Neutrino reactions in deuterium in the elementary-particle model and the axial-vector-current form factor

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The differential cross sections for the reactions $\nu_\mu + d \to p + p + \mu^-$ and $\bar{\nu}_e + d \to n + n + e^+$ are calculated in the forward direction from threshold to $E_\nu = 10$ GeV. The differential cross sections are found to be of the form $d\sigma/d\Omega = \beta(E_\nu)E_\nu^2$, where $\beta(E_\nu)$ is a slowly increasing function. It is shown that most of the contributions to $d\sigma/d\Omega$ come from the region of small q^2 and that to a good approximation $F_A(q^2 \sim 0)$ can be obtained from $d\sigma/d\Omega$ at large E_ν.

I. INTRODUCTION

In the immediate[1] and near future[2] it is expected that ν_e and ν_μ beams from medium to high energy will become available as a tool for studying the weak interactions. Consequently, it is important to have available accurate theoretical calculations of the differential cross sections for neutrino reactions such as $\nu_\mu + d \to p + p + \mu^-$ and[3] $\bar{\nu}_e + d \to n + n + e^+$ based on the present $V-A$ theory so that comparison with experiment may be made and any deviations examined.

In this paper we present an elementary-particle-model[4] calculation for the differential cross section in the forward direction based on the above-mentioned reactions where E_ν, the incident neutrino energy, ranges from threshold to 10 GeV.

In the elementary-particle approach the form factors describing the matrix element of the weak vector current are obtained from the electromagnetic form factors via the CVC (conserved vector current) hypothesis. The axial-current form factors are usually obtained from β-decay data by making use of the PCAC (partially conserved axial-vector current) hypothesis and a result derived via the impulse approximation.

The advantage of the elementary-particle approach over the conventional impulse-approximation treatment for this type of problem is that the elementary-particle approach avoids the use of nuclear wave functions. The cross sections calculated by means of an impulse-approximation treatment sometimes depend sensitively on these wave functions which are in general not well known.

In Sec. II of this paper we discuss the form of the matrix elements of the weak currents for these reactions and give expressions for these matrix elements. In Sec. III we obtain differential cross sections in the forward direction for these reactions for E_ν from threshold to 10 GeV. Finally, in Sec. IV we compare these results with results obtained from various impulse-approximation calculations. An approximate expression for the weak axial-vector form factor at $q^2 = 0$, $F_A(d \to NN, q^2 = 0)$ is also given.

II. GENERAL FORMULATION

The transition matrix element for the processes $\nu_\mu + d \to \mu^- + p + p$ and $\bar{\nu}_e + d \to e^+ + n + n$ can be written to the lowest order in G as

$$M(\nu_\mu + d \to \mu pp) = \frac{G}{\sqrt{2}} \cos\theta_C \langle pp | J_\lambda(0) | d\rangle \times \bar{u}_\mu \gamma^\lambda (1-\gamma_5) u_{\nu_\mu}, \tag{1a}$$

$$M(\bar{\nu}_e + d \to e^+ nn) = \frac{G}{\sqrt{2}} \cos\theta_C \langle nn | J_\lambda^\dagger(0) | d\rangle \times \bar{v}_{\nu_e} \gamma^\lambda (1-\gamma_5) v_e, \tag{1b}$$

where G $(= 1.05\times 10^{-5}/m_p^2)$ is the weak coupling constant and θ_C $(\cos\theta_C = 0.98)$ is the Cabibbo angle. The matrix elements $\langle pp|J_\mu(0)|d\rangle$ and $\langle nn|J_\mu^\dagger(0)|d\rangle$ are related by a rotation of angle π about the y axis in isotopic spin space,[5] i.e.,

$$\langle pp|J_\mu(0)|d\rangle = \langle pp|e^{-i\pi T_2}e^{i\pi T_2}J_\mu(0)e^{-i\pi T_2}e^{i\pi T_2}|d\rangle = -\langle nn|J_\mu^\dagger(0)|d\rangle, \tag{2}$$

where T_2 is the y component of the isotopic spin vector operator. The weak-current matrix element $\langle nn|J_\mu^\dagger(0)|d\rangle$ can be shown[6] by the use of LSZ (Lehmann-Symanzik-Zimmermann) techniques to have the form

