Octet dominance in hyperon decays

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Expanding the hyperon decay transition amplitudes in terms of the eigenamplitudes of the direct channel and assuming that only the eigenamplitudes corresponding to physical particle states can contribute to the transitions, we are led to the hypothesis of octet dominance. This assumption also yields a few extra relations for s- and p-wave amplitudes which compare favorably with experiments.

I. INTRODUCTION

The current × current form of the effective Hamiltonian for weak hyperon decays, i.e.,

$$H_\omega^h = -\frac{G}{\sqrt{2}} J_\lambda \bar{J}_\lambda , \qquad (1)$$

with J_λ and $\bar{J}_\lambda$ as Cabibbo currents belonging to the same unitary octet, can contain terms transforming as the $\underline{1}$, $\underline{8}_s$, and $\underline{27}$ representations of SU_3. The antisymmetric coupling terms in the direct product $8 \otimes 8$ are excluded by the Bose-Einstein statistics. It has been suggested by many authors[1] that the effective Hamiltonian for these processes contains only a term belonging to the octet representation. This is the hypothesis of octet dominance—the SU(3) equivalent of the $\Delta I = \frac{1}{2}$ rule. It yields the Lee-Sugawara relation for the s-wave amplitudes in addition to the $\Delta I = \frac{1}{2}$ relations. Experimental evidence overwhelmingly favors these relations.

Out of the three terms mentioned above, the singlet can be dropped because it does not contribute to any of the observed hyperon decays. However, it is neither simple nor trivial to rule out the possibility of a $\underline{27}$ contribution to H_ω^h. Attempts have been made to consider H_ω^h transforming as a pure $\underline{27}$,[2] and as an admixture of $\underline{8}_s$ and $\underline{27}$.[3] The former approach (H_ω^h as pure $\underline{27}$) has yielded results in conflict with experiment. The results obtained from the latter consideration (H_ω^h as an admixture of $\underline{8}_s$ and $\underline{27}$) have not been tested experimentally.

We think that at the present level of experimental precision there is no escape from the hypothesis of octet dominance. In this paper we analyze the dynamical considerations that can lead to this hypothesis. Expanding the hyperon decay transition amplitudes in terms of the eigenamplitudes of the direct channel we show that a single assumption—*that only the eigenamplitudes corresponding to the physically observed particle states ($\underline{1}$, $\underline{8}$, and $\underline{10}$) can contribute to the transitions*—makes all the $\underline{27}$ contributions vanish. The only other assumption made in obtaining this result is that of CP invariance, which is not seriously doubted for the hyperon decays.

An encouraging feature of the analysis is that this assumption leads to the following relations among amplitudes:

$$\sqrt{3}\, A(\Sigma_0^+) = A(\Lambda_-^0) + 2A(\Xi_-^-) , \qquad (2)$$

$$-(\tfrac{3}{2})^{1/2} A(\Sigma_-^-) = 3A(\Lambda_-^0) + 3A(\Xi_-^-) , \qquad (3)$$

$$B(\Lambda_-^0) = (\tfrac{3}{2})^{1/2}[\tfrac{4}{3} B(\Sigma_+^+) - \sqrt{2}\, B(\Sigma_0^+)] , \qquad (4)$$

$$2B(\Sigma_+^+) = (\tfrac{1}{6})^{1/2}[7B(\Lambda_-^0) + 5B(\Xi_-^-)] . \qquad (5)$$

As shown in the text of this paper, their agreement with experiment is quite good. Incidentally, we also get the relation

$$-\Delta_B\Lambda = \Delta_B\Xi = (\tfrac{3}{2})^{1/2}\Delta_B\Sigma \quad (\text{for } p \text{ wave}) , \qquad (6)$$

similar to the one derived by Riazuddin and Fayyazuddin[9] using current algebra, etc. We need a much smaller number of assumptions for this.

In Sec. II we give an outline of the method adopted, and the relevant assumptions are spelled out. Section III gives in detail the analysis and results for parity-violating (s-wave) amplitudes. Section IV deals similarly with parity-conserving (p-wave) amplitudes. In the Appendix complete expansions for the transition amplitudes are listed.

