Analysis of some observable effects of weak neutral currents coupled to charged leptons

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We have analyzed the parity-violating effects in the processes $e + \bar{e} \to k_1 + k_2 + X$, $k_1 + k_2 \to e + \bar{e} + X$, and $e + k_1 \to e + k_2 + X$, where $e, \bar{e}$ are the electron, positron states, k_1, k_2 are the hadron states, and X is the inclusive hadron state, due to the possible coupling of weak neutral currents to charged leptons. We estimate that for gauge theories of the weak interaction, the parity-violating asymmetry parameter for the distribution in $e + \bar{e} \to \pi^+ + \pi^- + \pi^0$ may be of the order of 5% in the resonance region of the A_1 meson and experimentally observable.

The need for understanding weak neutral currents has become imperative with two recent developments: One is the possibility of obtaining a unified renormalizable theory of electromagnetic and weak interactions with gauge fields[1] which in most cases imply weak neutral currents. The other is the tentative evidence for these currents in the observation[2] of a large number of events of the type

$$\nu(\bar{\nu}) + N \to \nu(\bar{\nu}) + \text{hadrons}, \tag{1}$$

where N is the nucleon. The neutral currents, if they exist, would also play an important role in the understanding of mass differences within multiplets and some rare weak decays. However, for a complete understanding of weak neutral currents, the confirmation of neutral currents in (1) would have to be supplemented by a knowledge of the nature of the coupling of these currents to charged leptons. Unfortunately, the analysis of the coupling to the charged leptons is made difficult by the presence of electromagnetic coupling, which at low energies is larger than the weak coupling by a factor of about 10^3.

There are two possible ways of making the effect of the coupling of weak neutral currents to charged leptons relatively important. One is to study high-energy processes where the energies involved are comparable to or greater than the large masses involved in the weak interaction and where electromagnetic and weak interaction strengths are expected to become of the same order. For example, at energies around 10 GeV, the weak neutral-current effects are expected to be about 10% of the electromagnetic-current effects.

Another way to enhance the effect of weak currents is to consider processes involving resonances such as the A_1 or B (assumed to be 1^+) which couple to the weak neutral currents but not to electromagnetic currents. The electromagnetic effect can be further suppressed by choosing a kinematic region forbidden by electromagnetic interaction. We explicitly demonstrate these features for the process

$$e + \bar{e} \to \pi^+ + \pi^- + \pi^0, \tag{2}$$

and show that the weak parity-violating effects are about 5% and may be experimentally observable.

We first consider the general reactions

$$e + \bar{e} \to k_1 + k_2 + X, \tag{3}$$

$$k_1 + k_2 \to e + \bar{e} + X, \tag{4}$$

$$e + k_1 \to e + k_2 + X, \tag{5}$$

where k_1 and k_2 stand for two strongly interacting particles with masses m_1 and m_2, and momenta k_1 and k_2, and X is the "inclusive" hadron state. We present the general kinematic results for the weak neutral-current effects for these processes and point out the critical variables most suited for the study of parity violation. In the latter part we give a detailed analysis of the electromagnetic and weak contributions to the reaction (2) in the A_1 region, and show that for the momenta of the π^+ and π^- nearly parallel in the center-of-mass (c.m.) frame of $e\bar{e}$, the cross section for the reaction is experimentally observable and the parity-violating effect is about 5% in terms of the asymmetry parameter which we define later.

In our analysis, a scalar intermediate boson (i.e., Higgs boson) will not give any asymmetry in the distributions discussed.

