Large $\pi\pi$ scattering lengths and broken chiral symmetries

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In order to allow for a large $\pi\pi$ scattering length $a_0^{(0)}$, a reducible model of chiral-symmetry breaking is considered. Our model yields reasonable possible values for the width of the decay $\eta'\to\eta\pi\pi$. A sum rule connecting chiral and dilatational symmetry breaking is furthermore fulfilled in the model.

I. INTRODUCTION

There are indications that the s-wave, $I=0$, $\pi\pi$ scattering length $a_0^{(0)}$ is large as compared to the Weinberg-prediction $a_0^{(0)}=0.16m_\pi^{-1}$. A short description of the situation—following the review in Ref. 1—has been given in the Appendix. In this paper we will assume that $a_0^{(0)}$ actually is large and positive and shall do calculations with the two values

$$a_0^{(0)}\sim 1.0m_\pi^{-1}, \tag{1.1a}$$

$$a_0^{(0)}\sim 0.6m_\pi^{-1}. \tag{1.1b}$$

As can be seen from the Appendix, the value of Eq. (1.1b) is very plausible on experimental grounds. We find it interesting to clarify further the effects of a large $a_0^{(0)}$ by also doing the calculation for the value in (1.1a). Since almost any current prejudice tends to keep $a_0^{(0)}$ near the Weinberg value, we have allowed for the possibility that (1.1a) is correct in the off-shell equations (2.1a) and (2.3).

If the order of magnitude of $a_0^{(0)}$ given in Eq. (1) is really correct, it presents a serious problem to all irreducible models of chiral-symmetry breaking investigated in the literature so far. Namely, the irreducible [of SU(3)⊗SU(3) and parity] models $(3,\bar{3})\oplus(\bar{3},3)$, (8, 8), and $(6,\bar{6})\oplus(\bar{6},6)$ yield[2-5] $m_\pi a_0^{(0)}=0.16$, -0.34, and -0.06, respectively. In order to accommodate a large $a_0^{(0)}$ it is therefore necessary to investigate still other irreducible models of chiral-symmetry breaking, or else to allow the symmetry-breaking Hamiltonian density $u(x)$ to contain parts belonging to different irreducible representations of SU(3)⊗SU(3) and parity. We shall investigate the possibility that u belongs to the representation $(3,\bar{3})\oplus(\bar{3},3)\oplus(8,8)$, i.e., has the form

$$u=u_0+cu_8+zS_0+\sqrt{3}\,S_8. \tag{1.2}$$

In the above, u_0 and u_8 (S_0 and S_8) belong to the $(3,\bar{3})\oplus(\bar{3},3)$ [(8, 8)] representation of SU(3)⊗SU(3); the index (0 or 8) denotes the transformation property under SU(3) (singlet or eighth component of an octet), and c and z are numerical constants.[6] We have defined

$$S_0=\sum_{\alpha=1}^{8}S_s^{\alpha\alpha}, \tag{1.3a}$$

$$S_8=\sum_{\alpha,\beta=1}^{8}d_{8\alpha\beta}S_s^{\alpha\beta}. \tag{1.3b}$$

The u given in (1.2) is the most general chiral-symmetry-breaking Hamiltonian density from the $(3,\bar{3})\oplus(\bar{3},3)\oplus(8,8)$ representation conserving parity, isospin, and strangeness and yielding the Gell-Mann–Okubo mass formulas for baryons and mesons. Since these mass formulas are accurate up to a few percent, we may neglect from the outset any contribution of an SU(3) 27 to the Hamiltonian density. Except for contributions of the (8, 1) ⊕ (1, 8) (which must be small), the model defined by Eq. (1.2) is the simplest mixed model in the sense that the number of basic operators of any other mixed model is greater. [Our model has 18 + 64 = 82 operators; a $(3,\bar{3})\oplus(\bar{3},3)\oplus(6,\bar{6})\oplus(\bar{6},6)$ has 18 + 72 = 90.] This is a rather weak argument in favor of our ansatz in Eq. (1.2), though we believe our methods to also be applicable to other mixed models. Incidentally a u as given in (1.2) has already been proposed in the literature with motivations different from ours.[7,8]

Without further theoretical input it appears to be impossible to obtain any testable prediction of such a model. Being forced by the large $a_0^{(0)}$ to introduce a mixed model, one has then the possibility, however, of incorporating further theoretical ideas which, in the case of irreducible models investigated so far, were excluded by the data. The initially distasteful presence of more parameters in a mixed-model theory is, then, compensated for by its added flexibility. In the present paper, we shall determine the essential constants of our mixed model and obtain some predictions which can be compared with experiment.

