

## A "Cabibbo" theory for leptons and the neutrino masses

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We suggest a complete analogy between the lepton current and the Cabibbo theory for hadronic weak interactions. Both neutrinos couple to both the muon and the electron, so at least one of the neutrinos is required to be massive. The two neutrinos have the same type of interactions and are distinguishable only by their masses, like the muon and the electrons. In this theory muon number (or electron number) may be violated. All possible types of such processes are considered. In particular  $\mu \rightarrow e\gamma$  and the decay of the more massive neutrino,  $\nu_\mu \rightarrow \nu_e\gamma$ , are calculated in detail. We find a lifetime for  $\nu_\mu$  larger than  $1.5 \times 10^{-3}$  sec.

### I. INTRODUCTION

Previous attempts at constructing an analogy between the weak interactions of quarks and leptons<sup>1</sup> have been disturbed by the problem that whereas proton quarks couple to a combination of neutron and  $\lambda$  quarks, electrons and muons each couple only to one particle (their respective neutrino). Usually a multiplet of quarks is constructed containing  $\tilde{\mathcal{U}}$  and  $\tilde{\lambda}$  quarks, which are related to the physical  $\mathcal{U}$  and  $\lambda$  quarks by the Cabibbo rotation<sup>2</sup>

$$\begin{pmatrix} \tilde{\mathcal{U}} \\ \tilde{\lambda} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \mathcal{U} \\ \lambda \end{pmatrix}, \quad (1.1)$$

where  $\theta$  is the Cabibbo angle, so that the weak current couples the proton quark to  $\tilde{\mathcal{U}}$  [and in SU(4) theories the charmed quark,  $\mathcal{C}'$ , couples to  $\tilde{\lambda}$ ]. However, when an analogous lepton multiplet is constructed there is no such rotation.

In this paper we postulate a complete analogy with the hadrons in which the electrons and muon couple to  $\tilde{\nu}_e$  and  $\tilde{\nu}_\mu$ , respectively, where

$$\begin{pmatrix} \tilde{\nu}_e \\ \tilde{\nu}_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \quad (1.2)$$

The weak leptonic charged current is therefore given by

$$J_\mu^+ = \bar{e}\gamma^\mu \frac{1}{2}(1 - \gamma^5)\tilde{\nu}_e + \bar{\mu}\gamma^\mu \frac{1}{2}(1 - \gamma^5)\tilde{\nu}_\mu, \quad (1.3)$$

and this current occurs in the Lagrangian in the form

$$\mathcal{L} = gJ_\mu^+ W_\mu^- + \text{H.c.} + \dots, \quad (1.4)$$

where  $W^\mu$  is the charged intermediate vector boson.  $\nu_e$  and  $\nu_\mu$  are defined to be eigenstates of the mass operator. But if  $\nu_e$  and  $\nu_\mu$  have equal mass, then  $\tilde{\nu}_e$  and  $\tilde{\nu}_\mu$  are also eigenstates with equal mass. In this case the only physical way of distinguishing between orthogonal combinations of

the neutrinos is by picking out the particular orthogonal combinations  $\tilde{\nu}_e$  and  $\tilde{\nu}_\mu$  which couple to the electron and muon, respectively, so that  $\nu_e$  and  $\nu_\mu$  have no physical significance. Equation (1.2) is only meaningful if the neutrinos have different mass (i.e., they cannot both be massless), so any physically measurable consequence of this postulate must be proportional to the mass difference  $m_{\nu_\mu} - m_{\nu_e}$ .

In this scheme [Eqs. (1.2)–(1.4)] there is only one conserved additive lepton number,  $L$ ,

$$L = L_e + L_\mu, \quad (1.5)$$

where  $L_e = +1$  for  $e^-, \nu_e$ ,  $L_e = -1$  for  $e^+, \bar{\nu}_e$ , and  $L_e = 0$  for  $\mu^-, \mu^+, \nu_\mu, \bar{\nu}_\mu$ . Similarly  $L_\mu = +1$  for  $\mu^-, \nu_\mu$ ,  $L_\mu = -1$  for  $\mu^+, \bar{\nu}_\mu$ , and  $L_\mu = 0$  for  $e^-, e^+, \nu_e, \bar{\nu}_e$ . However, the muon number ( $L_\mu$ ) and the electron number ( $L_e$ ) are not separately conserved.

