Regge analysis of deep-inelastic lepton-nucleon scattering

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The asymptotic behavior of the structure functions $F_2^{ep}(x)$ and $F_2^{\nu p}(x)$ is studied in a quark-parton model. It is shown that the structure functions have the expected Regge behavior. The calculated Regge residues agree well with those obtained from pion-nucleon cross sections by an extrapolation in Q^2 via a weak form of generalized scaling. The total photoabsorption cross section is also reproduced in this scheme, once the over-all scale is fixed.

I. INTRODUCTION

The high-energy behavior of total cross sections for various strong-interaction processes has been described successfully in Regge phenomenology. Usually, the asymptotic energy dependence is given in terms of a few Regge poles. Attempts have also been made to give a similar description of the asymptotic behavior of the cross sections for weak and electromagnetic interactions. The recent developments in deep-inelastic lepton-hadron scattering phenomena present a wide field where Regge phenomenology, if successful, may play a very useful role. One will have a simple and unified description of the high-energy behavior of all the three types of interaction. Moreover, the regularities observed may be useful in the formulation of the underlying theories. The scaling behavior observed in the deep-inelastic processes has already led to the formulation of many simple models for the substructure of the nucleon. The quark-parton models, in particular, have become very popular. Landshoff and Polkinghorne[1] and Kuti and Weisskopf[2] have worked out the details of a particular model, which will be referred to as the LPKW model. The model has been successful in describing the general features of the experimental results except near the threshold ($\omega \sim 1$) of the deep-inelastic region. In an earlier paper[3] (henceforth referred to as I), we have shown that the defects of the model can be remedied by introducing a correlation among the valence quarks.[4] We considered a simple modification: an increase in the momentum probability distribution for a paired valence quark by a constant η and a decrease by the same quantity in the distribution for an unpaired one. The modified model gave better agreement with the experimental results, confirming tentatively the idea of a correlation within the framework of the model. Our approach in I, though simple, does not lead to a basis convenient for application of the Regge phenomenology. In the LPKW model, the structure functions $F_i(\omega)$ are assumed to have Regge asymptotic behavior in the deep Regge limit, i.e., in the limit $\nu \to \infty$, $Q^2 \to \infty$, and $\omega = 2M\nu/Q^2$ also very large. But when the structure functions obtained in I are expressed in terms of the variable $x = 1/\omega$, one gets terms $\sim x^{(n+1)/2}$ (with $n = 0, 1, 2, \ldots,$) coming from the contribution of the valence quarks and terms $\sim x^n$ coming from the core contribution. Looking at the energy dependence, these will correspond to the Regge trajectory exchanges of intercepts $\alpha(0) = \frac{1}{2}, 0$ and their daughters (in both $I = 0$ and $I = 1$ exchanges) in addition to the Pomeron exchange. The undesirable terms can, however, be eliminated by introducing the correlation more carefully. We assume that the momentum probability distribution for a valence quark is given by

$$dP_v(x) \sim \frac{\sqrt{x}\,(1 \pm \epsilon x)}{(x^2 + \mu^2/P^2)^{1/2}}\ , \tag{1.1}$$

where x is the fraction of the total momentum P of the nucleon carried by the valence quark of mass μ. The positive sign is to be chosen if the quark is paired, and the negative sign if it is not. The momentum distributions $dP_c(x)$ for core quarks and $dP_g(x)$ for gluons will be assumed to be the same as in the original Kuti-Weisskopf model.[2] The calculation for the structure functions follows the method outlined in I, and the results are given in the next section.