II. GENERALITIES

We start with the general current × current weak Hamiltonian for hyperon decays, containing both $\underline{8}_s$ and $\underline{27}$ parts. The tensor structure of the Hamiltonian can be written as[4]

$$H_\omega^h \sim T^{(8)}_{(1,1/2,-1/2)} + T^{(27)}_{(1,1/2,-1/2)} + \sqrt{5}\, T^{(27)}_{(1,3/2,-1/2)} . \qquad (7)$$

The hyperon decay process $B \to B'\pi$ we write as

$$S + B \to m \to B' + \pi . \qquad (8)$$

The spurion S has the same tensor structure as the Hamiltonian [Eq. (7)] and is such that strong quantum numbers in Eq. (8) are conserved.

The transition amplitudes $X(B \to B'\pi)$ are expanded in terms of the reduced amplitudes of the direct channel by using the SU(3) Clebsch-Gordan coefficients corresponding to the projection of initial and final states on the eigenstates $|m\rangle$.[5] These reduced amplitudes are defined as

$$\begin{aligned} a_m &= \langle B'\pi|m\rangle\langle m|8_s|8_B\rangle, \\ b_m &= \langle B'\pi|m\rangle\langle m|27_s|8_B\rangle. \end{aligned} \tag{9}$$

In all we have eight a's, i.e., a_{27}, a_{10}, a_{10^*}, $a_{8_{11}}$, $a_{8_{12}}$, $a_{8_{21}}$, $a_{8_{22}}$, a_1 and six b's, i.e., b_{27_1}, b_{27_2}, b_{10}, b_{10^*}, b_{8_1}, b_{8_2}. Only the direct channel is taken into account because this alone is sufficient at low energy.

Eight hyperon decays are considered (listed in the Appendix). Seven of these are the traditional (observed) nonleptonic decays,

$$\Lambda^0_-,\ \Lambda^0_0,\ \Sigma^+_+,\ \Sigma^+_0,\ \Sigma^-_-,\ \Xi^-_-,\ \Xi^0_0.$$

The eighth is the nonphysical transition $\Sigma^0 \to n\pi^0(\Sigma^0_0)$. It becomes significant to consider this transition once the $\Delta I = \frac{1}{2}$ restriction is relaxed and it is possible to have four independent Σ decays.

The consequences of CP invariance are derived by writing the crossed-channel

$$B' + \bar{S} \to B + \bar{\pi}$$

transition amplitudes $X(B' \to B\bar{\pi})$ and then applying the conditions

$$A(B \to B'\pi) = -A(B' \to B\bar{\pi}) \tag{10}$$

for parity-violating (s-wave) amplitudes, and

$$B(B \to B'\pi) = B(B' \to B\bar{\pi}) \tag{11}$$

for parity-conserving (p-wave) amplitudes. This reduces the number of independent reduced amplitudes to six for the parity-violating case and seven for the parity-conserving case.

The crucial assumption of retaining only those eigenamplitudes in the direct channel which correspond to the observed particle states $|m\rangle$ imposes the condition

$$\begin{aligned} b_{27,10^*} &= 0 \quad \text{(i)}, \\ a_{27,10^*} &= 0 \quad \text{(ii)}. \end{aligned} \tag{12}$$

It should be noticed that the condition (i) of (12) alone gives the octet dominance.

III. PARITY-VIOLATING (s-WAVE) AMPLITUDES

The CP invariance condition (10) gives the following relations among the reduced matrix elements:

$$\begin{aligned} b_{27_1} &= -\tfrac{1}{108}(17b_{10} + 14b_{10^*}), \\ b_{27_2} &= -\tfrac{3}{2}(b_{10} - b_{10^*}), \\ b_{8_1} &= \tfrac{1}{54}(37b_{10} + 40b_{10^*}), \\ b_{8_2} &= \tfrac{1}{180}(-403b_{10} + 800b_{10^*}), \end{aligned} \tag{13}$$

$$\begin{aligned} a_{27} &= \tfrac{1}{72}[18(a_{10} - a_{10^*}) - 36(a_{8_{11}} - a_{8_{22}}) - 31b_{10} \\ &\qquad - 16b_{10^*}], \\ a_1 &= \tfrac{1}{72}[-54(a_{10} - a_{10^*}) + 540(a_{8_{11}} - a_{8_{22}}) + 737b_{10} \\ &\qquad + 464b_{10^*}], \\ a_{8_{12}} &= \tfrac{1}{1080}[270(a_{10} + a_{10^*}) - 427b_{10} + 20b_{10^*}], \\ a_{8_{21}} &= \tfrac{1}{1080}[270(a_{10} + a_{10^*}) - 295b_{10} - 100b_{10^*}]. \end{aligned} \tag{14}$$

