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Effects of diffractive channels on hadronic cross sections

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We consider the effects of diffractive channels on total, elastic, and diffractive cross sections in a model in which an elementary particle scatters on a composite system of A constituents. We find that the effect of diffractive channels is to lower σ_{el}/σ_T below the value it would have were no diffractive channels present. We find both σ_{el}/σ_T and $(\sigma_{el}+\sigma_{dif})/\sigma_T \leq \frac{1}{2}$. In the limit $A\to\infty$, we find $\sigma_{el}/\sigma_T \to \frac{1}{2}$ and $\sigma_{dif}/\sigma_T \to 0$, for both the optical and the overlap cases discussed in the text.

I. INTRODUCTION

It is known that a high-energy particle may interact with a target particle or system of particles to produce a new state while no quantum numbers are exchanged. Such interactions, which are nearly energy-independent, are termed diffraction excitation or diffraction dissociation of the projectile.[1] Since these processes form a considerable part of strong-interaction cross sections, there is great interest in understanding and obtaining estimates for them. Once diffractive production is a theoretical consideration, then the effect of these channels in the elastic scattering amplitude must also be understood. In other words, diffraction dissociation can play an important role in absorption models. Furthermore, one must deal with the composite nature of hadrons in the treatment of high-energy reactions; diffraction dissociation in the scattering of the hadronic components ("partons") themselves may be important. Of course multiple-scattering effects are not unrelated to absorption models.

Recently, Pumplin[2] used an eikonal model together with a diagonalization assumption (about which we shall say more below) to study the effect of diffractive channels in the elastic scattering from nuclei. If we denote σ_T, σ_{el}, and σ_{dif} as the total, elastic, and projectile diffraction dissociation cross section, respectively, and if it is assumed that the target is left in its ground state, he found

$$\frac{\sigma_{el}+\sigma_{dif}}{\sigma_T} \leq \frac{1}{2}\,. \tag{1.1}$$

In a very different vein, Skard and Fulco[3] have incorporated diffractive production into an exact s-channel unitary multiperipheral model. They find

$$\frac{\sigma_{el}}{\sigma_T} = \frac{1}{2I(s)}\,, \tag{1.2}$$

where $I(s)$ increases with s not faster than a power of a logarithm. In particular if σ_T is asymptotically constant, then $I(s)\sim \ln^2 s$. On the other hand, if the Froissart bound is saturated so that $\sigma_T \sim \ln^2 s$, the $I(s)$ is asymptotically constant. Unfortunately, the magnitude of $I(s)$ is not determined, nor is σ_{dif} computed in this work. However, $I(s)=1$ in the absence of diffractive channels. Along these lines we should also mention the field-theory work of Cheng and Wu[4] who, by satisfaction of two-body unitarity, saturate the Froissart bound. In their model, which does not include the effects of diffractive excitation, absorption is total, so that $\sigma_{el}/\sigma_T = \frac{1}{2}$.

We should note that although both σ_{el} and σ_T are known[5,6] to be reduced by the introduction of diffractive channels (essentially because the target becomes more transparent), the behavior of the ratio σ_{el}/σ_T is not determined by these simple arguments. In addition, as new diffractive channels are opened, the behavior of $(\sigma_{el}+\sigma_{dif})/\sigma_T$ will be determined by a competition between the increased transparency and the growth of σ_{dif}.

In this paper we consider a model in which there is, in addition to the elastic channel, one diffractive channel. We calculate σ_T, σ_{el}, and σ_{dif} for the scattering of an elementary projectile on a composite system having a large number, A, of constituents. Thus, we take explicit account of the composite nature of hadrons. (The extension to composite-composite scattering is straightforward, but messy.) Our results will, of course, apply to nuclear scattering, but we shall refer to the constituents as "partons" to emphasize the more general nature of the result. We shall study the cross sections in a dynamical approximation corresponding to partons whose size (measured by the width of the parton-parton scattering amplitude) is small compared to the geometric size of the composite. This is the optical limit,[7] which can apply to nuclear scattering, or to hadrons composed of pointlike partons. We also study the dynamical approximation in which the parton size is large compared to the composite size. This is the overlap limit, which could plausibly appear when the Froissart bound is saturated for the composite system, such saturation being due in turn to saturation in the parton scattering.[8]

Roughly speaking, calculation of σ_{el} with our formalism involves the parton projectile propagating within the composite target by means of elastic scattering or by conversion to a diffractive excitation with subsequent reconversion before leaving the target. The excitation may scatter elastically before reconverting [see Fig. 1(a)]. Similarly, the calculation of σ_{dif} involves such a sequence of events with the exception that the last conversion into a diffractively excited state does not reconvert, as in Fig. 1(b). We could imagine two ways to handle this problem. In the first, we simply sum all the possible graphs. Note that in such graphs, simple microscopic causality requirements suggest that the diffractive excitation be created before reconversion. The second method involves rotation of the physical states to two new states which do not couple, propagate them by elastic scattering through the composite system, and rotate back to the physical amplitudes; this method is the one used by Pumplin. We find that in the second method the eikonal requirement is not met. Fortunately, however, the disagreement between the two methods seems to be quantitative rather than qualitative. This is especially important because for technical reasons we are only able to compute by the first method to $O(G^2/\sigma^2)$, where σ is the parton-parton total cross section and G is the strength of the parton-diffractive excitation.

Our results can be summarized as follows. As one might expect, on intuitive grounds, inclusion of the diffractive channel decreases the elastic cross section. In the optical case, σ_{el}/σ_T and $(\sigma_{el}+\sigma_{dif})/\sigma_T$ are both less than $\frac{1}{2}$, and approach $\frac{1}{2}$ from below as $A\sigma/2\pi R^2 \to \infty$, where R is the radius of the composite system. Note that in this limit σ_{dif}/σ_T vanishes. These results are also true in the overlap limit, except the appropriate limit becomes $A\to\infty$. In other words, our results indicate that for a finite number of constituents σ_{el}/σ_T does not equal $\frac{1}{2}$; moreover, $(\sigma_{el}+\sigma_{dif})/\sigma_T$ is also less than $\frac{1}{2}$. In the total absorption limit σ_{dif} vanishes compared to σ_{el}.

In Sec. II we discuss the differences between the direct calculation of graphs (the "ordered" case) and the diagonalization technique (the "unordered" case) described above. Sections III and IV contain calculations and results for the various cross sections for the unordered and ordered cases, respectively. We conclude the paper with a discussion in Sec. V.

