It is already known that consistency can be achieved in the πN (CEX)[11] and in the $\pi\Delta$ (CEX)[12] processes with, in the latter case, a prediction in agreement with experiment.[13] We now summarize how that problem is solved in the $\pi + \rho \to \pi + \rho$ (CEX) process.[9]

With m and M the pion and the ρ masses, respectively, the kinematics of the process

$$\pi(p_1) + \rho(p_2) \to \pi(p_3) + \rho(p_4)$$

are described by

$$\begin{aligned} s &= (p_1 + p_2)^2\,, \\ t &= (p_1 - p_3)^2\,, \\ S &= \{[s-(M+m)^2][s-(M-m)^2]\}^{1/2}\,, \\ T &= [t(t-4M^2)]^{1/2}\,. \end{aligned} \tag{3}$$

There are four independent helicity amplitudes in each channel. Denoting them by $T^s_{\lambda\mu}$ for $T^s_{0\lambda;0\mu}$ and $T^t_{\lambda\mu}$ for $T^t_{00;\lambda\mu}$, our choice of independent amplitudes is, for the s channel,

$$\begin{aligned} &T^s_{11} = T^s_{-1\,-1}\,, \quad T^s_{1\,-1} = T^s_{-11}\,, \\ &T^s_{10} = -T^s_{-10} = -T^s_{01} = T^s_{0\,-1}\,, \quad T^s_{00}\,, \end{aligned} \tag{4}$$

and for the t channel,

TABLE I. Crossing matrix for $\pi\rho \rightarrow \pi\rho$.

	$\sqrt{2}\,T^s_{11}$	$2T^s_{10}$	$\sqrt{2}\,T^s_{1\,-1}$	T^s_{00}
$\sqrt{2}\,T^t_{11}$	$-\frac{1}{2}\sin^2 X$	$\sin X \cos X$	$-\frac{1}{2}(1+\cos^2 X)$	$-(1/\sqrt{2})\sin^2 X$
$2T^t_{10}$	$\sin X \cos X$	$\sin^2 X - \cos^2 X$	$-\sin X \cos X$	$\sqrt{2}\,\sin X \cos X$
$\sqrt{2}\,T^t_{1\,-1}$	$-\frac{1}{2}(1+\cos^2 X)$	$-\sin X \cos X$	$-\frac{1}{2}\sin^2 X$	$(1/\sqrt{2})\sin^2 X$
T^t_{00}	$(1/\sqrt{2})\sin^2 X$	$-\sqrt{2}\,\sin X \cos X$	$-(1/\sqrt{2})\sin^2 X$	$-\cos^2 X$

$$T^t_{11} = T^t_{-1\,-1}\,, \quad T^t_{1\,-1} = T^t_{-1\,1}\,, \qquad (5)$$
$$T^t_{10} = T^t_{01} = -\,T^t_{-1\,0} = -\,T^t_{0\,-1}\,, \quad T^t_{00}\,.$$

Then, with the conventions of Cohen-Tannoudji, Morel, and Navelet[14] (CTMN), the helicity crossing matrix is given by

$$T^s_{\lambda\mu}(s,t) = e^{i\pi\mu} \sum_{\lambda'\mu'} d^1_{\mu'\mu}(X_2)\, d^1_{\lambda'\lambda}(X_4) T^t_{\lambda'\mu'}\,, \qquad (6)$$

and is displayed in Table I, where

$$\cos X = \cos X_2 = -\cos X_4 = t(s + M^2 - m^2)/ST\,.$$

The contribution of a resonance R with spin J and mass M_R to an s-channel helicity amplitude can be written as

$$T^s_{\lambda\mu} = (2J+1)\, d^J_{\lambda\mu}(\theta_s) \frac{\Gamma_p R_{\lambda\mu}}{M_R - \sqrt{s} - i\Gamma/2}\,, \qquad (7)$$

with Γ_p proportional to the coupling constant. One easily sees that

$$R_{11} = R_{1\,-1} = 1\,, \qquad (8)$$
$$R_{10} = R_{00} = 0$$

for the natural-parity resonances. However, the unnatural-parity ones may couple in two different partial waves, so that there remains one free parameter; accordingly, we write

$$R_{11} = R_{1\,-1} = \cos^2\alpha\,,$$
$$R_{10} = \sin\alpha\cos\alpha\,, \qquad (9)$$
$$R_{00} = \sin^2\alpha$$

(thus we see that in this case, with the notation of (1), one has $F_0 \sim \sin\alpha$ and $F_1 \sim \cos\alpha$ for the decay of such a resonance in the $\pi\rho$ channel).

