Quark-parton model with pair correlation

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A relativistic quark-parton model is used to study the deep-inelastic lepton-hadron scattering phenomena. It is assumed that there is some correlation among the valence quarks, at least near the threshold of the deep-inelastic region. The assumption has been motivated by the observed threshold behavior of the structure functions of the nucleon. The inclusion of the pair correlation leads to an improved agreement between the quark-parton results and the experimental data. The question of the validity of the model has also been studied.

I. INTRODUCTION

The quark-parton model[1-4] has been fairly successful in the study of deep-inelastic lepton-hadron scattering phenomena. The model has been able to provide a basic structure from which many useful relations and inequalities have been derived.[2] The details of the model, however, remain to be worked out. Landshoff and Polkinghorne[3] and Kuti and Weisskopf[4] have already given a definite shape to the quark-parton ideas. This will be referred to as the LPKW model. Kuti and Weisskopf made definite assumptions about the probability distributions of the longitudinal momenta among the partons in a nucleon and, combining these with the expected Regge and threshold behaviors of the structure functions, succeeded in reproducing some of the characteristic features of the experimental results. The model is simple and interesting, but its success has been partial. The model was criticized also because the predicted value for the sum rule

$$\int_1^\infty \frac{d\omega}{\omega}\left[F_2^{ep}(\omega)-F_2^{en}(\omega)\right]=0.33\,, \tag{1.1}$$

was considered much higher than the preliminary experimental value, which was about 0.19. The sum rule gives a direct test of the basic assumption of the LPKW model that only the valence quarks contribute to the isospin of the nucleon. The large discrepancy, therefore, was considered a serious drawback for the model. However, Bloom[5] has recently reported new data from SLAC for values of ω up to 20. The new data combined with the usual Regge extrapolation give for the sum rule (1.1) the number 0.27 with an unestimated error. This gives a new outlook for the LPKW model. We have made a separate analysis with the recent data given by Bodek *et al.*[6] While we confirm the observation of Bloom that the right-hand side of the sum rule (1.1) is larger than what was determined earlier, we also observe a more disturbing feature. The experimental data do not show the tendency to extrapolate to the expected Regge behavior. This makes it difficult to calculate the contribution coming from large values of ω. If future experiments (with higher values of Q^2) confirm the results of Bodek *et al.*, this will pose another serious difficulty for the LPKW model. We will, however, overlook this difficulty (see Sec. III for a detailed discussion). The purpose of this paper is to study various aspects of the LPKW model. We will modify the basic assumptions so that the major defects of the model can be remedied. We have adopted the following procedure. Since the quark-parton model gives a simple picture of the structure of the nucleon, it is easy to see what modifications are required so that the essential features of the experimental results are reproduced. The information obtained from some of the experimental results can be fed back into the formulation of the model itself, and if this leads to a satisfactory agreement with all the available data, one may try to understand the significance of the modifications introduced.

The LPKW model in its present form has some serious defects. For example, it predicts for the ratio

$$y(x)\equiv\frac{F_2^{en}(x)}{F_2^{ep}(x)}\underset{x\to 1}{\longrightarrow}\tfrac{2}{3}\,, \tag{1.2}$$

with $x=1/\omega$, while the experimental data give a much smaller value. In fact, the experimental value may even tend to the lower limit of Nachtmann's inequality,[2]

$$4\geq y(x)\geq\tfrac{1}{4}\,. \tag{1.3}$$

The value for $y(x\to 1)$ imposes a lower bound on the threshold ratio of the neutrino-production structure functions through the Paschos inequality,[7]

$$R(x)\geq\tfrac{1}{2}+\tfrac{1}{2}\frac{6-9y}{4y-1}\,, \tag{1.4}$$

where

$$R(x) \equiv F_2^{\nu n}(x)/F_2^{\nu p}(x). \tag{1.5}$$

If $y(x\to 1)$ is close to $\frac{1}{4}$, $R(x\to 1)$ should become very large. Such a threshold behavior is indeed necessary if an extremely slow convergence of the Adler sum rule is considered unlikely. However, the results in (1.2) indicate that the LPKW model will not give rise to such a behavior of $R(x)$. In short, the model is inconsistent with the electroproduction data (and possibly with the neutrino-production data) in the region of large x.

