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Calculation of anomalous magnetic moments of nucleons

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A sideways dispersion relation is used to calculate the anomalous magnetic moments of nucleons. The absorptive parts for all energies are obtained as extrapolations of the one-pion–one-nucleon intermediate state in the elastic region having proper threshold behavior and an assumed Regge behavior at high energies. The calculated values are $\mu_S = -0.06$ and $\mu_V = 1.81$.

I. INTRODUCTION

One of the spectacular successes of perturbation calculations in quantum electrodynamics is in predicting the anomalous magnetic moment of the electron to a high degree of accuracy. The same technique was first applied by Case[1] in 1949 to calculate the anomalous magnetic moments of nucleons. It was found that the results are quite different from the experimental values. The reason was the general failure of the Dyson-Feynman type of perturbation expansion for strong interactions. Attempts were then made to use dispersion relations in "photon mass."[2] Assuming the dominance of two-pion states, the isovector part could be calculated to be $\mu_V \simeq 1.65$ with a fair degree of reliability. (We specify magnetic moments in nuclear magnetons throughout.) The calculation of the isoscalar part was, most often, conveniently ignored.

In answering a challenge of Feynman, Drell and Pagels[3] used a dispersion relation in electron mass and showed that the Schwinger correction,[4] $\alpha/2\pi$, of the electron magnetic moment can be calculated quite easily in terms of the exact low-energy Thomson limit to Compton scattering of photons by electrons.[5] They also calculated the anomalous magnetic moments of nucleons for which the Kroll-Ruderman theorem provided the necessary low-energy "anchor."

In a continuing effort in this direction, we report here the results of a calculation of the anomalous magnetic moments of nucleons from Bincer's sideways dispersion relation,[6] assumed valid without subtractions, using some ideas of modern analytic approximation and extrapolation theory.[7] Specifically, the scalar and vector form factors will be given by the relations

$$F_2^{S,V}(W^2) = \frac{1}{\pi}\int_{(m+m_\pi)^2}^{\infty} \frac{\mathrm{Im}F_2^{S,V}(W'^2)dW'^2}{W'^2 - W^2}, \qquad (1)$$

where

$$F_2^S(W^2) = \tfrac{1}{2}[F_2^p(W^2) + F_2^n(W^2)],$$

$$F_2^V(W^2) = \tfrac{1}{2}[F_2^p(W^2) - F_2^n(W^2)],$$

and the anomalous parts are given by $\mu_S = F_2^S(m^2)$ and $\mu_V = F_2^V(m^2)$.

Considered as functions of W^2, the form factors are assumed to be analytic functions in the entire complex W^2 plane with a cut extending from $(m+m_\pi)^2$ to ∞, where $(m+m_\pi)^2$ is the threshold for the production of the one-pion–one-nucleon intermediate state. The inelastic region starts from $(m+2m_\pi)^2$. Below this inelastic threshold the absorptive part is given exactly by relating it to the Chew-Goldberger-Low-Nambu (CGLN)[8] invariant amplitudes ($A^{\pm,0}$) of photoproduction,

$$\mathrm{Im}F_2^S(W^2) = \frac{m|\vec{k}|}{8\pi W(W^2-m^2)} \times \int_{-1}^{+1} d(\cos\theta) g(l-p')\cdot q\,\mathrm{Re}(3A^0), \qquad (2)$$

$$\mathrm{Im}F_2^V(W^2) = \frac{m|\vec{k}|}{8\pi W(W^2-m^2)} \times \int_{-1}^{+1} d(\cos\theta) g(l-p')\cdot q\,\mathrm{Re}(2A^- + A^+). \qquad (3)$$

W and $\vec{k}$ are the c.m. energy and pion momentum as shown in Fig. 1 and θ is the scattering angle.

If Eqs. (2) and (3) are used for all $s = W^2$, the nucleon pole term with $g^2/4\pi = 14.6$ gives the perturbation values, $\mu_V = 2.17$ and $\mu_S = -1.65$. These are far from the experimental result. The reason for this has been pointed out by Drell and Pagels. Near threshold the imaginary parts of the scalar and vector amplitudes behave, in the limit $(m_\pi/m) \to 0$, as

$$\mathrm{Im}F_2^S(W^2) \simeq -\frac{g^2}{4\pi}\frac{(W^2-m^2)^2}{4m^4}, \qquad (4)$$

$$\mathrm{Im}F_2^V(W^2) \simeq \frac{g^2}{4\pi}\frac{W^2-m^2}{2m^2}. \qquad (5)$$