We are left with only six independent reduced matrix elements: b_{10}, b_{10^*}, a_{10}, a_{10^*}, $a_{8_{11}}$, $a_{8_{22}}$. The s-wave transition amplitudes (A's) in terms of these parameters are listed below:

$$\begin{aligned} -\Lambda^0_- &= -\tfrac{2640}{270}b_{10} + \tfrac{780}{270}b_{10^*} + \tfrac{810}{270}a_{10} - \tfrac{270}{270}a_{10^*} + \tfrac{1080}{270}a_{8_{22}}, \\ \sqrt{2}\,\Lambda^0_0 &= \tfrac{3535}{270}b_{10} + \tfrac{6580}{270}b_{10^*} + \tfrac{810}{270}a_{10} - \tfrac{270}{270}a_{10^*} + \tfrac{1080}{270}a_{8_{22}}, \\ -(\tfrac{3}{2})^{1/2}\Sigma^+_+ &= -\tfrac{1925}{90}b_{10} + \tfrac{3100}{90}b_{10^*} - \tfrac{900}{90}a_{8_{11}} + \tfrac{540}{90}a_{8_{22}}, \\ -(\tfrac{3}{2})^{1/2}\Sigma^-_- &= -\tfrac{160}{90}b_{10} - \tfrac{2800}{90}b_{10^*} - \tfrac{90}{90}a_{10} \\ &\qquad - \tfrac{450}{90}a_{10^*} - \tfrac{900}{90}a_{8_{11}} + \tfrac{900}{90}a_{8_{22}}, \\ \sqrt{3}\,\Sigma^+_0 &= -\tfrac{2860}{90}b_{10} - \tfrac{100}{90}b_{10^*} - \tfrac{90}{90}a_{10} \\ &\qquad - \tfrac{450}{90}a_{10^*} + \tfrac{360}{90}a_{8_{22}}, \\ (\tfrac{3}{2})^{1/2}\Sigma^0_0 &= \tfrac{2895}{180}b_{10} + \tfrac{5100}{180}b_{10^*} + \tfrac{90}{180}a_{10} \\ &\qquad + \tfrac{450}{180}a_{10^*} + \tfrac{1800}{180}a_{8_{11}} - \tfrac{1440}{180}a_{8_{22}}, \\ \sqrt{2}\,\Xi^0_0 &= -\tfrac{6185}{270}b_{10} - \tfrac{3140}{270}b_{10^*} - \tfrac{270}{270}a_{10} \\ &\qquad + \tfrac{810}{270}a_{10^*} - \tfrac{1080}{270}a_{8_{22}}, \\ -\Xi^-_- &= -\tfrac{10}{270}b_{10} + \tfrac{2660}{270}b_{10^*} - \tfrac{270}{270}a_{10} + \tfrac{810}{270}a_{10^*} - \tfrac{1080}{270}a_{8_{22}}. \end{aligned} \tag{15}$$

At this stage we already expect one relation among the seven physical amplitudes, and this is the celebrated relation derived by Suzuki,[6] Bailin,[7] and Rosen *et al.*,[7]

$$\Delta_A\Lambda = -\Delta_A\Xi, \tag{16}$$

where $\Delta\Lambda$, $\Delta\Xi$, and $\Delta\Sigma$ stand for the deviations from the corresponding $\Delta I = \frac{1}{2}$ rule as defined in Ref. 4.

