PHYSICAL REVIEW D VOLUME 10, NUMBER 9 1 NOVEMBER 1974

Scaling deviations for neutrino reactions in asymptotically free field theories*

A. Zee,† F. Wilczek, and S. B. Treiman
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540
(Received 30 May 1974)

Several aspects of deep-inelastic neutrino scattering are discussed in the framework of asymptotically free field theories. We first consider the growth behavior of the total cross sections at large energies. Because of the deviations from strict scaling which are characteristic of such theories, the growth need not be linear. However, upper and lower bounds are established which rather closely bracket a linear growth. We next consider in more detail the expected pattern of scaling deviation for the structure functions and, correspondingly, for the differential cross sections. The analysis here is based on certain speculative assumptions. The focus is on qualitative effects of scaling breakdown as they may show up in the x and y distributions. The last section of the paper deals with deviations from the Callan-Gross relation.

I. INTRODUCTION

A considerable theoretical industry has built up around the idea of Bjorken scaling, which received its first experimental support in the SLAC-MIT experiments on electroproduction.[1] Subsequent confirmation, in part, has come from the observation[2] that the total cross sections for neutrinos and antineutrinos on nucleons appear to grow linearly with energy beyond a few GeV. A simple and highly successful physical picture of the scaling phenomenon is provided by the well-known parton model.[3] In its field-theoretic transcription, this model amounts to the assumption of canonical dimensions for the twist-2 operators that appear in the Wilson expansion of a product of currents. It has recently become clear, however, at least in the framework of renormalizable field theory, that the dimensions can be canonical for all the relevant operators only in the absence of interactions. Strict scaling, therefore, if it were to persist, would represent a major theoretical paradox. On the other hand, departures from scaling, if they develop in a sufficiently patterned way, could also be informative about the structure of the underlying theory.

So far, the closest that one has come to strict scaling is with a special class of theories, based on non-Abelian gauge symmetry. Theories of this class possess the property of asymptotic freedom[4] and lead to certain characteristic patterns of scaling breakdown.[5-7] In the present paper we discuss some of the observational implications, especially in the context of neutrino reactions. One issue concerns the dependence of total neutrino cross sections on energy. This is taken up in Sec. II, where upper and lower bounds are derived on the growth rate. The arguments employed in this section involve very little in the way of extra assumptions going beyond those implied by asymptotic freedom. It is found that the growth, while it need not be exactly linear once strict scaling breaks down, cannot depart too greatly from linearity. In the present context, asymptotically free theories make their most definite predictions for the large-q^2 behavior of the moments of the structure functions. Section III is concerned with converting this information into predictions about the large-q^2 behavior of the structure functions themselves. Issues of nonuniformity arise here in going from one to the other, so the discussion in Sec. III is based on frankly speculative procedures. The aim, however, is to assess qualitatively how the breakdown of scaling could reveal itself in certain aspects[8] of the differential cross section. In particular, one is led to expect what could be a substantial change with energy in the shapes of the x and y distributions. Section IV deals with a somewhat different subject, namely, corrections to the Callan-Gross relation.[9] However, this section also provides a brief review of asymptotic freedom, and it contains some comments on the nonuniformity issues mentioned above. Throughout the entire discussion we ignore possible deviations from scaling which would arise from the propagator term of a weak vector boson. If the mass were very large the effects would not be noticeable at present energies, but in any case the necessary modification could easily be made. In Secs. III and IV the discussion is implicitly restricted to strangeness- and charm-conserving neutrino reactions.

II. BOUNDS

We focus on the neutrino reactions and their structure functions $F_i(\omega,q^2)$, $i=1,2,3$. Here q^2 is the negative of the invariant momentum transfer and $\omega = 2m\nu/q^2$ is the Bjorken scaling variable. Strict scaling would imply that the $F_i(\omega, q^2)$ approach finite limits as $q^2 \to \infty$, for fixed ω. However, we are contemplating the possibility of departures from scaling; according to present thinking such departures are expected to take on their most characteristic shape when expressed in terms of the large-q^2 behavior of the *moments* of the structure functions,

$$F_i^{(n)}(q^2) = \int_1^\infty d\omega\, \omega^{-n-2} F_i(\omega, q^2). \tag{1}$$

For the asymptotically free theories under discussion the moments are predicted to display logarithmic deviations from scaling. Namely, for q^2 large enough (how large may, in general, depend on the order n of the moment) the predicted asymptotic behavior is

$$F_i^{(n)}(q^2) \sim b_n(i)\left(\ln\frac{q^2}{\mu^2}\right)^{-a_n(i)}, \tag{2}$$

where μ is a scale parameter not specified by the theory. The coefficients $b_n(i)$ are similarly unspecified, but the exponents $a_n(i)$ are definite and characteristic of the underlying theory. They can be computed explicitly, given the gauge group and the quark content of the theory.

Actually, there will, in general, be several different operators of spin $n+2$ in the Wilson expansion, each making a contribution to the right-hand side of Eq. (2), each with its own characteristic coefficient $b_n(i)$ and $a_n(i)$. For every n, it is the contribution with the smallest exponent that ultimately dominates at large q^2, and it is this contribution that is understood to be represented by the right-hand side of Eq. (2).

Equation (2) describes the leading term in an expansion in inverse powers of $\ln q^2$ and q^2. It would be tempting to try to reconstruct the full structure function $F_i(\omega, q^2)$, for large q^2, by supposing that the correction terms in each moment are uniformly small, for all n, when q^2 exceeds some n-independent value. We shall in fact succumb to this temptation later on, but it is clear that any such procedure is highly speculative. At the present stage of theoretical understanding the only firm predictions that follow from the ideas of asymptotic freedom are those embodied in Eq. (2). Thus a sharp test of asymptotic freedom requires the difficult experimentation involved in extracting from the data the individual moments, as functions of q^2. A more modest experimental objective is the study of total neutrino and antineutrino cross sections as a function of energy. As is well known, strict scaling implies a linear growth with energy, at large energies, and indeed this kind of behavior is what is indicated by existing data. The question arises as to the growth properties that are to be expected for theories of the sort under present discussion. This is our first topic. We will see that both upper and lower bounds can be set on the growth rate, on the basis of the moment properties discussed above. It turns out that the bounds rather closely bracket a linear growth behavior.

The integral in Eq. (1) is presumed to converge for all $n \geq 0$. It therefore defines the $F_i^{(n)}$ as functions of complex n, regular for $\mathrm{Re}\,n > 0$. For the present discussion we shall adopt the one additional assumption that the analytically continued moment functions are regular for all $\mathrm{Re}\,n > -n_0$, where n_0 is some small, but nonvanishing, positive number, independent of q^2. For given n, at large enough q^2, the $F_i^{(n)}$ are given by Eq. (2). The exponent functions $a_n(i)$ that occur in that equation can be explicitly computed and turn out to be regular for all $\mathrm{Re}\,n > -1$. We are assuming that the coefficients function $b_n(i)$ are also regular, at least for $\mathrm{Re}\,n > -n_0$. In the following discussion we will be concerned with real values of n in the vicinity of $n = 0$.

