Factorizable model for massive current processes*

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On the basis of intuition and experience obtained in applying multiperipheral models to hadronic processes we generalize these ideas and techniques to massive current reactions. The details of the exchanges and cutoffs are not important and most of our results depend only on factorization properties of these amplitudes. A relation for the structure functions of the various current processes is obtained. A comparison is made to other models for these hard-current reactions.

I. INTRODUCTION

Reactions involving currents with large momentum squared, q^2, are of considerable interest as they may probe pointlike constituents of matter. The rates for all these processes have observed or conjectured scaling behavior. It is this scaling behavior which indicates the possible point structure.

A variety of mechanisms has been proposed to account for this scaling. The original, the one responsible for even looking for such a behavior, was the picture of Feynman[1] and of Bjorken and Paschos[2] which suggested that all hadronic matter is composed of point partons. Though originally applied to deep-inelastic electron scattering, it has been extended to other reactions.[3] Closely analogous is the study of these processes based on assumptions involving the singularity structure of products of currents with near-light-cone separations.[4] Concrete realizations in terms of familiar field theories have been proposed[5]; among these is the study of ladder or multiperipheral graphs.[6] Dual-model and massive-quark variants of the parton model have been considered.[7] The consistency of these pictures with usual ideas of Regge pole exchange or Mueller-Regge description[8] has been shown. In all the above the scaling hypotheses are built into the models with varying degrees of naturalness.

In this article we propose a new multiperipheral-like model for these processes involving hard currents. The details of the exchange are not as important as the assumptions of the factorizability of the relevant amplitudes and inherent short-range correlations. Unlike the multiperipheral models of Refs. 5 and 6, this model permits the production of particles in the entire allowed rapidity gap. As in the previous models scaling is forced; secondary terms, describing the approach to scaling, may be included, though we have not done so in this publication. In the language of Ref. 8 what we have in mind is a specific mechanism for the photon fragmentation region.[9]

In line with the observation that, at least in purely hadronic processes, transverse momenta are limited, we shall carry this limitation to an extreme and take all incident and produced particles to be collinear. Again, for hadronic processes this has not proven to be a hindrance as most of the qualitative and many quantitative features of more detailed calculations are obtained in the approximation of setting all transverse momenta to zero.[10] The vectorial nature of the currents is treated in a full four-dimensional manner.

As mentioned earlier, the details of the nature of the produced particles or exchanges is not as important as certain general features of the resulting factorization of various amplitudes into hadronic and current components. Due to these factorization properties parameter-free relations among electron-positron annihilation into hadrons, deep-inelastic electron scattering, and massive μ-pair production (or their generalization to inclusive production) are obtained in Sec. VI. It is possible that these relations are of a more general nature and will survive the particular details of this or other models in which such relations obtain. We shall discuss the correspondence of our model to other ones mentioned previously.

With respect to scaling there is a caveat. The theoretical conjecture that the ratio of the cross sections for $e^+ + e^- \to$ hadrons to that for $e^+ + e^- \to \mu^+ + \mu^-$ should be a constant and the accompanying inclusive cross sections should exhibit definite scaling behavior has not been borne out by experiments at presently accessible energies.[11] If one wishes to preserve this model as well as all the other models one may employ the customary out of claiming that present experiments are not at sufficiently high an energy to test scaling. A more probable rescue of any of these ideas is to invoke several mechanisms responsible for the hard-current processes. One of these mechanisms (the untangling from the others is not clear) is a scaling one. One should note that as scaling is built into these models, a violation of scaling could likewise be put in an *ad hoc* manner. Our discussion will be presented assuming the usual scaling properties.