Even though the scalar part is very small near

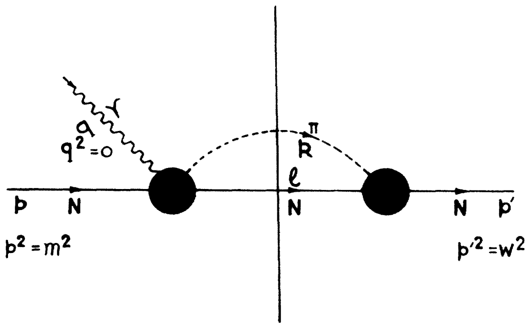

FIG. 1. Pion-nucleon intermediate-state contribution to the absorptive parts of the nucleon form factors.

threshold, it overtakes the vector part for larger values of W^2. Good values for the magnetic moments are obtained if one cuts off the dispersion integral at $s=(1.5M)^2$. This suggests that soon after the inelastic threshold, multiparticle-production effects damp the amplitude. For large s, the Born term is known not to give the correct behavior of the physical amplitudes $A^{\pm,0}$. They are given either by Regge theory or in actuality, by experiment.[9] So a cutoff becomes physically acceptable.

Ademollo, Gatto, and Longhi[10] have also calculated the anomalous magnetic moments by using the sideways dispersion relation using Born terms, a soft πNN vertex, and low-energy contributions from N^* to $\mathrm{Im}F_2^{S,V}$. They have also taken rescattering into consideration. Even then, the values obtained by them are $\mu_S=-0.17$, $\mu_V=1.0$. In this paper we try to present a theory that unifies the Drell and Pagels concept of dominance of threshold contribution to absorptive parts and the importance of taking the exact threshold behavior of amplitudes with the remark of Ademollo, Gatto, and Longhi that the amplitudes exhibit damping at high energies.

The plan of the paper will be as follows: In Sec. II we discuss the inputs used in our calculation. Information about the threshold properties and justification for an assumed high-energy behavior of the amplitude are given in this section. In Sec. III details of the calculation using the technique of conformal mapping are reported. The results are discussed in Sec. IV.

II. INPUTS FOR CALCULATION

A. Perturbation results

As mentioned before, the lowest-order perturbation calculations were first made by Case. The two Feynman diagrams used are shown in Fig. 2. One has to carry out mass and charge renormalization according to well-established procedures of quantum electrodynamics. In the context of sideways dispersion relations, the perturbation results are obtained by calculating $\mathrm{Im}F_2^{S,V}(W^2)$ from the equations using

$$A^{\pm}=\frac{eg}{2}\left(\frac{1}{s-m^2}\pm\frac{1}{u-m^2}\right), \tag{6}$$

$$A^{0}=\frac{eg}{2}\left(\frac{1}{s-m^2}+\frac{1}{u-m^2}\right), \tag{7}$$

and then evaluating the dispersion integral [Eq. (1)] for $W^2=M^2$. For a pion-nucleon coupling strength $g^2/4\pi\simeq 15$, one obtains $\mu_p=0.52$ and $\mu_n=-3.82$ as compared with the experimental values $\mu_p=1.79$ and $\mu_n=-1.91$. Even the ratio μ_p/μ_n, which to this order is independent of coupling strength, turns out to be 0.13 as compared with 1.07.

Since the photoproduction amplitudes are needed and several workers[11] have shown that $N^*(\frac{3}{2},\frac{3}{2})$ can contribute as much as 40% to the amplitude, we give these amplitudes for the low-lying resonances N^*, ρ, and ω as determined, for instance, by Ahmad and Fayyazuddin[12]:

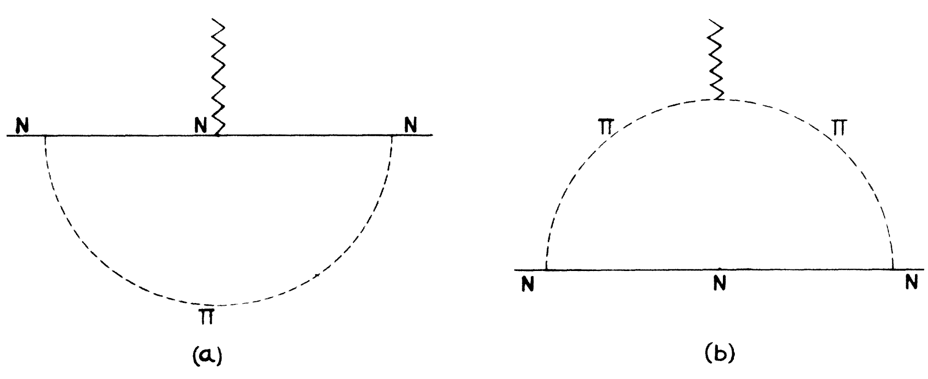

FIG. 2. Conventional contributions to the nucleon magnetic moments.