II. ORDERED vs UNORDERED METHODS OF SOLUTION

Our general guide to calculation is the Glauber theory. This theory, whose justification and general applicability have been amply discussed elsewhere,[9] requires that the dominant contributions to multiple scattering come from forward scattering of the parton constituents. This method, which assumes the simple additivity of phase shifts, leads to results which are similar to the eikonal approximation. On the other hand, we stated in the Introduction that a diffraction excitation, R^*, should be created before it scatters or reconverts to the original projectile. This, together with our remarks on Glauber theory, is seen to demand that if a projectile is changed to an excitation on a given parton target at z_1 and reconverted at z_2 (z is in the beam direction), then $z_1<z_2$ [see Fig. 1(a)]. The question of this ordering in z is the

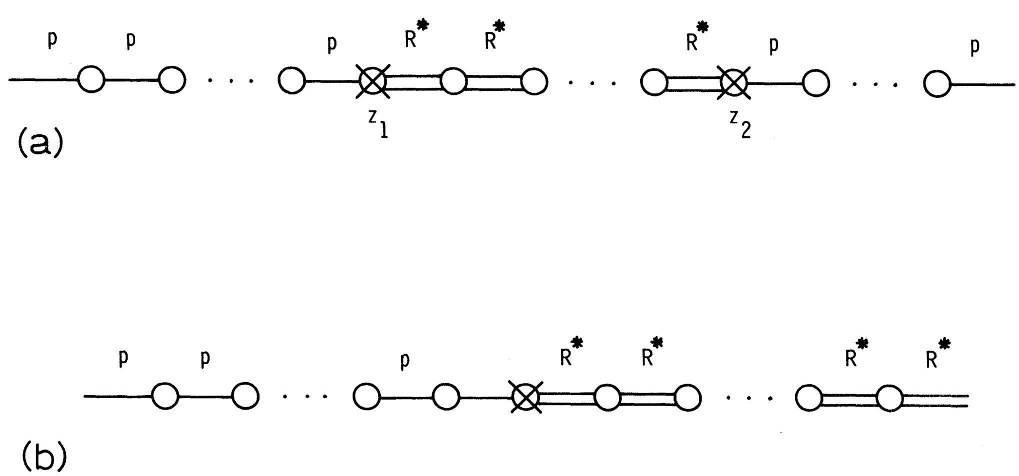

FIG. 1. (a) Multiple-scattering diagram with one diffractive channel in the elastic scattering amplitude. (b) Multiple-scattering diagram with one diffractive channel in the diffractive-dissociative amplitude.

one we wish to investigate more closely in this section.

Our procedure will be to assume that there is only one diffractive excitation R^* and compare the diagonalization procedure with the direct graph summation procedure in second order in scattering. Comparison to this order is sufficient to show the differences in the two procedures caused by the ordering requirement. As we develop this comparison we shall also discuss the application of the Glauber theory as well as derive some results which will be of use in later sections of this paper.

With these remarks let us consider the reaction $p+A\to p+A$, where p is a parton and A is the composite system. Denote by $F(\Delta^2)$ the elastic scattering amplitude for this process, with $\Delta^2=-t$ the momentum transfer.

$F(\Delta^2)$ is composed of two parts: one in which the incident parton elastically scatters with no intermediate states possible; the second in which the diffractive intermediate channel is open. We denote these terms by $F_0(\Delta^2)$ and $F_1(\Delta^2)$, respectively, so that

$$F(\Delta^2)=F_0(\Delta^2)+F_1(\Delta^2)\,. \tag{2.1}$$

We take as the ground-state density for the composite target system

$$|\psi|^2=\left(\frac{1}{\sqrt{\pi}\,R}\right)^{3A}\prod_{i=1}^{A}\exp[-(r_i^{\,2}/R^2)]\,, \tag{2.2}$$

where R characterizes the radius of the system. We take for the elastic parton-parton amplitude

$$f(q^2)=\frac{ip}{4\pi}\,\sigma\,\exp(-aq^2/2)\,, \tag{2.3}$$

where p is the incident momentum, σ is the total parton-parton cross section, a is the elastic width, and $-q^2$ is the parton-parton momentum transfer, and we have assumed that $\mathrm{Re}(f)=0$, appropriate to high-energy scattering. Then according[10] to Glauber theory

$$F_0(\Delta^2)=-\frac{ip}{2}\,(R'^2)\exp\left(\frac{\Delta^2R^2}{4A}\right)\sum_{k=1}^{A}\binom{A}{k}\frac{1}{k}\left(\frac{-\sigma}{2\pi R'^2}\right)^k \times\exp\left(\frac{-\Delta^2R'^2}{4k}\right)\,, \tag{2.4}$$

where

$$R'^2=R^2+2a\,. \tag{2.5}$$

To compute F_1, we need amplitudes involving the R^* diffractive state. We take the R^*-p elastic scattering amplitude as

$$g(q^2)=\frac{ip}{4\pi}\,\sigma^*\exp(-a^*q^2/2)\,, \tag{2.6}$$

and the R^* production amplitude to be

$$h(q^2)=\frac{ip}{4\pi}\,G\,\exp(-\bar{a}q^2/2)\,. \tag{2.7}$$

In writing this form for the production amplitude we have already made a high-energy approximation, since we have set the longitudinal momentum transfer $\Delta_{\min}=(m^2-m^{*2})/2E$ to zero. We shall also assume that $\sigma=\sigma^*$, and $\bar{a}=a^*=a$.

Trefil[5,11] was able to compute $F_1(\Delta^2)$ by individual graph computation to $O(G^2/\sigma^2)$. He used the so-called rim approximation, which presents no problem for us since it is exact for double scattering; he found

$$F_1(\Delta^2)=\frac{-ip}{\sqrt{\pi}}\,(R'^2)\left(\frac{G}{\sigma}\right)^2\exp\left(\frac{\Delta^2R^2}{4A}\right)\sum_{k=2}^{A}\binom{A}{k}\frac{1}{k}\left(\frac{-\sigma}{4\pi R'^2}\right)^k\exp\left(\frac{-\Delta^2R'^2}{4k}\right)\sum_{j=0}^{k-2}\sum_{l=0}^{k-j-2}\left[2\left(\frac{2}{\pi}\right)^{1/2}\right]^l\Gamma\left(\frac{l+1}{2}\right)\,. \tag{2.8}$$

While it is possible to reduce the triple sum to a double sum at $\Delta^2=0$, the remaining sums prove to be intractable for analytic purposes. For this reason we shall in Sec. IV turn to alternate means of calculation for the multiple-scattering graphs. Nevertheless, we can easily work out the $O(G^2)$ term, $k=2$. This is the simplest double-scattering term, corresponding to conversion and reconversion with no elastic scattering terms. In addition, Trefil computed the diffractive production amplitude $F_{\mathrm{dif}}(\Delta^2)$ for $p+A\to R^*+A$ to $O(G/\sigma)$ using the rim approximation (which again is exact up to second order in scattering). He found

$$F_{\mathrm{dif}}(\Delta^2)=-ipR'^2\frac{G}{\sigma}\exp\left(\frac{\Delta^2R^2}{4A}\right)\times\sum_{k=1}^{A}\binom{A}{k}\left(\frac{-\sigma}{4\pi R'^2}\right)^k\exp\left(\frac{-\Delta^2R'^2}{4k}\right)\,. \tag{2.9}$$

It is also trivial to extract the second-order scattering terms from this amplitude.

