Relationship between the isospin-even πN s-wave scattering length and the $J=0$ $N\bar{N}\to\pi\pi$ helicity amplitude

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Using dispersion relations we derive a simple relationship between the isospin even πN s-wave scattering length and the $N\bar{N}\to\pi\pi$ helicity amplitudes f^0_+. We note the use of the result in evaluating the intermediate-range nucleon-nucleon potential.

The fact that the isospin-even πN s-wave scattering length (a^+) is very small has been a source of fascination for particle theorists for some time now. In 1965, Adler[1] provided a neat explanation for the effect by using soft-pion techniques to show that

$$A^+(0,0)\simeq g^2/m,$$

where $A^+(\nu,t)$ is the usual isospin-even πN amplitude,[2] m is the nucleon mass, and $g^2/4\pi=14.4$. Thus there was shown to be a large cancellation between the A^+ and B^+ amplitudes at low energy and hence a small scattering length.

A curiously similar situation arises when an evaluation of the $J=0$, $N\bar{N}\to\pi\pi$ helicity amplitude[3] $f^0_+(t)$ is attempted. In particular it is found that $f^0_+(0)$ is hard to determine accurately due to the cancellation of two large terms.[4] In view of this it is tempting to suggest a relation between $f^0_+(0)$ and a^+. We show here that indeed this is the case.

We begin with the dispersion relation given by Höhler and Strauss,[2] i.e.,

$$\mathrm{Re}C^+(\nu,t)=C^+_N(\nu,t)+\frac{2}{\pi}\int_1^\infty\frac{d\nu'}{\nu'}\mathrm{Im}A^+(\nu',t)+\frac{2\nu^2}{\pi}\int_1^\infty\frac{d\nu'}{\nu'}\frac{\mathrm{Im}C^+(\nu',t)}{(\nu')^2-\nu^2},\qquad(1)$$

where we have used the usual notation[2] together with the definition

$$C^+_N(\nu,t)=-\frac{g^2}{m}\frac{\nu^2}{(\nu^2-\nu_B{}^2)(1-t/4m^2)}.\qquad(2)$$

Now we recall that

$$\mathrm{Re}f^0_+(0)=\frac{m^2}{8\pi}\int_{-1}^1 d\nu\,\mathrm{Re}C^+(\nu,0).\qquad(3)$$

Using (1) and recalling that

$$A^+(0,0)=\frac{2}{\pi}\int_1^\infty\frac{d\nu'}{\nu'}\mathrm{Im}A^+(\nu',0),\qquad(4)$$

we find that

$$\mathrm{Re}f^0_+(0)=\frac{m^2}{4\pi}A^+(0,0)-\frac{mg^2}{4\pi}+\frac{g^2}{16\pi m}+\frac{m^2}{4\pi^2}\int_1^\infty\frac{dw}{w}\sigma^+(w)(w^2-1)^{1/2}\left(-2+w\ln\frac{w+1}{w-1}\right),\qquad(5)$$

where we have used the optical theorem

$$\mathrm{Im}C^\pm(\nu,0)=k\sigma^\pm,$$

with k = pion lab momentum, w = pion lab energy, and

$$\begin{aligned}\sigma^+&=\tfrac{1}{2}(\sigma_-+\sigma_+),\\ \sigma^-&=\tfrac{1}{2}(\sigma_--\sigma_+),\\ \sigma_-&=\sigma(\pi^-p\to\pi^-p),\\ \sigma_+&=\sigma(\pi^+p\to\pi^+p).\end{aligned}\qquad(7)$$

Now we look back at the Höhler-Strauss relation and set $\nu=1$ to get

$$\mathrm{Re}C^+(1,0)=C^+_N(1,0)+A^+(0,0)+\frac{2}{\pi}\int_1^\infty\frac{d\nu'}{\nu'}\frac{\sigma^+(\nu')}{[(\nu')^2-1]^{1/2}}.\qquad(8)$$

We use this to eliminate $A^+(0,0)$ from (5) and obtain

$$\mathrm{Re}f^0_+(0)=\frac{g^2}{8\pi m}+\frac{m^2}{4\pi}\mathrm{Re}C^+(1,0)+\frac{m^2}{4\pi^2}\int_1^\infty dw\,\sigma^+(w)(w^2-1)^{1/2}\left(\ln\frac{w+1}{w-1}-\frac{2w}{w-1}\right),\qquad(9)$$

which, if we recall that

$$\mathrm{Re}C^{+}(1,0)=\frac{4\pi}{3m}(m+1)(a_1+2a_3) \tag{10}$$

(see Höhler and Strauss)[2] gives us the relation

$$\mathrm{Re}f_{+}^{0}(0)=\frac{g^2}{8\pi m}+m(m+1)a^{+}+\frac{m^2}{4\pi^2}\int_1^\infty dw\,\sigma^{+}(w)(w^2-1)^{1/2}\left(\ln\frac{w+1}{w-1}-\frac{2w}{w^2-1}\right). \tag{11}$$

Not only is this an interesting relation, but it also promises to be of some practical importance. Thus if $f_{+}^{0}(0)$ is known then a^{+} may be determined with good accuracy, due to the factor $m(m+1)$. Alternatively one may proceed to evaluate $f_{+}^{0}(0)$ from current experimental data on a^{+} and σ^{+}. Using the πN cross-section data quoted in Ref. 5 together with the value

$$a^{+}=0.00\pm0.04, \tag{12}$$

we find that

$$\mathrm{Re}f_{+}^{0}(0)=-2.4\pm1.0, \tag{13}$$

where most of the error is due to the a^{+} uncertainty. This value of $\mathrm{Re}f_{+}^{0}(0)$ can now be used in dispersion-relation calculations[6] of $f_{+}^{0}(t)$ in the range $4<t<40$. As is well known, the amplitude f_{+}^{0} in this range plays an important role in calculations of the intermediate-range nucleon-nucleon potential.[7]

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