The experimental status of the neutrino masses is not very good, so that we have only upper limits for them<sup>3</sup>:

$$m_{\nu_e} \leq 60 \text{ eV}, \quad m_{\nu_\mu} \leq 1.2 \text{ MeV}. \quad (1.6)$$

From this we see that although the assumption that the neutrinos have mass is not inconsistent with the experimental data, the possible masses are very small, so that the muon- (or electron-) number-violating processes have amplitudes which are much smaller than  $\tan\theta$ .

In Sec. II we calculate the decay process  $\mu \rightarrow e + \gamma$ , and by comparison with experiment<sup>4</sup> this gives a limit on the mass difference between the neutrinos. This process is chosen since it gives the lowest limit on this mass difference. Estimates are made for the processes  $\mu \rightarrow 3e$ ;  $\mu^- + Z \rightarrow e^- + Z$ , where  $Z$  is a nucleus; and  $\mu^+ e^- \rightarrow \mu^- e^+$ . In Sec. III we consider the decay of the more massive neutrino into the lighter neutrino and a photon. We get a lifetime of greater than  $10^{-2}$  sec (depending on the neutrino masses), so electromagnetic neu-

trino decay should be observable. Other experimental consequences are considered in Sec. IV. Section V presents a summary and conclusion.

II.  $\mu \rightarrow e\gamma$  AND RELATED PROCESSES

In the usual  $V-A$  theory with two massless neutrinos  $\mu \rightarrow e\gamma$  is strictly forbidden since the electron number and muon number are separately conserved. In a theory with one neutrino the ratio  $\Gamma(\mu \rightarrow e\gamma)/\Gamma(\mu \rightarrow e\nu\nu)$  has been estimated and found to be four orders of magnitude larger than the experimental limit,<sup>4</sup>

$$\frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e\nu\nu)} < 2.2 \times 10^{-8} . \tag{2.1}$$

Since we are using a renormalizable (but nevertheless model-independent) theory, and since the leading terms cancel, the remaining parts are finite and proportional to the neutrino mass difference. We neglect the effect of any interaction of leptons with Goldstone bosons since their coupling to leptons is of order  $G_F^{1/2}m_\mu$  and therefore negligible.<sup>5</sup> The contribution from diagram 1(a) is given by

$$M_a = eg^2 \sin\theta \cos\theta \epsilon^\mu \int \frac{d^4k}{(2\pi)^4} \left[ \langle \bar{e} | \gamma^\rho \frac{1-\gamma^5}{2} \frac{1}{\not{k} - m_{\nu_\mu}} \gamma^\rho \frac{1-\gamma^5}{2} \frac{1}{(\not{p} - \not{q}) - m_\mu} \gamma^\mu | \mu \rangle \frac{1}{(p-q-k)^2 - M_W^2} - (m_{\nu_\mu} \rightarrow m_{\nu_e}) \right], \tag{2.3}$$

where  $\epsilon^\mu$  is the polarization vector of the photon, and  $p$  and  $q$  are the momenta of the muon and photon, respectively. Assuming that the intermediate-vector-boson mass  $M_W$  is much larger than the lepton masses this gives

$$M_a = \frac{i}{2\sqrt{2}\pi^2} eG_F \frac{\sin\theta \cos\theta m_e^2}{m_\mu^2 - m_e^2} (m_{\nu_\mu}^2 - m_{\nu_e}^2) \epsilon^\mu \langle \bar{e} | \gamma^{\mu\frac{1}{2}}(1-\gamma^5) | \mu \rangle , \tag{2.4}$$

where  $G_F = g^2/(2\sqrt{2}M_W^2)$ .