If we define the matrix of amplitudes

$$M(\Delta^2)=\begin{bmatrix} F(p+A\to p+A) & F(p+A\to R^*+A) \\ F(R^*+A\to p+A) & F(R^*+A\to R^*+A)\end{bmatrix}\,, \tag{2.10}$$

then from Eqs. (2.4), (2.8), and (2.9) we have from Trefil's calculation to second order in the scattering

$$M(\Delta^2)|_{k=2} = C'(\Delta^2)\begin{bmatrix} \sigma^2+\frac{1}{2}G^2 & \sigma G \\ \sigma G & \sigma^2+\frac{1}{2}G^2 \end{bmatrix}, \quad (2.11)$$

where

$$C'(\Delta^2) = -\frac{ip}{4}R'^2 \exp\left(\frac{\Delta^2 R^2}{4A} - \frac{\Delta^2 R'^2}{8}\right)\binom{A}{2}\left(\frac{1}{2\pi R'^2}\right)^2. \quad (2.12)$$

Let us now examine the diagonalization method of calculation. Suppose instead of a parton, p, and an excitation, R^*, which are coupled, we transform to two new states $\bar{p}$ and $\bar{R}^*$ which are uncoupled. Physically what this means is that the production amplitudes of $\bar{p}+\bar{p}\to\bar{p}+\bar{R}^*$ vanish for the quasiparticle states. The immediate advantage of such a prescription in the multiple-scattering process is that it is not possible to have intermediate diffractive states, i.e., no production (and/or reconversion) can occur. Hence only elastic scattering of $\bar{p}$ and $\bar{R}^*$ can occur within the composite system. This is the procedure followed by Pumplin.[2]

To develop such a transformation we consider the matrix of the parton scattering amplitudes,

$$T(\Delta^2) = \begin{bmatrix} f(p+p\to p+p) & f(p+p\to R^*+p) \\ f(R^*+p\to p+p) & f(R^*+p\to R^*+p) \end{bmatrix} \quad (2.13)$$

$$= \frac{ip}{4\pi}\exp(-aq^2/2)\,\bar{T}, \quad (2.14)$$

where

$$\bar{T} = \begin{bmatrix} \sigma & G \\ G & \sigma^* \end{bmatrix}, \quad (2.15)$$

and we are for the moment assuming $\sigma^*\neq\sigma$. Then the matrix which diagonalizes $\bar{T}$ is

$$U = \begin{bmatrix} \dfrac{A'-2G+c}{2(c^2-2Gc)^{1/2}} & \dfrac{A'+2G-c}{2(c^2-2Gc)^{1/2}} \\ \dfrac{A'-2G-c}{2(c^2+2Gc)^{1/2}} & \dfrac{A'+2G+c}{2(c^2+2Gc)^{1/2}} \end{bmatrix}, \quad (2.16)$$

where

$$A' = \sigma - \sigma^*, \quad c = (A'^2+4G^2)^{1/2}. \quad (2.17)$$

The diagonal form of $\bar{T}$ is

$$\bar{T}' = \begin{pmatrix} \frac{1}{2}(\sigma+\sigma^*+c) & 0 \\ 0 & \frac{1}{2}(\sigma+\sigma^*-c) \end{pmatrix}. \quad (2.18)$$

Now that $\bar{T}$ is diagonalized we have defined the two new components which can scatter only elastically. The new quasiparticle states, $\bar{p}$ and $\bar{R}^*$ are related to the physical p and R^* states by

$$\begin{pmatrix} \bar{p} \\ \bar{R}^* \end{pmatrix} = U \begin{pmatrix} p \\ R^* \end{pmatrix}. \quad (2.19)$$

Since the new states are uncoupled we may take $\bar{T}'$ as the input into the Glauber theory to obtain the quasiparticle elastic scattering amplitudes on a composite system. We do not need to consider the inelastic, intermediate states and their related ordering; therefore we may use $F_0(\Delta^2)$ in Eq. (2.4) as our amplitude, with appropriate values for the parameters in that equation. Once the quasiparticle amplitudes are found, we can then transform the system matrix to find the elastic and diffractive amplitudes for the physical partons.

Denoting the matrix of the quasiparticle amplitudes for scattering on the composite by $M'(\Delta^2)$, we find

$$M'(\Delta^2) = -\frac{ip}{2}R'^2\exp\left(\frac{\Delta^2R^2}{4A}\right)\sum_{k=1}^{A}\binom{A}{k}\left(\frac{-1}{2\pi R'^2}\right)^k \exp\left(-\frac{\Delta^2R'^2}{4k}\right)\frac{1}{k}\begin{bmatrix}\sigma_+ & 0 \\ \sigma & \sigma_-\end{bmatrix}^k, \quad (2.20)$$

where we have now set $\sigma^*=\sigma$, and

$$\sigma_\pm = \sigma \pm G. \quad (2.21)$$

The transformation back to the unphysical amplitudes is

$$M(\Delta^2) = U^\dagger M'(\Delta^2)\,U, \quad (2.22)$$

and U is given by Eq. (2.15).

In this section we only want to write down the double-scattering terms, $O(\sigma^2, G\sigma, \text{ or } G^2)$, in $M(\Delta^2)$. This is easily accomplished, with the result

$$M(\Delta^2) = C'(\Delta^2)\begin{bmatrix} \sigma^2+G^2 & 2\sigma G \\ 2\sigma G & \sigma^2+G^2 \end{bmatrix}, \quad (2.23)$$

and $C'(\Delta^2)$ is given in Eq. (2.12).

The result, Eq. (2.23), derived from the diagonalization procedure, is to be compared with Eq. (2.11), derived from the exact calculation of multiple-scattering graphs. We note the discrepancy, which means the operator U which diagonalizes the matrix of elementary parton-parton amplitudes (T) will not diagonalize the matrix of the

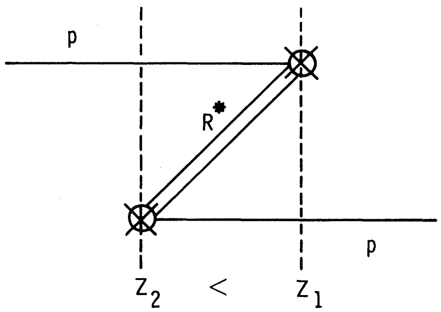

FIG. 2. Multiple-scattering diagram for double scattering with intermediate state, R^*. Such a graph is included in unordered models but neglected in ordered models.

parton-composite amplitudes. The source of the discrepancy is the extra factors the ordering introduces into the system amplitude. U would diagonalize both matrices if ordering effects were neglected in the exact calculation of M.

In terms of multiple scattering, ordering demands intermediate states to be created such that if z_1 is the production vertex and z_2 is the reconversion vertex then $z_1 < z_2$. The neglect of ordering *effectively* permits graphs like Fig. 2 because z_1 and z_2 are independently integrated from $-\infty$ to ∞. Inclusion of the ordering introduces a factor $\theta(z_1 - z_2)$ into the calculation and a simple factor of $\frac{1}{2}$ into terms involving G in second order in scattering. On the other hand, a direct graphical connection to the diagonalization procedure is possible if ordering is neglected.

Since both the ordered and unordered methods are used in the literature we treat both. We feel, however, that the ordered method is more consistent with the physics of the Glauber theory. Fortunately we see from Eqs. (2.11) and (2.23) that the differences are only quantitative, at least in second order. This is useful because we cannot compute the ordered case to all orders in the scattering, whereas we can for the unordered case.