$$A^{\pm} = \frac{eg}{2}\left(\frac{1}{s-m^2} \pm \frac{1}{u-m^2}\right)$$
$$\pm \begin{pmatrix} 2 \\ 1 \end{pmatrix} \frac{g_{\pi NN^*} g_{\gamma NN^*}}{12 m_\pi^2}\left[\left(2t + \frac{2}{3}\frac{m}{m^*}(s-m^2+2m_\pi^2) + \frac{2}{3}\frac{m^2}{m^{*2}}(s-m^2+m_\pi^2)\right)\frac{1}{m^{*2}-s} + (s-u)\right]$$
$$+ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \frac{teg_{\omega\pi\gamma}F_{2\omega NN}}{m_\omega^2 - t}, \tag{8}$$
$$A^0 = \frac{eg}{2}\left(\frac{1}{s-m^2} + \frac{1}{u-m^2}\right) - \frac{etg_{\rho\pi\gamma}F_{2\rho NN}}{m_\rho^2 - t}. \tag{9}$$

We shall assume that within the limited energy range of W^2 lying between the elastic and inelastic thresholds, the $\mathrm{Im}F_2^{S,V}(W^2)$ are exactly given by these amplitudes. Further, within this small range, we shall assume that the coupling strengths are given by their on-shell values. The above assumptions are quite appropriate for the present calculation. With rapid accumulation of experimental data, the values of these coupling strengths are getting known more and more accurately.

The $\mathrm{Im}F_2^{S,V}$ obtained by integrating Eqs. (2) and (3) with $A^{\pm,0}$ of Eqs. (8) and (9) are

$$\mathrm{Im}F_2^S(W^2) = \frac{g^2}{4\pi}\frac{3}{4}\left(\frac{x_+^2 - x_-^2}{2} + x_+ - x_- + \ln\frac{1-x_+}{1-x_-}\right)\frac{m^2}{W^2-m^2}$$
$$+ \frac{3m^2 g}{8\pi} g_{\rho\pi\gamma}F_{2\rho NN}\left[\frac{x_+^2-x_-^2}{2} - \frac{m_\rho^2(x_+-x_-)}{W^2-m^2} - \frac{m_\rho^2(m_\rho^2-m_\pi^2)}{(W^2-m^2)^2}\ln\frac{m_\rho^2-m_\pi^2+(W^2-m^2)x_-}{m_\rho^2-m_\pi^2+(W^2-m^2)x_+}\right], \tag{10}$$

$$\mathrm{Im}F_2^V(W^2) = -\frac{g^2}{4\pi}\frac{1}{4}\frac{m^2}{W^2-m^2}\left[-\tfrac{3}{2}(x_+^2-x_-^2) + x_+ - x_- + \ln\frac{1-x_+}{1-x_-}\right]$$
$$+ \frac{g_{\pi NN^*}g_{\gamma NN^*}}{3\pi^2}\frac{m^2 g}{8\pi e}\left\{\frac{x_+^2-x_-^2}{2}\left(1 - \frac{m}{3m^*} - \frac{m^2}{3m^{*2}}\right)\right.$$
$$+\left[W^2 + m_\pi^2 - m^{*2} + \frac{m}{3m^*}(m^{*2}+2m_\pi^2-m^2) + \frac{m^2}{3m^{*2}}(m^{*2}-m^2+m_\pi^2)\right]$$
$$\left.\times\left[\frac{m^{*2}-2m^2+W^2}{(W^2-m^2)^2}\ln\frac{m^{*2}-m^2+(W^2-m^2)(1-x_-)}{m^{*2}-m^2+(W^2-m^2)(1-x_+)} - \frac{x_+-x_-}{W^2-m^2}\right]\right\}$$
$$- \frac{3m^2 g}{8\pi} g_{\omega\pi\gamma}F_{2\omega NN}\left[\frac{x_+^2-x_-^2}{2} - \frac{m_\omega^2(x_+-x_-)}{W^2-m^2} - \frac{m_\omega^2(m_\omega^2-m_\pi^2)}{(W^2-m^2)^2}\ln\frac{m_\omega^2-m_\pi^2+(W^2-m^2)x_-}{m_\omega^2-m_\pi^2+(W^2-m^2)x_+}\right], \tag{11}$$

where

$$x_\pm = \frac{W^2+m_\pi^2-m^2}{2W^2} \pm \frac{1}{2W^2}\{[W^2-(m+m_\pi)^2][W^2-(m-m_\pi)^2]\}^{1/2}.$$

