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Pion-nucleon coupling constant

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The pion-nucleon coupling constant is determined by comparing the phase shifts obtained by using matrix Padé approximants with the Livermore phase-shift analysis. The calculations are based on one- and two-pion exchanges of the pseudoscalar interaction. Only phase shifts and energy regions which were expected to be dominated by the one- and two-pion exchanges were taken into account. A value of $g^2/4\pi = 13.1 \pm 0.4$ is recommended.

I. INTRODUCTION

In a previous work[1] the matrix Padé approximants (MPA's) of the S-matrix elements of the nucleon-nucleon scattering were calculated for total angular momentum $J \leq 4$ and for laboratory kinetic energy up to 425 MeV. The calculations were based on one- and two-pion exchanges of the pseudoscalar interaction. The coupling constant g was the only parameter of the interaction for which a value, consistent with experimental results, $g^2/4\pi = 15$ was assumed. In order to interpret the results properly, estimates were given for energy regions where the one- and two-pion exchanges should dominate. Within the limits of these regions it was found that the phase shifts obtained with the [1/1] MPA fitted the experimental data relatively well. In this work the above calculations are continued and the best value of the coupling constant is searched. For the basic experimental data set, experimental phase shifts are taken only for energy regions and partial waves for which the one- and two-pion exchanges are expected to dominate. As will be seen later, a rather deep minimum is obtained and the coupling constant appears with a rather small statistical error.

The pion-nucleon coupling constant, resulting from the nucleon-nucleon scattering, was evaluated previously in many works. For reviews see Refs. 2, 3, and 4, and for the latest works see Ref. 5. In this work the evaluation of the pion-nucleon coupling constant is based on the one- and two-pion exchanges (OPE and TPE). The

TABLE I. The Livermore phase shifts (Ref. 6), in degrees, used in this work.

Bar phase shifts	Lab kinetic energy in MeV 25	50	95	142	210	330	425
$\delta(^1D_2)$	0.74 ± 0.03	1.67 ± 0.10	3.71 ± 0.24	5.11 ± 0.16			
$\delta(^3D_2)$		9.36 ± 2.24	12.32 ± 3.23	22.54 ± 0.79			
$\delta(^3F_2)$		-0.02 ± 0.28					
ϵ_2		-1.63 ± 0.19					
$\delta(^1F_3)$				-2.02 ± 0.79	-4.80 ± 0.86	-6.37 ± 4.02	-4.91 ± 1.05
$\delta(^3D_3)$		1.50 ± 0.94	3.22 ± 1.22	2.52 ± 0.64			
$\delta(^3F_3)$		-0.38 ± 0.37	-0.91 ± 0.49	-2.05 ± 0.16	-2.58 ± 0.20	-3.58 ± 0.58	-3.25 ± 0.59
$\delta(^3G_3)$						-6.07 ± 1.91	-9.30 ± 1.35
ϵ_3				4.44 ± 0.40			
$\delta(^1G_4)$				0.62 ± 0.06	1.00 ± 0.10	1.22 ± 0.23	1.96 ± 0.34
$\delta(^3F_4)$		0.21 ± 0.15	0.89 ± 0.19	0.90 ± 0.12	2.02 ± 0.19	2.77 ± 0.23	3.34 ± 0.54
$\delta(^3G_4)$						11.24 ± 2.16	11.08 ± 1.08
$\delta(^3H_4)$					0.24 ± 0.21	1.12 ± 0.32	-0.32 ± 0.45
ϵ_4				-0.68 ± 0.03	-0.99 ± 0.09	-1.11 ± 0.28	-2.19 ± 0.34

purpose of these calculations is to get a more sensitive determination of the pion-nucleon coupling constant. As will be seen later, a significant improvement is obtained, as compared with the OPE, when the TPE contributions are added. In this work, in addition to the MPA's, similar calculations were performed with the aid of the K-matrix unitarization (KMU). As a further test of our calculations the pion mass was varied and its best value was determined by searching.

II. THE FITTING PROCEDURE

The set of experimental data was chosen according to the considerations of Ref. 1. Only partial waves with $L \geq 2$ and $J \leq 4$ are taken into account—among these only the partial waves which, according to the Livermore phase-shift analysis,[5] were different from the OPE phase shifts. The phase shifts of the D states and ϵ_3 were taken below lab energy of 200 MeV. The ${}^3D_1$ phase shift was not considered, as it is strongly affected by the ${}^3S_1$ state. The ${}^3F_2$ and ϵ_2 phase shifts are affected by the ${}^3P_2$ state; therefore they were considered only below lab energies of 60 MeV. The experimental data which were used in this analysis[6] are given in Table I. The value of