II. THE MODEL

A. Details

The multiperipheral model has been primarily applied to purely hadronic processes. The cross section for the production of n particles is the absorptive part of the forward amplitude of the diagram of Fig. 1. With the restriction to limited transverse momenta the maximum contribution to this processes occurs when the emitted particles are strongly ordered in rapidity.[12] Energy and longitudinal momentum are approximately conserved by the leading particles and thus the rapidities of the produced particles are restricted to lie between the rapidities of the incident particles. In the situation where one of the incident particles, or more specifically currents, has a mass, or four-momentum squared, large and of the order of the incident energies, the above restriction no longer holds; the rapidities of the produced particles extend beyond the current rapidity. Thus, the diagrams responsible for the reactions

$$\begin{aligned} &(a)\ e^+ + e^- \rightarrow \text{hadrons},\\ &(b)\ e + p \rightarrow \text{hadrons}, \qquad (1)\\ &(c)\ p + p \rightarrow \mu^+ + \mu^- + \text{hadrons} \end{aligned}$$

are given in Fig. 2. In the diagrams we have left off the leptonic part and just indicated the massive photon. Though this model is valid for all (electromagnetic and weak) current-induced processes, we will use the language of the electromagnetic reactions. To allow for the possibility that the region of rapidities adjacent to the photon may be governed by a different mechanism than the hadronic one we have indicated these exchanges with heavy lines. Arguments based on the parton model[9] or on Regge-Mueller diagrams[8] suggest that this region extends over a rapidity interval $\frac{1}{2}\ln(|q^2|/\mu^2)$ on either side of the photon rapidity. μ is the common mass of the produced particles. Certainly, we leave open the possibility that there is no difference in the mechanisms in the various regions.

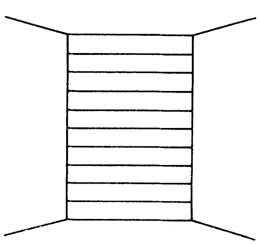

FIG. 1. Hadronic multiperipheral diagram.

B. Abstraction

We note that all the amplitudes of Fig. 2 are made up of two basic units indicated in Fig. 3. Figure 3(a) shows the usual kernel of a hadronic multiperipheral systems which we indicate by a rectangular block and denote by $\mathcal{B}$. The part associated with the photon is shown in Fig. 3(b) and will be drawn as an oval and denoted by $\mathcal{A}_{\mu\nu}$; the subscripts refer to the current indices. The three processes of Fig. 2 may now be represented by the abstracted diagrams of Fig. 4. In order to discuss inclusive processes for a particle emitted within the photon fragmentation region we need an analog of the kernel $\mathcal{B}$. This is indicated in Fig. 5 and we denote this kernel by $\mathcal{C}$. The diagrams governing the inclusive processes are now illustrated in Fig. 6. In Fig. 6(a) we show the process for inclusive $e^+ + e^-$ annihilation. In Fig. 6(b) the inclusive production of a particle in the photon-fragmentation region of deep-inelastic electron scattering is shown, while in Fig. 6(c) the same process for the particle emerging in the hadronic region is presented. Generalization to inclusive production of several particles is straightforward.

C. Kinematic variables

Aside from the tensor structure of $\mathcal{A}_{\mu\nu}$, to which we shall return, all the diagrams discussed previously will depend on scalars formed from the momenta entering or leaving the diagram. Likewise the momenta p_1 and p_2 will be large along

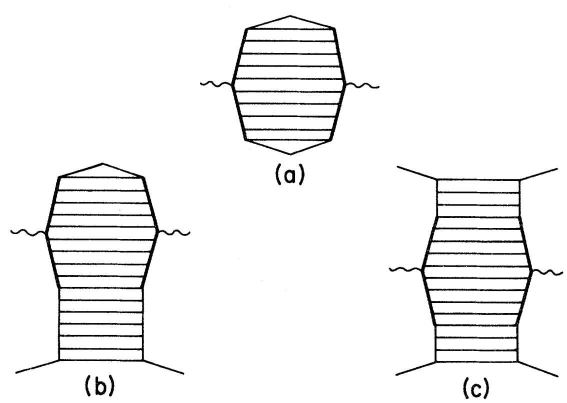

FIG. 2. Multiperipheral diagrams for processes involving massive currents: (a) $e^+ + e^-$ annihilation, (b) deep-inelastic electron scattering, (c) massive lepton pair production.