Our calculation will show that though ω makes a negligible contribution to the isovector part of the magnetic moment, the effect of ρ on the isoscalar part is appreciable.

B. Threshold behavior

We wish to emphasize that the exact threshold behavior of the amplitude for charged pion photoproduction is given, for $(m_\pi/m)\to 0$, by the renormalized perturbation Born term, according to the Kroll-Ruderman theorem.[13] For hard pions the corresponding threshold behavior is given by

$$\mathrm{Im}F_2^S(W^2) = -\frac{g^2}{4\pi}\frac{3}{2}\frac{m^3(W^2-W_{\mathrm{th}}^2)^{1/2}}{(m+m_\pi)^3(2m+m_\pi)}\left(\frac{m_\pi}{m}\right)^{3/2}, \tag{12}$$

$$\mathrm{Im}F_2^V(W^2) = \frac{g^2}{4\pi}\frac{1}{2}\frac{m^2(4m+m_\pi)(W^2-W_{\mathrm{th}}^2)^{1/2}}{(m+m_\pi)^3(m_\pi+2m)}\left(\frac{m_\pi}{m}\right)^{1/2}. \tag{13}$$

In the limit of $m_\pi/m\to 0$, these equations should reduce to the results of Eqs. (4) and (5). The equivalence of $\frac{2}{3}(W^2-m^2)/m^2$ to m_π/m should be noted. To see this more clearly, expansion of the absorptive parts is first made for small values of the variable $x=x_+-x_-$, obtaining

$$\mathrm{Im}F_2^S(W^2)=-\frac{g^2}{4\pi}\frac{3}{4}\frac{mm_\pi}{(m+m_\pi)(2m+m_\pi)}x, \qquad (14)$$

$$\mathrm{Im}F_2^V(W^2)=\frac{g^2}{4\pi}\frac{1}{4}\frac{m(4m+m_\pi)}{(2m+m_\pi)(m+m_\pi)}x. \qquad (15)$$

Taking the hard-pion limit at $W^2=W_{th}^2$ we obtain (12) and (13), and taking the soft-pion limit $m\pi/M\to 0$ followed by the threshold limit $W^2=M^2$ leads to Eqs. (4) and (5). Since it has been conjectured that the dominant contributions are from the threshold, the above considerations are important in choosing a correct variable for expansion of the magnetic form factors.

C. High-energy behavior

The nucleon Born terms lead to a logarithmic increase in energy for the absorptive parts. This is responsible for the large values of the scalar and vector anomalous moments obtained from perturbation theory. A convenient cutoff, therefore, leads to correct results. To obtain a more reliable high-energy behavior, one can try to use Eqs. (8) and (9) directly relating the large-energy behavior of the photoproduction amplitudes. The present status of experimental data listings did not enable us to make an estimate of the high-energy behavior of the real parts of A^+, A^0 for all allowed values of t needed for the integrations indicated in Eqs. (2) and (3). So we made the simple ansatz that the amplitudes are given by Regge theory. The following replacements were made[14]:

$$\frac{1}{t-m_\rho^2}\to -\alpha'\Gamma(1-\alpha_\rho(t))\,\frac{1-\cos\pi\alpha_\rho(t)}{2}s^{\alpha_\rho(t)-1}, \qquad (16)$$

$$\frac{1}{t-m_\omega^2}\to -\alpha'\Gamma(1-\alpha_\omega(t))\,\frac{1-\cos\pi\alpha_\omega(t)}{2}s^{\alpha_\omega(t)-1}, \qquad (17)$$

$$\frac{1}{u-m^2}\to -\alpha'\Gamma(\tfrac{1}{2}-\alpha_N(u))\times\frac{1+\cos\pi(\alpha_N(u)-\frac{1}{2})}{2}s^{\alpha_N(u)-1/2}, \qquad (18)$$

$$\frac{1}{u-m^{*2}}\to \alpha'\Gamma(\tfrac{3}{2}-\alpha_{N^*}(u))\times\frac{1-\cos\pi(\alpha_{N^*}(u)-\frac{1}{2})}{2}s^{\alpha_{N^*}(u)-3/2}. \qquad (19)$$

An absorption factor e^{bt} with $b\simeq 1.5$ was inserted for the ρ and ω terms so that only very forward angles contribute to the integration. We then assumed that the absorptive parts behave as $s^{-\alpha}$ for large s. This was found to be the case by plotting the absorptive parts against s on log-log graph paper as shown in Fig. 3. The values of α for the scalar and vector parts obtained from the plot are 0.89 and 0.35, respectively.

