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Strong anomalies in the partial conservation of the axial-vector current and the pseudoscalar-vector-vector vertex*

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Strong anomalies in the partial conservation of the axial-vector current involving the vector-meson nonet are used in an effective-Lagrangian approach to determine the over-all normalization of the pseudoscalar-vector-vector (*PVV*) vertex. The result is model-dependent. However, SU(3) symmetry of the *VPP* vertex leads to nonet symmetry for the *PVV* vertex and agrees with the experimental data.

Earlier this year, Freund and Nandi[1] suggested the interesting possibility of equivalence between the vector-meson-dominance method[2] and the fermion triangle[3] or the PCAC (partial conservation of the axial-vector current) anomaly[4] method of calculating the matrix element for $\pi^0 \rightarrow \gamma\gamma$. In this note we derive the pseudoscalar-vector-vector (PVV) vertex induced by the PCAC anomaly due to non-Abelian gauge fields.[5] The above-mentioned equivalence occurs naturally once the vector-meson dominance of the electromagnetic current is invoked.

Let ψ be a spinor representing N SU(3) triplets of quarks. The vector and axial-vector currents are given by

$$J_\mu^i(x) = \bar{\psi}\gamma_\mu(\Gamma_i/2)\psi + \cdots ,$$
$$J_\mu^{i5}(x) = \bar{\psi}\gamma_\mu(\Gamma_i/2)\gamma_5\psi + \cdots . \tag{1}$$

The Γ_i are nine $3N$-by-$3N$ matrices equivalent to the Gell-Mann λ_i matrices,[6] repeated N times along the main diagonal.[7] The ellipses refer to the possible contributions of boson fields which may or may not be included in the fundamental Lagrangian with ψ. According to the work of Ref. 5, closed fermion loops give rise to an anomaly in the conservation of the axial-vector current. Specifically, if external vector and axial-vector meson fields V_μ^i and A_μ^i couple via the interaction Lagrangian

$$\mathcal{L}_I = J_\mu^i V_i^\mu + J_\mu^{i5} A_i^\mu , \tag{2}$$

then

$$\partial^\mu J_\mu^i = D^i ,$$
$$\partial^\mu J_\mu^{i5} = D^{i5} + G^i . \tag{3}$$

The quantities D^i and D^{i5} are the divergences derivable by manipulation of the equations of motion, and G^i is the anomaly due to fermion loops given by[5]

$$G_i(V,A) = \frac{1}{4\pi^2}\epsilon_{\mu\nu\sigma\tau}\mathrm{tr}\left\{\frac{\Gamma_i}{2}\left[\tfrac{1}{4}V^{\mu\nu}V^{\sigma\tau} + \tfrac{1}{12}A^{\mu\nu}A^{\sigma\tau} + \tfrac{2}{3}i(A^\mu A^\nu V^{\sigma\tau} + V^{\mu\nu}A^\sigma A^\tau + A^\mu V^{\nu\sigma}A^\tau) - \tfrac{8}{3}A^\mu A^\nu A^\sigma A^\tau\right]\right\}, \tag{4}$$

where

$$V_\mu = \tfrac{1}{2}\Gamma_i V_\mu^i ,$$
$$A_\mu = \tfrac{1}{2}\Gamma_i A_\mu^i ,$$
$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu] - i[A_\mu, A_\nu] ,$$
$$A_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu A_\mu - i[V_\mu, A_\nu] - i[A_\mu, V_\nu] . \tag{5}$$

We suppose that the well-established pseudoscalar and vector-meson nonets are bound states and resonances formed by the N fundamental triplets. Let P_i and ϕ_i^μ be phenomenological fields representing these states. If the P_i and ϕ_i^μ are not allowed to participate in closed loops, then, as far as the fermion loops are concerned, P^i and ϕ_i^μ behave exactly like the external fields of Ref. 5. Thus one obtains Eq. (4) with V_i^μ replaced[8] by $g\phi_i^\mu$. We will assume in harmony with Eq. (5) that ϕ_i^μ couples universally to the U(3) current, as is appropriate for a gauge field. It is important to adhere to the tree approximation since closed loops involving ϕ_μ^i cause trouble through new divergence difficulties[9] and a strong violation of the Goldberger-Treiman relation.[9]

For simplicity, we do not include scalar mesons in the model; thus the P_i's transform according to a nonlinear realization[10] of U(3)×U(3).