II. STRUCTURE FUNCTIONS $F_2^{ep}(x)$ AND $F_2^{\nu p}(x)$

Following Kuti and Weisskopf, we first calculate $G_i(x)$, the probability that a parton of i type ($i = 0, 1, 2, 3$ for gluons and quarks of $\mathcal{P}$, $\mathfrak{N}$, and λ type, respectively) will carry a fraction x of the momentum of the nucleon. For a proton target, we have

$$G_i^p(x) = G_{iv}^p(x) + G_{ic}^p(x)\,, \tag{2.1}$$

$$G_{1v}^p(x) = \frac{35B}{16\sqrt{x}}(1+\epsilon x)(1-x)^3 \times\left[1 - \tfrac{1}{80}\epsilon^2(1-x)^2\right], \tag{2.2}$$

$$G_{2v}^{p}(x)=\frac{35B}{32\sqrt{x}}(1-\epsilon x)(1-x)^3 \times[1+\tfrac{1}{4}\epsilon(1-x)+\tfrac{1}{80}\epsilon^2(1-x)^2], \quad (2.3)$$

$$G_{ic}^{p}(x)=G_c(x) =\frac{B}{3x}(1-x)^{7/2} \times[1+\tfrac{1}{9}\epsilon(1-x)-\tfrac{1}{99}\epsilon^2(1-x)^2-\tfrac{1}{1287}\epsilon^3(1-x)^3], \quad (2.4)$$

and

$$G_{3v}^{p}=G_{0v}^{p}=0, \quad (2.5)$$

where

$$B=(1+\tfrac{1}{9}\epsilon-\tfrac{1}{99}\epsilon^2-\tfrac{1}{1287}\epsilon^3)^{-1}. \quad (2.6)$$

The probability functions $G_i^n(x)$ for a neutron target can be obtained from the above by considering an isospin rotation.

The structure functions can be given in terms of the G_i^p functions:

$$F_2^{ep}(x)=\tfrac{1}{9}[4xG_{1v}^p(x)+xG_{2v}^p(x)]+\tfrac{2}{3}xG_c(x), \quad (2.7)$$

$$F_2^{en}(x)=\tfrac{1}{9}[4xG_{2v}^p(x)+xG_{1v}^p(x)]+\tfrac{2}{3}xG_c(x), \quad (2.8)$$

$$F_2^{\nu p}(x)=2xG_{2v}^p(x)+2xG_c(x), \quad (2.9)$$

$$F_2^{\nu n}(x)=2xG_{1v}^p(x)+2xG_c(x), \quad (2.10)$$

where in (2.9) and (2.10) we have assumed that the Cabibbo angle is equal to zero. The constant ϵ is determined uniquely by the threshold ratio,

$$y(1)=\frac{F_2^{en}(x)}{F_2^{ep}(x)}, \text{ as } x\to 1 =\frac{6-2\epsilon}{9+7\epsilon}. \quad (2.11)$$

If $y(1)=\frac{1}{4}$, we have $\epsilon=1$. The value of $y(1)$ will be determined accurately in the near future. The present data[5] give $y(x=0.79)=0.36$ and the trend indicates that $y(1)$ should be still smaller. We shall consider any value of $y(1)$ between $\frac{1}{3}$ and $\frac{1}{4}$ as probable.

The calculated values for $F_2^{ep}(x)$ and $F_2^{ep}(x)-F_2^{en}(x)$ are in general agreement with the experimental results, except at low values of x. However, the available data[5] have large errors at small x and we may ignore this problem here. The comparison leads us to the conclusion that while a pair correlation is indeed necessary to fit the threshold region, no unique prescription could be given as to how to introduce it. One can, however, expect that the calculated expressions for the structure functions will correspond to the contributions only from the allowed trajectories and their daughters. This was precisely the motivation for the choice of the distribution (1.1). Moreover, since we have explicit expressions for the structure functions, we can study their small-x behavior and compare it with the asymptotic behavior of cross sections for pure hadronic processes. This will be taken up in the next section.