III. THE UNORDERED SOLUTION—BEHAVIOR OF σ_{el}, σ_T, σ_{dif}

We consider two cases mentioned previously, the optical and overlap limits. In each case the number of partons A is considered large. In the optical model we examine the cross section in the limit that σ vanishes and A grows large such that $A\sigma$ remains finite. In the overlap limit the sizes of the partons are large compared to the distribution of their centers. This means that a, the diffractive width of parton scattering, is large compared to R^2. Initially we state our results for all orders of G/σ and then to order G^2/σ^2 for comparison to the ordered solution.

A. Overlap limit

The matrix of the quasiparticle amplitudes is given by Eq. (2.20); we transform it to the physical system using Eqs. (2.22) and (2.15). The result is

$$M(\Delta^2) = -\frac{ip}{2}R'^2 \exp\left(\frac{\Delta^2 R^2}{4A}\right)\sum_{k=1}^{A}\binom{A}{k}\left(\frac{-1}{2\pi R'^2}\right)^k \frac{1}{2k}\exp\left(-\frac{\Delta^2 R'^2}{4k}\right)\begin{pmatrix}\sigma_+^k+\sigma_-^k & \sigma_+^k-\sigma_-^k\\ \sigma_+^k-\sigma_-^k & \sigma_+^k+\sigma_-^k\end{pmatrix}, \qquad (3.1)$$

where

$$R'^2 = 2a.$$

When $\Delta^2 = 0$, we require the sum

$$S_m = \sum_{k=1}^{A}\binom{A}{k}\frac{(-x)^k}{k^m}. \qquad (3.2)$$

This has been evaluated in the large-A limit in the Appendix of Ref. 6, and is given by

$$S_m = -B^m + \text{const} + O\left(\frac{1}{Ax}\right), \qquad (3.3)$$

where

$$B = \ln(Ax) + \gamma$$

and $\gamma = 0.577$, Euler's constant. Using Eqs. (3.3) and (3.1) together with the optical theorem, we find

$$\sigma_T = 4\pi a[B + \tfrac{1}{2}\ln(1 - y^2)], \qquad (3.4)$$

where we have defined

$$x = \frac{\sigma}{2\pi R'^2} = \frac{\sigma}{4\pi a} < 1 \qquad (3.5)$$

and

$$y = G/\sigma, \qquad (3.6a)$$

$$B = \ln(Ax) + \gamma. \qquad (3.6b)$$

Since $y < 1$, $\ln(1 - y^2)$ is negative. Therefore the effect of the diffractive channel is to decrease the total cross section. This is reasonable, since it has the effect of increasing the double scattering, or shadowing, term.

Direct computation of the electric cross section by integration of the elastic p-A amplitude, $M_{11}(\Delta^2)$, is difficult. We can, however, approximate it very well by noting the p-A amplitude is strongly peaked in the forward direction and defining an approximate form for $M_{11}(\Delta^2)$:

$$M_{11}^{\rm appr}(\Delta^2) = M_{11}(0)\exp(-a_{\rm el}\Delta^2/2), \tag{3.7}$$

where the width fits Eq. (3.1) at $\Delta^2=0$,

$$a_{\rm el} = \frac{-2}{|M_{11}(0)|}\left(\frac{d}{d\Delta^2}|M_{11}(\Delta^2)|\right)_{\Delta^2=0}. \tag{3.8}$$

Using this form, we have

$$\sigma_{\rm el} = \frac{\pi}{p^2}|M_{11}^{\rm appr}(0)|^2\frac{1}{a_{\rm el}}. \tag{3.9}$$

The evaluation of $a_{\rm el}$ is straightforward, requiring knowledge of S_m as in Eq. (3.2) for $m=2$. We then find the ratio of elastic to total cross section,

$$\frac{\sigma_{\rm el}}{\sigma_T} = \frac{\frac{1}{2}}{\{1+(1.65+\frac{1}{4}\xi^2)/[B+\frac{1}{2}\ln(1-y^2)]^2\}} \leq \frac{1}{2}, \tag{3.10}$$

where we also define

$$\xi = \ln\left(\frac{1+y}{1-y}\right). \tag{3.11}$$

We note that $\sigma_{\rm el}/\sigma_T$ approaches $\frac{1}{2}$ from below as A becomes large. For $y<1$, $\xi^2>0$ and $\ln(1-y^2)<0$, so that the effect of the inelastic states diminishes $\sigma_{\rm el}/\sigma_T$ from the value it would have if these states were omitted.

To compute the diffractive cross section we again note that the amplitude is forward peaked, and characterize the diffractive scattering amplitude $M_{12}(\Delta^2)$ as

$$M_{12}^{\rm appr}(\Delta^2) = M_{12}(0)\exp(-a_{\rm dif}\Delta^2/2), \tag{3.12}$$

$$a_{\rm dif} = \frac{-2}{|M_{12}(0)|}\left(\frac{d}{d\Delta^2}|M_{12}(\Delta^2)|\right)_{\Delta^2=0}. \tag{3.13}$$

This gives, for $\sigma_{\rm dif}$,

$$\sigma_{\rm dif} = \frac{\pi}{p^2}|M_{12}(0)|^2\frac{1}{a_{\rm dif}}, \tag{3.14}$$

so that we find

$$\frac{\sigma_{\rm dif}}{\sigma_T} = \frac{1}{16}\xi^2[B+\tfrac{1}{2}\ln(1-y^2)]^{-2} \tag{3.15}$$

and in addition

$$\frac{\sigma_{\rm el}+\sigma_{\rm dif}}{\sigma_T} = \frac{1}{2}\left[1+\frac{1.65+\xi^2/4}{[B+\frac{1}{2}\ln(1-y^2)]^2}\right]^{-1}\left[1+\frac{\xi^2\{1.65+[B+\frac{1}{2}\ln(1-y^2)]^2+\xi^2/4\}}{8[B+\frac{1}{2}\ln(1-y^2)]^4}\right] \leq \frac{1}{2}. \tag{3.16}$$

In the large-A limit, and assuming $y\neq 1$, the first term in Eq. (3.16) is $\sim(1+1.65/B^2)^{-1}$, while the second term $\sim 1+\xi^2/8B^2$. Then in this limit

$$\frac{\sigma_{\rm el}+\sigma_{\rm dif}}{\sigma_T} \simeq \frac{1}{2}\left[1+\frac{1}{B^2}\left(\frac{\xi^2}{8}-1.65\right)\right], \tag{3.17}$$

which approaches $\frac{1}{2}$ from below. If $y\simeq 1$, then ξ^2 is large and positive, so that Eq. (3.17) appears to be $>\frac{1}{2}$; however, in this case we would be required to keep the $\ln(1-y^2)$ terms in (3.16).

Note similarly that in the large-A limit the ratio $\sigma_{\rm dif}/\sigma_T$ *vanishes*, which is why both $\sigma_{\rm el}/\sigma_T$ and $(\sigma_{\rm el}+\sigma_{\rm dif})/\sigma_T$ can approach the value of $\frac{1}{2}$.