$\underline{e + \bar{e} \to k_1 + k_2 + X}$. The reaction may proceed via the photon or the weak intermediate vector boson. The corresponding lepton vertices are taken to be

$$V_\mu = e\bar{v}(p_2)\gamma_\mu u(p_1), \tag{6}$$

$$U_\mu = \bar{v}(p_2)\gamma_\mu(a - ib\gamma_5)u(p_1), \tag{7}$$

where the strengths of a and b depend on the model and the Weinberg mixing angle.[3] We will, for simplicity, take them to be of the order of e. We define

$$L_{\mu\nu} = \sum_{\text{spin av}} V_\mu V_\nu^* = \frac{e^2}{4m^2}(p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu} - p_1 \cdot p_2 g_{\mu\nu}) \,, \tag{8}$$

$$M_{\mu\nu} = \sum_{\text{spin av}} V_\mu U_\nu^* = \frac{a}{e} L_{\mu\nu} + N_{\mu\nu} \,, \tag{9}$$

$$N_{\mu\nu} = \frac{ibe}{4m^2}\epsilon_{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta} \,,$$

where m is the lepton mass, taken to be small, and the hadron structure functions

$$\begin{aligned} V_{\mu\nu} &= \sum_{X,\,\text{spin}} \langle k_1, k_2, X | J_\mu^e(0)|0\rangle\langle k_1,k_2,X|J_\nu^e(0)|0\rangle^* \frac{(2\pi)^9}{h_1 h_2}\delta^4(k_1+k_2+X-p) \\ &= \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)W_1 + \left(k_{1\mu} - \frac{k_1\cdot p}{p^2}p_\mu\right)\left(k_{1\nu} - \frac{k_1\cdot p}{p^2}p_\nu\right)W_2 \\ &\quad + \left(k_{2\mu} - \frac{k_2\cdot p}{p^2}p_\mu\right)\left(k_{2\nu} - \frac{k_2\cdot p}{p^2}p_\nu\right)W_3 + \epsilon_{\mu\alpha\beta\gamma}p_\alpha k_{1\beta}k_{2\gamma}\epsilon_{\nu\delta\epsilon\eta}p_\delta k_{1\epsilon}k_{2\eta}W_4 \,, \end{aligned} \tag{10}$$

$$\begin{aligned} U_{\mu\nu} &= \sum_{X,\,\text{spin}} \langle k_1, k_2, X | J_\mu^e(0)|0\rangle\langle k_1,k_2,X|J_\nu^{WV}(0)|0\rangle^* \frac{(2\pi)^9}{h_1 h_2}\delta^4(k_1+k_2+X-p) \\ &= \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)B_1 + \left(k_{1\mu} - \frac{k_1\cdot p}{p^2}p_\mu\right)\left(k_{1\nu} - \frac{k_1\cdot p}{p^2}p_\nu\right)B_2 \\ &\quad + \left(k_{2\mu} - \frac{k_2\cdot p}{p^2}p_\mu\right)\left(k_{2\nu} - \frac{k_2\cdot p}{p^2}p_\nu\right)B_3 + \epsilon_{\mu\alpha\beta\gamma}p_\alpha k_{1\beta}k_{2\gamma}\epsilon_{\nu\delta\epsilon\eta}p_\delta k_{1\epsilon}k_{2\eta}B_4 \\ &\quad + \left[\left(k_{1\mu} - \frac{k_1\cdot p}{p^2}p_\mu\right)\left(k_{2\nu} - \frac{k_2\cdot p}{p^2}p_\nu\right) - \left(k_{2\mu} - \frac{k_2\cdot p}{p^2}p_\mu\right)\left(k_{1\nu} - \frac{k_1\cdot p}{p^2}p_\nu\right)\right]B_5 \\ &\quad + \left(k_{1\mu} - \frac{p_\mu p\cdot k_1}{p^2}\right)\left(k_{1\nu} + \frac{p_\nu p\cdot k_1}{p^2}\right)B_6 + \left(k_{2\mu} - \frac{p_\mu p\cdot k_2}{p^2}\right)\left(k_{2\nu} + \frac{p_\nu p\cdot k_2}{p^2}\right)B_7 \,, \end{aligned} \tag{11}$$