The positive lesson one learns is that at high energy, the absorptive parts fall off with energy. Furthermore, the falloff is faster for $\mathrm{Im}F_2^S$ than it is for $\mathrm{Im}F_2^V$. The observed small value of μ_S is attributable to this large decrease with energy. The high-energy phase (like a Regge phase), however, cannot be reliably found from this simple ansatz and, instead, was determined from actual fits as will be discussed below. Since the asymptotic behavior is determined from the real part of the A's, the phases have to be chosen with some degree of reliability.

III. METHOD OF CALCULATION

A. Conformal mapping

With the above inputs, the problem at hand is to extrapolate to the inelastic region from a knowledge of the elastic region, with the assumed high-

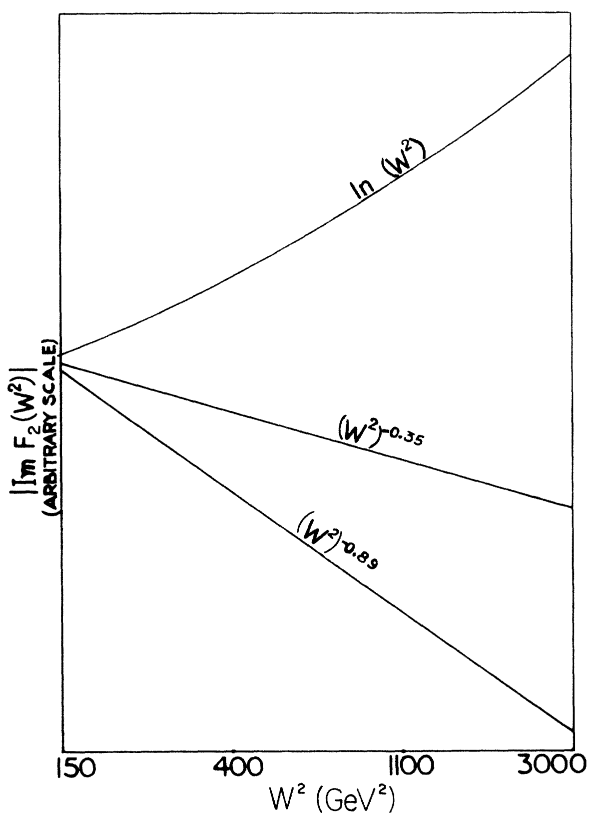

FIG. 3. Asymptotic nature of $\mathrm{Im}F_2^{S,V}(W^2)$.

energy constraint. Since this is a cut-to-cut extrapolation, we may follow Ciulli and Nenciu[15] and formally map the cut to form the boundary of a circle. The cut plane of analyticity will be within the circle of convergence, leading to the possibility of writing the form factors as a maximally converging series in this mapped variable. A convenient mapping could be

$$Y = \frac{W^2}{2W_{\text{th}}^2 - W^2}, \tag{20a}$$

$$Z = \frac{1}{Y}\left[1 - (1 - Y^2)^{1/2}\right]. \tag{20b}$$

However, at threshold this has only an $(s - s_{\text{th}})^{1/2}$, i.e., a square-root branch point, so that the matching with either Eq. (4) or (5) will not be accurate in the sense that the leading coefficient will be very small.[16] So we first open up the inelastic branch cut and then map the plane to the inside of a circle. The mapping variable is given, in terms of an adjustable parameter W_0^2, by the relations

$$Y = \frac{W^2 - W_0^2}{[(W^2 - W_{\text{th}}^2)(W^2 - W_{\text{in}}^2)]^{1/2} + [(W_0^2 - W_{\text{th}}^2)(W_0^2 - W_{\text{in}}^2)]^{1/2}}, \tag{21a}$$