It can easily be seen that a value of $y(x\to 1)$ lower than $\frac{2}{3}$ can be interpreted, within the framework of the quark-parton model, as an indication that in a proton, a $\mathcal{P}$-type valence quark has a greater probability for having a large momentum (which corresponds to $x\sim 1$) than an $\mathcal{N}$-type one. It is not easy to see why such a discrimination should exist. This may indicate that near the threshold $(x\sim 1)$ of the deep-inelastic region, there is some sort of pairing effect, which vanishes as x decreases, i.e., as ν increases when Q^2 is kept fixed. In the present paper we have incorporated this feature by making the following assumption: A paired valence quark has a greater probability for values of $x\sim 1$ than an unpaired one. Thus a $\mathcal{P}$-type quark in proton and an $\mathcal{N}$-type quark in neutron will have a greater probability for large momentum. Since we have no knowledge of the quark dynamics, we preferred to choose the simplest way of introducing the correlation effect. We have increased the Kuti-Weisskopf probability distribution function for a paired valence quark by a constant η and decreased that for an unpaired one by the same quantity. It is easy to see that this leads to the desired behavior near the threshold. The same feature can, of course, be interpreted in a different way. For example, it may be assumed that as $x\to 1$, the two wee valence partons in a proton (neutron) form an $I=0$ state leaving a $\mathcal{P}$-type ($\mathcal{N}$-type) quark to carry most of the momentum.[8] The exact mechanism can be understood only when a precise model for quark dynamics becomes available. Our approach here is entirely phenomenological. We shall determine the unknown parameter η from the observed ratio $y(x\to 1)$. Compared with the LPKW model, the present model gives, in general, better agreement with the experimental results. This, however, may not be taken as conclusive evidence for the existence of pair correlation.

The presentation of the paper is as follows. In Sec. II we shall summarize our basic assumptions and discuss the details of the model. In Sec. III the deep-inelastic electroproduction processes will be studied. In particular, the structure functions $F_2^{ep}(x)$ and $F_2^{en}(x)$, their difference, and the ratio near threshold will be evaluated and compared with the recent experimental data. In Sec. IV a similar study for the structure functions for neutrino-induced production will be made. The problem of the saturation of Adler sum rule will also be studied. Our conclusions will be summarized in Sec. V.

II. SINGLE-PARTICLE MOMENTUM DISTRIBUTION

In the LPKW model a nucleon is assumed to consist of three valence quarks and a core of virtual $q\bar{q}$ pairs carrying the quantum numbers of the vacuum. An indefinite number of neutral field particles (gluons) may also be present. The masses and the binding energies of the quarks will be assumed to be small in comparison to P, the momentum of the nucleon. We are interested in determining how the longitudinal momenta are distributed among the various constituents of the nucleon. If p_i is the longitudinal momentum of the ith parton, we define

$$x_i = \frac{p_i}{P}, \quad \sum x_i = 1. \tag{2.1}$$

The probability distribution of the fraction x among the core quarks will be assumed to be proportional to the phase space,[4] viz.,

$$dP_c(x) \sim \frac{dx}{(x^2+\mu^2/P^2)^{1/2}}, \tag{2.2}$$

where μ is the effective mass of the quarks. Since the mass will play no role in the subsequent developments, we shall put μ for the mass of the gluon also. We assume a similar distribution for the gluons, viz.,

$$dP_g(x) \sim \frac{\rho dx}{(x^2+\mu^2/P^2)^{1/2}}, \tag{2.3}$$

where ρ is a constant giving the relative abundance of the gluons compared with the core quarks.

In the present model the probability distribution for the valence quarks will depend on whether the quark is paired or not. Thus for a paired quark

$$dP_v(x;\text{paired}) \sim \left[\frac{x^{1-\alpha}}{(x^2+\mu^2/P^2)^{1/2}}+\eta\right]dx, \tag{2.4}$$

while for an unpaired one,

$$dP_v(x;\text{unpaired}) \sim \left[\frac{x^{1-\alpha}}{(x^2+\mu^2/P^2)^{1/2}}-\eta\right]dx, \tag{2.5}$$