Let us now turn to the cross section bounds for

$$\nu + N \to \mu^- + X$$

and the corresponding antineutrino reaction. Dropping at the outset certain kinematic corrections of order m/ϵ, we have for the differential cross section

$$\frac{\partial\sigma}{\partial x \partial q^2} = \frac{G^2}{\partial \pi x}\left[(1 - y + \tfrac{1}{2}y^2)F_2 - \tfrac{1}{2}F_L y^2 \mp y(1 - \tfrac{1}{2}y)xF_3\right], \tag{3}$$

where the upper sign in the last term refers to the neutrino reactions, the lower sign to the antineutrino reactions; and where

$$x = \omega^{-1} = q^2/2m\nu, \quad y = q^2/2mx\epsilon, \quad F_L = F_2 - 2xF_1. \tag{4}$$

To sufficient accuracy for our present purposes, we note the inequalities

$$F_2 \geq 2xF_1 \geq x|F_3|. \tag{5}$$

(i) Upper bound. Using the inequalities of Eq. (5), together with the inequality $1 - y + \frac{1}{2}y^2 < 1$ for $0 < y < 1$, we see that the total cross section (for the ν or $\bar{\nu}$ reactions) is bounded according to

$$\sigma < \frac{G^2}{2\pi}\int_0^{2m\epsilon} dq^2 \int_{q^2/2m\epsilon}^1 \frac{dx}{x} F_2(x, q^2). \tag{6}$$

Now introduce a positive parameter γ, in the range $0 < \gamma < n_0$, and observe that

$$\int_{q^2/2m\epsilon}^1 \frac{dx}{x} F_2 < \left(\frac{2m\epsilon}{q^2}\right)^{1-\gamma} \int_0^1 \frac{dx}{x^\gamma} F_2, \tag{7}$$

an inequality that follows from the positivity of F_2. According to Eq. (2), for *all* $q^2>0$ and all $n>-n_0$ we have the bound

$$\int_1^\infty d\omega\, \omega^{-n-2}F_2=\int_0^1 dx\, x^n F_2<B(n)\left[\ln\left(\frac{q^2+\bar{q}^2}{\mu^2}\right)\right]^{-a_n}, \tag{8}$$

where the $B(n)$ are unknown constants, independent of q^2, and where the parameter $\bar{q}^2$, with $\bar{q}^2>\mu^2$, has been supplied to guarantee that the moment exists for all $q^2\geqslant 0$. Define

$$f(-n)=-a_n\,. \tag{9}$$

Then

$$\sigma<\frac{G^2}{2\pi}B(-\gamma)\int_0^{2m\epsilon}dq^2\left(\frac{2m\epsilon}{q^2}\right)^{1-\gamma}\left[\ln\left(\frac{q^2+\bar{q}^2}{\mu^2}\right)\right]^{f(\gamma)}$$
$$<\frac{G^2}{2\pi\gamma}B(-\gamma)2m\epsilon\left[\ln\left(\frac{2m\epsilon+\bar{q}^2}{\mu^2}\right)\right]^{f(\gamma)}, \tag{10}$$

for all γ in the interval $0<\gamma<n_0$. We now invoke the result that the leading spin-2 ($n=0$) operator in the Wilson expansion is the stress tensor, an SU(3) singlet with canonical dimensions. This implies that a_n vanishes at $n=0$ and becomes negative for $n<0$. For the range of γ involved in Eq. (10), this means that $f(\gamma)$ is positive. However, $f(\gamma)$ can be made arbitrarily small by allowing γ to approach zero as closely as one wishes. We therefore conclude that σ/ϵ grows with energy less rapidly than $(\ln\epsilon)^\delta$, for δ positive but arbitrarily small.

(ii) Lower bound. From the inequalities of Eq. (5) we see that

$$\sigma>\frac{G^2}{2\pi}\int_0^{2m\epsilon}dq^2\int_{q^2/2m\epsilon}^1\frac{dx}{x}(1-y)^2F_2(x,q^2)\,. \tag{11}$$

The expression on the right-hand side can in turn be bounded from below if we shrink the range of integration. In particular, let us, say, double the lower limit on the x integral, so that $(1-y)^2>\frac{1}{4}$. Moreover, let us replace the upper limit on the q^2 integral by $2m\epsilon/[\ln(\epsilon/m)]^\gamma$, where γ is some positive parameter. Next observe that

$$\int_{q^2/m\epsilon}^1\frac{dx}{x}F_2>\int_{q^2/m\epsilon}^1\frac{dx}{x^\alpha}F_2$$
$$>\int_0^1\frac{dx}{x^\alpha}F_2-\left(\frac{q^2}{m\epsilon}\right)^\beta\int_0^1\frac{dx}{x^{\alpha+\beta}}F_2, \tag{12}$$

where $\alpha>0$, $\beta>0$, and $\alpha+\beta<n_0$. We now invoke the bound in Eq. (8) and require, in the notation of Eq. (9), that

$$f(\alpha+\beta)-f(\alpha)<\gamma\beta\,. \tag{13}$$

Then it readily follows that

$$\sigma/\epsilon>C(\alpha)/[\ln(\epsilon/m)]^{\gamma-f(\alpha)}, \tag{14}$$

where $C(\alpha)$ depends on the parameter α but not on the energy ϵ. The inequality of Eq. (14) holds for all positive values of γ, α, and β, subject to Eq. (13) and to $\alpha+\beta<n_0$. We therefore seek to minimize the quantity

$$P=\gamma-f(\alpha) \tag{15}$$

within these constraints. From familiar inequalities on moments of a positive function one has that

$$\frac{\partial f(\alpha)}{\partial\alpha}>0\,,\quad \frac{\partial^2 f(\alpha)}{\partial\alpha^2}>0\,. \tag{16}$$

Thus, for fixed α one minimizes γ within the constraint of Eq. (13) by letting β approach zero. Then from Eq. (13) it follows that

$$\gamma>\frac{\partial f(\alpha)}{\partial\alpha}\,. \tag{17}$$

It remains therefore to minimize

$$P(\alpha)=\frac{\partial f(\alpha)}{\partial\alpha}-f(\alpha) \tag{18}$$

with respect to α in the range $0<\alpha<n_0$. Since in this section, conservatively, we allow for the possibility that n_0 may be small, we shall simply set $\alpha=0$. Recalling that $f(0)=0$, we therefore have the bound

$$\sigma/\epsilon>\text{constant}[\ln(\epsilon/m)]^{-\bar{P}}\,,$$

where

$$\bar{P}=\left.\frac{\partial f}{\partial\alpha}\right|_{\alpha=0}. \tag{19}$$

The function $f(\alpha)$, which is related to the exponent function a_n by Eq. (9), can be computed explicitly, given the gauge group and quark content of the underlying theory. For definiteness we adopt the theory of Refs. 5–7 based on the color group SU(3)′ and containing three quark triplets. For this theory one finds

$$\bar{P}=1.35\,. \tag{20}$$

To summarize, we find (for neutrinos or antineutrinos) that σ/ϵ is bracketed at large energies within the limits

$$D\left(\ln\frac{\epsilon}{m}\right)^{-\bar{P}}<\frac{\sigma}{\epsilon}<\frac{1}{\delta}C\left(\ln\frac{\epsilon}{m}\right)^{\delta}, \tag{21}$$

where δ is an arbitrarily small positive constant and where $\bar{P}$, which depends on the structure of the underlying theory, is a constant of order unity; for SU(3)′ the value is given by Eq. (20). In deriving these bounds we have made the mild assumption that the moment function $F_2^{(n)}(q^2)$ can be continued a small but finite distance to the left of $n=0$, for all q^2. For the rest the results depend solely on Eq. (2), which represents the characteristic prediction of asymptotic freedom. The key technical fact that made it possible to achieve such close bounds is the fact that $a_n=0$ for $n=0$.