$$\begin{aligned} A_{\mu\nu} &= \sum_{X,\,\text{spin}} \langle k_1, k_2, X | J_\mu^e(0)|0\rangle\langle k_1,k_2,X|J_\nu^{WA}(0)|0\rangle^* \frac{(2\pi)^9}{h_1 h_2}\delta^4(k_1+k_2+X-p) \\ &= \epsilon_{\mu\alpha\beta\gamma}p_\alpha k_{1\beta}k_{2\gamma}(k_{1\nu}A_1 + k_{2\nu}A_2 + p_\nu A_3) + \epsilon_{\mu\nu\alpha\beta}p_\alpha(k_{1\beta}A_4 + k_{2\beta}A_5) \\ &\quad + \epsilon_{\nu\alpha\beta\gamma}p_\alpha k_{1\beta}k_{2\gamma}\left[\left(k_{1\mu} - \frac{k_1\cdot p}{p^2}p_\mu\right)A_6 + \left(k_{2\mu} - \frac{k_2\cdot p}{p^2}p_\mu\right)A_7\right] \,, \end{aligned} \tag{12}$$

where J^e, J^{WV}, J^{WA} are the electromagnetic, weak vector, and weak axial-vector currents, respectively, $p = p_1 + p_2$, W_i, B_i, and A_i are functions of $s = p^2$, $M^2 = X^2$, $s_1 = (p - k_1)^2$, $s_2 = (p - k_2)^2$, $s_3 = (k_1 + k_2)^2 = s + M^2 + m_1^2 + m_2^2 - s_1 - s_2$, and $h_i = m_i / k_{i0}$ for fermions and $1/2k_{i0}$ for bosons. In expression (11), the B_6 and B_7 terms are not present if the weak vector current is conserved. The cross section for the reaction (3) is given by

$$\sigma = (2\pi)^2 \frac{m^2}{[(p_1\cdot p_2)^2 - m^4]^{1/2}} \int d^4k_1 \int d^4k_2 \delta(k_1^2 - m_1^2)\delta(k_2^2 - m_2^2) 2k_{10} 2k_{20} h_1 h_2 \times \frac{1}{(2\pi)^7 s}\,\text{Re}\left[\frac{1}{s}L_{\mu\nu}V^{\mu\nu} + \frac{2}{m_W^2}\left(\frac{a}{e}L_{\mu\nu}A^{\mu\nu} + N_{\mu\nu}U^{\mu\nu}\right)\right] \,, \tag{13}$$

where m_W is the mass of the neutral intermediate vector boson, and where we have retained only the leading parity-violating term from the weak interaction. The integrals can be simplified in the c.m. system of $e\bar{e}$ to the form

$$\sigma = \frac{2(2\pi)^2}{s}\frac{\pi}{16 s (2\pi)^7}\int ds_1 \int ds_2 \int dM^2 \int_{-1}^{+1} d\cos\theta \int_{-\pi}^{\pi} d\phi\, 2k_{10} 2k_{20} h_1 h_2 \Delta\sigma \,,$$

where $\Delta\sigma = \Delta\sigma_1 + \Delta\sigma_2$,

$$\Delta\sigma_1 = \frac{e^2}{s^2}\Big[-\tfrac{1}{4}sW_1 + \tfrac{1}{2}s(k_{10}^2 - |\vec{k}_1|^2\sin^2\theta\cos^2\beta_1 - \tfrac{1}{4}m_1^2)W_2 + \tfrac{1}{2}s(k_{20}^2 - |\vec{k}_2|^2\sin^2\theta\cos^2\beta_2 - \tfrac{1}{4}m_2^2)W_3 + \tfrac{1}{8}s^2|\vec{k}_1|^2|\vec{k}_2|^2\sin^2\theta_{12}\sin^2\theta\, W_4\Big],$$

$$\Delta\sigma_2 = -\frac{2e}{sm_W^2}\tfrac{1}{8}(s^{3/2}|\vec{k}_1||\vec{k}_2|)\sin\theta_{12}\cos\theta\sin\theta\,\mathrm{Re}[a|\vec{k}_1|\cos\beta_1(A_1+A_6)+a|\vec{k}_2|\cos\beta_2(A_2+A_7)] + \frac{e}{m_W^2}|\vec{k}_1||\vec{k}_2|\sin\theta_{12}\cos\theta\,\mathrm{Re}(ibB_5). \tag{14}$$