Now applying condition (i) of Eq. (12) we see that all the b's become zero [Eq. (13)]. This is octet dominance for the parity-violating (pv) case. The assumption of observed particle intermediate states imposes condition [12(ii)] also and this reduces the number of parameters to only two. Consequently we get two more relations among the pv amplitudes in addition to the $\Delta I = \frac{1}{2}$ relations and the Lee-Sugawara triangle. These additional relations are

$$\sqrt{3}A(\Sigma_0^+) = A(\Lambda_-^0) + 2A(\Xi_-^-)$$

$$(-2.7158) \qquad (-2.4950)\,, \tag{17}$$

$$-(\tfrac{3}{2})^{1/2}A(\Sigma_-^-) = 3A(\Lambda_-^0) + 3A(\Xi_-^-)$$

$$(-2.2768) \qquad (-1.4250)\,. \tag{18}$$

The experimental values are shown in parentheses.[8] On imposing $A(\Sigma_+^+)=0$, we get a one-parameter solution. This condition yields the strong vertex D/F ratio as

$$D/F = 1.34\,, \tag{19}$$

a result not inconsistent with the experimental value.[8]

IV. PARITY-CONSERVING (p-WAVE) AMPLITUDES

The CP invariance condition (11) gives

$$b_{27_1} = \tfrac{1}{14}(b_{10} + b_{10*})\,,$$

$$b_{27_2} = \tfrac{1}{2}(b_{10} - b_{10*} - 2b_{8_2})\,, \tag{20}$$

$$b_{8_1} = 0\,,$$

$$a_{10} = a_{10*} = -(a_{8_{12}} + a_{8_{21}})\,,$$

$$a_{27} = -\tfrac{1}{6}(3a_{8_{11}} + a_{8_{22}})\,, \tag{21}$$

$$a_1 = -\tfrac{1}{2}(25a_{8_{11}} + 3a_{8_{22}})\,.$$

Substituting Eqs. (20) and (21) in the transition amplitudes we get the CP-invariant parity-conserving amplitudes in terms of the seven independent parameters b_{10}, b_{10*}, b_{8_2} and $a_{8_{11}}$, $a_{8_{12}}$, $a_{8_{21}}$, $a_{8_{22}}$:

$$-B(\Lambda_-^0) = \tfrac{1}{21}(28a_{8_{22}} + 84a_{8_{12}} + 168a_{8_{21}} - 240b_{10} + 12b_{10*} + 126b_{8_2})\,,$$

$$\sqrt{2}B(\Lambda_0^0) = \tfrac{1}{21}(28a_{8_{22}} + 84a_{8_{12}} + 168a_{8_{21}} + 420b_{10} - 168b_{10*} - 84b_{8_2})\,,$$

$$-(\tfrac{3}{2})^{1/2}B(\Sigma_+^+) = \tfrac{1}{21}(14a_{8_{22}} - 126a_{8_{12}} + 126a_{8_{21}} - 210a_{8_{11}} - 96b_{10} - 12b_{10*} - 42b_{8_2})\,,$$

$$-(\tfrac{3}{2})^{1/2}B(\Sigma_-^-) = \tfrac{1}{21}(-70a_{8_{22}} + 126a_{8_{12}} + 126a_{8_{21}} - 210a_{8_{11}} + 156b_{10} + 240b_{10*} + 210b_{8_2})\,,$$

$$\sqrt{3}B(\Sigma_0^+) = \tfrac{1}{21}(-84a_{8_{22}} + 252a_{8_{12}} - 288b_{10} - 288b_{10*} - 378b_{8_2})\,, \tag{22}$$

$$(\tfrac{3}{2})^{1/2}B(\Sigma_0^0) = \tfrac{1}{21}(28a_{8_{22}} - 126a_{8_{21}} + 210a_{8_{11}} + 90b_{10} + 216b_{10*} + 126b_{8_2})\,,$$

$$-B(\Xi_-^-) = \tfrac{1}{21}(28a_{8_{22}} - 84a_{8_{12}} - 168a_{8_{21}} + 12b_{10} - 240b_{10*} - 126b_{8_2})\,,$$

$$\sqrt{2}B(\Xi_0^0) = \tfrac{1}{21}(28a_{8_{22}} - 84a_{8_{12}} - 168a_{8_{21}} - 168b_{10} + 420b_{10*} + 84b_{8_2})\,.$$

No relation among the seven physical amplitudes is expected here. However, the condition

$$b_{27_1} = b_{27_2} = 0 \tag{23}$$

gives the relation

$$-\Delta_B\Lambda = \Delta_B\Xi = (\tfrac{3}{2})^{1/2}\Delta_B\Sigma\,. \tag{24}$$

This is similar to a relation derived by Fayyazuddin and Riazuddin[9] using current algebra and assuming $f=\frac{2}{5}$.