What emerges from all this is that the total cross sections are predicted to grow asymptotically in a way that cannot be too different from linear. In this particular respect the departure from strict scaling (which leads to a linear growth at large energies) is expected to be very mild. On the other hand, deviations from scaling have a chance to be more substantial for the structure functions themselves, in their detailed dependences on q^2 at each ω. To proceed further, however, one has to introduce new assumptions that go beyond Eq. (2). We shall introduce these in the following section, and attempt there to follow out some of the qualitative implications.

III. THE STRUCTURE FUNCTIONS

The discussion in this section, which is addressed to the properties of the structure functions at large q^2, will be based on a highly speculative assumption. Namely, let us suppose that the moments $F_i^{(n)}(q^2)$ are well represented by the asymptotic expression on the right-hand side of Eq. (2) once q^2 exceeds a certain limit, call it q_0^2, where q_0^2 is *independent* of n; i.e., let us suppose that the asymptotic behavior described in Eq. (2) is uniform in n. At the present stage of theoretical understanding this is to be regarded as a frankly phenomenological conjecture[10]; we shall return to warnings and comments later on. For the present, let us see what follows. In general, the inverse to Eq. (1) is given by

$$F_i(\omega,q^2)=\frac{1}{2\pi i}\int_{-i\infty+c}^{i\infty+c} dn\,\omega^{n+1}F_i^{(n)}(q^2)\,, \tag{22}$$

where the contour runs to the right of all singularities of $F_i^{(n)}$. What we are assuming now, for $q^2 \geqslant q_0^2$, is that the $F_i^{(n)}$ can be replaced by the expression on the right-hand side of Eq. (2). The exponent functions $a_n(i)$ can be explicitly computed and are known to be regular for all $\mathrm{Re}\,n>-1$. We shall assume that the $b_n(i)$ are similarly regular for $\mathrm{Re}\,n>-1$. Altogether, then, we are assuming for $q^2\geqslant q_0^2$ that $F_i^{(n)}$ is regular in the region $\mathrm{Re}\,n>-1$ and well approximated there by the right-hand side of Eq. (2).

If we are given the structure functions for some value of the momentum transfer in the above asymptotic region, say at the value q_0^2, we could compute the moments $F_i^{(n)}(q_0^2)$ and thereby the coefficients $b_n(i)$ in Eq. (2). From our assumptions it then follows for all $q^2>q_0^2$ that

$$F_i^{(n)}(q^2)=F_i^{(n)}(q_0^2)\lambda^{-a_m(i)}\,, \tag{23}$$

where

$$\lambda=\frac{\ln(q^2/\mu^2)}{\ln(q_0^2/\mu^2)}\,. \tag{24}$$

In principle the full structure functions $F_i(\omega,q^2)$ can now be computed for all $q^2>q_0^2$ on the basis of Eqs. (22) and (23). The practical implementation of this procedure, even apart from questions about the underlying assumption on which it is based, requires the "input" information $F_i(\omega,q_0^2)$, and also requires evaluation of the complicated integral of Eq. (22). The practical difficulty arises, in part, from the fact that the $a_n(i)$ are complicated functions of n (digamma functions are involved). There are no issues of principle here, but the situation calls for numerical approximations. Let us first deal with these. For definiteness we take the underlying theory to be based on the gauge group SU(3)$'$, with three quark triplets. Moreover, let us concentrate on the structure functions averaged over proton and neutron targets.

For the structure function F_2, the relevant exponent function $a_n(2)$ has the following key properties[5]:

(i) It vanishes at $n=0$, reflecting the fact that the stress tensor has canonical dimensions,[11]

$$a_0(2)=0\,.$$

(ii) For large n, $a_n(2)$ grows like

$$a_n(2)\sim \ln n\,.$$

(iii) The exponent function develops a pole[12] at $n=-1$,

$$a_n(2)\underset{n\to -1}{\simeq}\frac{-a}{n+1}\,.$$

This can be traced back to the presence of vector gluons in the underlying theory.

On the basis of the exact results given in Ref. 5, we adopt the following approximate expression for $a_n(2)$:

$$\begin{aligned} a_n(2)\approx{}& -1.478\ln(n+1)+2.071\ln(n+2)\\ &-0.1024-1.333/(n+1)\,. \end{aligned} \tag{25}$$

For large n the exact $a_n(2)$ grows like $A\ln n+B+O(1/n)$. The approximation adopted in Eq. (25) gets the coefficients A and B right. It also incorporates the exact residue for the pole at $n=-1$, and it satisfies $a_0(2)=0$.

The exponent function $a_n(3)$, relevant for F_3, has properties similar to those of $a_n(2)$, though with different numerical coefficients: a $\ln n$ growth for large n, a zero at $n=0$, and a pole at $n=-1$. The following expression incorporates these key features and represents a reasonable approximation to the exact results:

$$a_n(3) \approx 0.5926 \ln(n+2) - 0.1144 - 0.2963/(n+1). \tag{26}$$

We shall have some comments to make in the next section about the longitudinal structure function F_L. For the purposes of this section, however, we accept that F_L/F_2 for electroproduction is already small compared to unity in the q^2 region of the SLAC-MIT experiments.[1] Moreover, the ratio is predicted to vanish as $q^2 \to \infty$, in the model under discussion as well as in the simple quark-parton model. We shall suppose that F_L/F_2 is also already small at modest values of q^2 in the case of the neutrino reactions.