Similarly,

$$M_b = \frac{i}{2\sqrt{2}\pi^2} eG_F \frac{\sin\theta \cos\theta}{m_\mu^2 - m_e^2} m_\mu^2 (m_{\nu_\mu}^2 - m_{\nu_e}^2) \epsilon^\mu \langle \bar{e} | \gamma^{\mu\frac{1}{2}}(1-\gamma^5) | \mu \rangle . \tag{2.5}$$

The contribution from diagram 1(c) is given by

$$M_c = eg^2 \sin\theta \cos\theta \epsilon^\mu \int \frac{d^4k}{(2\pi)^4} \left[ \langle \bar{e} | (\not{k} + 2\not{q} - \not{p}) \not{k} \gamma^{\mu\frac{1}{2}}(1-\gamma^5) | \mu \rangle \right. \\ \left. + \langle \bar{e} | \gamma^\rho \not{k} \gamma^{\rho\frac{1}{2}}(1-\gamma^5) | \mu \rangle (q - 2p - 2k)^\mu + \langle \bar{e} | \gamma^\mu \not{k} (\not{q} + \not{p} - \not{k}) \frac{1}{2}(1-\gamma^5) | \mu \rangle \right] \\ \times \{ (\not{k} - m_{\nu_\mu}) [(p-k)^2 - M_W^2] [(p-k-q)^2 - M_W^2] \}^{-1} - (m_{\nu_\mu} \rightarrow m_{\nu_e}) . \tag{2.6}$$

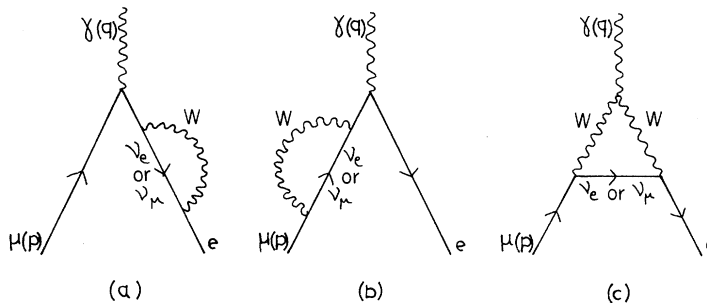


FIG. 1. Diagrams contributing in lowest order to the process  $\mu \rightarrow e\gamma$ .

The diagrams contributing to the amplitude for  $\mu \rightarrow e\gamma$  in our theory are shown in Fig. 1. It can be seen that these lowest-order contributions are one-loop diagrams. Furthermore, in each diagram [(a), (b), and (c)] both types of neutrino arise. The couplings of these neutrinos to the electron and the muon, given by Eqs. (1.2)–(1.4), are such that the leading-order contributions from these diagrams cancel. We assume that the photon  $A_\mu$  and the intermediate vector boson  $W_\mu$  are gauge bosons, so the electromagnetic Hamiltonian may be written

$$H_{em} = eA^\mu (\bar{e}\gamma_\mu e + \bar{\mu}\gamma_\mu \mu) + e[(\partial_\mu A_\nu - \partial_\nu A_\mu)W_+^\mu W_-^\nu + (\partial_\mu W_\nu^- - \partial_\nu W_\mu^-)A^\mu W_+^\nu + (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)W_-^\mu A^\nu] . \tag{2.2}$$

Making the same approximation as in Eqs. (2.4) and (2.5) this gives

$$M_c = \frac{-3i}{2\sqrt{2}\pi^2} eG_F \sin\theta \cos\theta (m_{\nu_\mu}^2 - m_{\nu_e}^2) \times \epsilon^\mu \langle \bar{e} | \gamma^{\mu\frac{1}{2}} (1 - \gamma^5) | \mu \rangle. \quad (2.7)$$