where $\alpha \equiv \alpha(0)$ is the intercept of the relevant Regge trajectory. For the problems under consideration, a common value $\alpha = \frac{1}{2}$ will be sufficient. The distributions (2.4) and (2.5) have been chosen for the following reasons. We have no knowledge of the quark dynamics, but the experimental data for $y(x)$ as $x \to 1$ suggest, within the framework of the quark-parton model, that there is some correlation among the valence quarks. It is, therefore, natural, as a first step, to introduce a marginal difference of the type shown and study the consequences of this modification on the entire spectrum of theoretical results for lepton-hadron scattering. A simple-minded approach like this is not likely or intended to yield quantitative agreement with the experimental data. Our purpose here is to check if by incorporating this new feature, the results of the model get modified in the right direction. Note that as $x \to 0$, the distributions (2.4) and (2.5) become identical, as in the original model. However, the over-all normalization factor depends on η, and the behaviors of the structure functions at various ranges of the values of x are interdependent. It is one of our objectives to study how an assumed threshold behavior affects the asymptotic behavior and vice versa. This can be done by varying the parameter η. Notwithstanding the simplicity of our choice, we have obtained fairly good agreement with the experimental results. Perhaps, more significant is the observation that the introduction of the correlation does not lead to any contradiction (or serious disagreement) with any of the experimental results.

Even though a correlation is assumed, we shall, for simplicity, still consider the nucleon as a stream of free particles and use the impulse approximation to calculate the structure functions. Thus the pairing effect is supposed to be taken care of by our *ad hoc* choice of different probability distribution functions. We first define the functions $G_i^{p,n}(x)$, which give the probability that in a proton/neutron a particular parton of type i ($i=0,1,2,3$, for gluons and quarks of $\mathcal{P}$, $\mathcal{N}$, and λ type, respectively) has a fraction x of the total momentum. The following relations follow from SU_3 considerations:

$$G_1^p(x)=G_2^n(x)\,,\quad G_2^p(x)=G_1^n(x)\,,$$
$$G_3^p(x)=G_3^n(x)\,,\quad G_0^p(x)=G_0^n(x)\,. \tag{2.6}$$

On splitting the probability function G_i into two parts,

$$G_i^{p,n}(x)=G_{iv}^{p,n}(x)+G_{ic}^{p,n}(x)\,, \tag{2.7}$$

we will calculate separately the two terms, giving the valence and the core contributions, respectively. Note that the simple relation of Kuti and Weisskopf,

$$G_{1v}^p(x)=2G_{2v}^p(x)\,,$$

is lost because of the modification. However, the following results still hold:

$$G_{1v}^p(x)=G_{2v}^n(x)\,,\quad G_{2v}^p(x)=G_{1v}^n(x)\,. \tag{2.8}$$

The function $G_{1v}^p(x)$ is given by

$$G_{1v}^p(x)=2N\frac{[x^{1/2}+\eta(x^2+\mu^2/P^2)^{1/2}]}{(x^2+\mu^2/P^2)^{1/2}}$$
$$\times\sum_{l=0,1,2,\ldots}\;\sum_{k_i=0,2,4,\ldots}\frac{\rho^l(\frac{1}{3})^{k_1+k_2+k_3}}{l!k_1!k_2!k_3!}\int[x_1^{1/2}+\eta(x_1^2+\mu^2/P^2)^{1/2}][x_2^{1/2}-\eta(x_2^2+\mu^2/P^2)^{1/2}]$$
$$\times\delta\Big(1-x-\sum_{j=1}^{n-1}x_j\Big)\prod_{j=1}^{n-1}\frac{dx_j}{(x_j^2+\mu^2/P^2)^{1/2}}\,, \tag{2.9}$$

where the k_i's give, in a particular configuration of n partons, the number of quark pairs of i type in the core, and l gives the number of gluons present. The symbol N stands for the normalization constant. The calculation can be carried out by the method of Kuti and Weisskopf and some additional simple manipulations. In the limit $P\to\infty$, we obtain

$$G_{1v}^p(x)=2N(1+\eta\sqrt{x}\,)x^{-1/2}(1-x)^{\rho+1}\times\left[\frac{\pi}{\Gamma(\rho+2)}-\eta^2(1-x)\frac{1}{\Gamma(\rho+3)}\right]. \tag{2.10}$$