What we do next is to saturate the direct channel by resonances, cross over to the t channel, and look for a concentration of zeros in the neighborhood of $t = -0.6$ GeV^2.

Clearly, for the natural-parity resonances (namely those which are on the ω-A_2 degenerate trajectories), one has no free parameter and one has to see if they reproduce a dip in the differential cross section at the required value of t. Such a differential cross section is shown in Fig. 1 (for the A_2), where it is seen that it is indeed the case (this fact can be shown[9] to be true for all the resonances on the ω-A_2 trajectories).

For the unnatural-parity resonances, one can display the zeros of the t-channel amplitudes as a function of the parameter α and see how duality constrains it. This is shown in Fig. 2 for a $2^{-+}(1640)$ meson. One sees that there are, *a priori*, three possible solutions of the duality problem: $\cos\alpha = 0.6, 0, -0.6$. However, looking at the differential cross sections induced by these three solutions, it turns out that $\cos\alpha = 0.6$ is favored (as can be seen from Fig. 3) because it

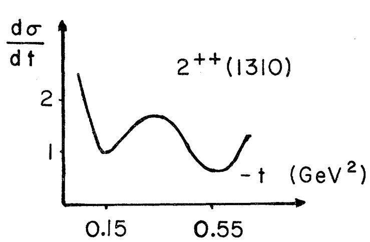

FIG. 1. Differential cross section induced by the A_2 resonance in the s channel.

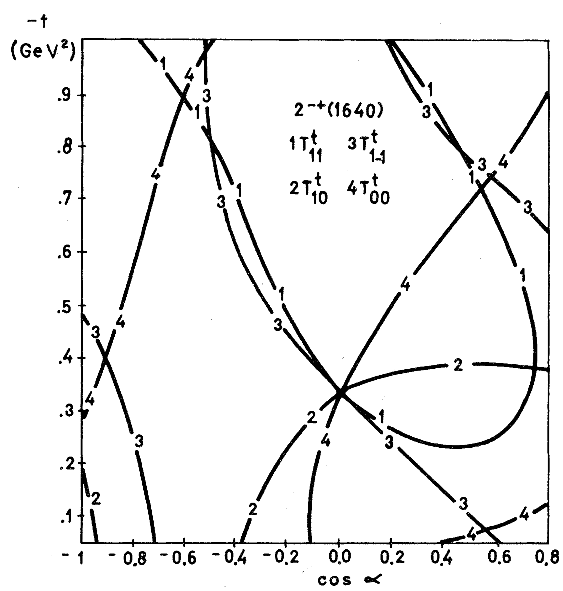

FIG. 2. Zeros of the t-channel amplitudes for a $2^{-+}(1640)$ in the s channel.

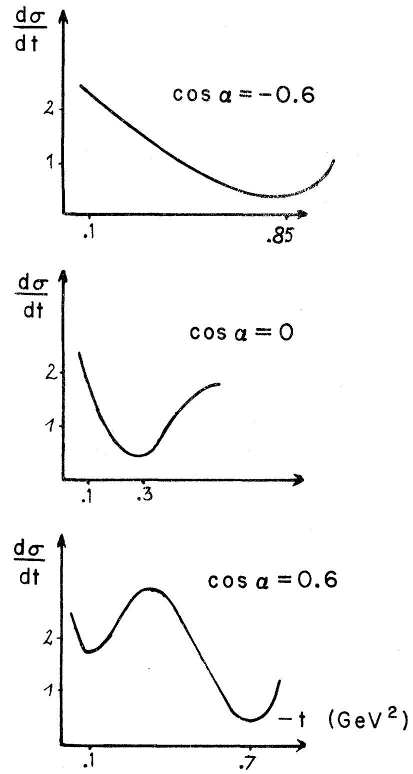

FIG. 3. Differential cross sections for the three solutions of Fig. 2.

induces a more pronounced dip at the required t value, together with another at $t=-0.1\ \mathrm{GeV}^2$ which remains unexplained but is also present in theoretical analyses of πN (see Ref. 11) and $\pi\Delta$ (see Ref. 12) processes. One can show[9] that the same result holds for each resonance of the π-A_1 trajectories.