$$\langle nn|J_\mu^\dagger(0)|d\rangle = \bar{u}_\alpha \bar{u}_\beta (\mathbb{C}_\mu^\nu(p_1,p_2,d))_{\alpha\beta}\xi_\nu, \tag{3}$$

where $(\mathbb{C}_\mu^\nu(p_1,p_2,d))_{\alpha\beta}$ is a 4×4 matrix with the property that

$$(\mathbb{C}_\mu^\nu(p_1,p_2,d))_{\alpha\beta} = -(\mathbb{C}_\mu^\nu(p_2,p_1,d))_{\beta\alpha}, \tag{4}$$

and where p_1 and p_2 are the 4-momenta of the two neutrons, d is the deuteron 4-momentum, and ξ^ν is the deuteron polarization vector.

The matrix elements of the vector current and axial-vector current $\langle nn|V_\mu|d\rangle$ and $\langle nn|A_\mu|d\rangle$, respectively, are each described by 24 form factors in general.[6] In the case of the vector current, the hypothesis of the conserved vector current yields 6 conditions so that 18 independent form factors are necessary for the description of the matrix element of the vector current. It is known, however, that two form factors are sufficient to describe the matrix element of the vector current in an impulse-approximation treatment which is accurate in principle to about 10%.[7] By arguments based on magnitude it is possible to reduce the 18 form factors to 2 so that these matrix elements agree with those calculated by use of the impulse approximation to the lowest order.[6] Thus the matrix element of the vector current is found to be

$$\langle nn|V_\mu^\dagger(0)|d\rangle = \eta\bar{u}(p_1)\left(\frac{F_1}{M_d^2}\epsilon_{\mu\nu\rho\sigma}\xi^\nu Q^\rho d^\sigma + \frac{F_2}{M_d}\gamma^\nu\epsilon_{\nu\rho\sigma\mu}\xi^\rho q^\sigma\right)\gamma_5 v(p_2), \tag{5}$$

where $\eta = [M^2/(E_1E_2)]^{1/2}(2\pi)^{-1/2}(2d_0)^{-1/2}$, M being the nucleon mass, E_1 and E_2 the neutron energies, and d_0 being the deuteron energy, and where F_1 and F_2 are functions of the three scalar variables Q^2, q^2, $P\cdot d$, with

$$Q_\mu = (p_1+p_2)_\mu, \quad P_\mu = (p_1-p_2)_\mu, \quad q_\mu = Q_\mu - d_\mu. \tag{6}$$

The deuteron mass is denoted by M_d.

The form factors F_1 and F_2 are found from photodisintegration data ($\gamma + d \to n + p$) and electrodisintegration data ($e + d \to e + n + p$) via the CVC, particularly by use of the commutation relation

$$[I^-, J_\mu^{(3)}(0)] = [I^-, J_\mu^{\text{em}}(0)] = V_\mu^\dagger(0). \tag{7}$$

Equation (7) leads to the relation

$$\langle nn|V_\mu^\dagger(0)|d\rangle = \sqrt{2}\,\langle np|J_\mu^{\text{em}}(0)|d\rangle \tag{8}$$

so that from Eq. (5)

$$F_1 = \sqrt{2}\,F_a \text{ and } F_2 = \sqrt{2}\,F_b, \tag{9}$$

where F_a and F_b are the electromagnetic form factors which appear in the matrix element of the electromagnetic current

$$\langle np|J_\mu^{\text{em}}(0)|d\rangle = \eta\bar{u}(p_1)\left(\frac{F_a}{M_d^2}\epsilon_{\mu\nu\rho\sigma}\xi^\nu Q^\rho d^\sigma + \frac{F_b}{M_d}\gamma^\nu\epsilon_{\nu\rho\sigma\mu}\xi^\rho q^\sigma\right)\gamma_5 v(p_2). \tag{10}$$