Wess and Zumino have calculated the terms in the effective Lagrangian implied by the anomaly of Eq. (4) in the case of the exponential realization[11]:

$$\mathcal{L}_I = -\frac{1}{F_\pi}\int_0^1 dt\, P^i\left[\exp\left(-\frac{t}{F_\pi}\int d^4x\, P^j y_j\right)\right] \times G_i(V,A)\,. \quad (6)$$

The quantity F_π is the charged-pion decay constant ($\simeq$ 94 MeV), and y_i is a functional differential operator for axial-vector transformations:

$$y_i(x) = \partial^\mu \frac{\delta}{\delta A_\mu^i(x)} - f_{ijk} V_\mu^j \frac{\delta}{\delta A_\mu^k(x)} - f_{ijk} A_\mu^j(x) \frac{\delta}{\delta V_\mu^k(x)}\,. \quad (7)$$

The desired PVV vertex follows from the first term in the exponential operator since higher-order terms involve two or more P's:

$$\mathcal{L}^{PVV} = \frac{-Ng^2}{16\pi^2 F_\pi}\epsilon_{\mu\nu\sigma\tau} d_{ijk} P^i \partial^\mu \phi_j^\nu \partial^\sigma \phi_k^\tau\,. \quad (8)$$

Anticipating $N=3$, the PVV vertex is now determined by relatively well-known quantities.[12] In a quark model, it would be unnatural for the 1^{--} mesons to mix according to the current-mixing formalism[13] and the 0^{-+} to mix according to the mass-mixing formalism.[13] We adopt mass mixing for both. In that case the VPP and PVV couplings are U(3)-invariant and we take $g = g_{\rho\pi\pi}$.

In terms of the ω-ϕ mixing angle θ one finds

$$g_{\pi\rho\omega} = -Ng^2\cos(\theta_c - \theta)/8\pi^2 F_\pi \quad (9)$$

and

$$g_{\pi\rho\phi} = -Ng^2\sin(\theta_c - \theta)/8\pi^2 F_\pi\,, \quad (10)$$

where $\theta_c = 35.3$ is the canonical Okubo nonet or SU(6) mixing angle.

In contrast to Ref. 1 we have not yet used the hypothesis of the vector-meson dominance (VMD) of the electromagnetic current. In fact the present approach is incompatible[9] with the algebra of fields. Thus when we adopt VMD of the electromagnetic current we do so in its pole-theory from, not as a field-current identity.[13] The vector-meson-photon couplings appropriate to mass mixing are

$$G_{\rho\gamma} = em_\rho^2/g\,,$$
$$G_{\omega\gamma} = (em_\omega^2/g)(\sin\theta/\sqrt{3})\,, \quad (11)$$
$$G_{\phi\gamma} = (em_\phi^2/g)(\cos\theta/\sqrt{3})\,.$$

Equations (9) and (11) lead immediately, with $\theta = \theta_c$, to the ansatz of Ref. 1:

$$g_{\pi^0\gamma\gamma}^{\mathrm{VMD}} = 2g_{\pi\rho\omega}e^2/3g^2 = -N\alpha/3\pi F_\pi\,. \quad (12)$$

This is the triangle value for $N=3$.

It is apparent that the calculation of the PVV vertex is model-dependent. For example, by introducing symmetry breaking in the VPP vertices, as in the current-mixing model, one derives symmetry breaking in PVV. Another possibility is that $g \neq g_{\rho\pi\pi}$, since many chiral models allow for a discrepancy[10] between the two.[14] We should point out, however, that the result is independent of the specific nonlinear realization used for the P's.

If Eq. (6) is expanded to second order in P, a contact interaction of the $VPPP$ form[11,15] is found. This result is even more model-dependent than the one for PVV. Not only is the answer for $VPPP$ modestly affected by the above considerations, but also by whether one associates a term $-c\partial_\mu P^i$ with a part of A_μ^i or not. This amounts to the assertion that the pseudoscalar mesons couple to ψ via pseudovector coupling. Whether this is true or not depends on the detailed dynamics of the fundamental Lagrangian. We shall return to questions of model dependence in a more detailed study. For the present we examine meson decay rates that do not involve $VPPP$.

In Table I, VPP, $Vl\bar{l}$, and PVV decay rates have

TABLE I. Meson decay rates for $g^2/4\pi = 3.27$, $\theta = 37.3°$, $\gamma = -10°$, and $F_\pi = 94$ MeV.