Similar calculations give

$$G_{2v}^p(x)=N(1-\eta\sqrt{x}\,)x^{-1/2}(1-x)^{\rho+1}\times\left[\frac{\pi}{\Gamma(\rho+2)}+2\eta(1-x)^{1/2}\frac{\sqrt{\pi}}{\Gamma(\rho+\frac{5}{2})}+\eta^2(1-x)\frac{1}{\Gamma(\rho+3)}\right] \tag{2.11}$$

and

$$G_{1c}^p(x)=G_{2c}^p(x)=G_{3c}^p(x)=\frac{1}{3\rho}G_{0c}^p(x)=G_c(x)\,, \tag{2.12}$$

where

$$G_c(x)=\tfrac{1}{3}Nx^{-1}(1-x)^{\rho+3/2}\times\left[\frac{\pi^{3/2}}{\Gamma(\rho+\frac{5}{2})}+\eta(1-x)^{1/2}\frac{\pi}{\Gamma(\rho+3)}-\eta^2(1-x)\frac{\sqrt{\pi}}{\Gamma(\rho+\frac{7}{2})}-\eta^3(1-x)^{3/2}\frac{1}{\Gamma(\rho+4)}\right]. \tag{2.13}$$

The constant N is determined from the normalization condition

$$N=\left[\frac{\pi^{3/2}}{\Gamma(\rho+\frac{5}{2})}+\eta\frac{\pi}{\Gamma(\rho+3)}-\eta^2\frac{\sqrt{\pi}}{\Gamma(\rho+\frac{7}{2})}-\eta^3\frac{1}{\Gamma(\rho+4)}\right]^{-1}. \tag{2.14}$$

These results will be used to calculate the electroproduction and neutrino-induced production structure functions in the deep-inelastic region.

III. ELECTROPRODUCTION STRUCTURE FUNCTIONS

The kinematics of the deep-inelastic electroproduction processes has been given in the literature[1-4] and need not be repeated here. For our purpose, it is sufficient to note that in the scaling region the impulse approximation gives

$$\nu W_2^{ep,en}(Q^2,\nu)\equiv F_2^{ep,en}(x)=x\sum_{i=1}^{3}e_i{}^2G_i^{p,n}(x), \tag{3.1}$$

where $x=Q^2/2M\nu$ and the e_i's are the charges on the quarks. The quantity x defined above must be equal to the fraction x occurring in (2.1). The relation (3.1) permits one to calculate $F_2^{ep,en}(x)$ from the expressions (2.11), (2.12), and (2.13), once the parameters ρ and η are known. The value of ρ can be determined from the expected correspondence of the threshold behavior $\sim(1-x)^{\rho+1}$ of the structure functions with the Q^2 dependence of the elastic form factors of the nucleon in the asymptotic region.[9] This gives $\rho=2$. To determine η we proceed as follows. The ratio of the two structure functions $F_2^{en}(x)$ and $F_2^{ep}(x)$ is given by

$$y(x)=\frac{F_2^{en}(x)}{F_2^{ep}(x)}=\frac{4G_{2v}^p(x)+G_{1v}^p(x)+6G_c(x)}{4G_{1v}^p(x)+G_{2v}^p(x)+6G_c(x)}, \tag{3.2}$$

which in the limit $x\to1$ reduces to

$$y(x)\underset{x\to1}{\longrightarrow}\frac{6-2\eta}{9+7\eta}. \tag{3.3}$$

The experimental value[6] for $y(x\sim0.8)$ is about 0.36 and this may extrapolate to a still lower value for $x\to1$. The value $y(x\to1)=0.33$ may, therefore, be considered a reasonable choice. This gives $\eta=0.7$. All subsequent calculations in this paper will be made with this value of η. However, we will study also the consequences of varying the parameter η. From (3.3) it is obvious that an increase in the value of η will give a lower value for $y(x\to1)$. The limiting value $\eta=1$ corresponds to $y(x\to1)=0.25$, the lower limit of Nachtmann's inequality.