The form factors F_a and F_b are found to factorize via an argument based on the impulse approximation as follows[6]:

$$F_i(Q^2, q^2, P\cdot d) = f_i(q^2)F(Q^2, P\cdot d), \quad i = a, b. \tag{11}$$

In the laboratory frame (where the deuteron is at rest) one finds,[6] using photodisintegration[8] and electrodisintegration[9] data,

$$F^2(Q^2, P\cdot d) = K(\theta)a(2\pi)^2(M_d^2/p_1q_0e^2), \tag{12}$$

with

$$K(\theta) = [f_1 + f_2(1-\cos\theta) + f_3\sin^2\theta\cos\theta + f_4\sin^2\theta], \tag{13}$$

where

$$f_1 = 6.4,$$

$$f_2 = 6.4[1 - f_4/(f_3+f_4)],$$

$$f_3 = 1.122 + \frac{0.6988\times10^{-4}}{(|\vec{q}|/M_d - 0.01495)^2 + 0.507\times10^5},$$

$$f_4 = \frac{839.5}{1 + 1.27\times10^5(|\vec{q}|/M_d)^2},$$

$$a = 2.57\times10^{-9}/\text{MeV}^2,$$

and where the form factors f_a and f_b are normalized such that $f_a(q^2=0) - f_b(q^2=0) = 1$. From electrodisintegration data we find[4]

$$|f_a(q^2) - f_b(q^2)| = \frac{1}{(1-q^2/M^2)^2}, \quad M = 224 \pm 25 \text{ MeV} \tag{14}$$

in the spacelike region $q^2 \le 0$.

The matrix element of the axial-vector current is found by the use of arguments based on dispersion theory[10] and by comparison with impulse-approximation results[6] to be

$$\langle nn|A_\mu^\dagger(0)|d\rangle = \eta\bar{u}(p_1)\left(F_A\xi_\mu + F_P\frac{\xi\cdot Qq_\mu}{M_d^2}\right)\gamma_5 v(p_2). \tag{15}$$

The form factor $F_A(Q^2, q^2, P\cdot d)$ is obtained by the use of arguments based on the impulse approximation and is found[6] to be

$$F_A(Q^2, q^2, P\cdot d) = \frac{F(Q^2, P\cdot d)[f_a(q^2) - f_b(q^2)]\sqrt{2}\,F_A^I(0)}{F_V(0, n\to p) + F_M(0, n\to p)}, \tag{16}$$

where $F(Q^2, P\cdot d)$ was defined in Eq. (11) and[11] $F_V(0, n\to p) = 1$, $F_M(0, n\to p) = 3.70$, and[12] $F_A^I(0) = 1.23 \pm 0.01$.

We obtain F_P from F_A by making use of the PCAC hypothesis following an argument due to Nambu.[13] The result found is

$$F_P = -M_d^2F_A/(q^2 - m_\pi^2), \tag{17}$$

where m_π is the pion mass. Thus, Eq. (15) may be written

$$\langle nn|A_\mu^\dagger(0)|d\rangle = \eta F_A(q^2, Q^2, P\cdot d) \times \bar{u}(p_1)\left(\xi_\mu - \frac{q_\mu \xi\cdot Q}{(q^2 - m_\pi^2)}\right)\gamma_5 v(p_2). \quad (18)$$

Thus by making use of Eq. (2) we have obtained the hadronic part of the weak-current matrix element for the reactions $\nu_\mu + d \to p + p + \bar{\mu}$ and $\bar{\nu}_e + d \to n + n + e^+$. We note again that we have merely sketched the derivation of Eqs. (10)–(18) as the details can be found in the paper listed in Ref. 6.

III. DIFFERENTIAL CROSS SECTIONS FOR THE NEUTRINO REACTIONS

Using Eqs. (1a) and (1b) as well as Eqs. (15) and (18), we obtain the following results for the square of the transition matrix elements M_1 and M_2 of the reactions $\nu_\mu + d \to p + p + \mu^-$ and $\bar{\nu}_e + d \to n + n + e^+$, respectively:

$$|M_1|^2 = \frac{2}{m_\mu m_\nu m^2}\left\{\vec{Q}^2\mu_0^2(F_1 - F_2)^2 + (n_1\cdot n_2 + m^2)F_A^2\left[(3\mu_0\nu - \mu\nu\cos\theta) + \frac{(2\vec{Q}^2 m_\mu E_\nu - q\cdot\mu\vec{Q}\cdot\vec{\nu} - q\cdot\nu\vec{Q}\cdot\vec{\mu})}{(q^2 - m_\pi^2)} + \frac{q\cdot\mu q\cdot\nu\vec{Q}^2 - \vec{Q}^2 q^2\nu\cdot\mu}{(q^2 - m_\pi^2)^2}\right]\right\} \quad (19a)$$