We shall assume lowest order in SU(3) breaking and Goldstone axial-symmetry breaking for the vacuum expectation values of the σ terms. Furthermore, the (8, 8) necessarily breaks SU(2) $\otimes$ SU(2) and we shall assume this to be the only SU(2)$\otimes$ SU(2)-breaking present. That is to say, we put $c=-\sqrt{2}$. These assumptions are essential for all of our conclusions.

The derivation of a sum rule in Sec. III assumes that the current divergences and δ [an SU(3) $\otimes$ SU(3)-invariant scale-noninvariant contribution to the Hamiltonian density] have dimensions l_u and l_δ, respectively. Our test of the sum rule in that section assumes furthermore that δ is a c number and that $1 \leqslant l_u \leqslant 3$. In deriving a prediction for the width $\eta' \to \eta\pi\pi$ in Sec. IV we shall require the low-energy theorems for pions to be correct, which, as is well known, is in agreement with the above assumptions only if $l_u = 2$.

II. THE $\pi\pi$ SCATTERING LENGTHS AND PARAMETERS OF THE MODEL

As has been shown in Ref. 2, the $\pi\pi$ scattering lengths with isospin 0 and 2, respectively, can be written as

$$a_0^{(0)} = \frac{1}{96\pi m_\pi}\left(5A + 16\frac{m_\pi^2}{f_\pi^2}\right), \tag{2.1a}$$

$$a_0^{(2)} = \frac{1}{96\pi m_\pi}\left(2A - 8\frac{m_\pi^2}{f_\pi^2}\right). \tag{2.1b}$$

In the above, only A depends upon the model of chiral-symmetry breaking. The two possible values for $a_0^{(0)}$ which we chose in (1.1a) and (1.1b) yield for A

$$A \sim 23 m_\pi^2/f_\pi^2 \quad (a_0^{(0)} = m_\pi^{-1}) \tag{2.2a}$$

and

$$A \sim 13 m_\pi^2/f_\pi^2 \quad (a_0^{(0)} = 0.6 m_\pi^{-1}), \tag{2.2b}$$

respectively. We have taken $f_\pi^2 = 0.44 m_\pi^2$.

The theory requires A to satisfy, to leading order,

$$A(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) = -(1/8f_\pi^4)\{\langle\Omega|[Q_a^5,[Q_b^5,[Q_c^5,[Q_d^5,u]]]]|\Omega\rangle + (23 \text{ other terms symmetrizing in } a,\ b,\ c, \text{ and } d)\}, \tag{2.3}$$

where a, b, c, and d run 1, 2, 3. It is straightforward to compute A in the $(3,\bar{3})\oplus(\bar{3},3)$, (8, 8), or $(6,\bar{6})\oplus(\bar{6},6)$ models of chiral-symmetry breaking. We shall demonstrate the calculation in our model using the result, quite easily obtainable from (2.3), that

$$A = -(4/15 f_\pi^4)\langle\Omega|(3b_2^2 - 4b_2)u|\Omega\rangle, \tag{2.4}$$

where b_2 is the SU(2)$\otimes$ SU(2) Casimir operator

$$\sum_{a=1}^{3}(Q_a^+Q_a^+ + Q_a^-Q_a^-)$$

(see Ref. 9). Now, as we have chosen the $(3,\bar{3})\oplus(\bar{3},3)$ part of u to be an SU(2)$\otimes$ SU(2) scalar, it is set equal to zero by the operator b_2. As for the (8, 8) part, Eqs. (1.2), (1.3a), and (1.3b) enable us to write

$$zS_0 + \sqrt{3}\,S_8 = (z+1)\sum_{p=1}^{3} S_s^{pp} + (z-\tfrac{1}{2})\sum_{r=4}^{7} S_s^{rr} + (z-1)S_s^{88}. \tag{2.5}$$