From Eq. (20) it is apparent that the b's will vanish if we apply the condition [12(i)] and octet dominance results. Our assumption regarding the eigenstates $|m\rangle$ also imposes [12(ii)] and this reduces the number of parameters to two. Besides the $\Delta I=\frac{1}{2}$ rules two more relations result:

$$B(\Lambda_-^0) = (\tfrac{3}{2})^{1/2}[\tfrac{4}{3}B(\Sigma_+^+) - \sqrt{2}B(\Sigma_0^+)]$$

$$(10.6440) \qquad (11.1093), \tag{25}$$

$$2B(\Sigma_+^+) = \frac{1}{\sqrt{6}}[7B(\Lambda_-^0) + 5B(\Xi_-^-)]$$

$$(38.1560) \qquad (44.3606)\,. \tag{26}$$

The numbers in parentheses are, as before, experimental values.[8] In addition we also get $B(\Sigma_-^-)=0$. Relation (26) is then similar to the Lee-Sugawara relation and is at least equally well satisfied experimentally.

V. CONCLUSION

The assumption that only physical particle states can contribute to the transition amplitudes in the direct channel has led to octet dominance. This is a dynamical assumption which may well be true. We think that this assumption is plausible because of the following:

(1) The relations (17) to (19) and (25) to (26) are quite consistent with the available experimental data.

(2) The relation (26) for the parity-conserving amplitudes is similar to the Lee-Sugawara relation which does not follow naturally from the symmetry arguments. Since this relation is well satisfied experimentally, it may now replace the Lee-Sugawara triangle for the p wave.

(3) In a quark model,[10] where the baryons are built out of three quarks, the only possible baryon states belong to the representations $\underline{1}$, $\underline{8}$, and $\underline{10}$ of SU(3).

It may be noted that in some recent papers[11] the $\Delta I=\frac{1}{2}$ rule has been shown to be a consequence of some dynamical assumptions within the quark model.

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APPENDIX

The complete transition amplitudes in terms of all the 14 reduced matrix elements are listed below:

$$-X(\Lambda^0_-)=\tfrac{1}{15}a_{27}-\frac{1}{\sqrt{30}}a_{10^*}+\tfrac{1}{10}(\tfrac{3}{2})^{1/2}a_{8_{11}}+\frac{1}{2\sqrt{30}}a_{8_{12}}+\frac{3}{2\sqrt{30}}a_{8_{21}}+\frac{\sqrt{6}}{12}a_{8_{22}}$$
$$+\tfrac{2}{15}(\tfrac{1}{14})^{1/2}b_{27_1}-\tfrac{1}{3}(\tfrac{3}{10})^{1/2}b_{27_2}-\frac{5}{6\sqrt{5}}b_{10}-\frac{1}{6\sqrt{5}}b_{10^*}+\tfrac{1}{5}b_{8_1}+\frac{1}{3\sqrt{5}}b_{8_2}\,,$$

$$X(\Lambda^0_0)=\frac{1}{15\sqrt{2}}a_{27}-\frac{1}{\sqrt{60}}a_{10^*}+\frac{\sqrt{3}}{20}a_{8_{11}}+\frac{1}{2\sqrt{60}}a_{8_{12}}+\frac{3}{2\sqrt{60}}a_{8_{21}}+\frac{\sqrt{3}}{12}a_{8_{22}}$$
$$-\tfrac{7}{20}(\tfrac{1}{7})^{1/2}b_{27_1}+\tfrac{1}{4}(\tfrac{3}{5})^{1/2}b_{27_2}+\frac{5}{3\sqrt{10}}b_{10}-\frac{1}{6\sqrt{10}}b_{10^*}+\frac{1}{5\sqrt{2}}b_{8_1}+\frac{1}{3\sqrt{10}}b_{8_2}\,,$$