Our next task, then, is to settle on the input information $F_2(\omega, q_0^2)$ and $F_3(\omega, q_0^2)$. To be safely asymptotic we would like to have these for "large enough" q_0^2. Detailed structure function information for the neutrino reactions is, however, still lacking. Apart from everything else, therefore, we cannot at present proceed in a really quantitative way. However, in order to see *qualitatively* what kinds of effects are to be expected in the present framework, we adopt the following illustrative hypothesis. Let us suppose that $q_0^2 \approx 5$ GeV^2 is already just sufficiently asymptotic so that, for electroproduction, we can employ the SLAC-MIT results for $F_2(\omega, q_0^2)$. We may then employ a simple parton model (from whose predictions we are expecting substantial departures only at much larger q^2) to translate this into the F_2 structure function for neutrino reactions at q_0^2. The details of one such approach and fit are discussed for example by Albright and Jarlskog.[13] We shall adopt a slightly modified version[14] of their Eq. (3.8c) to represent the neutrino structure function F_2, averaged over protons and neutrons, at $q_0^2 \approx 5$ (GeV)^2. Concerning $F_3(\omega, q_0^2)$ we make use of the fact that at CERN energies (where departures from scaling are presumably still small) the cross section ratio,[2] $\sigma^\nu/\sigma^{\bar\nu} = 2.6 \pm 0.2$, is fairly close to its upper bound, $\sigma^\nu/\sigma^{\bar\nu} \leq 3$. The bound corresponds to $F_3 = -\omega F_2$. It will simplify matters, and will perhaps not be too misleading for our qualitative purposes, if we accept this relation at the reference momentum transfer q_0^2. In any case, asymptotic freedom implies at very large q^2 that $|F_3|/\omega F_2 \to 0$, hence that $\sigma^\nu/\sigma^{\bar\nu} \to 1$ as $\epsilon \to \infty$. Our input hypotheses merely help us to get started on this road. When better starting information becomes available one will be in a position to do a more serious and qualitative extrapolation into the asymptotic region than is now possible.

Given the approximations of Eqs. (25) and (26), which are reasonably good, and given the input structure functions, which are perhaps only illustrative, one can now work out the structure functions for all $q^2 > q_0^2$ on the basis of Eqs. (22) and (23). This has to be done numerically, and a number of technical comments are assembled in the Appendix. The qualitative behavior of the structure functions at large q^2 can be inferred rather directly from the properties of the exponent functions, as has already been discussed in the literature. Consider $F_2(x, q^2)$, for example, where for convenience we now work with the variable $x = \omega^{-1}$ in the place of ω. Since $a_0(2) = 0$ it is obvious that $\int_0^1 F_2\,dx$, the area under the F_2 curve, must become independent of q^2 in the large-q^2 region. However, the shape of the curve changes with changing q^2. The behavior near threshold,[15] i.e., near $x = 1$, is clearly governed by the large-n properties of the exponent function. Since $a_n(2)$ grows, logarithmically, with n, it follows for increasing q^2 that F_2 should vanish increasingly rapidly as $x \to 1$. On the other hand, the behavior as $x \to 0$ is governed by the pole that $a_n(2)$ develops at $n = -1$. At large q^2, this leads to an unbounded growth[12,16] as $x \to 0$, proportional to $\exp\{2[a \ln\lambda(\ln x^{-1})]^{1/2}\}$, where a is a constant and λ is proportional to $\ln q^2$. The rate of growth as $x \to 0$ increases with increasing q^2. It is obvious that these properties of F_2 are all shared also by the structure function F_3.

We shall not present here the results of our detailed computations of the structure functions themselves. It will probably be some time before F_2 and F_3 can be experimentally determined in detail, as functions of x and large q^2. Moreover, in particular for F_2, the behavior near $x = 1$ and $x = 0$ has already been discussed in the literature. Instead, we shall display the structure functions in what are effectively partially integrated forms. Namely, we consider the partially differential cross sections, $\partial\sigma/\partial y$ and $\partial\sigma/\partial x$, obtained by integrating $\partial^2\sigma/\partial x \partial y$ over one or the other of the two variables. For given beam energy ϵ this requires knowledge of the structure functions for all q^2 up to the kinematic limit $2m\epsilon$. The preceding discussion, given the basic assumptions adopted for this section, deals only with the asymptotic region $q^2 > q_0^2$, where, ideally, q_0^2 should be taken "large enough." In practice we are supposing that q_0^2 somewhere in the SLAC-MIT region will do; we have somewhat optimistically taken $q_0^2 \approx 5$ GeV^2. To discuss the differential cross sections we must also know the cross sections for $q^2 < q_0^2$. Here we rely on the observation that scaling seems in fact to hold well enough for modest q^2, say for $q_1^2 < q^2 < q_0^2$, where q_1^2 is perhaps of order 1 GeV^2. It is stretching things, however, to suppose that the transition from scaling to asymptotic behavior sets in sharply, for all x, at some particular q_0^2. Nevertheless, we are forced to this assumption. This introduces certain artifacts in the final results, especially for low beam energies ϵ where both the $q^2 < q_0^2$ and $q^2 > q_0^2$ regions are making comparable contributions to the cross section. These effects, however, become less serious as one goes to large energies. There is also the problem of scaling breakdown at the other end, for $q^2 < q_1^2$. The low-q^2 region ($q^2 < q_1^2$) contributes significantly to the cross sections even for $2m\epsilon$ substantially larger than q_1^2. It has always been something of a puzzle, therefore, even when strict scaling is assumed to hold beyond $q_1^2 \approx 1$ GeV^2, why the total cross sections become so nearly linear in ϵ already at a few GeV. These uncertainties about scaling breakdown at low q^2 make themselves felt in our computations here, although the effects become unimportant for large beam energies. In practice we have simply cut off all q^2 integrations below $q_1^2 = 1.0$ GeV^2. For all of these reasons we restrict ourselves to large energies ϵ. For the remaining parameter, the scale μ^2, we take $\mu^2 = 0.5m^2$, where m is the proton mass.

(i) The y distribution. From Eqs. (3) and (4), and ignoring the longitudinal structure function F_L, we have

$$\frac{\partial\sigma^{(\nu,\bar\nu)}}{\partial y} = \frac{G^2 m\epsilon}{\pi}\left[(1-y+\tfrac{1}{2}y^2)H(\epsilon y) \pm y(1-\tfrac{1}{2}y)K(\epsilon y)\right],$$

$$H(\epsilon y) = \int_0^1 dx\, F_2(x, q^2 = 2m\epsilon y x), \tag{27}$$

$$K(\epsilon y) = -\int_0^1 dx\, xF_3(x, q^2 = 2m\epsilon y x).$$

If strict scaling were to hold, both H and K would be constants and the total cross sections σ^ν and $\sigma^{\bar\nu}$ would grow linearly with energy. Moreover, with $xF_3 = -F_2$ it would follow that $H = K$, which implies that $\sigma^\nu/\sigma^{\bar\nu} = 3$, $\partial\sigma^\nu/\partial y = \text{constant}$, and $\partial\sigma^{\bar\nu}/\partial y \sim (1-y)^2$. Except for the question of scaling breakdown at very small q^2, these are precisely the results that we are adopting as input for small energies ($2m\epsilon < q_0^2 = 5.0m^2$). As we go up in energy, departures begin to develop, since we are assuming onset of asymptotic behavior for $q^2 > q_0^2$. The functions H and K begin to acquire a dependence on the argument ϵy. The behavior at small y still comes exclusively from the scaling region, whereas the large-y behavior ($y \to 1$) reflects contributions from q^2 in the asymptotic region. With increasing energy ϵ the transition moves increasingly towards small values of y. The functions H and K in this region are sensitive to our assumption that there *is* a sharp transition from scaling to asymptotic behavior. They both undergo variations in this region but then become smooth and slowly varying functions for larger values of y.