This decomposition makes it clear that[10]

$$b_2 u = 4(z+1)S_s^{pp} + \tfrac{3}{2}(z-\tfrac{1}{2})S_s^{rr}, \tag{2.6}$$

from which

$$(3b_2^2 - 4b_2)u = 40(z+1)S_s^{pp} + \tfrac{15}{4}(z-\tfrac{1}{2})S_s^{rr}. \tag{2.7}$$

The leading-order SU(3) statement

$$\langle S_s^{\alpha\beta}\rangle_0 = \delta^{\alpha\beta}y, \tag{2.8}$$

with y a constant, enables us to write

$$A = (-6/f_\pi^4)(6z+5)y. \tag{2.9}$$

Now the important constants of our model are z, y, and $\langle u_0\rangle_0$. We may consider (2.9), with the values of A given in Eqs. (2.2), to give one relation between these quantities. Two other relations are

$$f^2 m_\pi^2 = -6(2z+1)y, \tag{2.10a}$$

$$f^2 m_K^2 = -6(2z-\tfrac{1}{2})y - \langle u_0\rangle_0, \tag{2.10b}$$

which follow from Goldstone vacuum symmetry breaking for the axial part of SU(3)$\otimes$ SU(3). Here we have set $f_\pi = f_K = f$, where f_π, for example, is defined by

$$\langle\pi_a|\partial A_b|\Omega\rangle = m_\pi^2 f_\pi \delta_{ab}.$$

A third relation

$$f^2 m_\eta^2 = -6(2z-1)y - \tfrac{4}{3}\langle u_0\rangle_0 \tag{2.10c}$$

is not independent of the first two; taken together they yield the Gell-Mann–Okubo mass formula.

Solving Eqs. (2.2), (2.9), and (2.10) we find the solutions

$$a_0^{(0)} \sim 1.0 m_\pi^{-1} \Rightarrow \langle u_0 \rangle_0 / m_\pi^2 f_\pi^2 \sim -27, \quad y/m_\pi^2 f_\pi^2 \sim -1.7, \tag{2.11a}$$

and

$$z \sim -0.45;$$

$$a_0^{(0)} \sim 0.6 m_\pi^{-1} \Rightarrow \langle u_0 \rangle_0 / m_\pi^2 f_\pi^2 \sim -20, \quad y/m_\pi^2 f_\pi^2 \sim -0.9, \tag{2.11b}$$

and

$$z \sim -0.41.$$

We have thus calculated the important parameters of the theory. They require that

$$\langle u \rangle_0 / m_\pi^2 f_\pi^2 = \langle u_0 \rangle_0 / m_\pi^2 f_\pi^2 + 8zy/m_\pi^2 f_\pi^2 \tag{2.12a}$$

$$\sim -21 \quad (a_0^{(0)} m_\pi \sim 1) \tag{2.12b}$$

$$\sim -17.3 \quad (a_0^{(0)} m_\pi \sim 0.6). \tag{2.12c}$$

It is worthwhile to mention that in determining z from $a_0^{(0)}$ we have $z = 0$ for $a_0^{(0)} = 0.31 m_\pi^{-1}$ and $z = \infty$ [i.e., no SU(3)-violating (8, 8) part of $u(x)$] at $a_0^{(0)} = 0.24 m_\pi^{-1}$. There is, however, no peculiar behavior of the observables we compute at these points since zy does not develop a singularity at $a_0^{(0)} = 0.24 m_\pi^{-1}$. In the next section we use the fact that this vacuum expectation value is alternatively obtainable through the low-energy theorems of broken scale invariance to provide a test of our model.

III. A SUM RULE TESTING THE PREDICTIONS OF EQS. (2.12)

In this section we shall make essential use of our assumption that the current divergences have dimension.[11] It then follows that $u(x)$ has the same dimension, l_u. Namely, in the present model $u(x)$ can be written as[9,12]

$$u(x) = i(\tfrac{13}{48}) \sum_{a=1}^{16} [Q^a, \partial^\mu J_\mu^a] + i(-\tfrac{1}{64}) \sum_{a,b=1}^{16} [Q^b, [Q^b, [Q^a, \partial^\mu J_\mu^a]]]. \tag{3.1}$$

The charges are assumed to have a dimension and current algebra then implies that this dimension is zero. Then $u(x)$ is seen to have the same dimension as the current divergences by commuting Eq. (3.1) with the dilatation charge and using the Jacobi identity.