and

$$|M_2|^2 = \frac{2}{m_e m_\nu m^2}\left\{\vec{Q}^2 E_e^2(F_1 - F_2)^2 + (n_1\cdot n_2 + m^2)F_A^2\left[(3E_e\nu - p_e\nu\cos\theta) + \frac{(2\vec{Q}^2 m_e E_\nu - q\cdot e\vec{Q}\cdot\vec{\nu} - q\cdot\nu\vec{Q}\cdot\vec{e})}{(q^2 - m_\pi^2)} + \frac{(q\cdot e q\cdot\nu\vec{Q}^2 - \vec{Q}^2 q^2\nu\cdot e)}{(q^2 - m_\pi^2)^2}\right]\right\}, \quad (19b)$$

where m_e, E_e, $e^\mu = (E_e, \vec{e})$, are the electron mass, energy, and four-momentum, respectively, $\nu^\mu = (E_\nu, \vec{\nu})$ is the neutrino four-momentum, and $\mu^\mu = (E_\mu, \vec{\mu}) = (\mu_0, \vec{\mu})$ is the muon four-momentum. From Eqs. (12), (19a), and (19b) we see that the transition matrix element is large at $q_0 = 0$. For this case we have

$$E_\nu \cong E_\mu = (\vec{\mu}^2 + m_\mu^2)^{1/2}, \quad (20)$$

where $\vec{\mu}$ is the muon three-momentum. Solving for $|\vec{\mu}|$ one sees that

$$|\vec{\mu}| = (E_\nu^2 - m_\mu^2)^{1/2} \cong E_\nu\left|1 - \frac{1}{2}\frac{m_\mu^2}{E_\nu^2}\right| \quad (21)$$

so that as E_ν becomes large

$$|\vec{\mu}| \approx |E_\nu| \equiv \nu \equiv |\vec{\nu}|. \quad (22)$$

Thus since $\vec{Q} = \vec{\nu} - \vec{\mu}$ and since we are considering reactions in the forward direction

$$|\vec{Q}| = |\vec{\nu}| - |\vec{\mu}| \approx 0 \quad (23)$$

in the region of large values of $|M_1|$ or $|M_2|$. We have also verified directly by computer computation that the differential cross sections are dominated by their values near $q_0 = 0$. Therefore the vector current makes very little contribution to $|M_1|$ and $|M_2|$ because the vector form factors F_1-F_2 are multiplied by $\vec{Q}^2$ [see Eqs. (19a) and (19b)] and almost the entire contribution comes from axial-vector current.

The terms in Eqs. (19a) and (19b) proportional to the induced pseudoscalar form factor are also found to be quite small. These terms are identified by the factor $(q^2 - m_\pi^2)$ appearing to the first or second power in their denominators. These

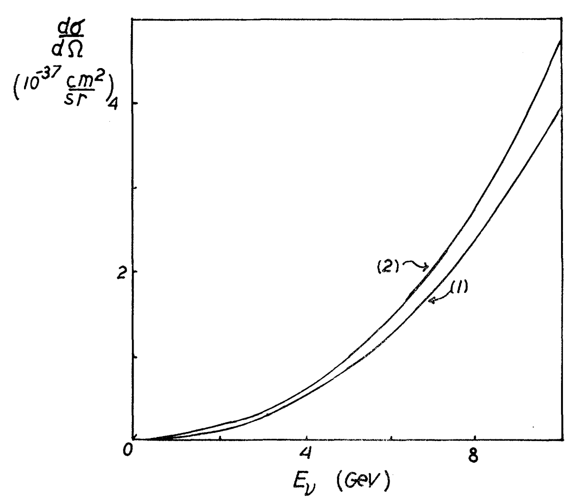

FIG. 1. Plot of the differential cross sections in the forward direction for the reactions $\nu_\mu + d \to p + p + \mu^-$ [curve (1)] and $\bar{\nu}_e + d \to n + n + e^+$ [curve (2)] as functions of E_ν.

contributions are found to be on the order of 2–3%.