Here all the variables are in the c.m. system of $e\bar{e}$;

$$k_{i0} = \frac{s+m_i^2-s_i}{2s^{1/2}},\quad |\vec{k}_i|^2 = k_{i0}^2 - m_i^2,$$

$$\cos\theta_{12} = \frac{2k_{10}k_{20}+s_1+s_2-s-M^2}{2|\vec{k}_1||\vec{k}_2|}$$

(θ_{12} being the angle between $\vec{k}_1$ and $\vec{k}_2$), $\vec{k}_1\times\vec{k}_2$ is taken as the z axis and $-(\vec{k}_1+\vec{k}_2)$ as the x axis, θ and ϕ are the polar and azimuthal angles of $\vec{p}_1$; ϕ_1 and ϕ_2 are the azimuthal angles of $\vec{k}_1$ and $\vec{k}_2$, respectively, and $\beta_i = \phi_i - \phi$. The ranges of integration are, for M^2: $(\sqrt{s}-m_1-m_2)^2$ and 0; for s_1: $(\sqrt{s}-m_1)^2$ and $(M+m_2)^2$; for s_2:

$$M^2+m_1^2+2(k_{10}X_0 \pm |\vec{k}_1||\vec{X}|),$$

where

$$X_0 = \frac{s_1+M^2-m_2^2}{2s_1^{1/2}},\quad k_{10} = \frac{s-m_1^2-s_1}{2s_1^{1/2}}.$$

Note that changing θ to $\pi-\theta$ leaves $\Delta\sigma_1$ unchanged but changes the sign of $\Delta\sigma_2$, so that we have parity-violating asymmetry about the $(\vec{k}_1, \vec{k}_2)$ plane. We define an asymmetry parameter

$$A = \frac{\sum_{\theta_0<\theta<\theta_1}\Delta\sigma - \sum_{\pi-\theta_0>\theta>\pi-\theta_1}\Delta\sigma}{\sum_{\theta_0<\theta<\theta_1}\Delta\sigma + \sum_{\pi-\theta_0>\theta>\pi-\theta_1}\Delta\sigma} = \frac{\sum_{\theta_0<\theta<\theta_1}\Delta\sigma_2}{\sum_{\theta_0<\theta<\theta_1}\Delta\sigma_1}, \tag{15}$$

which gives a measure of parity violation in the kinematic region of interest for $\theta_0<\theta<\theta_1$. One could also discuss asymmetry in the azimuthal angle with respect to the z axis taken along $-(\vec{k}_1+\vec{k}_2)$ and the y axis along $\vec{k}_1\times\vec{k}_2$; here the asymmetry is the difference in the distribution of events at ϕ and $2\pi-\phi$.

Later, we will discuss in detail the asymmetry for the $\pi^+\pi^-\pi^0$ final state.

$k_1+k_2 \to e+\bar{e}+X$. The kinematics of this reaction is quite similar to the one just discussed. The cross section is given by

$$\sigma = (2\pi)^2\frac{(2k_{10}h_1)(2k_{20}h_2)}{[(k_1\cdot k_2)^2-m_1^2m_2^2]^{1/2}}\frac{4m^2}{(2j_1+1)(2j_2+1)} \times\int d^4p_1 d^4p_2\,\delta(p_1^2-m^2)\delta(p_2^2-m^2)\frac{1}{(2\pi)^7p^2}\mathrm{Re}\left[\frac{1}{p^2}L_{\mu\nu}V^{\mu\nu} + \frac{2}{m_W^2}\left(\frac{a}{e}L_{\mu\nu}A^{\mu\nu}+N_{\mu\nu}U^{\mu\nu}\right)\right], \tag{16}$$