If one assumes that the chiral-invariant scale symmetry-breaking Hamiltonian density $\delta(x)$ has dimension l_δ then it follows that[13]

$$i\langle [Q_D, T_\mu{}^\mu(0)] \rangle_0 = (l_u - l_\delta)(4 - l_u)\langle u \rangle_0. \tag{3.2}$$

We shall assume at this point that δ is a c number ($l_\delta = 0$). The reader should notice that this attractive assumption *cannot* be made for any one of the irreducible models mentioned in the Introduction since it contradicts the experimental values of the meson-nucleon σ terms.[5,14] In order to agree with the soft-pion theorems we will assume $l_u = 2$ in the following section. At present we shall only use

$$1 \le l_u \le 3. \tag{3.3}$$

In the above, $Q_D \equiv \int d^3x\, x^m T_{0m}(0, \vec{x})$ is the dilatation charge as defined in terms of a certain symmetric energy-momentum tensor $T_{\mu\nu}$.

It is easily seen that any intermediate state contributes negatively to $i\langle [Q_D, T_\mu{}^\mu(0)] \rangle_0$. Equations (3.2) and (3.3) then show that at least the sign of $\langle u \rangle_0$ in Eqs. (2.12) is correct. Allowing only integral values for l_u, the factor $l_u(4 - l_u)$ is 3 or 4; consequently an estimate of $i\langle [Q_D, T_\mu{}^\mu](0) \rangle_0$ will give us a value for $\langle u \rangle_0$ to compare with Eqs. (2.12). Such estimates exist in the literature, using either a single ϵ-meson state, or two ϵ resonances, to dominate the matrix element. In the case of a single ϵ meson of mass m_ϵ and width Γ one finds[13,15]

$$i\langle [Q_D, T_\mu{}^\mu(0)] \rangle_0 = (-3/32\pi)[m_\epsilon^2 + (l_u - 2) m_\pi^2] \times (m_\epsilon^2 - 4 m_\pi^2)^{1/2} \Gamma^{-1} \approx (-3/32\pi)(m_\epsilon^5/\Gamma). \tag{3.4}$$

This result presumably has a rather large uncertainty (see, however, the discussion at the end of this section) due to the assumptions that go into the derivation and due to the appearance of m_ϵ^5/Γ.

With the limiting values of $m_\epsilon = 700$ MeV and $\Gamma = 600$ MeV we find from (3.4)

$$\langle u \rangle_0 / f_\pi^2 m_\pi^2 = -10. \tag{3.5}$$

This certainly agrees with Eqs. (2.12) within the errors since use of $m_\epsilon = 721$ MeV already yields

$$\langle u \rangle_0 / f_\pi^2 m_\pi^2 = -18. \tag{3.6}$$

Thus as a test of the order of magnitude our mixed model agrees with the prediction of ϵ-meson dominance.

Phenomenologically, rather than by ϵ saturation, the expression in (3.4) should be computed directly from the $\pi\pi$ phase shifts. No satisfactory attempt presently exists in the literature. In a preliminary study (using experimental[16] $\pi\pi$ phases up to 1 GeV) we find agreement with our present results. Namely, the number which is finally obtained in Ref. 17 for $m_\pi^2 f_\pi^{-2} \langle u \rangle_0$ is -19. Moreover, there is good agreement with y and $\langle u_0 \rangle_0$ taken separately.

IV. THE WIDTH $\eta' \to \eta\pi\pi$ IN THE MIXED MODEL

Theoretical estimates of the width Γ of the decay $\eta' \to \eta\pi\pi$ have been obtained in the $(3,\bar{3})\oplus(\bar{3},3)$, $(8,8)$, and $(6,\bar{6})\oplus(\bar{6},6)$ models of chiral-symmetry breaking.[18-20] Experimentally an upper bound of 2.0 MeV has been set[21] for this width,[22] and one would expect a Γ of about 1 MeV. If one takes $l_u = 2$ from the soft-pion theorem, one finds,[18] 0.07 MeV as upper limit on Γ in the $(3,\bar{3})\oplus(\bar{3},3)$. This value appears to be unacceptably low. [For $l_u=3$, the upper limit in the $(3,\bar{3})\oplus(\bar{3},3)$ is 0.25 MeV.] The $(8,8)$ and the $(6,\bar{6})\oplus(\bar{6},6)$, on the other hand, yield values of Γ of the expected order of magnitude. For details and for a discussion of the uncertainties of these estimates the reader is referred to the original literature. It is our present purpose to obtain an estimate for Γ, similarly, in the mixed model.