We also present for completeness the case in which G/σ approaches unity. This means that diffractive production is the only process which can occur in parton-parton scattering. Considering Eqs. (3.1), (3.10), and (3.15) in this limit we obtain after some algebra

$$\sigma_T \xrightarrow[y\to 1]{} \pi a(B+\ln 2), \tag{3.18a}$$

$$\frac{\sigma_{\rm el}}{\sigma_T} \xrightarrow[y\to 1]{} \frac{1}{4}, \tag{3.18b}$$

$$\frac{\sigma_{\rm dif}}{\sigma_T} \xrightarrow[y\to 1]{} \frac{1}{4}. \tag{3.18c}$$

Therefore $(\sigma_{\rm el}+\sigma_{\rm dif})/\sigma_T$ approaches $\frac{1}{2}$.

In order to compare our results to the ordered case, which will be calculated to $O(G^2/\sigma^2)$, we write Eqs. (3.4), (3.10), (3.15), and (3.16) keeping only terms of this order:

$$\sigma_T = 4\pi a(B-\tfrac{1}{2}y^2), \tag{3.19a}$$

$$\frac{\sigma_{\rm el}}{\sigma_T} = \frac{1}{2}\left(1+\frac{1.65}{B^2}\right)^{-1}\left[1-\frac{y^2(1+1.65/B)}{B^2+1.65}\right], \tag{3.19b}$$

$$\frac{\sigma_{\rm dif}}{\sigma_T} = \frac{1}{4}\frac{y^2}{B^2}, \tag{3.19c}$$

$$\frac{\sigma_{\rm el}+\sigma_{\rm dif}}{\sigma_T} = \frac{1}{2}\left(1+\frac{1.65}{B^2}\right)^{-1} \times\left[1-\frac{y^2(B^2+3.30B-3.30-2.72/B^2)}{2B^2(B^2+1.65)}\right]. \tag{3.19d}$$

The qualitative behavior of Eqs. (3.19) is no different from that of the results we found for all orders in G/σ. This will be important in gauging the usefulness of these results composed to the ordered case.

B. Optical limit

For the sake of completeness, we write down the optical limit ($\sigma\to 0$, $A\to\infty$, $A\sigma$ finite) for the unordered calculation presented in Sec. III A. The reader wishing to find the discussion of ordering may proceed immediately to Sec. IV.

We can immediately recover the usual optical-limit results for no intermediate states by setting $R'^2=R^2$ in Eq. (2.4). This is because $\sigma\to 0$ implies $a\to 0$ as well. This would give us, for the elastic scattering of the quasistates,

$$F_{\rm opt}(\Delta^2)=\frac{ip}{2}R^2x'\sum_{k=1}^{\infty}\frac{(-x')^{k-1}}{kk!}\exp\left(\frac{-\Delta^2R^2}{4k}\right). \tag{3.20}$$

Note that as $A\to\infty$, we set $\binom{A}{k}\simeq A^k/k!$. [Equation (3.20) can alternatively be derived by starting from a continuum form of Eq. (2.4); see Ref. 12.] In this equation we have set

$$x'=\frac{A\sigma}{2\pi R^2}, \tag{3.21}$$

which can range freely in its magnitude. A rotation back to the physical states gives us the amplitude $M_{11}(\Delta^2)$ for $p+A\to R^*+A$. We have

$$M_{11}(A^2)=\tfrac{1}{2}\left(F_{\rm opt}(\Delta^2)|_{\sigma=\sigma_+}+F_{\rm opt}(\Delta^2)|_{\sigma=\sigma_-}\right), \tag{3.22}$$

$$M_{12}(A^2)=\tfrac{1}{2}\left(F_{\rm opt}(\Delta^2)|_{\sigma=\sigma_+}-F_{\rm opt}(\Delta^2)|_{\sigma=\sigma_-}\right). \tag{3.23}$$

To compute the cross sections we use the same techniques as in Sec. III A. For example, for the total cross section we require

$$M_{11}(0)=-\tfrac{1}{4}ipR^2[T_1(x'_+)+T_1(x'_-)], \tag{3.24}$$

where

$$x'_\pm=\frac{A\sigma_\pm}{2\pi R^2}, \tag{3.21'}$$

and we define the sums

$$T_m(z)=\sum_{k=1}^{\infty}\frac{(-z)^k}{k^m k!}. \tag{3.25}$$

We know[13] that

$$T_1(x'_\pm)=-\gamma-\ln x'_\pm-E_1(x'_\pm), \tag{3.26}$$

where $E_1(z)$ is the exponential integral function. For the elastic and differential cross sections we define approximations as in Eqs. (3.7)–(3.9) or (3.12)–(3.14). Calculation of the appropriate slopes requires

$$\left.\frac{d|M_{11}^{\rm opt}(\Delta^2)|}{d\Delta^2}\right|_{\Delta^2=0}=\frac{ip}{4}R^2\left(\frac{R^2}{4}\right)[T_2(x'_+)+T_2(x'_-)], \tag{3.27}$$

and these sums are also known in various limits (see Appendix).

Since $x'_\pm$ can be small or large, we consider these cases separately. First, let us study x' large and $|G|\ll\sigma$, so that $x'_\pm$ are both large. Since $E_1(x)\sim e^{-x}/x$ for large x,

$$T_1(x'_\pm)\simeq-\gamma-\ln x'_\pm, \tag{3.28}$$

which by the optical theorem and Eq. (3.24) gives

$$\sigma_T=2\pi R^2[B+\tfrac{1}{2}\ln(1-y^2)], \tag{3.29}$$

where B and y are defined by Eqs. (3.6) and x is now defined in the optical limit by

$$x=\frac{\sigma}{2\pi R'^2}\simeq\frac{\sigma}{2\pi R^2}. \tag{3.5'}$$

Note that (3.29) is just the same result as we obtained in the overlap case, Eq. (3.4), under the interchange of R^2 and $2a$. As in the overlap case, σ_T is decreased by the intermediate states.

For $\sigma_{\rm el}/\sigma_T$ and $\sigma_{\rm dif}/\sigma_T$ we require $T_2(x'_\pm)$ for large $x'_\pm$. We have in this limit

$$T_2(x'_\pm)\simeq-\tfrac{1}{2}(\ln x'_\pm+\gamma)^2-0.807. \tag{3.30}$$

Using this result we find again results as in the overlap case with $2a\to R^2$. Thus $\sigma_{\rm el}/\sigma_T$, $\sigma_{\rm dif}/\sigma_T$, and $(\sigma_{\rm el}+\sigma_{\rm dif})/\sigma_T$ are given by Eqs. (3.10), (3.15), and (3.16) with $2a\to R^2$. The same qualitative remarks concerning the effect of the diffractive excitation apply. Of course this will also be true for the $(G/\sigma)^2=y^2$ order approximation to these results.