some direction. The momenta transverse to that direction will be limited and in the context of the present approximation set equal to zero. By convention we consider the various momenta entering or leaving the diagrams as indicated in Fig. 3 and Fig. 5. Thus

$$\mathcal{B}(p_1, p_2) = B(2p_1 \cdot p_2/\mu^2),$$
$$\mathcal{C}(p_1, p_2) = C(2p_1 \cdot p_2/\mu^2). \quad (2)$$

μ is the common mass of the emitted particles. The tensor $\mathcal{A}_{\mu\nu}$ can depend on the four scalars formed from p_1, p_2, and q. However, in the approximation of treating all momenta as collinear and μ as small, only three of these are independent;

$$\mathcal{A}_{\mu\nu}(p_1, p_2, q) = A_{\mu\nu}\left(\frac{2p_1 \cdot q}{q^2}, \frac{2p_2 \cdot q}{q^2}, q^2\right). \quad (3)$$

Scaling will determine the dependence on the third variable.

D. Tensor structure of $\mathcal{A}_{\mu\nu}$

Current conservation requires $\mathcal{A}_{\mu\nu}$ to be proportional to $(q^2 g_{\mu\nu} - q_\mu q_\nu)$ and dyadics formed from the

$$\left(p_i - q\,\frac{p_i \cdot q}{q^2}\right)_\mu .$$

In the approximation of treating all processes as collinear and all momenta as large, these conserved vectors are proportional to one another. With these restrictions

$$\mathcal{A}_{\mu\nu}(p_1, p_2; q) = -(q^2 g_{\mu\nu} - q_\mu q_\nu) A_1\left(\frac{2p_1 \cdot q}{q^2}, \frac{2p_2 \cdot q}{q^2}; q^2\right)$$
$$+\frac{1}{2}\left[\left(p_1 - q\,\frac{p_1 \cdot q}{q^2}\right)_\mu \left(p_2 - q\,\frac{p_2 \cdot q}{q^2}\right)_\nu + \left(p_1 - q\,\frac{p_1 \cdot q}{q^2}\right)_\nu \left(p_2 - q\,\frac{p_2 \cdot q}{q^2}\right)_\mu\right]$$
$$\times A_2\left(\frac{2p_1 \cdot q}{q^2}, \frac{2p_2 \cdot q}{q^2}; q^2\right). \quad (4)$$

The A_i's satisfy the symmetry

$$A_i(x, y; q^2) = A_i(y, x; q^2). \quad (5)$$

III. APPLICATIONS

Before discussing the individual examples that we shall study in detail, let us repeat the conventions and assumptions.

(i) All amplitudes will be built out of the hadronic and current kernels discussed in the previous section.

(ii) All transverse momenta will be set identically equal to zero.

(iii) Energy and longitudinal momentum conservation will be taken care of by the leading particles.

(iv) In addition to the kernels $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ we need certain coupling constants. Let g and $\bar{g}$ represent the coupling of a produced particle to a hadronic ($\mathcal{B}$) or current ($\mathcal{A}$ and $\mathcal{C}$) kernels respectively. Likewise, let G_h represent the coupling of $\mathcal{B}$ to one of the incident hadrons and G_γ the effects of performing the phase-space integration over one of the leading particles in $\mathcal{A}$ or $\mathcal{C}$.

(v) For notational convenience we introduce

$$\Pi_\mu(p) = p_\mu - q_\mu\, p \cdot q/q^2, \quad (6)$$

for any vector p. q will be understood to be the momentum of the current relevant for each process.

A. $e^+ + e^- \to$ hadrons

This process [Fig. 4(a)] is obtained from

$$\overline{W}_{\mu\nu}(g) = (2\pi)^6 \sum_n \langle 0|J_\mu|n\rangle\langle n|J_\nu|0\rangle \delta^4(q - p_n)$$
$$= -\tfrac{2}{3}\pi(g_{\mu\nu} - q_\mu q_\nu/q^2) q^2 R(q^2), \quad (7)$$

where $R(q^2)$ is the ratio of the cross section for this process to that for $e^+ + e^- \to \mu^+ + \mu^-$.