For Γ we shall use the expression

$$\Gamma = \xi \left| \frac{m_\pi^2}{f_\pi^2} \frac{\sigma_{\eta\eta'}}{m_\pi^2} \right|^2, \tag{4.1}$$

with $\sigma_{\eta\eta'}$ given by

$$\sigma_{\eta\eta'} = \tfrac{1}{3} i \left\langle \eta \left| \sum_{p=1}^{3} [Q_A^p, \partial^\mu A_\mu^p] \right| \eta' \right\rangle \tag{4.2a}$$

$$= \tfrac{1}{3}\langle \eta | (2b_2)u | \eta' \rangle \tag{4.2b}$$

using the notation of Sec. II. In the above,

$$9 \text{ keV} < \xi (\approx 15 \text{ keV}) < 28 \text{ keV}. \tag{4.3}$$

Equations (4.1)–(4.3) follow from assuming a linear extrapolation in the Dalitz plot of the decay and standard current algebra. From Eq. (2.6) now,

$$\sigma_{\eta\eta'} = \tfrac{1}{3}\langle \eta | 8(z+1)S_s^{pp} + 3(z-\tfrac{1}{2})S_s^{rr} | \eta' \rangle. \tag{4.4}$$

It is convenient to write the $|\eta\rangle$, $|\eta'\rangle$ states as a mixture of SU(3) singlet ($|\eta_0\rangle$) and octet ($|\eta_8\rangle$) states. Thus

$$|\eta\rangle = p|\eta_8\rangle + q|\eta_0\rangle,$$
$$|\eta'\rangle = -q|\eta_8\rangle + p|\eta_0\rangle. \tag{4.5}$$

Substituting into Eq. (4.4), and using the result of the low-energy theorems assumed here that

$$\langle \eta_0 | S_s^{\alpha\beta} | \eta_8 \rangle = 0 \tag{4.6}$$

[which follows from

$$\langle \eta_0 | S_s^{\alpha\beta} | \eta_8 \rangle = -(i/f) \sum_{\rho,\sigma=1}^{8} C_{\rho\sigma}^{\alpha\beta} \langle \eta_0 | S_s^{\rho\sigma} | \Omega \rangle, \tag{4.7}$$

where $C_{\rho\sigma}^{\alpha\beta}$ is some constant tensor, and the absence of an SU(3) scalar, pseudoscalar operator in the (8, 8)], one obtains

$$\sigma_{\eta\eta'} = \tfrac{1}{3} pq \{ \langle \eta_0 | 8(z+1)S_s^{pp} + 3(z-\tfrac{1}{2})S_s^{rr} | \eta_0 \rangle - \langle \eta_8 | 8(z+1)S_s^{pp} + 3(z-\tfrac{1}{2})S_s^{rr} | \eta_8 \rangle \}. \tag{4.8}$$

The first term may be rewritten as

$$(3pq/4)(2z+1)\langle \eta_0 | S_0 | \eta_0 \rangle, \tag{4.9}$$

using SU(3) and the definition (1.3a). The value of the matrix element $\langle \eta_0 | S_0 | \eta_0 \rangle$ being ill-determined, it is important to realize that the factor $(2z+1)$ is close to zero. Consequently, the precise value of $\langle \eta_0 | S_0 | \eta_0 \rangle$ is not as decisive for the final answer as one might think. We shall estimate it presently.

For the second term of (4.8) we use the low-energy theorems to see that

$$\langle \eta_8 | S_s^{pp} | \eta_8 \rangle = 0, \tag{4.10a}$$

and

$$\langle \eta_8 | S_s^{rr} | \eta_8 \rangle = m_\pi^2 / (z+\tfrac{1}{2}). \tag{4.10b}$$

Thus (4.9) can be written as

$$\sigma_{\eta\eta'} = pq\{ -[(z-\tfrac{1}{2})/(z+\tfrac{1}{2})] m_\pi^2 + \tfrac{3}{4}(2z+1)\langle \eta_0 | S_0 | \eta_0 \rangle \}. \tag{4.11}$$