To get the total cross sections we have to integrate over all y in the interval 0–1, and this includes the problematic transition region. For small energies the results are sensitive to the choice of cutoff and to artifacts associated with the transition region. Once large energies are reached, roughly $\epsilon \gtrsim 50$ GeV, the behavior becomes smooth. Indeed, to within the numerical accuracy of the computation σ^ν/ϵ is then essentially constant up to the highest energies (≈ 350 GeV) that we have considered; $\sigma^{\bar\nu}/\epsilon$ rises very slowly toward σ^ν/ϵ. Eventually, as $\epsilon \to \infty$, $\sigma^\nu/\sigma^{\bar\nu}$ must approach unity—for the present model but also in fact for any interacting field theory. This is because the singlet operators in the Wilson expansion must have smaller dimensions than the corresponding nonsinglet operators, owing to positivity. However, for asymptotically free theories the approach to unity is very slow, reflecting the fact that departures from scaling are only logarithmic. Thus, we find that the ratio $\sigma^\nu/\sigma^{\bar\nu}$, which was equal to 3 in the scaling region, has dropped only by about 10% at $\epsilon = 200$ GeV. Because of the transition region artifacts, however, we cannot be too precise about this number. What are less sensitive, at large energies, are the differential cross sections $\partial\sigma^\nu/\partial y$ and $\partial\sigma^{\bar\nu}/\partial y$ at large values of y. For $\epsilon = 200m$ and $\frac{1}{2} < y < 1$, these are displayed in Figs. 1 and 2. For $\partial\sigma^{\bar\nu}/\partial y$ in particular, we show for comparison the input curve $(1-y)^2$ which obtains at low energies. The changed behavior reflects the fact that H and K, though they are slowly vary-

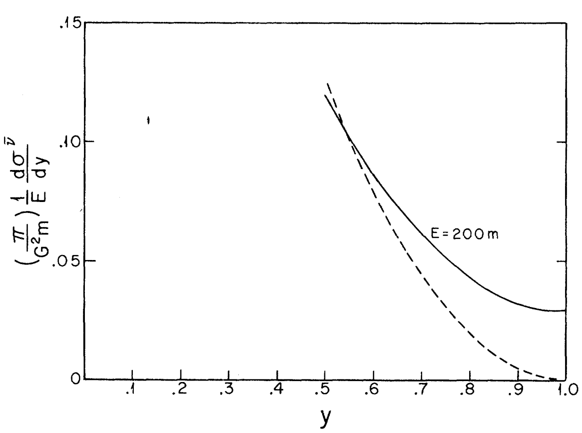

FIG. 1. A plot of $(\pi/G^2m)(1/E)d\sigma^{\bar\nu}/dy$ for $E = 200m$ (solid line). For comparison we have plotted (dashed line) the function $0.50(1-y)^2$ which is what $(\pi/G^2m)(1/E) \times d\sigma^{\bar\nu}/dy$ would be if scaling held with the structure function given by Albright and Jarlskog (Refs. 13 and 14).

ing in y away from the small y region, are no longer equal in magnitude away from small y.

(ii) The x distribution. As was discussed earlier, with increasing values of q^2 we expect the structure functions to fall off increasingly rapidly as $x \to 1$, and to grow increasingly rapidly as $x \to 0$. For large energies ϵ, which allow for contributions from large values of q^2, something of this comes through in the x distributions $\partial\sigma^\nu/\partial x$ and $\partial\sigma^{\bar{\nu}}/\partial x$. This is especially the case so far as the $x \to 1$ behavior is concerned. Unfortunately, since for given x, q^2 cannot exceed $2m\epsilon x$, the small-x effects in the structure functions are somewhat washed out in the cross sections $\partial\sigma/\partial x$. Nevertheless, for large energies the effects are visible. The results are shown in Figs. 3 and 4, for $\epsilon = 50m$ and $250m$.

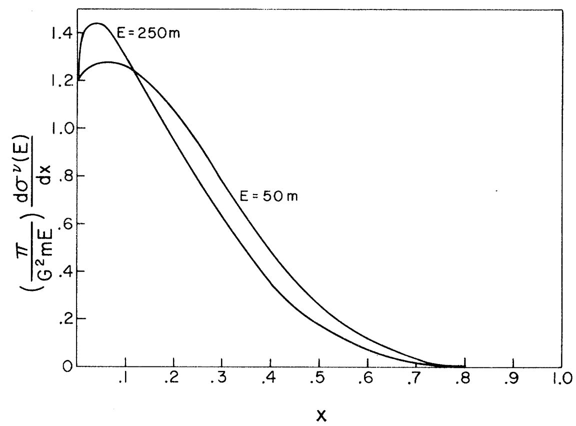

FIG. 3. A plot of $(\pi/G^2mE)d\sigma^\nu(E)/dx$ as a function of x for two different energies ($E = 50m$ and $E = 250m$.)

IV. CORRECTIONS TO THE CALLAN-GROSS RELATION

In our discussion of the differential cross sections we have ignored possible contributions from the longitudinal structure function $F_L = F_2 - 2xF_1$. For electroproduction the ratio F_L/F_2 is known to be small already in the SLAC-MIT region. Moreover, both for electroproduction and for the neutrino reactions, asymptotically free field theories and the quark-parton model both agree that this ratio must go to zero as $q^2 \to \infty$: This is the Callan-Gross relation.[9] However, although the effects arising from F_L may indeed be small, it is nevertheless interesting to try to detect its contributions experimentally. Owing to the absence of the photon propagator this may be easier to do at large q^2 in the neutrino reaction than in electroproduction. In this section we shall consider the large-q^2 properties of the ratio F_L/F_2 in the context of asymptotic freedom. This will also provide an opportunity to briefly review some of the ideas of asymptotic freedom.[17]

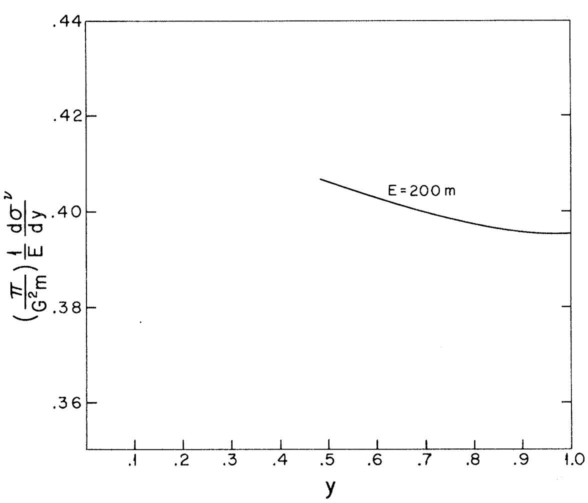

FIG. 2. A plot of $(\pi/G^2m)(1/E)d\sigma^\nu/dy$ for $E = 200m$. Note the suppressed zero. We see that $(1/E)d\sigma^\nu/dy$ is practically a constant from $y = \frac{1}{2}$ to 1, which is what scaling would predict.