$$-X(\Sigma^+_+)=\tfrac{2}{15}(\tfrac{2}{3})^{1/2}a_{27}-\frac{1}{3\sqrt{5}}a_{10}+\frac{1}{3\sqrt{5}}a_{10^*}-\tfrac{3}{10}a_{8_{11}}-\frac{1}{\sqrt{20}}a_{8_{12}}+\frac{1}{\sqrt{20}}a_{8_{21}}+\tfrac{1}{6}a_{8_{22}}$$
$$+\tfrac{1}{10}(\tfrac{1}{21})^{1/2}b_{27_1}+\tfrac{1}{2}(\tfrac{1}{5})^{1/2}b_{27_2}-\tfrac{2}{3}(\tfrac{2}{15})^{1/2}b_{10}+\tfrac{1}{6}(\tfrac{2}{15})^{1/2}b_{10^*}+\tfrac{2}{5}(\tfrac{2}{3})^{1/2}b_{8_1}+\tfrac{2}{3}(\tfrac{2}{15})^{1/2}b_{8_2}\,,$$

$$-X(\Sigma^-_-)=\tfrac{1}{3}(\tfrac{2}{3})^{1/2}a_{27}-\frac{1}{\sqrt{5}}a_{10}+\tfrac{3}{2}(\tfrac{1}{21})^{1/2}b_{27_1}-\tfrac{5}{6}(\tfrac{1}{5})^{1/2}b_{27_2}+\tfrac{1}{2}(\tfrac{2}{15})^{1/2}b_{10}\,,$$

$$X(\Sigma^+_0)=\tfrac{1}{5}(\tfrac{1}{3})^{1/2}a_{27}-\frac{2}{3\sqrt{10}}a_{10}-\frac{1}{3\sqrt{10}}a_{10^*}+\frac{3}{10\sqrt{2}}a_{8_{11}}+\frac{1}{\sqrt{40}}a_{8_{12}}-\frac{1}{\sqrt{40}}a_{8_{21}}-\frac{1}{6\sqrt{2}}a_{8_{22}}$$
$$-\tfrac{11}{20}(\tfrac{2}{21})^{1/2}b_{27_1}+\tfrac{7}{12}(\tfrac{2}{5})^{1/2}b_{27_2}-\tfrac{4}{3}(\tfrac{1}{15})^{1/2}b_{10}-\tfrac{1}{6}(\tfrac{1}{15})^{1/2}b_{10^*}-\tfrac{2}{5}(\tfrac{1}{3})^{1/2}b_{8_1}-\tfrac{2}{3}(\tfrac{1}{15})^{1/2}b_{8_2}\,,$$

$$X(\Sigma^0_0)=-\tfrac{7}{30}(\tfrac{2}{3})^{1/2}a_{27}+\frac{2}{3\sqrt{5}}a_{10}-\frac{1}{6\sqrt{5}}a_{10^*}+\tfrac{3}{20}a_{8_{11}}+\frac{1}{2\sqrt{20}}a_{8_{12}}-\frac{1}{2\sqrt{20}}a_{8_{21}}-\tfrac{1}{12}a_{8_{22}}$$
$$-\tfrac{1}{20}(\tfrac{1}{21})^{1/2}b_{27_1}-\tfrac{1}{4}(\tfrac{1}{5})^{1/2}b_{27_2}+\tfrac{1}{2}(\tfrac{2}{15})^{1/2}b_{10}+\tfrac{3}{4}(\tfrac{2}{15})^{1/2}b_{10^*}+\tfrac{3}{10}(\tfrac{2}{3})^{1/2}b_{8_1}+\tfrac{1}{2}(\tfrac{2}{15})^{1/2}b_{8_2}\,,$$

$$X(\Xi^0_0)=-\frac{\sqrt{2}}{15}a_{27}+\frac{1}{\sqrt{60}}a_{10}+\frac{1}{\sqrt{60}}a_{10^*}-\frac{\sqrt{3}}{10}a_{8_{11}}-\frac{1}{\sqrt{60}}a_{8_{21}}+\tfrac{7}{10}(\tfrac{1}{7})^{1/2}b_{27_1}-\tfrac{1}{2}(\tfrac{1}{15})^{1/2}b_{27_2}-\tfrac{1}{2}(\tfrac{2}{5})^{1/2}b_{10}$$
$$+\tfrac{1}{2}(\tfrac{2}{5})^{1/2}b_{10^*}-\tfrac{2}{5}(\tfrac{1}{2})^{1/2}b_{8_1}\,,$$

$$-X(\Xi^-_-)=-\tfrac{2}{15}a_{27}+\frac{1}{\sqrt{30}}a_{10}+\frac{1}{\sqrt{30}}a_{10^*}-\tfrac{1}{5}(\tfrac{3}{2})^{1/2}a_{8_{11}}-\frac{1}{\sqrt{30}}a_{8_{21}}-\tfrac{1}{10}(\tfrac{1}{14})^{1/2}b_{27_1}+\tfrac{3}{4}(\tfrac{2}{15})^{1/2}b_{27_2}-\tfrac{1}{6}(\tfrac{1}{5})^{1/2}b_{10}$$
$$-\tfrac{2}{3}(\tfrac{1}{5})^{1/2}b_{10^*}+\tfrac{4}{15}b_{8_1}\,.$$