Thus, using Eqs. (12), (14), (16), (19a), and (19b) we obtain the differential cross sections for the reactions $\nu_\mu + d \to p + p + \mu^-$ and $\bar{\nu}_e + d \to n + n + e^+$ in the forward direction.[14] The results are given in Fig. 1. It is somewhat more convenient to work with the quantities $E_\nu^{-2} d\sigma/d\Omega_\mu$. These are shown over the range of E_ν from threshold to 10 GeV in Fig. 2. These results can be represented for values of E_ν roughly in the region $0.5\ \mathrm{GeV} \le E_\nu \le 10$ GeV by expressions of the form

$$\frac{1}{E_\nu^2}\frac{d\sigma}{d\Omega}(\nu_\mu d \to pp\mu^-) = a(E_\nu),$$
$$\frac{1}{E_\nu^2}\frac{d\sigma}{d\Omega}(\bar{\nu}_e d \to nne^+) = b(E_\nu), \tag{24a}$$

where $a(E_\nu)$ and $b(E_\nu)$ are slowly increasing functions of neutrino energy. We tabulate $a(E_\nu)$ and $b(E_\nu)$ in Table I and note that the rate of increase falls with increasing[15] E_ν. This is consistent with impulse-approximation calculations,[16] which indicate that the differential cross sections at high incident neutrino energy should increase linearly as E_ν^2. We again note that to the order of G to which we are calculating, the differential cross section for the processes $\nu_e + d \to p + p + e^-$ and $\bar{\nu}_e + d \to n + n + e^+$ are identical (see footnote 3).

IV. CONCLUSION

The differential cross section for the reaction $\nu_\mu + d \to p + p + \mu^-$ has been calculated[14] by the use of various forms of the impulse approximation over a range of incident neutrino energy from 0.4 GeV to 10 GeV. These results are plotted in Fig. 2. As can be seen from curves 3 and 4 in Fig. 2 in the impulse-approximation-based results increase less rapidly and are somewhat lower at higher values of E_ν than are those obtained via an elementary-particle-model calculation.

The percentage differences in the quantity are about −11% at 1 GeV, 30% at 5 GeV, and 44% at 10 GeV and tend to become constant at higher E_ν. They are not, however, particularly significant due to the strong dependence of the differential cross section [see Eq. (12)] on the exact form of $F(Q^2, Q\cdot d)$ at $q_0 = 0$. Using available electrodisintegration data[7] near threshold we found a best fit was obtained when the factor $1/q_0$ in Eq. (12) was replaced by $a/q_0 + b/q_0^3$, where $a = 0.282$ and $b = 20.56\ \mathrm{MeV}^2$. The values obtained for $E_\nu^{-2} d\sigma/d\Omega$ depend strongly on these parameters. But the available data are not sufficient to determine the factor completely since for these neutrino reactions [see Eq. (23)] $|\vec{q}|$ is also small when q_0 is small, but none of the available electrodisintegration data has been taken at small $|\vec{q}|$. Instead, for the available data $|\vec{q}|$ is typically about 40 to 100 MeV, so that extrapolation was necessary. Therefore, what is clearly necessary are additional electrodisintegration experiments in the appropriate low q_0 and $|\vec{q}|$ range, so that $F(Q^2, Q\cdot d)$ may be more accurately obtained. At present we can only conclude that the impulse-approximation results and the elementary-particle-model results are compatible.

The fact that the scattering matrix element [Eq. (19a)] is dominated by F_A, particularly $F_A(q^2=0)$ can be used to obtain an expression for $F_A(0)$ in terms of $E_\nu^{-2} d\sigma/d\Omega$, which is in principle measurable and should become available in the not distant future:

$$F_A(0) \approx \left(\frac{1}{E_\nu^2}\frac{d\sigma}{d\Omega}\right)^{1/2} \frac{3.82}{\beta(E_\nu)}, \tag{25}$$

where $\beta(E_\nu)$ is $a(E_\nu)$ or $b(E_\nu)$ and we have made use of Eqs. (24), (19a), and (19b). Obviously the value of E_ν to be used in Eq. (25) is the largest one available so that from Eqs. (23), (19), and (12),

TABLE I. Values for the slowly varying functions $a(E_\nu)$ and $b(E_\nu)$ in the range of $0.5 \le E_\nu \le 10$ GeV, where $E_\nu^{-2} d\sigma/d\Omega(\nu_\mu d \to pp\mu^-) = a(E_\nu)$ and $E_\nu^{-2} d\sigma/d\Omega(\bar{\nu}_e d \to nne^+) = b(E_\nu)$.