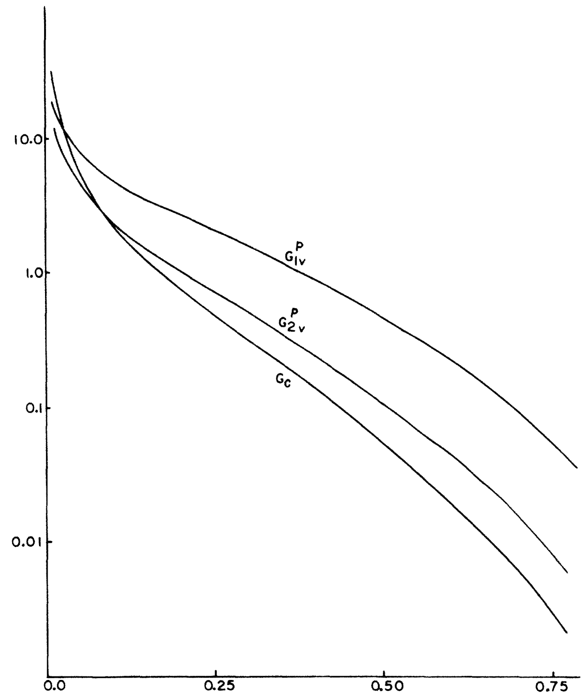

FIG. 1. Calculated values for the probability functions.

In Fig. 1 we have plotted the values of the probability functions $G_{1v}^p(x)$, $G_{2v}^p(x)$, and $G_c(x)$ calculated for $\rho=2$ and $\eta=0.7$. Figure 2 gives the values of $F_2^{ep}(x)$. The contributions coming from the valence quarks and core are shown separately. The curve for $F_2^{ep}(x)$ agrees well with the experimental data. We have, for definiteness, compared our results with the empirical expression given by Miller *et al.*,[10] viz.,

$$F_2^{ep}(x)=(1-x)^3[1.274+0.5989(1-x)-1.675(1-x)^2]. \tag{3.4}$$

The agreement with (3.4) is very good except for the range $x<0.15$, where our model gives higher values. Some features of our results may be mentioned. While the core contribution is a decreasing function of x, the valence contribution shows a maximum around $x\sim0.13$. Over a wide range of x values our results are in slightly better agreement with (3.4) than what is obtained with the LPKW model.

In Fig. 3 we have plotted the calculated values of

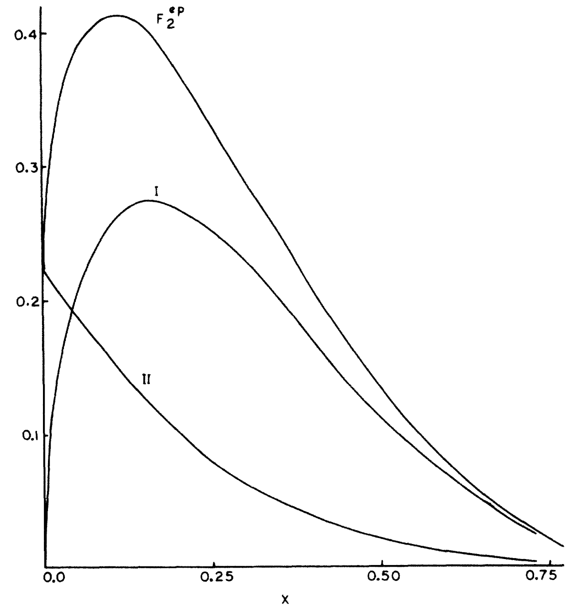

FIG. 2. Theoretical values of the structure function F_2^{ep}. The curves I and II give contributions from the valence quarks and the core quarks, respectively.

$[F_2^{ep}(x) - F_2^{en}(x)]$ against x. The experimental points have been taken from the paper of Bodek *et al.*[6] It may be noted that the agreement is good for large x. For small x, our results are systematically higher than the observed values. This needs some explanation. It may be pointed out that the experimental data for small values of x correspond to low values of $Q^2 \sim 1$ GeV^2. The assumption of the model that Q^2 is much larger than the masses and the binding energies of the quarks does not hold here and the impulse approximation is not valid. A decisive test of the model can be made only when data with higher values of Q^2 are available.