In order to remove the irrational numbers occurring in the above relations and to make them look decent, the a's and b's have been redefined as below:

$$\tfrac{1}{60}a_{27}\to a_{27}\,,\qquad \tfrac{1}{60}(\tfrac{1}{14})^{1/2}b_{27_1}\to b_{27_1}\,,$$
$$\frac{1}{2\sqrt{30}}a_{10}\to a_{10}\,,\qquad \tfrac{1}{8}(\tfrac{2}{15})^{1/2}b_{27_2}\to b_{27_2}\,,$$
$$\frac{1}{2\sqrt{30}}a_{10^*}\to a_{10^*}\,,\qquad \tfrac{1}{12}(\tfrac{1}{5})^{1/2}b_{10}\to b_{10}\,,$$
$$\tfrac{1}{20}(\tfrac{3}{2})^{1/2}a_{8_{11}}\to a_{8_{11}}\,,\qquad \tfrac{1}{12}(\tfrac{1}{5})^{1/2}b_{10^*}\to b_{10^*}\,,$$
$$\frac{1}{4\sqrt{30}}a_{8_{12}}\to a_{8_{12}}\,,\qquad \tfrac{1}{6}(\tfrac{1}{5})^{1/2}b_{8_2}\to b_{8_2}\,,$$
$$\frac{1}{4\sqrt{30}}a_{8_{21}}\to a_{8_{21}}\,,\qquad \tfrac{1}{30}b_{8_1}\to b_{8_1}\,,$$
$$\frac{\sqrt{6}}{24}a_{8_{22}}\to a_{8_{22}}\,,$$
$$\frac{\sqrt{3}}{4}a_1\to a_1\,.$$

The redefined parameters have been used in Secs. III and IV.

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[1]M. Gell-Mann, Phys. Rev. Lett. 12, 155 (1964); H. Sugawara, Prog. Theor. Phys. 31, 213 (1964); B. W. Lee, Phys. Rev. Lett. 12, 83 (1964).
[2]K. C. Gupta and R. Majumdar, Nuovo Cimento 37, 1801 (1965); D. Bailin, *ibid.* 38, 1342 (1965).
[3]M. Suzuki, Phys. Rev. 137, B1602 (1965); S. Rosen, S. Pakvasa, and E. C. G. Sudarshan, Phys. Rev. 146, 1118 (1966); S. Pakvasa, R. Graham, and S. Rosen, *ibid.* 149, 1200 (1966).
[4]R. E. Marshak, Riazuddin, and C. Ryan, *Theory of Weak Interactions* (Wiley-Interscience, New York, 1969); E. Fischbach, Phys. Rev. 170, 1398 (1968).
[5]M. Gourdin, *Unitary Symmetry* (North-Holland, Amsterdam, 1967), p. 136; M. O. Taha, Phys. Rev. 169, 1182 (1968); M. O. Taha, *ibid.* 171, 1481 (1968).
[6]M. Suzuki, Phys. Rev. 137, B1602 (1965).
[7]D. Bailin, Nuovo Cimento 38, 1342 (1965); Rosen *et al.*, Ref. 3.
[8]Experimental values given by Marshak *et al.* (Ref. 4) are used.
[9]Fayyazuddin and Riazuddin, Nuovo Cimento 47, 222 (1967).
[10]M. Gell-Mann, Phys. Lett. 8, 214 (1964); G. Zweig, CERN Reports Nos. 8182/TH/401 and 8419/TH/412 (unpublished).
[11]M. Gronau, Phys. Rev. Lett. 28, 188 (1972); Phys. Rev. D 5, 118 (1972); 5, 1877(E) (1972); K. Sebastian and C. A. Nelson, *ibid.* 8, 3144 (1973).