Let us first recall how parton-model relations among structure functions are partially recovered in an asymptotically free theory. We adhere closely to the notations of Ref. 5; for simplicity we restrict ourselves at first to SU(3) nonsinglet structure functions. The analysis presented in Ref. 5, which is based on the work[18] of Wilson, Callan, and Symanzik, leads to relations of the form

$$\int_0^1 dx\, x^n F(x,q^2) = C^{(n)}\left(\frac{q^2}{\mu^2}, g\right) M_n\,, \tag{28}$$

where F is a generic structure function, M_n is the matrix element of the operator of spin $n+2$ appearing in the Wilson expansion, and C_n is the Fourier transform of the coefficient of this operator. The parameter μ is a reference momentum at which the coupling constant g is defined. The function $C^{(n)}$ satisfies a renormalization group equation

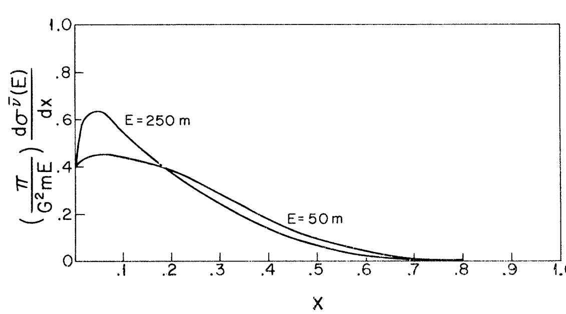

FIG. 4. A plot of $(\pi/G^2mE)d\sigma^{\bar{\nu}}(E)/dx$ as a function of x for two different energies ($E = 50m$ and $E = 250m$).

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}-\gamma_n(g)\right]C^{(n)}=0\,, \tag{29}$$

whose solution is

$$C^{(n)}\left(\frac{q^2}{\mu^2},g\right)=C^{(n)}(1,\bar g)\exp\left[-\int_0^{\ln(g^2/\mu^2)}\gamma_n(\bar g(x))\,dx\right]. \tag{30}$$

The effective coupling constant $\bar g(x)$ is defined through

$$\frac{d\bar g}{dx}=\beta(\bar g(x))\,,\quad \bar g(0)=g\,. \tag{31}$$

In an asymptotically free theory $\bar g(\ln(q^2/\mu^2))\to 0$ as $q^2\to\infty$, so that on the right-hand side of Eq. (30) the Wilson coefficient $C^{(n)}(1,\bar g)$ approaches its free-field value. *In this sense* one recovers the algebraic relations of the parton model, such as the Callan-Gross relation $F_2-2xF_1=F_L=0$. Deviations from scaling, which formed the subject of the previous sections, come, of course, from the exponential factor in Eq. (30).

By evaluating $C^{(n)}(1,\bar g)$ to the next order in perturbation theory one obtains corrections to the parton-model relations. A simple calculation involving the graphs of Fig. 5 [in fact only 5(a) gives a nonvanishing contribution] leads to the (quark operator) result

$$\frac{C_L^{(n)}(\text{quark})}{C_2^{(n)}(\text{quark})}=\frac{F_L^{(n)}(q^2)}{F_2^{(n)}(q^2)}=\frac{\bar g^2}{16\pi^2}C_2(R)\frac{4}{n+3}\,, \tag{32}$$

where $C_2(R)$ is the quadratic Casimir operator for the representation of the quarks. For the colored quark model we have $C_2(R)=\frac{4}{3}$. It should be emphasized again that Eq. (32) refers to the SU(3) nonsinglet combinations of structure functions, e.g., the proton-neutron difference. The left-hand side of Eq. (32) is an experimentally defined quantity and provides a direct determination of the effective coupling constant as a function of q^2. The smallness of $\bar g$ is required for self-consistency of our expansions, in the large-q^2 region that we are considering. One can now invert Eq. (32), at fixed q^2, to obtain

$$F_L(\omega,q^2)=4C_2(R)\frac{\bar g^2}{16\pi^2}\,\frac{1}{\omega^2}\int_1^\omega d\omega'\omega'F_2(\omega',q^2)\,, \tag{33}$$

where we have switched to $\omega=x^{-1}$. In this way we see that

$$\frac{F_L(\omega,q^2)}{F_2(\omega,q^2)}\sim K_1\frac{\bar g^2}{16\pi^2}\,,\quad \omega\to\infty$$
$$\frac{F_L(\omega,q^2)}{F_2(\omega,q^2)}\sim K_2(\omega-1)\frac{\bar g^2}{16\pi^2}\,,\quad \omega\to 1 \tag{34}$$

where K_1 and K_2 are constants which may be computed from Eq. (33) if we know the growth properties of F_2 as a function of ω. For instance if $F_2\to\omega^{-1/2}$ as $\omega\to\infty$, as expected from Regge arguments for the proton-neutron difference, then $K_1=\frac{8}{3}C_2(R)$ ($K_1=\frac{32}{9}$ for the colored quark model). If $F_2(\omega-1)^{P(q^2)}$ as $\omega\to 1$, then $K_2=4C_2(R)/(P+1)$. For q^2 in the region of several GeV^2 experiment suggests that $P\approx 3$. With more reliable input data one could try a global fit based on Eq. (33).

The q^2 dependence of $F_L(\omega,q^2)/F_2(\omega,q^2)$ is, of course, also determined by Eq. (33). Once q^2 is large enough so that $\bar g^2/8\pi^2$ is small compared to unity we expect

$$\bar g^2\to A\left(\ln\frac{q^2}{\mu^2}\right)^{-1}\,, \tag{35}$$

where the constant A is computable. For the colored quark model $A=\frac{8}{9}\pi^2$. Since $\bar g$ is an experimental quantity, Eq. (35) therefore permits an experimental determination of the parameter μ^2. In the renormalization-group formalism μ^2 is of course an arbitrary parameter, but we might conventionally define it by requiring a good fit to Eqs. (34) and (35). Defined in this way, μ^2 is a fundamental parameter, which describes the rate at which the strong interactions "turn off" in the deep Euclidean region.