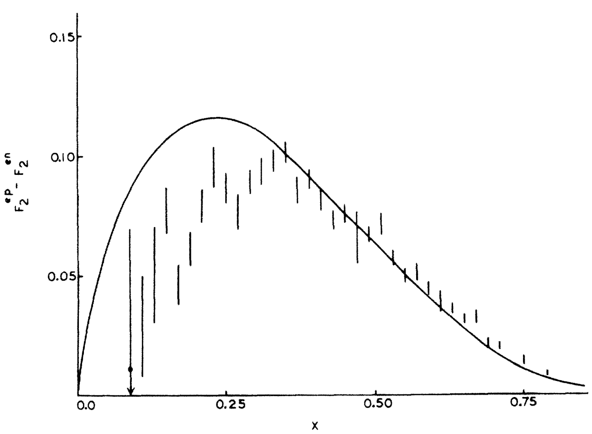

FIG. 3. The difference of the structure functions $[F_2^{ep}(x) - F_2^{en}(x)]$. The experimental points have been taken from the paper of Bodek *et al.* (Ref. 6).

The results of Bodek *et al.*[6] point toward a more serious problem. The data do not show the expected Regge behavior. This can clearly be seen by comparing the theoretical and the experimental values for

$$V(x) = \frac{1}{x}[F_2^{ep}(x) - F_2^{en}(x)] .$$

It appears that for small x, the experimental values for $V(x)$ exhibit a slow decrease with x and not a rise of the type $1/\sqrt{x}$. If the situation does not improve at higher Q^2 this will create a problem for the model. The simple correspondence of the momentum distribution functions (2.4) and (2.5) for the valence quarks with Regge phenomenology may have to be dropped in that case.

Since the area under the curve for $V(x)$ is always 0.33 in this model, we can easily study the effect of altering the parameter η. As $\eta \to 0$, one gets back the Kuti-Weisskopf results with $y(x\to 1) = 0.67$. With an increase in η, the threshold behavior improves, but the area under $V(x)$ gained thereby for large x is compensated by a decrease in the value of $V(x)$ for smaller x. The behaviors for small and large values of x are thus interconnected in a simple way by the normalization condition of the probability functions, G_i.

IV. NEUTRINO INDUCED REACTIONS

As in the case of electroproduction, we assume that the lepton is scattered incoherently by the constituent quarks. We first consider the difference of the structure functions,

$$2U(x) = F_2^{\nu n}(x) - F_2^{\nu p}(x), \tag{4.1}$$

the combination that appears in the Adler sum rule. The present model gives (with $\rho = 2$)

$$U(x) = Nx^{1/2}(1-x)^3\cos^2\theta \times [\tfrac{1}{6}\pi + \tfrac{1}{2}\pi\eta\sqrt{x} - \tfrac{32}{105}\eta(1-x)^{1/2} - \tfrac{1}{8}\eta^2(1-x) + \tfrac{32}{105}\eta^2\sqrt{x}\,(1-x)^{1/2} - \tfrac{1}{24}\eta^3\sqrt{x}\,(1-x)], \tag{4.2}$$

where θ is the Cabibbo angle. The constant N is given by (2.14). The ratio of the structure functions is given by

$$R(x) = \frac{G_{1v}^{p}(x)\cos^2\theta + G_c(x)}{G_{2v}^{p}(x)\cos^2\theta + G_c(x)} , \tag{4.3}$$

where $G_c(x)$ is the probability that any core particle has a momentum x. As $x \to 1$, the valence-quark contribution dominates and we get

$$R(x) \underset{x\to 1}{\longrightarrow} \frac{2+2\eta}{1-\eta} . \tag{4.4}$$

For $\eta = 0.7$, the ratio $R(x \to 1)$ is about 11.3. This may be compared with the inequality (1.4). For $y(x \to 1) = 0.33$, the lower limit given by the inequality is 5, whereas the LPKW model gives the value as 2. Note that as η increases to its maximum value 1, $y(x \to 1)$ takes its minimum value 0.25 and $R(x \to 1)$ tends toward infinity.

For 90% saturation of the Adler sum rule, we must integrate up to $\omega \sim 124$ in the present model with $\eta = 0.7$. In the LPKW model one needs ω values up to 478. With the modified distribution functions (2.4) and (2.5), the values for the quantity $U(x)$ are increased from the corresponding values in the LPKW model in the intermediate region of x, leading to an earlier saturation of the sum rule. It may be mentioned here that models with high threshold ratio $R(x \to 1)$ do not necessarily lead to an early saturation of the sum rule. The threshold region does not contribute significantly to the Adler integral.