where j_1, j_2 are the spins of particles k_1 and k_2, and $L_{\mu\nu}$, $V_{\mu\nu}$, $A_{\mu\nu}$, $N_{\mu\nu}$ are defined in (8)–(12). In terms of the variables $s=(k_1+k_2)^2$, $M^2=X^2$, $s_1=(k-p_1)^2$, $s_2=(k-p_2)^2$, $s_3=s+M^2+2m^2-s_1-s_2$, $p=p_1+p_2$, and $k=k_1+k_2$, the integrals simplify in the c.m. system of k_1, k_2 to the form

$$\sigma = \frac{(2\pi)^2k_{10}k_{20}h_1h_2}{[(k_1\cdot k_2)^2-m_1^2m_2^2]^{1/2}}\frac{\pi}{(2j_1+1)(2j_2+1)16s(2\pi)^7} \times\int ds_1\int ds_2\int dM^2\int_{-1}^{+1}d\cos\theta\int_{-\pi}^{\pi}d\phi\,\Delta\sigma,$$

where

$$\Delta\sigma = \Delta\sigma_1 + \Delta\sigma_2,$$

$$\Delta\sigma_1 = \frac{4e^2}{s_3^2}\Big[-s_3W_1 + (2k_1\cdot p_1k_1\cdot p_2 - \tfrac{1}{2}s_3m_1^2)W_2 + (2k_2\cdot p_1k_2\cdot p_2 - \tfrac{1}{2}s_3m_2^2)W_3 + (\tfrac{1}{2}s_3|\vec{p}|^2 - 2p_{10}^2p_{20}^2\sin^2\theta_{12})s|\vec{k}_1|^2\sin^2\theta\, W_4\Big], \tag{17}$$

$$\Delta\sigma_2 = -\frac{8es^{1/2}}{s_3m_W^2}p_{10}p_{20}|\vec{k}_1|\sin\theta_{12}\sin\theta\sin\phi\,\mathrm{Re}[ak_1\cdot(p_1-p_2)(A_1+A_6)+ak_2\cdot(p_1-p_2)(A_2+A_7)+ibB_5].$$

Here,

$$k_1\cdot p_i = p_{i0}(k_{10} - |\vec{k}_1|\cos\theta\cos\theta_i - |\vec{k}_1|\sin\theta\sin\theta_i\cos\phi),$$

$$k_2\cdot p_i = p_{i0}(k_{20} + |\vec{k}_2|\cos\theta\cos\theta_i + |\vec{k}_2|\sin\theta\sin\theta_i\cos\phi),$$

$$k_{10} = \frac{s+m_1^2-m_2^2}{2\sqrt{s}},\quad k_{20} = \frac{s+m_2^2-m_1^2}{2\sqrt{s}},$$

$$p_{i0} = \frac{s-s_i}{2\sqrt{s}},\quad \cos\theta_i = -\frac{p_{10}+p_{20}-s_3/2p_{i0}}{|\vec{p}|},$$

$$|\vec{p}| = [(p_{10}+p_{20})^2 - s_3]^{1/2},$$

$$\cos\theta_{12} = 1 - \frac{s_3}{2p_{10}p_{20}}.$$

All the variables are in the c.m. system of k_1, k_2, with $-(\vec{p}_1+\vec{p}_2)$ taken as the z axis and $\vec{p}_1\times\vec{p}_2$ as the y axis, θ and ϕ are the polar and azimuthal angles of $\vec{k}_1$, and θ_i are the polar angles of $\vec{p}_i$. In these expressions, we have neglected the lepton mass. The ranges of integration are, for M^2: s and 0; for s_1: s and M^2; and for s_2: $M^2+2(p_{10}X_0 \pm |\vec{p}_1||\vec{X}|)$, where

$$X_0 = \frac{s_1+M^2}{2\sqrt{s_1}},\quad p_{10} = \frac{s-s_1}{2\sqrt{s_1}}.$$

Note that for $\phi\to 2\pi-\phi$, $\Delta\sigma_1$ remains unchanged, while $\Delta\sigma_2$ changes its sign, so that we have parity-violating asymmetry about the $(\vec{p}_1, \vec{p}_2)$ plane. We define

$$B = \frac{\sum_{\phi_1>\phi>\phi_0}\Delta\sigma - \sum_{2\pi-\phi_1<\phi<2\pi-\phi_0}\Delta\sigma}{\sum_{\phi_1>\phi>\phi_0}\Delta\sigma + \sum_{2\pi-\phi_1<\phi<2\pi-\phi_0}\Delta\sigma} = \frac{\sum_{\phi_1>\phi>\phi_0}\Delta\sigma_2}{\sum_{\phi_1>\phi>\phi_0}\Delta\sigma_1}, \tag{18}$$

which gives a measure of parity violation in the region of interest for $\phi_1>\phi>\phi_0$.