In our previous discussion of the asymptotic inversion assumptions we had to express strong caveats about the uniformity in n of the onset of asymptotic behavior for the moments. The ques-

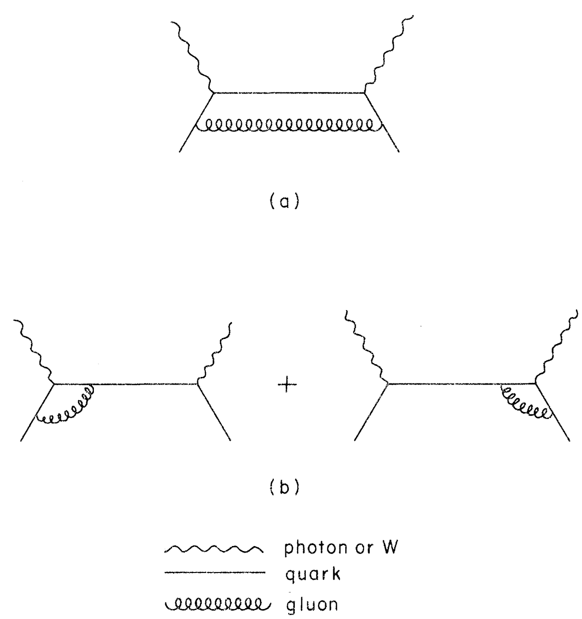

FIG. 5. Feynman diagrams contributing corrections to the Wilson coefficients for fermion operators. The graphs (b) do not modify the Callan-Gross relation.

tion boiled down to whether one can trust perturbation theory for the anomalous dimensions γ_n, especially with respect to the growth at large n and the singularity at $n=-1$. Concerning the former in particular, there is the danger that higher-order terms in perturbation theory lead to corrections which increase with n. In the present context we must ask whether we can trust perturbation theory for $C_L^{(n)}/C_2^{(n)}$. Here we believe that we are on firmer ground, for the following reasons:

(a) In each order of perturbation theory the leading contribution to $C_2^{(n)}$ as $n\to\infty$ is given by vertex correction graphs, as displayed in Fig. 6(a). These give no contributions to $C_L^{(n)}$. However, the graphs of Fig. 6(b), down by exactly one power of n as $n\to\infty$, do contribute to $C_L^{(n)}$. There is no obvious nonuniformity, therefore, and the $n\to\infty$ behavior in Eq. (32) may therefore be realistic even beyond lowest order in perturbation theory.

(b) Graphs involving exchange of two gluons do not contribute to the ratio $C_L^{(n)}/C_2^{(n)}$. Order by order in perturbation theory, therefore, it seems that there are no singularities to the right of $n=-2$. Even if the sum over all orders produces a moving singularity (as q^2 varies), since the effective coupling constant at large q^2 is small, such a singularity should not move much to the right of $n=-2$. This is relevant because Regge arguments suggest that $F_2^{(n)}$ has a singularity at $n=-\frac{3}{2}$ (for the nonsinglet case under discussion). Therefore, the $x\to 0$ behavior of $F_L(x,q^2)$, obtained from the inversion of

$$\int_0^1 dx\, x^n F_L(x,q^2)=\int_0^1 dx\, x^n F_2(x,q^2)C_L^{(n)}/C_2^{(n)}\,, \tag{36}$$

will be dominated by the singularity of $F_2^{(n)}$. This means that the behavior predicted by Eq. (34) is not sensitive to the singularity structure of $C_L^{(n)}/C_2^{(n)}$ and should therefore be reliable.

For SU(3) singlet structure functions the analysis is more complicated,[5] and the results are weaker. Here one has contributions from the gluon operators in the light-cone expansion. Their coefficients vanish to zeroth order in $\bar{g}$, but in order $\bar{g}^2$ we have to consider the graphs of Fig. 7. It turns out that only graph 7(a) gives a nonvanishing contribution to $C_L^{(n)}$. In fact, the gluon contribution leads to

$$\frac{C_L^{(n)}(\text{gluon})}{C_2^{(n)}(\text{quark})}=\frac{\bar{g}^2}{16\pi^2}\,C_2(G)\frac{16}{(n+3)(n+4)}\,.$$

We see that for large n the gluon contributions are negligible compared to the purely quark contributions, Eq. (32). Thus the $\omega\to 1$ prediction of Eq. (34) applies for the singlet as well as the nonsinglet case. The $\omega\to\infty$ prediction is also unchanged, in form, but the coefficient K_1 is no longer determined.

V. CONCLUSIONS

Our discussion of deviations from scaling, for deep-inelastic neutrino reactions in the context of asymptotically free theories, has been at two levels. Concerning the growth properties of the total ν and $\bar{\nu}$ cross sections, we could set lower and upper bounds without recourse to serious as-

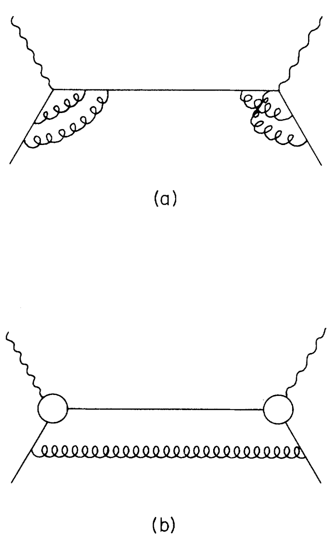

FIG. 6. Representative high-order Feynman diagrams controlling the n behavior of corrections to the Wilson expansion. (a) A typical leading contribution to $C_2^{(n)}$. This graph gives no contribution to $C_L^{(n)}$. (b) A leading contribution to $C_L^{(n)}$. The bubbles represent radiatively corrected vertices.

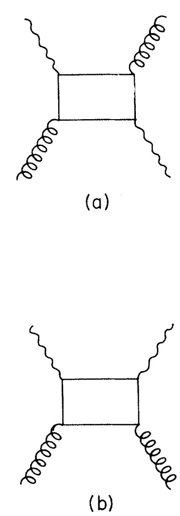

FIG. 7. Feynman diagrams contributing corrections to the Wilson coefficients for gluon operators. The graphs (b) do not modify the Callan-Gross relation.

sumptions going beyond the basic features of asymptotic freedom. The bounds closely bracket a linear growth, so that in this respect deviations from scaling are predicted to be small.

In order to treat the structure functions in more detail, and thereby the differential cross sections, we had to invoke uniformity assumptions of a speculative character. Given these, we are led to expect substantial deviations from scaling in the structure functions at large q^2. The effects are somewhat washed out in the partially integrated cross sections but are still visible there, especially for the x distributions as they change shape with beam energy.

It is natural to ask how these results compare with expectations for other possible mechanisms of scaling breakdown. In this connection it is especially interesting to contemplate a situation where the strong interactions are governed by an Abelian rather than a non-Abelian gauge theory.[19] Of course Abelian theories are not asymptotically free. That is, if there is a fixed point, it is not at the origin of coupling constant space. The anomalous dimensions, which are determined at the fixed point, cannot therefore be reliably gotten by perturbation theory—even if we knew where the fixed point is located. Just for orientation, however, suppose that the effective coupling constant at the fixed point is very small, so that lowest-order perturbation theory can be used. In that case the anomalous dimensions would have the same general properties as in the non-Abelian case. The chief difference is that the analog of Eq. (2) would contain q^2/μ^2 in place of $\ln(q^2/\mu^2)$; the scaling deviations, that is, would go like inverse powers of (q^2/μ^2) rather than inverse powers of $\ln(q^2/\mu^2)$. For the structure functions and differential cross sections, therefore, the general trends would resemble those of the non-Abelian case, but the effects would be greatly magnified.

There is another mechanism of possible scaling breakdown for neutrino processes that has been discussed in the literature.[20] The idea there is to modify the parton model solely through endowing the partons with form factors. The trends can be seen in the paper by Barger.[20]

ACKNOWLEDGMENTS

We would like to thank C. G. Callan and M. L. Goldberger for helpful discussions, E. Wilczek for patience and a helpful suggestion, and V. Fitch for the use of his computer.