The total matrix element adding Eqs. (2.4), (2.5), and (2.7) is

$$M = \frac{-\sqrt{2}i}{\pi^2} eG_F \sin\theta \cos\theta (m_{\nu_\mu}^2 - m_{\nu_e}^2) \times \epsilon^\mu \langle \bar{e} | \gamma^{\mu\frac{1}{2}} (1 - \gamma^5) | \mu \rangle. \quad (2.8)$$

This S-matrix element is squared and summed over spins, giving the decay rate (neglecting  $m_e^2/m_\mu^2$ )

$$\Gamma(\mu \rightarrow e\gamma) = \frac{\alpha}{\pi^4} G_F^2 \sin^2\theta \cos^2\theta m_\mu (m_{\nu_\mu}^2 - m_{\nu_e}^2)^2, \quad (2.9)$$

where

$$\alpha = \frac{e^2}{4\pi}.$$

From the known decay rate for  $\mu \rightarrow e\nu_e\nu_\mu$  (Ref. 6) we get

$$\frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e\nu\nu)} = \frac{192\alpha}{\pi} \left( \frac{m_{\nu_\mu}^2 - m_{\nu_e}^2}{m_\mu^2} \right)^2 \sin^2\theta \cos^2\theta. \quad (2.10)$$

Comparing this with the experimental upper limit (2.1) and using  $\sin\theta = 0.2$  (the Cabibbo angle),<sup>7</sup>

$$\frac{m_{\nu_\mu}^2 - m_{\nu_e}^2}{m_\mu^2} \leq 6 \times 10^{-4}. \quad (2.11)$$

If  $m_{\nu_\mu} \gg m_{\nu_e}$  then we obtain an upper limit for the mass of  $\nu_\mu$

$$m_{\nu_\mu} < 2.6 \text{ MeV}. \quad (2.12)$$

This is consistent with the experimental limit (1.6). However, by substituting the experimental upper limit for the mass of the muon-type neutrino, we predict the ratio

$$\frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e\nu\nu)} \leq 10^{-9}. \quad (2.13)$$

The processes  $\mu^- \rightarrow e^- e^- e^+$  and  $\mu^- + Z \rightarrow e^- + Z$  are related to the decay amplitude for  $\mu \rightarrow e\gamma$  by attaching an electromagnetic current to the photon, e.g.,

$$\mu^- \rightarrow e^- + \gamma(\text{virtual}) \rightarrow e^- + e^- + e^+.$$

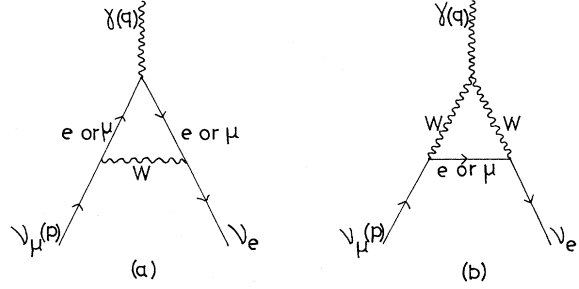


FIG. 2. Diagrams contributing in lowest order to the process  $\nu_\mu \rightarrow \nu_e \gamma$ .

Consequently we find that the branching ratios

$$\frac{\Gamma(\mu \rightarrow 3e)}{\Gamma(\mu \rightarrow e\nu\nu)} \quad \text{and} \quad \frac{\sigma(\mu^- + Z \rightarrow e^- + Z)}{\sigma(\mu^- + Z \rightarrow \nu + \dots)}$$

are of order  $\alpha$  times the branching ratio  $\Gamma(\mu \rightarrow e\gamma)/\Gamma(\mu \rightarrow e\nu\nu)$  (i.e.,  $< 10^{-11}$ ). This is well within the experimental limit,<sup>8,9</sup>

$$\frac{\Gamma(\mu \rightarrow 3e)}{\Gamma(\mu \rightarrow e\nu\nu)} < 1.3 \times 10^{-7}$$

and

$$\frac{\sigma(\mu^- + Z \rightarrow e^- + Z)}{\sigma(\mu^- + Z \rightarrow \nu + \dots)} < 2.2 \times 10^{-7}.$$