It may be pointed out that the parity-violating effect may be enhanced by studying the reaction for $s_3\approx m_A^2$, where m_A is the A_1 mass, since the A_i amplitudes are larger in this region.

$e+k_1\to e+k_2+X$. The cross section for this process is given by

$$\sigma = (2\pi)^2\frac{(2k_{10}h_1)(2k_{20}h_2)}{[(k_1\cdot p_1)^2-m_1^2m^2]^{1/2}}\frac{2m^2}{2j_1+1}\times\int d^4k_2\,d^4p_2\,\delta(k_2^2-m_2^2)\delta(p_2^2-m^2)\frac{1}{(2\pi)^7p^2}\mathrm{Re}\left[\frac{1}{p^2}L_{\mu\nu}V^{\mu\nu}+\frac{2}{m_W^2}\left(\frac{a}{e}L_{\mu\nu}A^{\mu\nu}+N_{\mu\nu}U^{\mu\nu}\right)\right], \tag{19}$$

where j_1 is the spin of particle k_1, and the tensors $L_{\mu\nu}$, $V_{\mu\nu}$, $N_{\mu\nu}$, $U_{\mu\nu}$ are as in (8)–(12) except that now $p=p_1-p_2$. Here, the dependence of the differential cross section on the Treiman-Yang angle[4] ϕ is the most revealing. In the c.m. system of the final hadron state, i.e., $\vec{p}_1+\vec{k}_1-\vec{p}_2=0$, the cross section is

$$\sigma = \frac{(2\pi)^2(2k_{10}h_1)(2k_{20}h_2)}{[(k_1\cdot p_1)^2-m_1^2m^2]^{1/2}}\frac{1}{(2j_1+1)(2\pi)^7}\frac{\pi}{16(s-m_1^2)}\int ds_1\int dt\int dM^2\,\frac{|\vec{k}_2|}{\sqrt{s_1}}\int_{-1}^{+1}d\cos\theta\int_0^{2\pi}d\phi\,\Delta\sigma, \tag{20}$$

where $s=(p_1+k_1)^2$, $t=q^2=(p_1-p_2)^2$, $M^2=X^2$, $s_1=(q+k_1)^2$, $\Delta\sigma=\Delta\sigma_1+\Delta\sigma_2$,

$$\Delta\sigma_1 = \frac{e^2}{t^2}[tW_1+(2p_1\cdot k_1p_2\cdot k_1+\tfrac{1}{2}m_1^2t)W_2 + (2p_1\cdot k_2p_2\cdot k_2+\tfrac{1}{2}m_2^2t)W_3 - s_1|\vec{k}_1|^2|\vec{k}_2|^2\sin^2\theta_{12}(2|\vec{p}_1|^2\sin^2\theta_1\sin^2\phi+\tfrac{1}{2}t)W_4],$$

$$\Delta\sigma_2 = \frac{2e\sqrt{s_1}}{tm_W^2}|\vec{k}_1||\vec{k}_2|\sin\theta\sin\theta_1\sin\phi\,\mathrm{Re}[ak_1\cdot(p_1+p_2)(A_1+A_6)+ak_2\cdot(p_1+p_2)(A_2+A_7)+2ibB_5],$$

where

$$k_2\cdot p_2 = k_2\cdot p_1 - k_2\cdot p,$$

$$k_1\cdot p_1 = \tfrac{1}{2}(s-m_1^2),\quad k_1\cdot p_2 = \tfrac{1}{2}(s+t-s_1),$$