Decay mode	Model (keV)	Experiment (keV)
1. $\rho \to \pi\pi$	171×10^3	$(150\pm10)\times10^3$
2. $K^* \to K\pi$	48.9×10^3	$(49.8\pm1.1)\times10^3$
3. $\phi \to K^+K^-$	2.05×10^3	$(1.96\pm0.14)\times10^3$
4. $\phi \to K_L K_S$	1.35×10^3	$(1.45\pm0.12)\times10^3$
5. $\rho \to l\bar{l}$	4.2	7 ± 1.4
6. $\omega \to l\bar{l}$	0.52	0.76 ± 0.17
7. $\phi \to l\bar{l}$	1.17	1.25 ± 0.15
8. $\pi \to \gamma\gamma$	7.47×10^{-3}	$(7.75\pm0.93)\times10^{-3}$
9. $\omega \to \pi\gamma$	893	870 ± 61
10. $\phi \to \pi\gamma$	2.6	<15
11. $\rho \to \pi\gamma$	94	<750
12. $K^{*\pm} \to K^\pm\gamma$	54	<80
13. $K^{*0} \to K^0\gamma$	220	
14. $\eta \to \gamma\gamma$	0.364	0.352 ± 0.135
15. $\phi \to \eta\gamma$	177	126 ± 47
16. $\eta' \to \gamma\gamma/\eta' \to \rho\gamma$	0.0506	0.0693 ± 0.0120
17. $\omega \to \eta\gamma$	6.2	<50
18. $\phi \to \eta'\gamma$	0.77	
19. $\eta' \to \gamma\gamma$	6.1	<19
20. $\eta' \to \omega\gamma$	12	<80
21. $\rho \to \eta\gamma$	56	

been collected.[16,17] We do not take g from $\Gamma(\rho\to\pi\pi)$ alone, since this rate has a large uncertainty. The values of $g^2/4\pi$ and θ are adjusted to give the best fit to the strong VPP rates $\rho\to\pi\pi$, $K^*\to K\pi$, $\phi\to K^+K^-$, and $\phi\to K_LK_S$ (lines 1–4). We obtain $g^2/4\pi=3.27$ and $\theta=37.3°$ with a χ^2 of 6. The large value of $g^2/4\pi$ is chiefly a consequence of the precise $K^*\to K\pi$ rate. Thus, these parameters are derived from the experimental VPP rates without using the main result of this paper, Eq. (8), or VMD for that matter. Note that from Eq. (9) we now obtain $m_\omega{}^2 g_{\pi\rho\omega}{}^2/4\pi=13.4$.

The rates listed in lines 5–21 of Table I are now predicted,[18] given values for the η-η' mixing angle γ and F_π. For γ we adopt the conventional value[16,19] $-10°$. Only the rates listed in lines 14–21 depend on γ. There is some uncertainty whether to use the experimental value for F_π (94 MeV) or the Goldberger-Treiman value (86 MeV). By adopting 94 MeV we might underestimate all the PVV coupling constants by 10%. In addition it is not at all obvious that $F_{\eta'}=F_\eta=F_\pi$, which is implicit in our nonet-symmetric PVV vertex.

The overall agreement is quite good, perhaps better than expected in view of the theoretical uncertainties. The χ^2 for the 12 measured rates in Table I is 16, and all the experimental upper bounds are satisfied. The quality of the fit may best be appreciated by comparing with an alternative model with the same number of parameters. If one assumes current mixing for the vector mesons and mass mixing for the pseudoscalars the SU(3)-invariant VPP couplings are multiplied by m_V/m_ρ and the PV_1V_2 couplings by $m_{V_1}m_{V_2}/m_\rho{}^2$, where m_V is a vector-meson mass and m_ρ is the ρ mass. The best χ^2 that can be achieved by variation of $g^2/4\pi$, θ, and γ is 38 for the 12 measured rates. However, this solution implies $\Gamma(\phi\to\pi\gamma)=72$ keV, violating the experimental upper limit 15 keV.

As mentioned before, this alternative model is incompatible in spirit with the quark model. Our conclusion is that various general quark-model notions such as anomalies, formation of meson nonets, and unbroken symmetry for vertices are well supported by the present study.

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[6]We also include $\lambda_0=(\frac{2}{3})^{1/2}I$ to incorporate nonet symmetry.

[7]Naturally $N=3$ is expected. See Y. Nambu and M. Y. Han, Phys. Rev. D 10, 674 (1974), for a critical study of the wide variety of nine-quark models.

[8]Technically, the object that replaces V^i_μ is $g\Delta^{ij}_{\mu\nu}(x-y)\times S^\nu_j(y)$, where $\Delta^{ij}_{\mu\nu}$ is the vector-meson propagator and $S^\nu_j(y)$ its source or sink. See J. Schwinger, *Particles and Sources* (Gordon and Breach, New York, 1969).

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[12]Any Lagrangian model that involves the PVV vertex would have terms similar to Eq. (8) and consequently imply a strong violation of PCAC. The relation of the PVV problem and PCAC violation by the vector mesons was first brought out by R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Lett. 27B, 657 (1968). In this paper two overall PVV coupling constants were adjusted to fit the data. Here we calculate the PVV coupling constants in terms of known quantities.

[13]See, for example, N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967) for the distinction between current mixing and mass mixing.