In our model we obtain

$$\overline{W}_{\mu\nu}(g) = \tfrac{1}{2}\int G_\gamma \mathcal{A}_{\mu\nu}(p_1, p_2; q) G_\gamma \delta(E_1 + E_2 - q_0)$$
$$\times \delta(p_1 - p_{2\parallel})\delta^2(p_{2\perp}) d\vec{p}_1 d\vec{p}_2 \quad (8)$$

where the longitudinal direction is taken to be along $\vec{p}_1$. Using Eqs. (2), (4), and (7) we find

$$R(q^2) = \tfrac{1}{2} G_\gamma [3A_1(1, 1; q^2) - \tfrac{1}{4}A_2(1, 1; q^2)] G_\gamma. \quad (9)$$

The factor $\frac{1}{2}$ is due to the fact that the integration over p_1 is restricted to half of the sphere.

B. $e^+ + e^- \to$ hadron $(\vec{p}) + \cdots$

Inclusive annihilation is determined by Fig. 6(a) and obtained from

$$\overline{W}^{(\mathrm{incl})}_{\mu\nu}(p,q)=(2\pi)^6\frac{E_p}{\mu}\sum_n \langle 0|J_\mu|\vec{p},n\rangle \times\langle n,\vec{p}|J_\nu|0\rangle\delta^4(q-p_n-p)$$
$$=-(g_{\mu\nu}-q_\mu q_\nu/q^2)\overline{W}_1(p,q)+\Pi_\mu(p)\Pi_\nu(p)\overline{W}_2(p,q)/\mu^2\,. \tag{10}$$

With the assumptions listed previously we obtain

$$\overline{W}^{(\mathrm{incl})}_{\mu\nu}(p,q)=\frac{2}{q^2}G_\gamma\,\mathcal{A}_{\mu\nu}(p_1,p;q)\bar{g}^2\,\mathcal{C}(p,p_2)G_\gamma\,, \tag{11}$$

where $p=(E_p,p)$, $p_1=(q/2,\ q/2)$, $p_2=(q/2,\ -q/2)$ and $q=(q,0)$ are collinear vectors. Introducing $\omega=2p\cdot q/q^2$, the fraction of the maximum momentum carried off by the produced particle, we find

$$\overline{W}_1(p,q)=\frac{2}{\mu}G_\gamma C(1/\omega)\bar{g}^2A_1(\omega,1;q^2)G_\gamma\,,$$
$$\overline{W}_2(p,q)=-2\frac{\mu}{\omega q^2}G_\gamma C(1/\omega)\bar{g}^2A_2(\omega,1,q^2)G_\gamma\,. \tag{12}$$

We have assumed that $1/\omega$ is large.

C. $e+p\to e+$hadrons

This is the classic of all deep-inelastic processes governed by Fig. 4(b) and

$$W_{\mu\nu}(p,q)=(2\pi)^6\frac{E}{\mu}\sum_n\langle p|J_\mu|n\rangle\langle n|J_\nu|p\rangle\delta^4(p+q-p_n)$$
$$=-(g_{\mu\nu}-q_\mu q_\nu/q^2)W_1(p,q)+\Pi_\mu(p)\Pi_\nu(p)W_2(p_1q)/\mu^2\,. \tag{13}$$

Using the conventional variables $Q^2=-q^2$, $p\cdot q=\mu\nu$, and $\omega=2\mu\nu/Q^2$ we obtain

$$W_{\mu\nu}(p,q)=\frac{2}{\mu\omega Q^2}G_\gamma\,\mathcal{A}_{\mu\nu}(p,-k;q)\mathcal{B}(k,p)G_h\,. \tag{14}$$