III. REGGE EXPANSION OF THE STRUCTURE FUNCTIONS

As $x\to 0$, the calculated expressions for the structure functions can be rewritten as

$$F_2^{ep}(x)=g_P^e+g_{P'}^e\sqrt{x}+g_{A_2}^e\sqrt{x}+\cdots, \quad (3.1)$$

$$F_2^{\nu p}(x)=g_P^\nu+g_{P'}^\nu\sqrt{x}+g_\rho^\nu\sqrt{x}+\cdots, \quad (3.2)$$

where

$$g_P^e=\tfrac{1}{3}g_P^\nu=\tfrac{2}{9}, \quad (3.3)$$

$$g_{P'}^e=\tfrac{5}{18}g_{P'}^\nu =\frac{175B}{576}(3+\tfrac{1}{4}\epsilon-\tfrac{1}{80}\epsilon^2), \quad (3.4)$$

$$g_{A_2}^e=\tfrac{1}{6}g_\rho^\nu =\frac{35B}{192}(1-\tfrac{1}{4}\epsilon-\tfrac{3}{80}\epsilon^2). \quad (3.5)$$

These may be compared with a similar expansion one makes for the pion-nucleon scattering cross section, viz.,

$$\sigma_t^{\pi^-p}=g_P^\pi+g_{P'}^\pi\nu^{-1/2}+g_\rho^\pi\nu^{-1/2}, \quad (3.6)$$

where[6]

$$g_P^\pi=21.3,\quad g_{P'}^\pi=14.4,\quad g_\rho^\pi=3.2, \quad (3.7)$$

ν is the laboratory energy of the pion in GeV, and the cross section is expressed in mb.

Let us first consider the Pomeron contributions. Langacker and Suzuki[7,8] have recently suggested that

$$\sigma_t^{\pi^\pm p}(\nu\to\infty)=\tfrac{3}{4}\pi f_\pi^{-2}F_2^{ep}(x\to 0), \quad (3.8)$$

where $\sqrt{2}f_\pi=0.96m_\pi$ is the pion decay constant. The result depends on a number of assumptions, including PCAC (partially conserved axial-vector current) and the equality between $F_{2V}(x)$ and $F_{2A}(x)$ and exact SU_3 symmetry. In our model, the Pomeron contribution comes entirely from the $q\bar{q}$ core. The right-hand side of (3.8) comes out to be 21.5 mb, which may be compared with the value of g_P^π given in (3.7). As noticed earlier,[9] the excellent agreement should be considered as accidental, in view of the approximate nature of the assumptions that go into the derivation of the relation (3.8).

Comparison of other residues will require the knowledge of the explicit relation between the variables ν and x. However, one may compare the relative contributions of the leading non-Pomeron trajectories. Thus, in πN scattering

$$\frac{g_\rho^\pi}{g_{P'}^\pi}=\frac{3.2}{14.4}=0.22\,, \tag{3.9}$$

while in our model

$$\frac{g_\rho^\nu}{g_{P'}^\nu}\simeq \tfrac{1}{3}(1-\tfrac{1}{3}\epsilon-\tfrac{1}{180}\epsilon^2) = 0.22\,, \tag{3.10}$$

for $\epsilon=1$. Encouraged by this result, we now try to establish an empirical relation between the variables x and ν by equating

$$\frac{g_\rho^\nu}{g_P^\nu}x^{1/2}=\frac{g_\rho^\pi}{g_P^\pi}\nu^{-1/2}\,. \tag{3.11}$$

This gives

$$\omega=\frac{1}{x}\simeq 50\nu\,, \tag{3.12}$$

where ν is expressed in GeV. The left-hand side of (3.11) is calculated in our model, while the right-hand side is obtained from the results fitted to the experimental data.[6] The relation (3.12) is consistent with the possibility of a generalized scaling variable

$$\omega'=\frac{2M\nu+M'^2}{Q^2+a^2}\,, \tag{3.13}$$

with $a^2=0.04$ GeV^2. The quantity M'^2 is not determined here, but it is not important for our purpose. We are interested in the region of large ν and ω'. The extrapolation suggested by (3.13) may be looked upon, in the language of Suzuki,[10] as representing a weak form of generalized scaling. Thus the functions $F_2(x)$ can be extrapolated in Q^2 down to $Q^2=0$ only for small values of x. One can now write down the following relations ($\epsilon=1$):

$$F_2^{\nu p}(x)=\frac{4}{\pi}f_\pi^{\,2}(g_P^\pi+g_{P'}^\pi\nu^{-1/2}+g_\rho^\pi\nu^{-1/2}+\text{daughter terms})\,, \tag{3.14}$$

$$F_2^{ep}(x)=\frac{4}{3\pi}f_\pi^{\,2}(g_P^\pi+\tfrac{5}{6}g_{P'}^\pi\nu^{-1/2}+\tfrac{1}{2}g_\rho^\pi\nu^{-1/2}+\text{daughter terms})\,, \tag{3.15}$$

where ν is related to x through (3.12). The daughter terms are important in $F_2^{\nu p}$ and F_2^{ep}.