[14]Further sources of model dependence in the PCAC-anomaly PVV calculations are possible at the level of either the quark currents or different styles of symmetry breaking. This may be observed in the anomaly discussions of the $\pi^0\to\gamma\gamma$, $\eta\to\gamma\gamma$, $\eta'\to\gamma\gamma$ problem given by S. Okubo [in *Symmetries and Quark Models,* edited by Ramesh Chand (Gordon and Breach, New York, 1970), p. 59] and by S. L. Glashow, R. Jackiw, and S. S. Shei [Phys. Rev. 187, 1916 (1969)].

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[17]All the data are from Ref. 16 with the exception of $\eta\to\gamma\gamma$. Here the new $\eta\to\gamma\gamma$ measurement of A. Brow-

man *et al.* [Phys. Rev. Lett. 32, 1067 (1974)], which is approximately a factor of 3 smaller than the previous result, is combined with the result tabulated in Ref. 16.

[18]Formulas for the PVV coupling constants may be obtained from those listed by L. H. Chan, L. Clavelli, and R. Torgerson [Phys. Rev. 185, 1754 (1969)]. In Eqs. (86a) to (86m) substitute $h_0=-g^2N/16\pi^2F_\pi$ and set the mass-weighting factors $m_0^2/m_{V_1}m_{V_2}$ equal to unity.

[19]Only $|\gamma|$ is determined by the pseudoscalar masses. The negative sign fits the data much better. For $\gamma=10°$, the χ^2 of the 12 measured rates is 115.

PHYSICAL REVIEW D VOLUME 10, NUMBER 9 1 NOVEMBER 1974

Unitarity and analyticity bounds on inclusive cross sections

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Upper bounds on inclusive cross sections are derived using unitarity and general analyticity assumptions. The bounds do not contain unknown constants and can be tested experimentally. The bounds are saturated to within logarithmic factors of the energy if the inclusive cross sections scale at high energy.

There has been some recent interest in the study of constraints on inclusive cross sections imposed by unitarity and analyticity.[1-3] The important investigation of Tiktopoulos and Treiman in Refs. 1 and 2 places bounds on partially integrated inclusive cross sections. Their results are that

$$\int dp\, p^2 f(p,0,s) < c s^{3/2}(\ln s)^4, \tag{1}$$

$$F(\cos\theta) \equiv 2\pi\int dp\, p^2 f(p\cos\theta, p\sin\theta, s) < c' s^{7/6}(\ln s)^{10/3}(\sin\theta)^{-2/3}|\cos\theta|^{-1/3}, \tag{2}$$

where $f(p_L, p_T, s)$ is the invariant inclusive cross section which is defined by

$$f(p_L, p_T, s) = E\frac{d\sigma}{d^3p}, \tag{3}$$

and which satisfies the energy sum rule

$$\int_{-1}^{1} F(\cos\theta)d(\cos\theta) = s^{1/2}\sigma, \tag{4}$$

where σ is the total cross section.

We would like to point out in this paper that the bounds of Eqs. (1) and (2) can easily be improved and that upper bounds for the constants c and c' can also be provided. We also derive some new inequalities for the derivatives of $F(\cos\theta)$ with respect to $\cos\theta$ [denoted $F'(\cos\theta)$].

The derivation of our bounds starts with the partial-wave-expanded form of the inclusive cross section[2]

$$F(\cos\theta) = \pi(2ps^{1/2})^{-1}\sum_{j,j',m}(2j+1)(2j'+1) \times d_j^{m0}(\theta)d_{j'}^{m0}(\theta)A_{jj'}^m, \tag{5}$$

where $A_{jj'}^m$ is related to the imaginary part of the elastic-scattering partial-wave amplitude by

$$\sum_m A_{jj'}^m = 4s\,\mathrm{Im}f_j, \tag{6}$$

and where $A_{jj'}^m$ also satisfies the triangular inequality,

$$(A_{jj'}^m)^2 \le (A_{jj}^m)(A_{j'j'}^m). \tag{7}$$

Tiktopoulos and Treiman[2] use Eqs. (5)–(7) to obtain the bound,

$$F(\cos\theta) \le 2\pi s^{1/2}p^{-1}\left[\sum_j (2j+1)M_j(\theta)(\mathrm{Im}f_j)^{1/2}\right]^2, \tag{8}$$

where $M_j(\theta)$ is defined by

$$M_j(\theta) = \max_m |d_j^{m0}(\theta)|. \tag{9}$$

The same methods allow us to derive bounds on $F'(\cos\theta)$. Differentiating Eq. (5) with respect to $\cos\theta$ and using Eqs. (6) and (7) we find

$$F'(\cos\theta) \le 4\pi s^{1/2}p^{-1}\left[\sum_j (2j+1)M_j(\theta)(\mathrm{Im}f_j)^{1/2}\right] \times\left[\sum_j (2j+1)N_j(\theta)(\mathrm{Im}f_j)^{1/2}\right], \tag{10}$$

where $N_j(\theta)$ is defined by