Again we assume ω large compared to the masses relevant to the problem. In the laboratory system

$$p=(\mu,0),$$
$$k=\left(\frac{\mu\omega}{2},\frac{\mu\omega}{2}-\frac{\mu}{\omega}\right),$$
$$q=\left(\frac{\omega Q^2}{2\mu},\frac{\omega Q^2}{2\mu}+\frac{\mu}{\omega}\right)$$

and

$$p_1=\left(\frac{\omega Q^2}{2\mu},\frac{\omega Q^2}{2\mu}\right).$$

The sign in front of k in $\mathcal{A}_{\mu\nu}$ reflects the convention we have chosen as to the direction of the momenta entering or leaving the various blocks. The deep-inelastic structure functions are

$$W_1(\nu,q^2)=-\frac{2}{\mu\omega}G_\gamma A_1(1,1;-Q^2)B(\omega)G_h\,,$$
$$W_2(\nu,q^2)=-\frac{2\mu}{Q^2\omega^3}G_\gamma A_2(1,1;-Q^2)B(\omega)G_h\,. \tag{15}$$

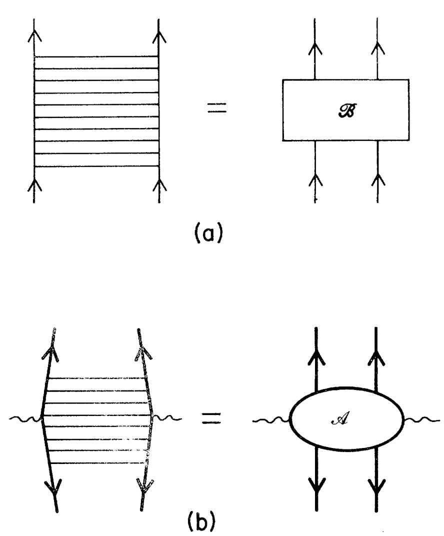

FIG. 3. Abstraction of multiperipheral diagrams. (a) Hadronic part, (b) photon-fragmentation part.

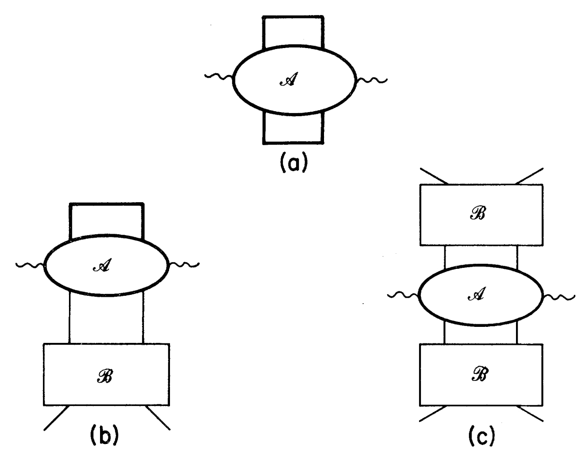

FIG. 4. Abstracted diagrams for massive current processes. (a) $e^+ + e^-$ annihilation, (b) deep-inelastic scattering, (c) massive lepton pair production.

For normalization purposes we note that the total hadronic cross section is

$$\sigma(s)=\frac{\mu^2 G_h B(s/\mu^2) G_h}{s^2}. \qquad (16)$$

D. $e+p\to e+$hadron $(p_1)+\cdots$

This inclusive reaction is illustrated in Figs. 5(b) and 5(c) and obtained through

$$W_{\mu\nu}^{(\text{incl})}(p,p_1;q)=(2\pi)^6\frac{E}{\mu}\frac{E_1}{\mu}\times\sum_n\langle p|J_\mu|p_1,n\rangle\langle p_1,n|J_\nu|p\rangle\times\delta^4(p+q-p_1-p_n). \qquad (17)$$