We end this section by noting that the process  $\mu^+ + e^- \rightarrow \mu^- + e^+$  can be calculated by squaring the amplitude for  $\mu^- \rightarrow e^- + \gamma$  so that the cross section is extremely small:

$$\frac{\sigma(\mu^+ + e^- \rightarrow \mu^- + e^+)}{\sigma(\mu^+ + e^- \rightarrow \mu^+ + e^-)} < 10^{-49}.$$

### III. THE DECAY OF THE NEUTRINO

Since the neutrinos are of different mass and both neutrinos interact with the electron and the muon, we expect that the more massive neutrino will decay into the lighter neutrino plus other decay products. The most probable decay mode is

$$\nu_\mu \rightarrow \nu_e + \gamma, \quad (3.1)$$

although with the present experimental limit for the mass of  $\nu_\mu$  the decay mode  $\nu_\mu \rightarrow \nu_e + e^+ + e^-$  is not excluded. If there exist other light particles (e.g., a real Goldstone boson) then there may be other possible modes of decay.

The diagrams contributing in lowest order to the amplitude for the decay of Eq. (3.1) are shown in Fig. 2. Again we see that these are one-loop diagrams. The contribution from diagram 2(a) is

$$M_a = -eg^2 \sin\theta \cos\theta \epsilon^\mu \int \frac{d^4 k}{(2\pi)^4} \left[ \langle \bar{\nu}_e | \gamma^\rho \frac{1 - \gamma^5}{2} \frac{1}{\not{p} - \not{k} - m_\mu} \gamma^\mu \frac{1}{\not{p} - \not{q} - \not{k} - m_\mu} \gamma^\rho \frac{1 - \gamma^5}{2} | \nu_\mu \rangle \frac{1}{k^2 - M_W^2} - (m_\mu \rightarrow m_e) \right], \quad (3.2)$$

where  $\epsilon^\mu$  is the polarization vector of the photon and  $p^\mu$  and  $q^\mu$  are the momenta of  $\nu_\mu$  and the photon, respectively. Using the approximation  $M_W \gg$  lepton masses this gives

$$M_a = \frac{i}{4\sqrt{2}\pi^2} eG_F \sin\theta \cos\theta (m_\mu^2 - m_e^2) \left[ \ln\left(\frac{M_W^2}{m_\mu^2}\right) - 1 \right] \epsilon^\mu \left\langle \bar{\nu}_e \left| \gamma^\mu \frac{1-\gamma^5}{2} \right| \nu_\mu \right\rangle. \quad (3.3)$$

Diagram 2(b) is similar to diagram 1(c), but with  $\mu(e)$  and  $\nu_\mu(\nu_e)$  interchanged. Therefore its contribution is, in analogy with Eq. (2.7),

$$M_b = \frac{-3i}{2\sqrt{2}\pi^2} eG_F \sin\theta \cos\theta (m_\mu^2 - m_e^2) \times \epsilon^\mu \langle \bar{\nu}_e | \gamma^\mu \frac{1-\gamma^5}{2} | \nu_\mu \rangle. \quad (3.4)$$

Adding Eqs. (3.3) and (3.4) to find the total matrix element, squaring, and summing over spins we get for the decay rate

$$\Gamma(\nu_\mu \rightarrow \nu_e \gamma) = \frac{\alpha G_F^2}{16\pi^4} \sin^2\theta \cos^2\theta [5 + \ln(M_W^2/m_\mu^2)]^2 \times (m_\mu^2 - m_e^2) \frac{m_{\nu_\mu}^2 - m_{\nu_e}^2}{E_{\nu_\mu}}. \quad (3.5)$$

In this case, the factor  $(m_{\nu_\mu}^2 - m_{\nu_e}^2)$  arises from the available phase space and not directly from the amplitude. Taking  $M_W \simeq 50$  GeV,<sup>10</sup>  $\sin\theta = 0.2$ , this expression gives

$$\Gamma(\nu_\mu \rightarrow \nu_e \gamma) \simeq 0.6 \times 10^{-18} \frac{m_{\nu_\mu}^2 - m_{\nu_e}^2}{E_{\nu_\mu}}. \quad (3.6)$$