E_ν (GeV)	0.5	1.0	2.5	5.0	7.5	10.0
$a(E_\nu)$ (10^{-38} cm^2/sr GeV^2)	0.222	0.245	0.296	0.342	0.364	0.393
$b(E_\nu)$ (10^{-38} cm^2/sr GeV^2)	0.210	0.256	0.317	0.375	0.422	0.471

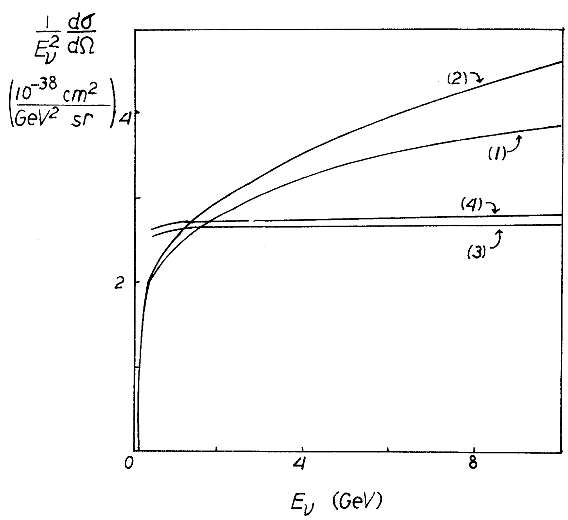

FIG. 2. Plot of the differential cross sections divided by E_ν^2 for (a) the reaction $\nu_\mu + d \to p + p + \mu^-$, and (b) the reaction $\bar{\nu}_e + d \to n + n + e_\nu^+$. Curve (2) refers to reaction (b), calculated by an elementary-particle treatment. Curves (3) and (4) refer to reaction (a) treated by an impulse-approximation calculation.

the approximation $q^2 = 0$ will be most accurate and will result in the best value for $F_A(0)$. This number is not presently available due to the fact that the deuteron does not undergo β decay but would be very useful for studying all semileptonic weak process in deuterium, in particular muon capture. Finally we note that if sufficiently good data were available for the electromagnetic form factors F_1-F_2 and the axial-vector form factor F_A, the reactions described in this paper could be used to test the PCAC hypothesis in the manner suggested by Adler.[17]

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[3]Actually the neutrino beam at LAMPF (Los Alamos Meson Physics Facility) is ν_e rather than $\bar{\nu}_e$, so that the reaction observed at that facility will be $\nu_e + d \rightarrow p + p + e^-$ rather than $\bar{\nu}_e + d \rightarrow n + n + e^+$. However, the differential cross section will be exactly the same to order G^2 for both processes, as can be seen by substituting electron variables into Eq. (19a) and comparing it with Eq. (19b). The reason for this is that by using CVC and charge symmetry we take final-state strong interactions into account but not final-state electromagnetic interactions. These interactions are not expected to be large, particularly in the forward direction at intermediate and large incident-neutrino energies (see Kelly and Überall, Ref. 16).

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[14]Calculations at angles other than 0° are obviously useful, and they will form part of a subsequent paper in which $d\sigma/dt$ at various incident neutrino energies will be calculated for a range of t, the momentum transfer squared.

[15]We make a remark concerning the differences in the differential cross sections for the processes $\nu_\mu + d \rightarrow p + p + \mu^-$ and $\bar{\nu}_e + d \rightarrow n + n + e^+$. As was noted, at high neutrino energy most of the contribution to the differential cross sections comes at $q^2 = 0$. However, the function $K(\theta)$ [Eq. (13)] is sensitive to $|\vec{q}|$ in the neighborhood of $|\vec{q}| \approx 0$. This, in turn, depends from Eq. (21) on the mass of the particle (muon or electron) being considered. Thus, although both cross sections have the same $E_\nu{}^2$ dependence, the constant is a little different for the two processes. It is likely that when more accurate determinations of $K(\theta)$ are possible, this difference (about 16% at high E_ν) will disappear.

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