Simpler expressions occur if one integrates over the azimuthal direction of the observed particle. In our situation, assuming zero transverse momenta, this introduces no further restrictions. Instead of the structure functions of Eq. (17) we introduce[13]

$$\begin{aligned}\mathcal{W}_{\mu\nu}&=\int\mu^2\frac{d^3p_1}{E_1}\delta(p\cdot p_1-\mu\kappa)\delta(q\cdot p_1-\mu\nu_1)W_{\mu\nu}^{(\text{incl})}\\&=-(g_{\mu\nu}-q_\mu q_\nu/q^2)\nu_1\mathcal{W}_1(\nu,\nu_1,\kappa,Q^2)\\&\quad+\Pi_\mu(p)\Pi_\nu(p)\mathcal{W}_2(\nu,\nu_1\kappa,Q^2)/\mu^2.\end{aligned} \qquad (18)$$

Anticipating scaling we introduce the variables $\omega_1=2\mu\nu_1/Q^2$. As the procedure within this model should be familiar by now, we shall just state the results. We have to distinguish, however, two regions in momentum of the produced particle.[8]

1. *Hadronic plateau*

$$\begin{aligned}\mathcal{W}_1&=-\frac{2}{\mu^2\omega\nu_1}\delta\left(\kappa-\frac{\omega\mu}{2\omega_1}\right)G_\gamma A_1(1,1;-Q^2)\\&\quad\times B(\omega_1)g^2B(\omega/\omega_1)G_h,\\\mathcal{W}_2&=-\frac{2}{Q^2\omega^3\nu_1}\delta\left(\kappa-\frac{\omega\mu}{2\omega_1}\right)G_\gamma A_2(1,1;-Q^2)\\&\quad\times B(\omega_1)g^2B(\omega/\omega_1)G_h.\end{aligned} \qquad (19)$$

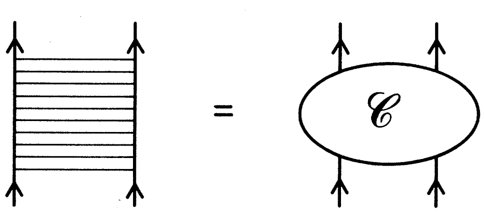

FIG. 5. Kernel in current-fragmentation region.

2. *Current plateau*

$$\begin{aligned}\mathcal{W}_1&=-\frac{2}{\mu^2\omega|\nu_1|}\delta\left(\kappa-\frac{\omega\mu}{2\omega_1}\right)G_\gamma A_1(1,\omega_1;-Q^2)\bar{g}^2\\&\quad\times C(1/\omega_1)B(\omega)G_h,\\\mathcal{W}_2&=-\frac{4\mu}{Q^4\omega^3}\delta\left(\kappa-\frac{\omega\mu}{2\omega_1}\right)G_\gamma A_1(1,\omega_1;-Q^2)\bar{g}^2\\&\quad\times C(1/\omega_1)B(\omega)G_h.\end{aligned} \qquad (20)$$

Equation (20) is valid for $\nu_1>0$. For $\nu_1<0$ we just replace $\delta(\kappa-\omega\mu/2\omega_1)$ by $\delta(\kappa-\omega_1)$. These δ functions reflect the restriction on transverse moments.

E. $p_1+p_2\to\mu^++\mu^-+$hadrons

This process has been previously treated in a similar model.[14] The structure functions are obtained from

$$V_{\mu\nu}(p_1,p_2;q)=(2\pi)^6\frac{E_1E_2}{\mu^2}\times\sum_n\langle p_1,p_2|J_\mu|n\rangle\langle n|J_\nu|p_1,p_2\rangle\times\delta^4(p_1+p_2-q-p_n). \qquad (21)$$

$V_{\mu\nu}$ may be built out of $-(g_{\mu\nu}-q_\mu q_\nu/q^2)$ and the dyadics formed out of $\Pi_\mu(p_1)$ and $\Pi_\nu(p_2)$. As in the previous discussion, the one-dimensional nature of our problem makes these two vectors proportional to one another. Thus we choose