We have evaluated the integrals (with $\theta = 0$)

$$I^{\nu n} = \int_0^1 F_2^{\nu n}(x)dx \,, \tag{4.5}$$

$$I^{\nu p} = \int_0^1 F_2^{\nu p}(x)dx \tag{4.6}$$

and their ratio

$$S = \frac{I^{\nu n}}{I^{\nu p}} \,. \tag{4.7}$$

These results are given in Table I. Note that S may or may not be equal to the ratio $\sigma^{\nu n}/\sigma^{\nu p}$, depending on the behavior of the structure functions, $F_3^{\nu n}(x)$ and $F_3^{\nu p}(x)$. However, we can assume that

$$-F_2^{\nu p,\,\nu n}(x) \leq xF_3^{\nu p,\,\nu n}(x) \leq 0 \,, \tag{4.8}$$

so that $\sigma^{\nu p,\,\nu n}$ could range from $I^{\nu p,\,\nu n}$ to $\frac{2}{3}I^{\nu p,\,\nu n}$. The calculated value $S = 2.04$ is consistent with our expectations.

The uncertainty regarding the structure functions $F_3^{\nu p,\,\nu n}(x)$ can be eliminated by considering the total ν and $\bar{\nu}$ cross section, viz.,

$$\sigma^{\nu p} + \sigma^{\bar{\nu} p} + \sigma^{\nu n} + \sigma^{\bar{\nu} n} = \frac{4}{3\pi} G^2 ME \int_0^1 [F_2^{\nu p}(x) + F_2^{\nu n}(x)]\,dx \,, \tag{4.9}$$

where E is the energy of the neutrino/antineutrino beam. The theoretical value for the integral is 0.93, while the preliminary data[11,12] give the value as 0.94 ± 0.14.

A reasonable estimate of the accuracy of the model is obtained by considering the following integrated quantities:

(a) The electroproduction data give for the integral

$$\tfrac{1}{2}\int [F_2^{ep}(x) + F_2^{en}(x)]\,dx = 0.14 \pm 0.02 \,, \tag{4.10}$$

while the model gives the value 0.14.

(b) The results given in (4.10) may be combined with the observed slopes of the neutrino cross sections to obtain for the ratio

$$\frac{\int [F_2^{ep}(x) + F_2^{en}(x)]\,dx}{\int [F_2^{\nu p}(x) + F_2^{\nu n}(x)]dx} = 0.30 \pm 0.06 \,, \tag{4.11}$$

while the theoretical value is 0.29.

(c) From the results in (4.10) and (4.11) one can calculate the fraction ϵ of the momentum carried by the gluons, which is given by

$$\epsilon = 1 + \tfrac{3}{4}\int [F_2^{\nu p}(x) + F_2^{\nu n}(x)]\,dx - \tfrac{9}{2}\int [F_2^{ep}(x) + F_2^{en}(x)]\,dx \,. \tag{4.12}$$

The experimental value for (4.12) is 0.46 ± 0.21, while the calculated value is 0.44.

It may be interesting to consider in this context the work of Sakurai, Thacker and Tuan,[13] and Tuan.[14] In an attempt to remedy some of the defects of the LPKW model, they suggested, as an ansatz,

$$U(x) = \gamma B(x; \tfrac{1}{2}, 3) + (1-\gamma)B(x; -\tfrac{1}{2}, 3) \,, \tag{4.13}$$

where

$$B(x; \alpha, \beta) = \frac{\Gamma(2+\beta-\alpha)}{\Gamma(1-\alpha)\Gamma(1+\beta)} x^{1-\alpha}(1-x)^{\beta} \,, \tag{4.14}$$

and γ is a parameter, which can take a value between 0.25 and 0.30. The prescription, of course, does not give any information about the sum $F_2^{\nu n}(x) + F_2^{\nu p}(x)$. Moreover, when combined with the empirical relation of Myatt and Perkins,[11] viz.,

$$\tfrac{1}{2}[F_2^{\nu n}(x) + F_2^{\nu p}(x)] = a(1-x^2)^3 \,, \tag{4.15}$$

with $a \sim 1$, the form (4.13) leads to a value of $S > 3$, which is higher than our results as well as the preliminary experimental value.[11,12]

V. DISCUSSION

We have presented in this paper a critical study of a quark-parton model where a correlation effect is incorporated as a basic assumption. We have obtained good agreement with the entire range of experimental data for the deep-inelastic lepton-hadron scattering. A comparative study of our results with the available experimental data has been given in Table I. Our major conclusions are as follows:

(a) The assumption of a pair correlation improves the agreement of the theoretical results with the experimental data. No inconsistency is observed anywhere. However, no explanation for

TABLE I. Comparison of the theoretical results with the experimental values.