$$Z = \frac{1}{Y}\left[1 - (1 - Y^2)^{1/2}\right]. \tag{21b}$$

The value of W_0^2 is chosen so that, using the equivalence of m_π/m to $\frac{2}{3}(W^2 - m^2)/m^2$,

$$\text{Im} Z_S \sim (W^2 - m^2)^2, \tag{22a}$$

$$\text{Im} Z_V \sim (W^2 - m^2) \tag{22b}$$

at threshold. We take $(W_0^S)^2 = (m + m_\pi)^2 - \frac{1}{10}(m/m_\pi)^3 m^2$ and $(W_0^V)^2 = (m + m_\pi)^2 - (m/m_\pi)m^2$. The sketch of the mapped plane is shown in Fig. 4(b) along with the W^2-plane analytic structure in Fig. 4(a). We see in this figure that the elastic region is a rising curve and that beyond $s = (m + 2m_\pi)^2$ the imaginary part falls off, vanishing at infinity as $s^{-1/2}$ as given by Eq. (21).

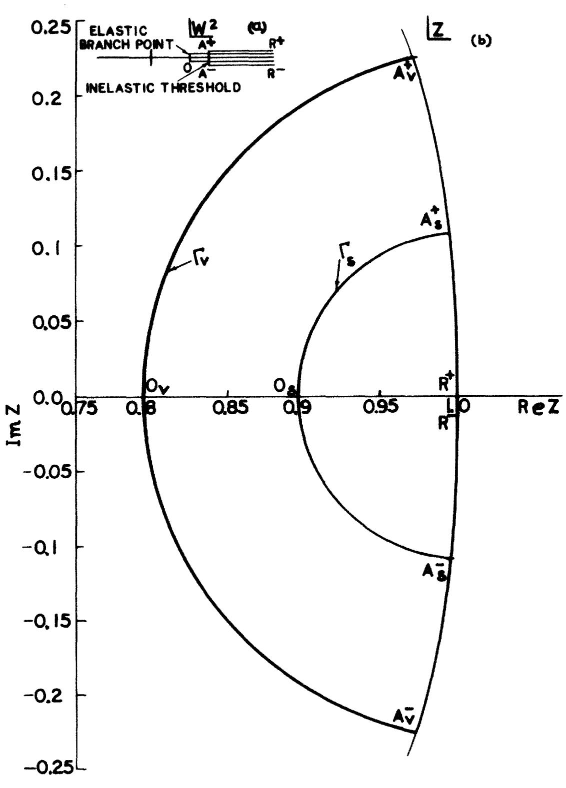

FIG. 4. (a) W^2-plane analyticity of $F_2^{S,V}(W^2)$ and (b) z-plane analyticity of $F_2^{S,V}(W^2)$.

Next we construct orthonormal polynomials in z, such that

$$\int_{\Gamma_{S,V}} \phi_m^*(z)\phi_n(z)\,|dz| = \delta_{mn}, \tag{23}$$

where

$$\phi_n(z) = \sum_{m=0}^{n} b_m^{(n)} z^m$$

and $\Gamma_{S,V}$ are the elastic contours. This can be easily done by using Schmidt's orthogonalization procedure. We then write the form factor as[17]

$$F_2(W^2) = \psi_R(z) \sum_n a_n\,\phi_n(z). \tag{24}$$

Here $\psi_R(z)$ is the Regge factor which in the variable z can be written as

$$\psi_R^S(z) = \left(\frac{1 - z}{1 - z^*}\right)^{\beta^S} [(1 - z)(1 - z^*)]^{\nu}, \tag{25a}$$

$$\psi_R^V(z) = \left(\frac{1 - z}{1 - z^*}\right)^{\beta^V} [(1 - z)(1 - z^*)]^{\nu}. \tag{25b}$$

The factors $\beta^{S,V}$ are the high-energy phase factors which were fixed by the matching procedure outlined below.