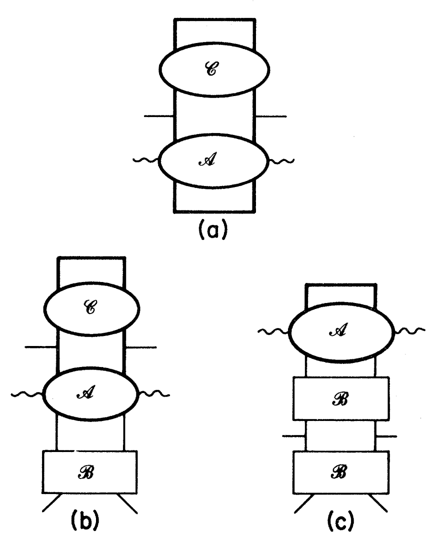

FIG. 6. Inclusive processes. (a) $e^++e^-\to$hadron $+\cdots$, (b) deep-inelastic production in current-fragmentation region, (c) deep-inelastic production in hadronic region.

$$V_{\mu\nu} = -(g_{\mu\nu} - q_\mu q_\nu/q^2)V_1 + \tfrac{1}{2}[\Pi_\mu(p_1)\Pi_\nu(p_2) + \Pi_\mu(p_2)\Pi_\nu(p_1)]\, V_2/\mu^2 . \quad (22)$$

Using the variables $\omega_i = 2p_i \cdot q/q^2$, $s = 2p_1 \cdot p_2$ ($s = q^2\omega_1\omega_2$), one finds

$$V_1(\omega_1, \omega_2; q_\perp) = \frac{2q^2}{\mu^2 s} G_h B(\omega_1) A_1(1, 1; q^2) \times B(\omega_2) G_h \delta^2(q_\perp),$$
$$V_2(\omega_1, \omega_2; q_\perp) = \frac{2}{s\,\omega_1\omega_2} G_h B(\omega_1) A_2(1, 1; q^2) \times B(\omega_2) G_h \delta^2(q_\perp). \quad (23)$$

It should be fairly clear by now how to proceed with any deep-inelastic scattering using the methods of this model.

IV. SCALING

In order to insure scaling we must make an assumption on the large-q^2 limit of $A_{1,2}(x, y; q^2)$. It may be noted that postulating a finite limit for these functions as q^2 increases ensures the expected scaling properties for all the structure functions discussed earlier. However, the A_i's may approach different limits for large spacelike or timelike q^2. This ambiguity may be settled by studying the crossing properties of these amplitudes. With this in mind we define a combination of A_1 and A_2 corresponding to coupling to longitudinal photons,

$$A_L(x, y; q^2) = -\tfrac{1}{4}xyA_2(x, y; q^2) + A_1(x, y; q^2) . \quad (24)$$

The crossing relations state that[6, 15]

$$A_1(x, y; q^2) = -A_1(x, y; -q^2) ,$$
$$A_L(x_1, y; q^2) = +A_L(x, y; -q^2) . \quad (25)$$

With scaling in mind we postulate

$$A_1(x, y; q^2) = \epsilon(q^2)A_1(x, y),$$
$$A_L(x, y; q^2) = A_L(x, y). \quad (26)$$

and list the scaling forms of some of the structure functions discussed earlier:

$$R = \tfrac{1}{2}G_\gamma[2A_1(1, 1) + A_L(1, 1)]\, G_\gamma , \quad (27a)$$

$$\mu \overline{W}_1 = \overline{F}_1(\omega) = 2G_\gamma C(1/\omega)\overline{g}^2 A_1(\omega, 1) G_\gamma , \quad (27b)$$

$$\nu \overline{W}_2 = \overline{F}_2(\omega) = \frac{4}{\omega} G_\gamma C(1/\omega)\overline{g}^2[A_L(\omega, 1) - A_1(\omega, 1)]\, G_\gamma , \quad (27c)$$

$$\mu W_1 = F_1(\omega) = \frac{2}{\omega} G_\gamma A_1(1, 1) B(\omega) G_h , \quad (27d)$$

$$\nu W_2 = F_2(\omega) = \frac{4}{\omega^2} G_\gamma [A_1(1, 1) + A_L(1, 1) B(\omega)]\, G_h , \quad (27e)$$