APPENDIX

We present here some of the details of our procedure for extrapolating the structure functions from one value of q^2 to higher values of q^2. This involves inversion of the moments, Eq. (23), with the a_n given by Eq. (25) or Eq. (26). Let us quote three relevant theorems on Mellin transforms:

(a) If

$$\int_1^\infty \frac{d\omega}{\omega^{n+2}} f_i(\omega) = g_i(n)\,,$$

then

$$\int_1^\infty \frac{d\omega}{\omega^{n+2}} f_1 * f_2 = g_1(n)\, g_2(n)\,,$$

where

$$f_1 * f_2 = \int_1^\omega \frac{d\omega'}{\omega'} f_1(\omega') f_2(\omega/\omega')\,.$$

(b) If $k>0$, $\nu>0$, then

$$\int_1^\infty \frac{d\omega}{\omega^{n+2}} \left(\frac{\ln\omega}{k}\right)^{(\nu-1)/2} I_{\nu-1}[2(k\ln\omega)^{1/2}] = \frac{1}{(n+1)^\nu} e^{k/(n+1)}\,,$$

where $I_{\nu-1}$ is the modified Bessel function of index $\nu-1$.

(c) If $f(1)=0$ and $f(\omega)/\omega^{n+1}\to 0$ as $\omega\to\infty$, then

$$\int_1^\infty \frac{d\omega}{\omega^{n+2}} \omega^{-\alpha+1} \frac{d}{d\omega} \omega^\alpha f(\omega) = (n+\alpha+1) \int_1^\infty \frac{d\omega}{\omega^{n+2}} f(\omega)\,.$$

The moment problem that we encounter is

$$F_n(q^2) = F_n(q_0^2)\lambda^{-a_n}\,, \tag{A1}$$

where, with the approximations that have been adopted, a_n has the form

$$a_n = \text{constant} + \sum_\beta \left(c_\beta \ln(n+\alpha_\beta+1) + \frac{d_\beta}{n+\alpha_\beta+1}\right). \tag{A2}$$

Our coefficients d_β are positive. If all the c_β were similarly positive we could invert λ^{-a_n} by repeated convolutions, using (a) and (b). One further convolution would then yield $F(\omega,q^2)$. Actually, the c_β are not all positive. However, if a given c_β is negative we can use (c) to write

$$F_n(q_0^2) = \frac{1}{n+\alpha_\beta+1} G_n(q_0^2;\ \alpha_\beta)\,, \tag{A3}$$

where

$$G_n(q_0^2;\alpha_\beta) = \int_1^\infty \frac{d\omega}{\omega^{n+2}} \omega^{-\alpha_\beta+1} \frac{d}{d\omega} \omega^{\alpha_\beta} F(\omega,q_0^2) \tag{A4}$$

is known from the input data [which satisfy the requirement that $F(1,q_0^2)=0$]. In this way we are led to consider the moment problem

$$F_n(q^2) = G_n(q_0^2; \alpha_\beta)\lambda^{-b_n} ,$$

where

$$b_n = a_n + (\ln\lambda)^{-1} .$$

The new problem has exactly the same structure as the original one, with $F_n(q_0^2) \to G_n(q_0^2; \alpha_\beta)$ and $a_n \to b_n$. By repeated use of this trick we can arrange (over some range of λ which is big enough for our needs) that the modified c_β are all positive. Indeed, with sufficient repetition we can arrange that the index ν encountered in (b) is always greater than unity. This last allows us to avoid modified Bessel functions of negative index. The latter are singular at the origin and would be a nuisance for numerical work.

---

*Work supported by the National Science Foundation under Grant No. GP-3078X, and by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-3072.
†Alfred P. Sloan Foundation Fellow.

[1]G. Miller *et al.*, Phys. Rev. D 6, 3011 (1972).

[2]D. H. Perkins, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 189. For more recent results see the proceedings of the Fourth International Conference on Neutrino Physics and Astrophysics, Downingtown, Pa., 1974 (unpublished).

[3]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972); J. D. Bjorken and E. Paschos, Phys. Rev. 185, 1975 (1969).

[4]D. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. D. Politzer, *ibid.* 30, 1346 (1973); G. 't Hooft (unpublished).

[5]D. Gross and F. Wilczek, Phys. Rev. D 9, 980 (1974).

[6]H. Georgi and H. D. Politzer, Phys. Rev. D 9, 416 (1974).

[7]D. Bailin, A. Love, and D. V. Nanopoulos, Nuovo Cimento Lett. 9, 501 (1974).

[8]Some of these results have already been briefly presented. See A. Zee, in proceedings of the Fourth International Conference on Neutrino Physics and Astrophysics, Downingtown, Pa., 1974 (unpublished).

[9]C. Callan and D. Gross, Phys. Rev. Lett. 22, 156 (1969).

[10]We thank R. Feynman for a discussion on this point. Calculations by M. Calvo [thesis, Princeton University (unpublished)] suggest that $\gamma_n(\bar{g}) = a_1 \ln n + a_2 \bar{g}^2 \ln^2 n + \ldots$, so the $n \to \infty$ limit is highly nonuniform.

[11]K. Wilson, Phys. Rev. 179, 1499 (1969); G. Mack, Phys. Rev. Lett. 25, 400 (1970); S. Cicciarello, R. Gatto, G. Sartori, and M. Tonin, Phys. Lett. 30B, 546 (1969).

[12]A. De Rújula, S. L. Glashow, H. D. Politzer, S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D 10, 1649 (1974).

[13]C. H. Albright and C. Jarlskog, CERN Report No. CERN-TH-1828, 1974 (unpublished).

[14]We change the last term in Eq. (3.8c) of Ref. 12 to $17.5(1-x)^7$; this has the effect of flattening F_2 as $x \to 0$.

[15]D. Gross, Phys. Rev. Lett. 32, 1071 (1974). See also A. De Rújula, *ibid.* 32, 1143 (1974); D. Gross and S. B. Treiman, *ibid.* 32, 1145 (1974).

[16]Several authors have discussed the large-ω limit in a manner similar to Ref. 12 but outside the context of asymptotically free theory: see V. N. Gribov and L. N. Lipatov, Yad. Fiz. 15, 781 (1972) [Sov. J. Nucl. Phys. 15, 438 (1972)], Eqs. 93 and ff. For further references see Ref. 12.

[17]The calculations in this section were done with W. Caswell and R. Shrock (unpublished). M. Calvo [thesis, Princeton University (unpublished)] has independently analyzed corrections to other parton-model sum rules as well as the Callan-Gross relation.

[18]K. Wilson, Phys. Rev. 179, 1499 (1969); C. Callan, Phys. Rev. D 5, 3203 (1972); K. Symanzik, Commun. Math. Phys. 23, 491 (1971).

[19]We thank S. Adler for emphasizing this question.

[20]M. S. Chanowitz and S. D. Drell, Phys. Rev. Lett. 30, 807 (1973); V. Barger, Phys. Lett. 49B, 43 (1974).