For a given n, the coefficients a_n were calculated by inverting an $(n \times n)$ matrix, obtained by equating $\text{Im}F_2^{S,V}(W^2)$ of Eq. (24) at n equally spaced s points in the elastic region with perturbation values of $\text{Im}F_2^{S,V}(W^2)$ as given by equations such as (10) and (11). This is done for values of $\beta^{S,V}$ around the expected Regge phase values for the asymptotic behavior obtained by the procedure outlined in Sec. II. Then a least-squares fit for the

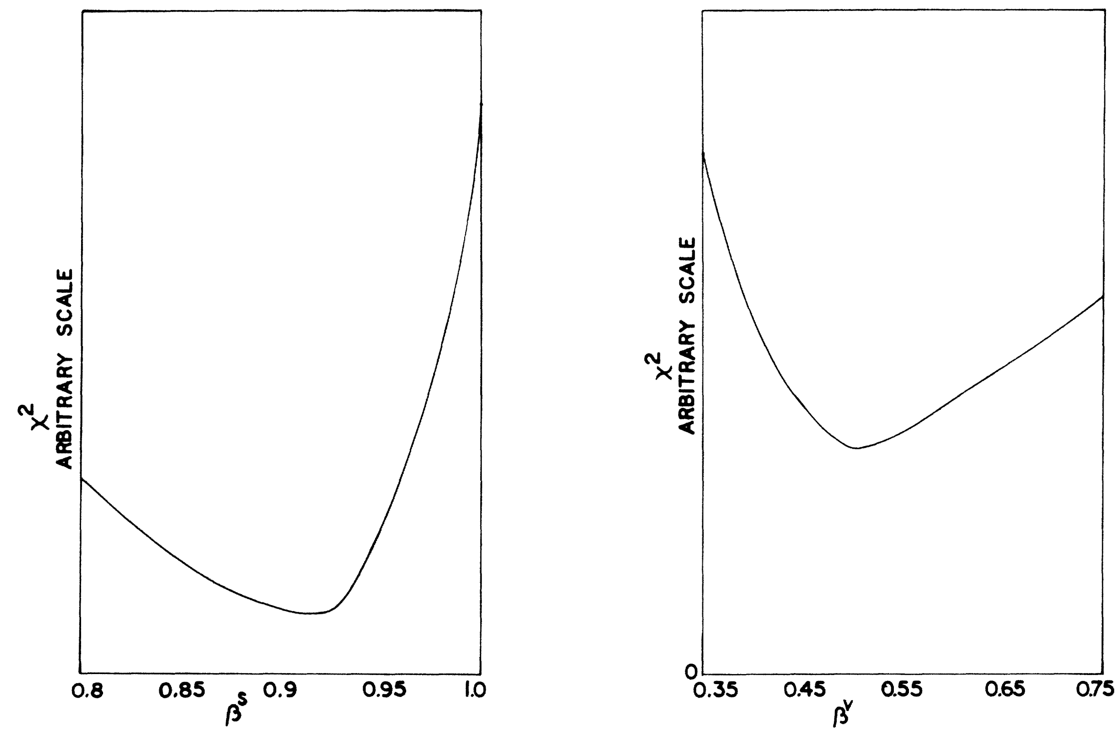

FIG. 5. χ^2 plot to determine the phase factors.