Next we can study $x'_\pm$ small compared to 1. Here we can only study low-order expansions in y, using known expansions for E_1 for small argument.[13] We find in this limit

$$\sigma_T=A\sigma[1-\tfrac{1}{4}x'(1+y^2)]. \tag{3.31}$$

The effect of the inelastic states decreases the total cross section as before. We also find

$$\frac{\sigma_{\rm el}}{\sigma_T}=\frac{x'}{4}\left[1-\frac{3x'}{8}(1+y^2)\right], \tag{3.32a}$$

$$\frac{\sigma_{\rm dif}}{\sigma_T}=\frac{x'}{4}y^2(1-x'). \tag{3.32b}$$

IV. THE ORDERED SOLUTION—BEHAVIOR OF $\sigma_{\rm el}$, σ_T, AND $\sigma_{\rm dif}$

As we discussed in Sec. II, the change of basis to the quasistates as in Sec. III does not take into account the ordering in $F_1(\Delta^2)$. While the correct ordering prescription to $O(y^2)$ is given in Eq. (2.8), the summations given there are intractable. On the other hand, $F_{\rm dif}(\Delta^2)$ does have a simple expression at high energies, as seen in Eq. (2.9). (We will sum this expression below.) As a consequence $\sigma_{\rm dif}$ can be easily computed.

We attack the problem of $F_1(\Delta^2)$ by returning to the z integration $I_z'(k)$ which gives rise to Eq. (2.8). These integrals are precisely the double sum at the end of Eq. (2.8) up to a factor of $(\sqrt{\pi}\ 2^{k-1})^{-1}$; i.e., if we turn to Ref. 5, we find

$$F_1(\Delta^2) = \frac{-ip}{2} R'^2 \left(\frac{G}{\sigma}\right)^2 \exp\frac{R^2\Delta^2}{4A} \times \sum_{k=2}^{A} \binom{A}{k} \frac{(-x)^k}{k} \exp\left(\frac{-\Delta^2 R'^2}{4k}\right) I_z'(k), \tag{4.1}$$

when x is given by Eq. (3.5). Rather than using Trefil's method for $I_z'(k)$, we shall reduce the calculation to a problem in counting; moreover, we will not be forced to make the rim approximation. In order to understand the techniques we use, we return to first principles in the Glauber theory.

In the standard Glauber formalism it is assumed that all the z dependence in the problem vanishes. Consequently, if we were to look at a particular multiple-scattering diagram we could label the constituents arbitrarily; i.e., they are indistinguishable. This is a property of the fact that the incident particle scatters only elastically. But ordering gives two constituents preferred positions in a multiple-scattering diagram [see Fig. 1(a)], the production vertex and the reconversion vertex. Our technique hinges on relating this case in which only two of the parton positions are fixed and the rest are unfixed to the case when all parton positions are fixed.

Consider a scattering graph in which the positions of all partons are fixed, as in Fig. 3. The corresponding integral is, for a Gaussian density of partons,

$$I = \left(\frac{1}{\sqrt{\pi}R}\right)^n \int_{-\infty}^{\infty} dz_n e^{-z_n^2/R^2} \int_{-\infty}^{z_n} dz_{n-1} e^{-z_{n-1}^2/R^2} \cdots \int_{-\infty}^{z_2} dz_1 e^{-z_1^2/R^2}. \tag{4.2}$$

Since all the integrands commute, the ordering operator is the simple product of the exponentials and we can write

$$I = \frac{1}{n!}\left(\frac{1}{\sqrt{\pi}R}\right)^n \int_{-\infty}^{\infty} dz_n \int_{-\infty}^{\infty} dz_{n-1} \cdots \int_{-\infty}^{\infty} dz_1 e^{-(z_n^2+z_{n-1}^2+\cdots+z_1^2)/R^2} = \frac{1}{n!}. \tag{4.3}$$

Once we know this, we are ready to calculate these diagrams, which include excitation of the projectile; we consider a specific example to exhibit how the counting is done, and then we give our result for $I_z'(k)$.

We consider $k=4$ scattering; the possible diagrams are given in Fig. 4. In Fig. 4(a) we understand that $z_1 < z_2$ since those are the production and reconversion vertices. Now all that we demand for z_3 and z_4 is that they follow z_2 with no fixed relative positions since these are elastic collisions. Therefore we may break Fig. 4(a) into two diagrams: one with $z_1<z_2<z_3<z_4$, the other with $z_1<z_2<z_4<z_3$. But each of these separately gives a factor of 1/4!, so that the total contribution to $I_2'(k)$ of Fig. 4(a) is 2/4!. In Fig. 4(b), we see all the nucleons are ordered since z_2 must be between z_1 and z_3, and z_4 must follow z_3. Its weight is 1/4!. Similarly we find for the sum of all the diagrams in Fig. 4

$$I_z'(k=4) = \tfrac{3}{8}.$$

Generalizing this procedure for general k, we set the denominator of $I_z'(k)$ to $k!$, then sum the numerators of each set of graphs. To calculate this sum, we note that of the k constituents in the chain of k-fold scattering in Fig. 5, one is a production vertex (labeled "1") and another is a recon-

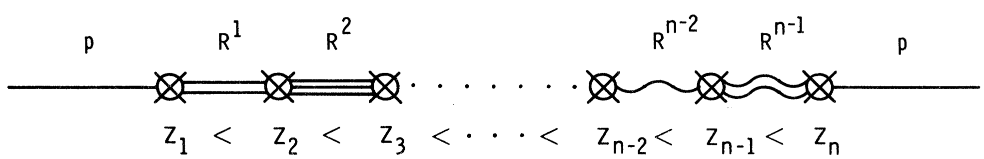

FIG. 3. Multiple-scattering diagram in which all parton positions are fixed since a new state is created at every position.

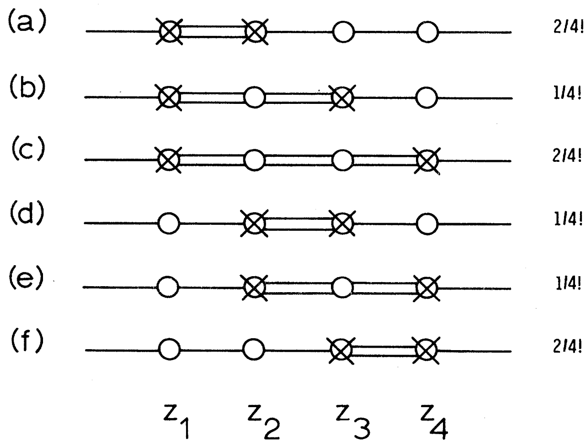

FIG. 4. Possible multiple-scattering diagrams for $k = 4$ scattering in the elastic amplitude with one intermediate channel.

version vertex (labeled "2"). The number, m, of intermediate elastic scatterings of the diffractive state ranges from zero to $k-2$, and there are $m!$ possible arrangements of the constituents in this region. For a given m, the number of elastic scatterings n which the initial state suffers ranges from zero to $k-m-2$. Once n is specified these are $n!$ ways of arranging the constituents prior to production, and $(k-m-n-2)!$ ways after reconversion.