Thus we conclude that duality predicts for the $\pi\rho$ decay of resonances on the π-A_1 trajectories, and in particular for the A_1 itself, the following ratio of helicity decay amplitudes:

$$\left(\frac{F_1}{F_0}\right)_{A_1\to\pi\rho} = \frac{\cos\alpha}{\sin\alpha} = \frac{3}{4}\ . \tag{10}$$

The explicit construction[6] of the transformation between constituent and current quarks in the free quark model has provided a prescription for abstracting (among other things) some algebraic properties of the axial charge. These properties combined with the PCAC hypothesis and the classification of the observed particles under $SU(6)_W$ allow one to compute the pionic transitions of the hadrons.[7,8]

It turns out that the pionic transitions from $\underline{35}(L=1)$ to $\underline{35}(L=0)+\pi$ are described in terms of two reduced matrix elements:

$$\begin{aligned} A &= \langle \underline{35}\,L=1 \| \underline{35}(8,3)W_z=0, L_z=0 \| \underline{35}\,L=0\rangle ,\\ B &= \langle \underline{35}\,L=1 \| \underline{35}(8,3)W_z=\mp1, L_z=\pm1 \| \underline{35}\,L=0\rangle , \end{aligned} \tag{11}$$

with the numbers in parentheses referring to SU(3) and $SU(2)_W$ representations. With this parametrization and assuming no strange quarks in the ω and "Zweig's rule" ($B \not\to \phi\pi$), one obtains[7,8] the following parametrization of helicity amplitudes F_λ:

$$\begin{aligned} F_1(A_1\to\pi\rho) &= \tfrac{1}{8}\sqrt{3}\,A + \tfrac{1}{12}\sqrt{6}\,B\ ,\\ F_0(A_1\to\pi\rho) &= \frac{1}{\sqrt{6}}\,B\ ,\\ F_1(B\to\omega\pi) &= -\tfrac{1}{6}\sqrt{3}\,B\ ,\\ F_0(B\to\omega\pi) &= -\tfrac{1}{8}\sqrt{6}\,A\ . \end{aligned} \tag{12}$$

Then (10) together with (12) leads to the prediction

$$\left(\frac{F_1}{F_0}\right)_{B\to\omega\pi} = 2\ ,$$

which, with the normalization (1), gives us[18]

$$|F_0|^2 = 0.11\ ,$$

in good agreement with the experimental number, Eq. (2).

In the same way, if we use Eq. (10) and the Melosh transformation for axial charge, we can predict the $\underline{35}(L=1)$ into $\underline{35}(L=0)+\pi$ decays given in Table II [some values of Γ (experimental) are from Refs. 15 and 16] with only one input.

Thus we have shown that, putting together the constituent-current quark picture[6-8] and the requirement of duality[9-12] as dynamical input, one predicts for the $B\to\omega+\pi$ decay a helicity amplitude ratio in good agreement with experimental data.

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TABLE II. Decays of $\underline{35}$ ($L=1$) mesons into $\underline{35}$ ($L=0$) $+\pi$.

Decay	Γ (theory) (MeV)	Γ (experimental) [a] (MeV)
$B(1235)\to\omega\pi$	150 (input)	150 ± 20 [b]
$A_2(1310)\to\pi\rho$	33	77 ± 20
$A_1(1070)\to\pi\rho$	235	200–400
$A_2(1310)\to\pi\eta$	8	16 ± 4
$f(1260)\to\pi\pi$	50	125 ± 25
$\delta(975)\to\pi\eta$	96	60^{+50}_{-30} [c]
$\sigma(760)\to\pi\pi$	635	~400

[a] See Ref. 15.
[b] See Ref. 1.
[c] See Ref. 16.

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Two-pion correlations in pp collisions at 102 GeV/c*

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We have measured two-pion correlations in rapidity and transverse momentum in pp collisions at 102 GeV/c. The correlations, which are highly sensitive to the charges of the pions, display a behavior consistent with that expected on the basis of the short-range-order hypothesis, and, when compared with a model which does not contain explicit correlations, suggest the presence of substantial dynamic correlations in the data.

Several studies of longitudinal- and transverse-momentum correlations between particles produced in high-energy collisions have recently appeared in the literature.[1] The results of these investigations of inclusive reactions have often been somewhat difficult to interpret because these experiments have suffered from either (1) the fact that correlations, particularly at low energies, tend to be strongly influenced by the requirements of momentum-energy conservation, or (2) the fact that the measurements did not provide a comparison of correlations between different types of particles. In this paper we discuss the nature of transverse and longitudinal correlations between two π^+, between two π^-, and between π^+ and π^- mesons produced in pp collisions at 102 GeV/c. Preliminary results from this experiment, conducted in the 30-in. bubble chamber at the Fermi National Accelerator Laboratory (FNAL), have been published elsewhere.[2] The present data are from a measurement of a complete sample of ≈ 1800 events (all topologies) and an additional sample of ≈ 1200 events from the <10-pronged topologies.

The specific reactions we consider are the following:

$$pp \to \pi^+\pi^- + \text{anything}, \tag{1}$$