$$k_2\cdot p_1 = p_{10}k_{20} - p_{10}|\vec{k}_2|(\cos\theta\cos\theta_1+\sin\theta\sin\theta_1\cos\phi),$$

$$p_0 = \sqrt{s_1}-k_{10},\quad p^2 = t,$$

$$k_{10} = \frac{s_1-t+m_1^2}{2\sqrt{s_1}},\quad k_{20} = \frac{s_1-M^2+m_2^2}{2\sqrt{s_1}},$$

$$p_{10} = \frac{s+t-m_1^2}{2\sqrt{s_1}},\quad p_{20} = \frac{s_1-s}{2\sqrt{s_1}},$$

$$\cos\theta_1 = \frac{s-m_1^2-2k_{10}p_{10}}{2|\vec{k}_1||\vec{p}_1|}.$$

Here, the z axis is along $\vec{k}_1$, $(\vec{p}_1\times\vec{p}_2)$ is along the

y axis, θ and ϕ are the polar and azimuthal angles of $\vec{k}_2$, and θ_1 is the polar angle of $\vec{p}_1$. The ranges of integration are, for M^2: $(\sqrt{s}-m_2)^2$ and 0; for s_1: s and $(M+m_2)^2$; for t: $-(s-m_1)^2(s-s_1)/s$ and 0. For $\phi\to 2\pi-\phi$, $\Delta\sigma_1$ remains unchanged while $\Delta\sigma_2$ changes sign so that we have parity violation about the $(\vec{p}_1,\vec{p}_2)$ plane. As before we define an asymmetry parameter C,

$$C=\frac{\sum_{\phi_1>\phi>\phi_0}\Delta\sigma-\sum_{2\pi-\phi_1<\phi<2\pi-\phi_0}\Delta\sigma}{\sum_{\phi_1>\phi>\phi_0}\Delta\sigma+\sum_{2\pi-\phi_1<\phi<2\pi-\phi_0}\Delta\sigma}=\frac{\sum_{\phi_1>\phi>\phi_0}\Delta\sigma_2}{\sum_{\phi_1>\phi>\phi_0}\Delta\sigma_1}\,, \tag{21}$$

which gives a measure of parity violation in the region of interest $\phi_1>\phi>\phi_0$.

We now consider the evaluation of the asymmetry parameter A for the reaction $e+\bar{e}\to\pi^++\pi^-+\pi^0$ near the A_1 region.

$e+\bar{e}\to\pi^++\pi^-+\pi^0$. The general forms for the coupling of π^+, π^-, and π^0, with momenta k_1, k_2, and k_3 respectively, to the photon and the intermediate vector boson are given by

$$\langle k_1k_2k_3|J^e_\mu(0)|0\rangle=\frac{f\epsilon_{\mu\alpha\beta\gamma}k_{1\alpha}k_{2\beta}k_{3\gamma}}{(2\pi)^{9/2}(8k_{10}k_{20}k_{30})^{1/2}}\,, \tag{22}$$

$$\langle k_1k_2k_3|J^{WV}_\mu+J^{WA}_\mu|0\rangle=\frac{f_1\epsilon_{\mu\alpha\beta\gamma}k_{1\alpha}k_{2\beta}k_{3\gamma}}{(2\pi)^{9/2}(8k_{10}k_{20}k_{30})^{1/2}}+\frac{k_{1\mu}g_1+k_{2\mu}g_2+k_{3\mu}g_3}{(2\pi)^{9/2}(8k_{10}k_{20}k_{30})^{1/2}} \tag{23}$$

so that for this case,

$$W_1=W_2=W_3=0\,,\qquad W_4=|f|^2\delta[(p-k_1-k_2)^2-\mu^2]\,, \tag{24}$$

and

$$\begin{aligned}&B_4=ff_1^*\delta[(p-k_1-k_2)^2-\mu^2]\,,\\&B_i=0,\quad\text{for } i\neq 4\\&A_1=f(g_1^*-g_3^*)\delta[(p-k_1-k_2)^2-\mu^2]\,,\\&A_2=f(g_2^*-g_3^*)\delta[(p-k_1-k_2)^2-\mu^2]\,,\\&A_3=fg_3^*\delta[(p-k_1-k_2)^2-\mu^2]\,,\\&A_i=0,\quad\text{for } i=4,5,6,7\end{aligned} \tag{25}$$