Therefore for general k,

$$I_z'(k) = N(k)/D(k), \tag{4.4}$$

where

$$D(k) = k! \tag{4.5}$$

and

$$N(k) = \sum_{m=0}^{k-2} m! \sum_{n=0}^{k-m-2} n!(k-m-n-2)! \, . \tag{4.6}$$

We may also write

$$I_z'(k) = \frac{1}{k(k-1)} \sum_{m=0}^{k-2} \frac{1}{\binom{k-2}{m}} \sum_{n=0}^{k-m-2} \frac{1}{\binom{k-m-2}{n}} \, . \tag{4.7}$$

We note that this method does not make use of the rim approximation (which is exact only for $k=2$ scattering).

Now that we have a simpler expression for $F_1(\Delta^2)$ we can attempt to calculate it in the overlap and optical limits, as in Sec. III.

A. Overlap limit

We must compute Eq. (4.1), with $x = \sigma/4\pi a \leq 1$ in this limit. To simplify our task we shall assume here that parton-parton scattering is completely absorptive, i.e., $x=1$ (see Ref. 8). In this limit

$$F_1(0) = -\tfrac{1}{2} ip(2a) y^2 V(A), \tag{4.8}$$

where

$$V(A) = \sum_{k=2}^{A} \binom{A}{k} \frac{(-1)^k}{k} I_z'(k) \, . \tag{4.9}$$

For $V(A)$, we find numerically that at small A ($A \sim 10$ to 40) it has a $\ln A$ form, while at large A it approaches a constant [see Fig. 6(a)]. We approximate $V(A)$ by

$$V(A) = c - d\left(\frac{\ln A}{1+A}\right), \tag{4.10}$$

where $c \simeq 5.04$ and $d = 12.96$. With this form we can extract the A dependence of σ_T, using $F_0(0)$, letting $x \to 1$, and employing the optical theorem,

$$\sigma_T = 4\pi a \left\{ B - y^2 \left[c - d\left(\frac{\ln A}{1+A}\right)\right]\right\}, \tag{4.11}$$

and B is now evaluated at $x=1$.

Again the effect of the inelastic states is to decrease the total cross section; the decrease is more than the $O(G^2/\sigma^2)$ unordered case [Eq. (3.19a) with $x=1$], as $V(A)$ approaches 5 for large A as compared to $\frac{1}{2}$ in Eq. (3.19a). To calculate the elastic cross section we use the methods of Sec. III together with another sum which is used to calculate the diffractive width,

$$W(A) = \sum_{k=2}^{A} \binom{A}{k} \frac{(-1)^k}{k^2} I_z'(k) \, . \tag{4.12}$$

$W(A)$ is displayed in Fig. 6(b), and it has the same general form as $V(A)$ except that it approaches a larger asymptotic value. We can approximate it by

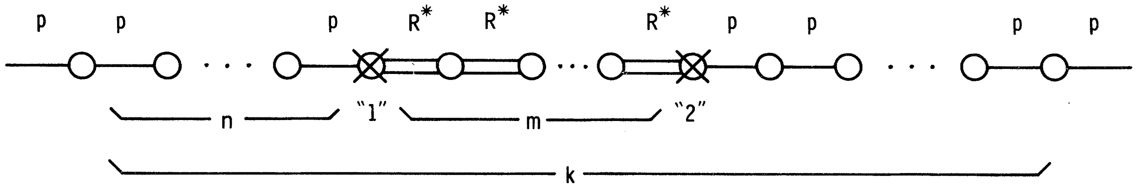

FIG. 5. Generalized k-fold scattering diagram in elastic amplitude with one intermediate channel (see text).

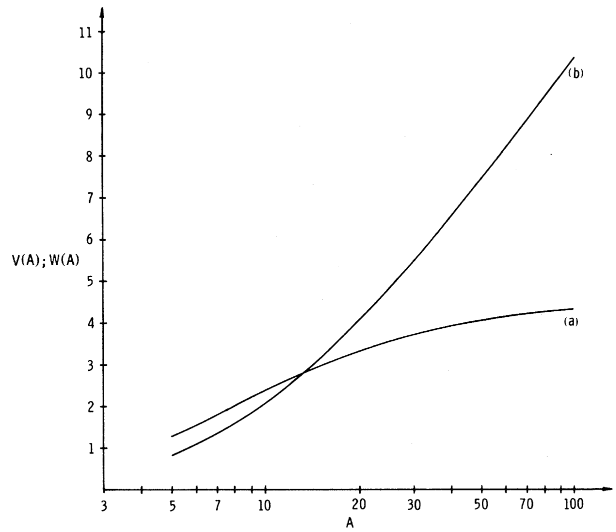

FIG. 6. (a) $V(A)=\sum_{k=2}^{A}\binom{A}{k}[(-1)^k/k]\,I_z'(k)$ as function of A. (b) $W(A)=\sum_{k=2}^{A}\binom{A}{k}[(-1)^k/k^2]\,I_z'(k)$ as function of A.

$$W(A)=c'-d'\left(\frac{\ln A}{1+A}\right), \tag{4.13}$$

where $c'\simeq 11.31$ and $d'\simeq 45.10$. Then $\sigma_{\rm el}/\sigma_T$ is found to be

$$\frac{\sigma_{\rm el}}{\sigma_T}=\frac{1}{2}\left(1+\frac{1.65}{B^2}\right)^{-1}\times\left[1-\frac{2y^2[VB-W+(1.65V/B)]}{B^2+1.65}\right]. \tag{4.14}$$

Comparing this to Eq. (3.19b) with $x=1$, the result of the $O(G^2/\sigma^2)$ unordered case, we see that both reduce to the same expression in the limit of no coupling, as they should. Furthermore, in the large-A limit

$$VB-W\simeq 5\ln A-11.3\,,$$

greater than unity, so again the effect of the inelastic states (as in the unordered case) diminishes $\sigma_{\rm el}/\sigma_T$ from the value it has when the inelastic states are omitted. We see that in the ordered case the effect is greater than in the unordered case, and the approach to $\frac{1}{2}$ is also slower as a function of A. The diffractive cross section can be computed for arbitrary x using Eq. (2.9) and the methods of Sec. III:

$$\frac{\sigma_{\rm dif}}{\sigma_T}=\frac{y^2}{B(B-\ln 2)}\,. \tag{4.15}$$

Again, comparing to the $O(G^2/\sigma^2)$ unordered case, Eq. (3.19c), the ordering increases $\sigma_{\rm dif}/\sigma_T$. We also exhibit $(\sigma_{\rm el}+\sigma_{\rm dif})/\sigma_T$ in the large-B limit, to $O(1/B^2)$,

$$\frac{\sigma_{\rm el}+\sigma_{\rm dif}}{\sigma_T}=\frac{1}{2}\left[1-\frac{1.65+2y^2(VB+1-W)}{B^2}\right]. \tag{4.16}$$

For comparison we rewrite the $O(G^2/\sigma^2)$ unordered case in this limit from Eq. (3.19d):

$$\frac{\sigma_{\rm el}+\sigma_{\rm dif}}{\sigma_T}=\frac{1}{2}\left(1-\frac{1.65+y^2/2}{B^2}\right). \tag{4.17}$$

The value of the ratio is smaller for the ordered case than for the unordered case. In both cases, $\frac{1}{2}$ is approached from below as $A\to\infty$. However, $(\sigma_{\rm el}+\sigma_{\rm dif})/\sigma_T$ will be different from $\frac{1}{2}$ for both as long as A remains finite. If A is large enough, $\sigma_{\rm dif}/\sigma_T\to 0$ as well.