For the experimental upper limit on  $m_{\nu_\mu}$  and  $m_{\nu_e}$  this gives a lifetime of about  $E_{\nu_\mu}$  sec, where  $E_{\nu_\mu}$  is the energy of the neutrino in GeV. For example, for neutrinos coming from pion decay,  $E_{\nu_\mu} \simeq 25$  MeV, we expect a lifetime

$$\tau = 0.025 \text{ sec}. \quad (3.7)$$

For less energetic neutrinos such events should be observable in the laboratory. Such events would be detected by the occurrence of photons along the predicted neutrino path.

#### IV. OTHER EXPERIMENTAL CONSEQUENCES

In this section we consider processes involving neutrinos in which muon number is violated. A typical process would be  $\nu_\mu + n \rightarrow e + p$ . However, in order to compare it with the existing experimental upper limit, we must consider the mechanism from which the neutrino was created. This is usually the process  $\pi \rightarrow \mu \nu_\mu$ . But in our scheme, the decay  $\pi \rightarrow \mu \nu_e$  has a branching ratio [compared with  $\Gamma(\pi \rightarrow \mu \nu_\mu)$ ] of  $\tan^2\theta$ . However, we distinguish between the two neutrinos by observing their interaction with hadrons and seeing whether a muon or an electron is produced. Thus, what one is really observing is the process

$$\pi \rightarrow \mu + \bar{\nu}_\mu \rightarrow n \rightarrow \mu \text{ (or } e) + p, \quad (4.1)$$

where  $\bar{\nu}_\mu$  is the combination  $\nu_\mu \cos\theta - \nu_e \sin\theta$ . Since we do not detect which neutrino occurs in this two-stage process we must add the amplitudes

$$\pi \rightarrow \mu + \nu_\mu \rightarrow n \rightarrow \mu \text{ (or } e) + p$$

and

$$\pi \rightarrow \mu + \nu_e \rightarrow n \rightarrow \mu \text{ (or } e) + p,$$

rather than adding the probabilities. It is for this reason that the muon-number-violating amplitude in the process (4.1) is proportional to the neutrino mass difference. For simplicity (and since the estimated result does not alter our previous calculation) we perform only an order-of-magnitude estimate of such a process, and estimate

$$\frac{\sigma(\nu_\pi + n \rightarrow e + p)}{\sigma(\nu_\pi + n \rightarrow \mu + p)} \sim \tan^2 2\theta \frac{(m_{\nu_\mu}^2 - m_{\nu_e}^2)^2}{(m_\pi^2 - m_\mu^2)^2}, \quad (4.2)$$

where  $\nu_\pi$  is the neutrino which comes from pion decay. This is well within the experimental limit.<sup>11</sup>

Another interesting process which has been recently discussed in relation to neutral currents is  $\nu_\mu + e \rightarrow \nu_\mu + e$ . In our scheme this can proceed through the charged weak currents with muon-number violation. As discussed above, if it were possible to identify the neutrino before its interaction, we would get a branching ratio

$$\frac{\sigma(\nu_\mu + e \rightarrow \nu_\mu + e)}{\sigma(\nu_e + e \rightarrow \nu_e + e)} = \tan^2\theta, \quad (4.3)$$

neglecting the possible contribution of neutral currents. However, since the neutrino produced in pion (or kaon) decay cannot be identified we estimate the measured branching ratio as in Eq. (4.2), which is negligible compared with the possible neutral-current contribution.<sup>12</sup>