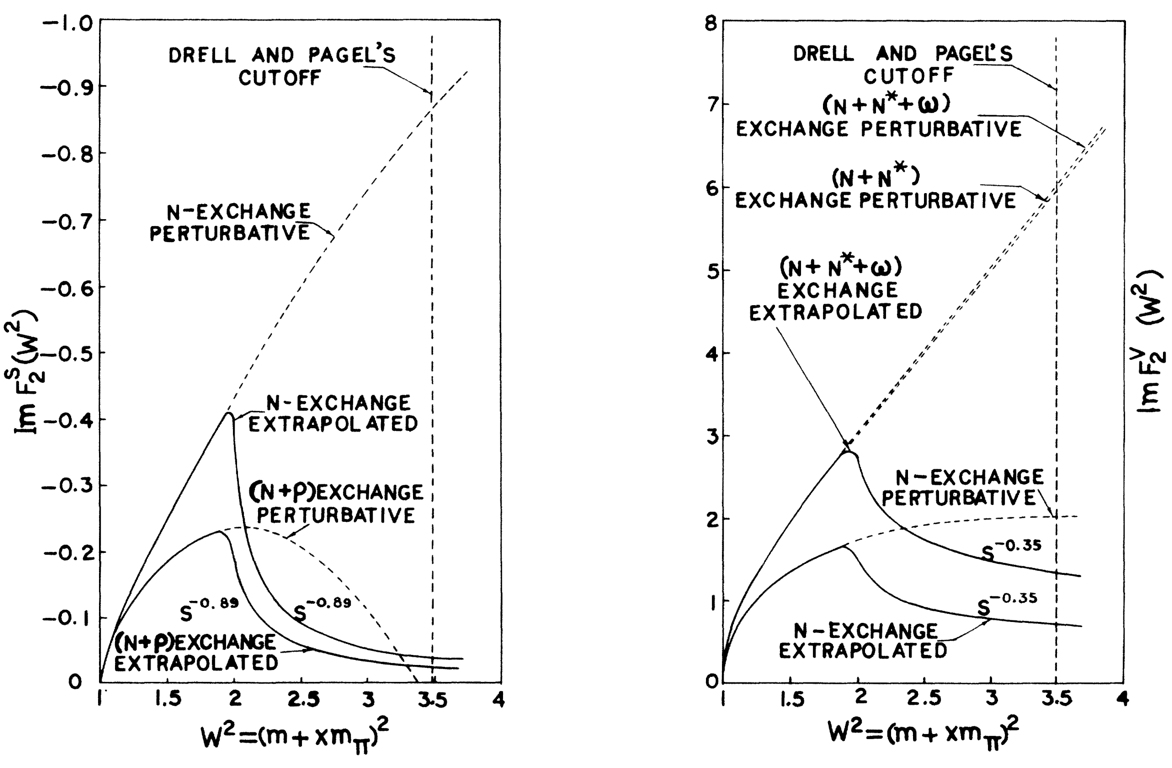

FIG. 6. Behavior of perturbative and extrapolated $\mathrm{Im}F_2^{S,V}(W^2)$ with W^2.

TABLE I. Estimated contributions from N, N^*, ρ, and ω exchanges.

Magnetic moments \ Exchanged particles	N	$N+N^*$	$N+N^*+(\rho \text{ or } \omega)$
μ_S	-0.0829	-0.0829	-0.0552
μ_V	1.0076	1.8026	1.8087

entire elastic region is done to find out which values of $\beta^{S,V}$ gave the best fit by making a χ^2-plane plot as shown in Fig. 5.[18] The phase values obtained are $\beta^V=0.5$, $\beta^S=0.92$. It was found that only four terms were enough for the calculation.[19] The results of the fits below the inelastic threshold and the subsequent extrapolation are shown in Fig. 6.

IV. RESULTS AND DISCUSSION

$\mathrm{Im}F_2^{S,V}$ as given in Eq. (24) are now assumed to extrapolate correctly into the inelastic region. Strong damping can be seen from Fig. 6, indicating that inelastic effects start predominating from fairly low energies. The dispersion integrals are evaluated and the results are shown in Table I. We see that the ρ contribution to the scalar form factor is quite appreciable, leading to a value very close to the experimental one. The vector form factor has a much larger contribution from N^* which is quite expected. Our final results are $\mu_S=-0.06$, $\mu_V=1.81$ compared with experimental values $\mu_S=-0.06$, $\mu_V=1.85$. It may be noted that these are the best theoretical values reported so far, giving $\mu_p=1.744$ and $\mu_n=-1.864$.

It is to be emphasized that we do not have any unknown parameters except the $g_{NN^*\pi}$, $g_{NN^*\gamma}$, $g_{NN\pi}$, $\gamma_{\rho\pi\pi}$, $\gamma_{\omega\pi\pi}$, and the Regge behavior of $s^{-0.89}$ for the scalar and $s^{-0.35}$ for the vector amplitudes. The near exact agreement with experiment is due to the fact that the more accurate information on the elastic region has been approximated by a converging set of polynomials which extrapolates correctly and smoothly into the unknown inelastic region and exhibits the assumed asymptotic behavior at high energies.

ACKNOWLEDGMENTS

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[16]Calculation with circular mapping following a procedure analogous to that outlined in the text yields $\mu_V=1.67$ and $\mu_S=-0.8$.
[17]The expression (24) for $F_2(s)$ may differ by an entire function having no discontinuity in the complex s plane.
[18]In finding χ^2, we have compared our theoretical values of $F_2^{S,V}$ given by Eq. (24) with perturbation values given by Eqs. (10) and (11). We have taken the error as

$$[A+B(W^2-W_{\mathrm{th}}{}^2)^{1/2}]\times \text{perturbation value}.$$

A and B are chosen such that the errors at the elastic and inelastic thresholds are 1% and 20% of the perturbation values, respectively.
[19]The a_n's obtained for the scalar and vector cases are as follows:

$$a_1^S=4.3847,\ a_2^S=-1.709,\ a_3^S=-0.5727,\ a_4^S=-0.3053;$$

$$a_1^V=-4.8043,\ a_2^V=-0.2694,\ a_3^V=-0.1042,\ a_4^V=0.0057.$$