B. Optical limit

As in Sec. III, we can go from the general form to the optical limit by setting $R'^2=R^2$, $A\to\infty$, and $\binom{A}{k}\to A^k/k!$. Then as in Eq. (3.20) we have from Eq. (4.1)

$$F_1(\Delta^2)=-\frac{ip}{2}R^2y^2\times\exp\left(\frac{\Delta^2R^2}{4A}\right)\sum_{k=2}^{\infty}\frac{(-x)^k}{k\,k!}\exp\left(\frac{\Delta^2R^2}{4k}\right)I_z'(k), \tag{4.18}$$

where x' is given by Eq. (3.21), $x'=A\sigma/(2\pi R^2)$.

When $\Delta^2=0$, the sum in Eq. (4.18) reduces to $V(Ax)$, $x=\sigma/2\pi R^2$, to order $1/A$ in Eq. (4.9). This means that the results of the optical limit are given to $O(1/A)$ accuracy by the results of the overlap limit. We need only replace $V(A)$ [and $W(A)$] by $V(x')$ [and $W(x')$], $B=\ln x'+\gamma$, and $a\to\frac{1}{2}R^2$. Since x' ranges from small to large, we can use our fits to $V(x')$ and $W(x')$, Eqs. (4.10) and (4.13), over this full range. Note that in particular for small x', the $\ln x'$ behavior dominates in V and W, while for large x' the constant behavior dominates. The expressions for σ_T, $\sigma_{\rm el}/\sigma_T$, $\sigma_{\rm dif}/\sigma_T$, and $(\sigma_{\rm el}+\sigma_{\rm dif})/\sigma_T$ are still given formally by Eqs. (4.11), (4.14), (4.15), and (4.16). For convenience we rewrite these expressions for $x'\ll 1$ and $x'\gg 1$. For $x'\ll 1$, rather than use the above equations, we return to Eqs. (2.9), (3.20), and (4.18) and methods of Sec. III. In the large-x' limit we keep terms to $O(1/B^2)$.

$x'\ll 1$:

$$\sigma_T\simeq A\sigma\left[1-\frac{x'}{4}\left(1+\frac{y^2}{2}\right)\right], \tag{4.19a}$$

$$\frac{\sigma_{\rm el}}{\sigma_T}\simeq\frac{x'}{4}\left[1-\frac{3x'}{8}\left(1+\frac{y^2}{2}\right)\right], \tag{4.19b}$$

$$\frac{\sigma_{\rm dif}}{\sigma_T} \simeq \frac{y^2}{8} x'(1-x'). \tag{4.19c}$$

$x' \gg 1$:

$$\sigma_T \simeq 2\pi R^2(B - y^2 c), \tag{4.20a}$$

$$\frac{\sigma_{\rm el}}{\sigma_T} \simeq \frac{1}{2}\left(1 - \frac{2cy^2}{B} - \frac{1.65 - 2cy^2 - c'}{B^2}\right), \tag{4.20b}$$

$$\frac{\sigma_{\rm dif}}{\sigma_T} \simeq \frac{y^2}{B^2}. \tag{4.20c}$$

The qualitative features of these results are the same as we have already seen for the optical limit in the unordered case to $O(y^2)$.

V. DISCUSSION

We have seen that in either case, ordered or unordered, the effect of the inelastic, intermediate states is to diminish σ_T, $\sigma_{\rm el}/\sigma_T$, $(\sigma_{\rm el}+\sigma_{\rm dif})/\sigma_T$ compared to their respective values if these states were not present. Also we have found $(\sigma_{\rm el}+\sigma_{\rm dif})/\sigma_T$ will approach $\frac{1}{2}$ only in the limit of a large number of constituents in the target. The statements made above are true both to $O(G^2/\sigma^2)$ and, when it is possible to compute, to all orders in G/σ. These statements do not depend on whether ordering is properly treated in the multiple-scattering process. The proper treatment of ordering numerically enhances the effect of the inelastic states, and in general adds more detail to the picture. However, the treatment of higher orders in G/σ remains a computational problem in that picture, whereas the neglect of ordering enables us to treat all orders of G/σ simultaneously.

We should mention that we have only included diffractive production processes in which the target remains intact in its ground state: coherent production. The inclusion of incoherent processes would increase $\sigma_{\rm dif}$, but the increase would be small compared to $\sigma_{\rm el}$.

With the assumptions of compositeness for hadrons along with the eikonal picture of scattering that the Glauber theory gives, we are able to understand the effect of the diffractive channels. If one admits to compositeness of a system then at high energies one must accept the possibility of nonvanishing intermediate states inside of it due to the multiple-scattering nature of the collision. We are led to a simple geometrical picture of scattering which agrees qualitatively with the results of the inclusion of diffractive effects in the field-theory model of Skard and Fulco[3] in that $\sigma_{\rm el}/\sigma_T < \frac{1}{2}$. However, Skard and Fulco have not calculated $\sigma_{\rm dif}/\sigma_T$ explicitly. In the geometrical picture, $\sigma_{\rm dif}/\sigma_T$ approaches zero. It then follows that that the relation $(\sigma_{\rm el}+\sigma_{\rm dif})/\sigma_T \leq \frac{1}{2}$ given in Ref. 3 is a trivial consequence of our results.

APPENDIX

We have defined in Eq. (3.25)

$$T_m(z) = \sum_{k=1}^{\infty} \frac{(-z)^k}{k^m k!}. \tag{A1}$$

Now

$$T_2(z) = \int_0^z dz' \frac{T_1(z)}{z}. \tag{A2}$$

Using Eq. (3.26) we can write for $T_1(z)$

$$T_1(z) = -\gamma - \ln z - E_1(z), \tag{A3}$$

where γ is Euler's constant and $E_1(z)$ is the exponential integral function. For $x<1$ we have[13]

$$E_1(x) \simeq -\ln(x) - \gamma + x - \tfrac{1}{4}x^2, \tag{A4}$$

and for $x>1$,

$$E_1(x) \simeq \frac{e^{-x}}{x}\left(\frac{x^2+2.33x+0.251}{x^2+3.33x+1.68}\right). \tag{A5}$$

Using Eqs. (A3)–(A5) in Eq. (A2) we find

$$T_2(z) = -\gamma \ln z - \tfrac{1}{2}\ln^2 z - 0.875 - \int_1^z dz' \frac{e^{-z'}}{z'}\left(\frac{z'^2+2.33z'+0.251}{z'^2+3.33z'+1.68}\right). \tag{A6}$$

The major contribution of the integrand in the last term comes from z' near unity. For $z \gg 1$, the integrand falls off rapidly, so that the integral is a constant for any $z>4$. It is equal to ~0.0977. After some algebra we obtain

$$T_2(z) = -\tfrac{1}{2}(\ln z + \gamma)^2 - 0.807, \tag{A7}$$

which is Eq. (3.30).

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