$$\int d^2q_\perp \mu^2 V_1 = \mathcal{V}_1(\omega_1\omega_2) = \frac{2}{\omega_1\omega_2} G_h B(\omega_1) A_1(1, 1) B(\omega_2) G_h , \quad (27f)$$

$$\int d^2q_\perp \frac{s}{2} V_2 = \mathcal{V}_2(\omega_1, \omega_2) = \frac{4}{\omega_1^2\omega_2^2} G_h B(\omega_1) \times [A_1(1, 1) - A_L(1, 1)]\, B(\omega_2) G_h . \quad (27g)$$

V. HIGH-ENERGY BEHAVIOR

The factorization assumption, crucial to this model, is expected to hold, at best, not only in the scaling region, but in the situation where all subenergies are large. In the language of Regge poles, this corresponds to retaining only the leading trajectory. Conventional wisdom would suggest for large ω both $B(\omega)$ and $C(\omega)$ vary as ω^2, while $A_i(\omega, 1)$ is finite for both large and small ω.

Considering the way the formulas for all the structure functions are presented, one may easily incorporate lower-lying trajectories by treating A_i, B, C, g^2, and $\overline{g}^2$ as matrices and G_h and G_γ as vectors.[10, 16] One then abandons strict factorization in favor of matrix factorization.

VI. AN ASYMPTOTIC RELATION

Factorization properties of amplitudes permit many relations among various processes. Within the context of this model there is an interesting ratio,

$$\lim_{\omega_i \to \infty} \frac{[F_1(\omega_1) + (\omega_1/2)F_2(\omega_1)][F_1(\omega_2) + (\omega_2/2)F_2(\omega_2)]}{6\mathcal{V}_1(\omega_1, \omega_2) - \mathcal{V}_2(\omega_1\omega_2)} = 2R . \quad (28)$$

As factorization may hold only for large ω_i, we expect Eq. (29) to be valid only in this limit. It is possible that such relations may be valid in more general contexts than the one of this model. More detailed discussion of Eq. (28) and a comparison with experiment are presented elsewhere.[17]

VII. RESTRICTIONS AND RELATIONS TO OTHER MODELS

The model discussed in this article is, by construction, consistent with the Regge-Mueller picture of massive current processes. As mentioned earlier it may be viewed as a special realization of the photon-fragmentation region. Likewise, it is consistent with the parton picture conditional on the assumption that the distribution of wee partons is universal and independent of the parton charge. Though the philosophy is different the results are similar to those obtained in the model of Ref. 7. We should note that in all these models parameters may be chosen to ensure the validity of Eq. (28).

The SU(3) singlet nature of the wee-parton region is obtained in this model trivially. As we have not introduced any quantum numbers all the kernels are naturally symmetric. Extending this work to include quantum numbers would necessitate the specific assumption that $A(1,1)$ transforms as an SU(3) singlet. To the extent that the wee region is responsible for diffraction and that diffraction is an SU(3) singlet, this assumption is valid.

Restrictions that pertain to other models may be built into this analysis. Choosing A_L or A_1 to be zero one could reproduce results of spin-$\frac{1}{2}$ or spin-zero parton models.

With the severe restrictions on the kernels A_i, B, and C, we may obliterate the distinction between hadronic and current parts of the inclusive spectra. There is preliminary evidence that such a distinction may not in fact exist.[19] The restrictions sufficient to insure this are

(i) $A_i(x,y)$ constant in x, y;

(ii) $g^2C(\omega)=\bar{g}^2B(\omega)$.

If instead of the second relation above one assumes $G_hB=G_\gamma C\bar{g}^2$ and takes $A_L=0$ then the asymptotic reciprocal relations

$$F_1(1/\omega)=\frac{1}{\omega}\,\bar{F}_1(\omega)\,,$$
$$F_2(1/\omega)=-\frac{1}{\omega^3}\,\bar{F}_2(\omega) \qquad (29)$$

will hold.[20] These relations are not natural to this model.

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