We do not intend to become too involved in a discussion of the η/η' mixing angle and shall simply quote the commonly accepted estimates of

$$|pq| \sim 0.17 - 0.23 \tag{4.12a}$$

and use

$$|pq| \sim 0.2. \tag{4.12b}$$

There finally remains the estimation of $\langle \eta_0 | S_0 | \eta_0 \rangle$. For this we shall assume that the contribution of the zS_0 part of the Hamiltonian density to the "η_0 mass" is less than the whole mass, but greater than zero. Taking also $z = -\frac{1}{2}$ for the purpose of this estimate, we write, then,

$$0 \le -\tfrac{1}{2}\langle \eta_0 | S_0 | \eta_0 \rangle \le m_{\eta'}^2 \tag{4.13a}$$

or, in round numbers,

$$0 \le -\langle \eta_0 | S_0 | \eta_0 \rangle \le 100 m_\pi^2. \tag{4.13b}$$

Now the two possible values of z in Eqs. (2.11) lead to

$$\sigma_{\eta\eta'} \sim pq(19 m_\pi^2 + 0.075\langle \eta_0 | S_0 | \eta_0 \rangle) \text{ for } z \sim -0.45, \tag{4.14a}$$

$$\sigma_{\eta\eta'} \sim pq(10 m_\pi^2 + 0.14\langle \eta_0 | S_0 | \eta_0 \rangle) \text{ for } z \sim -0.41. \tag{4.14b}$$

Setting $|pq| \sim 0.2$, as in (4.12b), and imposing the bounds (4.13b) one gets

$$\sigma_{\eta\eta'}/m_\pi^2 = \begin{cases} 3.8 \\ 2.4 \end{cases} \text{for } z\sim -0.45\,, \tag{4.15a}$$

$$\sigma_{\eta\eta'}/m_\pi^2 = \begin{cases} 2.0 \\ -0.8 \end{cases} \text{for } z\sim -0.41\,, \tag{4.15b}$$

as a result of using the lower bound of (4.13b) shown above in each case. We see that for $z \sim -0.45$ (which corresponds to $a_0^{(0)} m_\pi \sim 1.0$) we have obtained a fairly good estimate of $\sigma_{\eta\eta'}$ although we only know the order of magnitude of $\langle \eta_0 | S_0 | \eta_0 \rangle$. For $z\sim -0.41$ ($a_0^{(0)} m_\pi \sim 0.6$) there is a cancellation, however, and the result is consistent with zero if we really allow such a large upper bound in (4.13b). One might argue that it is unrealistic.

Finally, using (4.1) with $\xi \approx 15$ keV, one gets

$$0.4 \lesssim \Gamma \lesssim 1.1 \text{ MeV for } z\sim -0.45\,, \tag{4.16a}$$

$$0 \lesssim \Gamma \lesssim 0.3 \text{ MeV for } z\sim -0.41\,. \tag{4.16b}$$

Thus—contrary to the $(3,\bar{3})+(\bar{3},3)$—plausible values of Γ are possible in the present model.

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APPENDIX

In this appendix, we give a brief account of the determination of $a_0^{(0)}$ as reviewed in Ref. 1. We have also used the more elaborate analysis of Ref. 16.

Independent of the model of chiral-symmetry breaking, it follows from Eq. (2.1) that $(2a_0^{(0)} - 5a_0^{(0)})m_\pi \approx 0.54$. This relation is in good agreement with the data in Ref. 16. In order to determine $a_0^{(0)}$, one might first consider extrapolation of the $\pi\pi$ phases down from the region with $E_{\pi\pi} \approx (500-1000 \text{ MeV})$. Taking into account all available phase shifts, this method yields only rather weak restrictions according to Ref. 1 (where the original work has been quoted):

$$-0.05 \lesssim m_\pi a_0^{(0)} \lesssim 0.07\,.$$

Second, according to Laurens in Ref. 1, there are data on the $\pi\pi$ S wave from $\pi^0\pi^0$ and K_{l4} at low $E_{\pi\pi}$. These agree with each other and yield

$$m_\pi a_0^{(0)} \simeq 0.6\,.$$

It appears to the present authors that a larger value of $a_0^{(0)} m_\pi \simeq 0.8$ is not excluded by Fig. 5 of Laurens in Ref. 1. In view of this, our choice in Eq. (1.1a) still exaggerates somewhat. However, off-shell corrections might amount to a correction of that size.

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