where μ is the pion mass. The resulting cross section is

$$\sigma=\frac{1}{16s^2(2\pi)^4}\int ds_1\int ds_2\int_{-1}^{1}d\cos\theta\int_{-\pi}^{\pi}d\phi\Big\{e^2\tfrac{1}{8}|\vec{k}_1\times\vec{k}_2|^2\sin^2\theta|f|^2-\frac{e}{8m_W^2}s^{1/2}|\vec{k}_1\times\vec{k}_2|\sin2\theta\times\mathrm{Re}f[(g_1^*-g_3^*)a|\vec{k}_1|\cos(\phi_1-\phi)+(g_2^*-g_3^*)a|\vec{k}_2|\cos(\phi_2-\phi)]\Big\}\,, \tag{26}$$

where

$$|\vec{k}_1\times\vec{k}_2|^2=\frac{s_1s_2s_3}{4s}-\frac{s\mu^2}{4}+\frac{\mu^4}{2}-\frac{\mu^6}{4s}\,.$$

The scalar function f is estimated by saturating the vertex with the ω meson (the ϕ pole is neglected since its coupling to 3π is suppressed), so that we have

$$f=\frac{em_\omega^2\sin\theta_m}{\sqrt{3}\,(s-m_\omega^2)}f_0\sum_{i=1}^{3}\frac{1}{s_i-m_\rho^2+i\Gamma_i}\,, \tag{27}$$

where m_ω is the ω mass and θ_m is the mixing angle for the vector nonet. The constant f_0 can be estimated from the ω decay width and comes out to be $f_0^2/4\pi\approx 2/\mu^2$. Similarly the scalar functions g_i are estimated by saturating the axial-vector vertex with the A_1 meson pole, which gives

$$\begin{aligned}g_1&=\frac{em_A^2(g_0/f_\rho)}{(s-m_A^2)(s_2-m_\rho^2+i\Gamma_2)}\,,\\g_2&=\frac{em_A^2(g_0/f_\rho)}{(s-m_A^2)(s_1-m_\rho^2+i\Gamma_1)}\,,\\g_3&=-\frac{em_A^2(g_0/f_\rho)}{(s-m_A^2)}\left(\frac{1}{s_1-m_\rho^2+i\Gamma_1}+\frac{1}{s_2-m_\rho^2+i\Gamma_2}\right)\,,\end{aligned} \tag{28}$$

where m_A is the complex mass of A_1, g_0 is the product of the A_1 coupling to $\rho\pi$ and the ρ coupling to 2π, and where we have taken the coupling of the neutral intermediate boson to A_1 to be of the same form as that of a photon to a vector boson. The constant g_0 is obtained from the A_1 decay width and we get $(1/4\pi)(g_0/f_\rho)^2\approx 30\mu^2$. With these values, the numerical integration for the range $-\frac{1}{2}\pi<\phi$

$<\frac{1}{2}\pi$, $\theta<\frac{1}{4}\pi$, and $4\mu^2<s_3<10\mu^2$ yields

$$A\approx 4.5\%$$

and (29)

$$\sum\Delta\sigma\approx 10^{-34}\ \mathrm{cm}^2\ ,$$

which should be experimentally observable. Of course, as mentioned earlier, the actual numbers will depend upon the numerical values of a and b, which in turn depend upon the weak-interaction model and the mixing parameter. They are also rather rough estimates, depending on the ω, A_1 decay widths and pole dominance. However, we expect the general features of our analysis to remain valid and the numerical prediction to be meaningful. It would indeed be very interesting to verify the existence or the nonexistence of these parity-violating effects experimentally. The experiments are feasible.

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