Finally we consider the process  $K_L^0 \rightarrow \mu^+ e^-$ . This process should be compared with  $K_L^0 \rightarrow \mu^+ \mu^-$ , which is a second-order weak process. In theories involving a charmed quark  $\phi'$ , the amplitude for  $K_L^0 \rightarrow \mu^+ \mu^-$  is of order  $G_F^2 (m_{\phi'}^2 - m_\phi^2) m_K^2$ ,<sup>13</sup> where  $m_{\phi'}$ ,  $m_\phi$ , and  $m_K$  are the masses of the charmed quark, proton quark, and kaon, respectively. In this case the amplitude for  $K_L^0 \rightarrow \mu^+ e^-$  is of order  $G_F^2 (m_{\phi'} - m_\phi) (m_{\nu_\mu} - m_{\nu_e}) m_K^2 \sin\theta \cos\theta$  so we get a

branching ratio

$$\frac{\Gamma(K_L^0 \rightarrow \mu^+ e^-)}{\Gamma(K_L^0 \rightarrow \mu^+ \mu^-)} \sim \left( \frac{m_{\nu_\mu} - m_{\nu_e}}{m_{\phi'} - m_\phi} \right)^2 \sin^2 \theta \cos^2 \theta. \quad (4.4)$$

From  $m_{\phi'} \sim 5$  GeV,  $m_\phi \sim 1$  GeV, and  $m_{\nu_\mu} - m_{\nu_e} < 1$  MeV, this branching ratio is smaller than  $10^{-8}$  and is therefore negligible.

#### V. SUMMARY AND CONCLUSION

We suggest a scheme in which, in analogy with the Cabibbo theory for hadronic weak interactions, the electron (and muon) couple to a linear combination of two neutrinos, given by a mixing angle  $\theta$ , the Cabibbo angle. In order that this should be meaningful the two neutrinos had to have different masses so that both of them could no longer be massless. This leptonic interaction is described by Eqs. (1.2)–(1.4). In this scheme there is only one conserved additive lepton number, and muon number (or electron number) may be violated. This violation is proportional to  $\sin\theta$  and to the neutrino mass difference.

The decay  $\mu \rightarrow e\gamma$  was calculated in detail, since the experimental limit gave the lowest upper bound for the mass of the muon-type neutrino, which was still found to be larger than the experimental upper limit. A new feature of this theory, which is of major importance, is the allowed decay of the more massive neutrino into the lighter neutrino and a photon. This process was calculated

in Sec. III. We find a lifetime

$$\tau_{\nu_\mu} \gtrsim 1.5 \times 10^{-3} \text{ sec}. \quad (5.1)$$

This decay might be experimentally detectable.

Other processes in which muon (or electron) number is violated have been discussed. Although one might expect that the branching ratio of muon-number-violating to muon-number-conserving processes, such as

$$R = \frac{\sigma(\nu_\mu + n \rightarrow e + p)}{\sigma(\nu_\mu + n \rightarrow \mu + p)},$$

to be  $\tan^2\theta$ , one has to be aware of the preparation of the initial state in this interaction. If the neutrinos come from pion decay then both electron- and muon-type neutrinos occur and if one does not measure the mass of the outgoing neutrino one must add the amplitudes (rather than probabilities). This procedure gives a result which is proportional to the neutrino mass difference. If it were possible to devise more sensitive experiments in which the neutrino mass is measured, then one would be able to identify the neutrino, and in this case the branching ratio  $R$  would be  $\tan^2\theta$ .

We conclude by observing that this theory gives a complete analogy between hadronic and leptonic weak interactions. Furthermore, the two types of neutrinos,  $\nu_\mu$  and  $\nu_e$ , have the same interactions but are distinguished only by their masses in the same way as are the muon and the electron.

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<sup>5</sup>In gauge theories the coupling of the scalar particles to the leptons is given by  $F\bar{\psi}\phi\psi$ . By replacing  $\phi$  by its vacuum expectation value  $\langle\phi\rangle$ , we get the mass term for the leptons, so that  $F\langle\phi\rangle$  is the lepton mass, say

$m_\mu$ . Since  $M_W = g\langle\phi\rangle$ ,  $F \simeq (g/M_W)m_\mu \simeq G_F^{1/2}m_\mu$ .

<sup>6</sup>See, for example, R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969).

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