Suzuki[10] has already pointed out that a small value of $a^2\sim 0.03$ GeV^2 will give a slow convergence of the Adler neutrino sum rule. In the present model, for 90% saturation of the Adler sum rule one has to integrate up to ω_0 given by

$$\omega_0\simeq 478B^2(1-\tfrac{1}{4}\epsilon-\tfrac{3}{80}\epsilon^2)^2 \simeq 200\,, \tag{3.16}$$

for $\epsilon=1$, which agrees with the estimate of Suzuki. He recommends a larger value for a^2 and a correspondingly faster saturation of the sum rule. However, the quantitative agreement between the pion-nucleon and deep-inelastic processes we have sought to establish here will become worse if a^2 is given a larger value.

It may be interesting to see if the leading energy dependence of the total photoabsorption cross section $\sigma_t^{\gamma p}$ also follows from above considerations. We expect, in analogy to (3.15),

$$\sigma_t^{\gamma p}=K(g_P^\pi+\tfrac{5}{6}g_{P'}^\pi\nu^{-1/2}+\tfrac{1}{2}g_\rho^\pi\nu^{-1/2}+\text{daughter terms})\,, \tag{3.17}$$

where the cross section is expressed in μb and the photon energy ν in GeV. The proportionality constant K can be determined by comparing with the expected Pomeron contribution to the total photoabsorption cross section. Taking the Pomeron contribution to be 97.4,[11] we have $K=4.57$. This gives the P' and A_2 contributions as 54.9 and 7.3, which compare favorably with the values 55.0 ± 5.1 and 12.3 ± 2.3, determined experimentally.[11]

IV. DISCUSSIONS

The results of the preceding sections can be summarized as follows:

(1) A modification of the LPKW model leads to structure functions in good agreement with the experimental results of the SLAC-MIT group[12] and of Bodek *et al.*[5] The structure functions exhibit expected Regge asymptotic behavior.

(2) The Pomeron contributions to the πN scattering cross section and to the structure functions F_2^{ep} and $F_2^{\nu p}$ are connected by the relation (3.8) obtained by Langacker and Suzuki.

TABLE I. Values of the Regge residues. The pion-nucleon cross section is in mb and $\sigma_t^{\gamma p}$ is in μb. The energy is measured in GeV.

	Residues of the trajectories			
Quantity	g_P	$g_{P'}$	g_{A_2}	g_ρ
$\sigma_t^{\pi N}$	21.3	14.4		3.2
$F_2^{\nu p}$	0.667	3.22		0.708
F_2^{ep}	0.222	0.894	0.118	
$\sigma_t^{\gamma N}$	97.4	55.4	7.3	
$\sigma_t^{\gamma N}$(Exp.)[a]	97.4 ± 1.9	55.0 ± 5.1	12.3 ± 2.3	

[a]See Ref. 11.

(3) The ratio of the contributions of P' and ρ trajectories are equal in $\sigma_t^{\pi N}$ and $F_2^{\nu p}(x)$, as calculated in this model.

(4) A weak form of generalized scaling does permit an extrapolation in Q^2 of the structure functions down to $Q^2=0$. The extrapolation also reproduces the total photoabsorption cross section reasonably well, provided its over-all scale is fixed. The calculated values of different Regge residues relevant here are presented in Table I.

In conclusion, we have shown that the structure functions in deep-inelastic electroproduction and neutrino-induced reactions calculated from a quark-parton model have proper Regge asymptotic behavior. There is also quantitative agreement with comparable hadronic processes. The model can be tested in the near future when further experimental results become available.

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