Serial No.	Quantity	Theoretical $\eta = 0.7$	Experimental
1	$\int x G^p_{1v}(x) dx$	0.253	
	$\int x G^p_{2v}(x) dx$	0.086	
	$\int x G_c(x) dx$	0.074	
2	$I_p = \sum e_i^2 \int x G^p_i(x) dx$	0.17	0.17 ± 0.01 (Ref. 4)
	$I_n = \sum e_i^2 \int x G^n_i(x) dx$	0.12	~0.12 (Ref. 4)
3	$\frac{1}{2}\int [F_2^{ep}(x) + F_2^{en}(x)] dx$	0.14	~0.14 ± 0.02 (Ref. 7)
4	$F_2^{en}(x)/F_2^{ep}(x)$ for $x \to 1$	0.33	<0.36 (Ref. 6)
5	$F_2^{\nu n}(x)/F_2^{\nu p}(x)$ for $x \to 1$	11.3	
6	$\int F_2^{\nu p}(x) dx$	0.32	
7	$\dfrac{\int F_2^{\nu n}(x) dx}{\int F_2^{\nu p}(x) dx}$	2.04	
8	$\dfrac{3\pi}{4G^2 ME}(\sigma^{\nu p} + \sigma^{\bar{\nu} p} + \sigma^{\nu n} + \sigma^{\bar{\nu} n})$	0.97	0.94 ± 0.14 (Ref. 10)
9	$\dfrac{\int [F_2^{ep}(x) + F_2^{en}(x)] dx}{\int [F_2^{\nu p}(x) + F_2^{\nu n}(x)] dx}$	0.29	0.30 ± 0.06 (Ref. 7)
10	ϵ, the fraction of momentum carried by gluons	0.44	0.46 ± 0.21 (Ref. 7) (assuming quark-parton formula)
11	Value of ω for 90% saturation of Adler sum rule	124	

the correlation effect, which has been introduced here in an *ad hoc* manner, has been forwarded in this paper.

(b) The recent data of Bodek *et al.*[6] do not exhibit the expected asymptotic behavior. If the observed behavior [i.e., the function $V(x)$ falling as $x \to 0$ instead of rising] is confirmed by further experiments with higher Q^2, this will almost eliminate the LPKW model. The status of the sum rule (1.1) therefore, must be clearly determined before one could really look for a point-by-point agreement with the experimental results.

(c) With pair correlation, the threshold ratio $R(x \to 1)$ is large. Also one obtains an earlier saturation of the Adler sum rule. The possibility that the sum rule is not satisfied at all has recently been studied in great detail.[13-16] Within the framework of the quark-parton model, the Adler sum rule follows in a natural way (in the form of a normalization condition) and must be satisfied.

(d) The neutrino cross sections calculated are in agreement with the available experimental results.

(e) Although the conclusions are drawn from the results obtained on introducing a simple modification, these will generally be true even if the correlation is introduced in a more complicated way. The analysis has been kept simple and transparent so that the essential features of the model can be understood. An attempt to find a very good agreement with the experimental data is not called for until the problem mentioned in (b) is settled.[17]

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[17]After the completion of the work, we have come to know of a paper by R. McElhaney and S. F. Tuan, Phys. Rev. D 8, 2267 (1973), in which the Kuti-Weisskopf model has been modified following a suggestion by Friedman (see note added in proof in Ref. 4). McElhaney and Tuan have changed the $\mathfrak{N}$-type valence probability distribution according to

$$dP_v(x) \sim \frac{(1-x)\,x^{1-\alpha}dx}{(x^2+\mu^2/P^2)^{1/2}}\,,$$

with no change in the corresponding $\mathcal{P}$-type valence distribution. The calculated results, however, do not fit the new data well, unless the parameter g of the Kuti-Weisskopf model is also altered from 1 to $\frac{1}{2}$ and a daughter term is added. The modification assumes that $y(1)=\frac{1}{4}$ and $R(1)\to\infty$. The present model will also reproduce qualitatively these results if η is taken to be 1. In addition, it will give an early saturation of the Adler sum rule.