$$\chi^2 = \sum_i \left(\frac{\delta_i - \delta_{ei}}{\Delta_i}\right)^2$$

was minimized in order to get the best value of

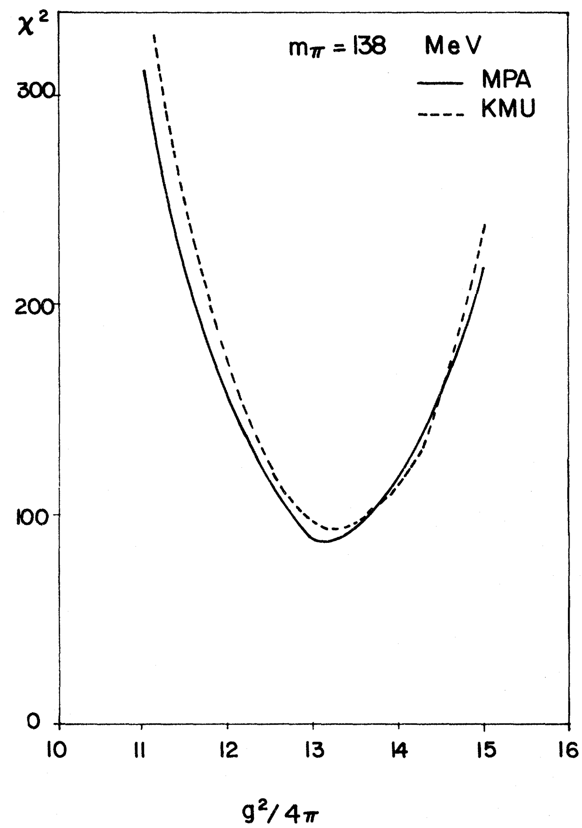

FIG. 1. χ^2 as a function of $g^2/4\pi$.

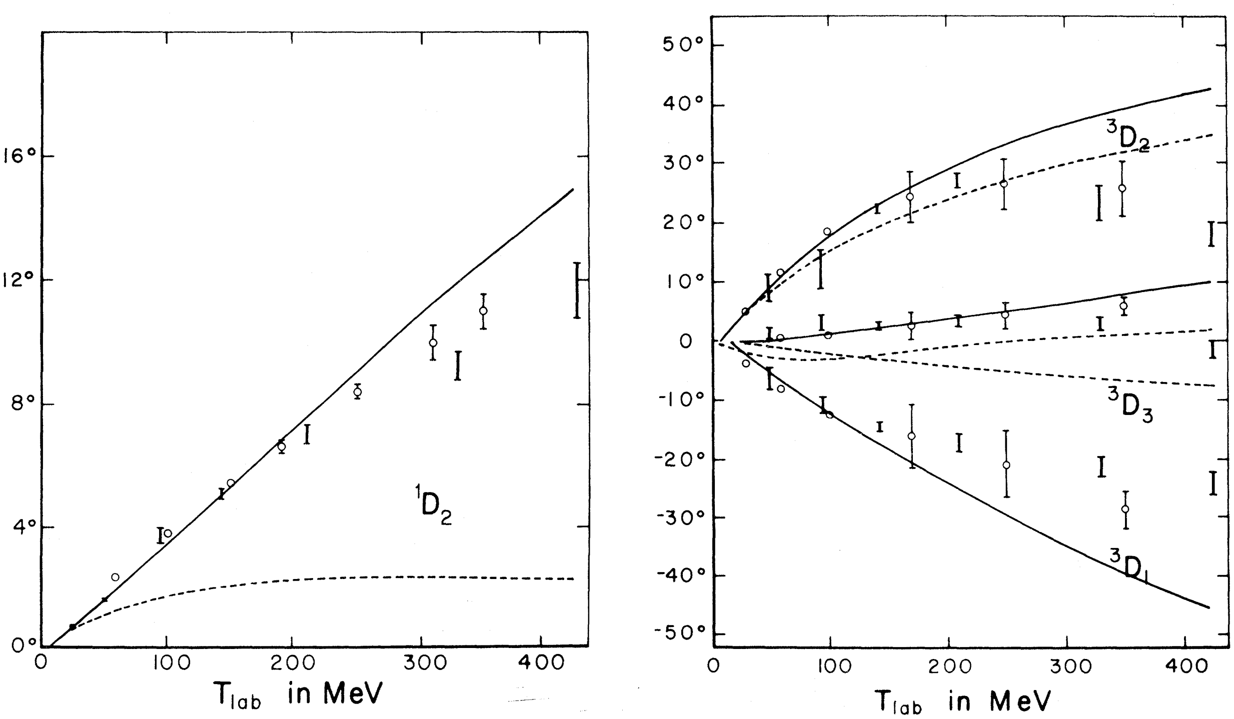

FIG. 2. *(Continued on following page.)*

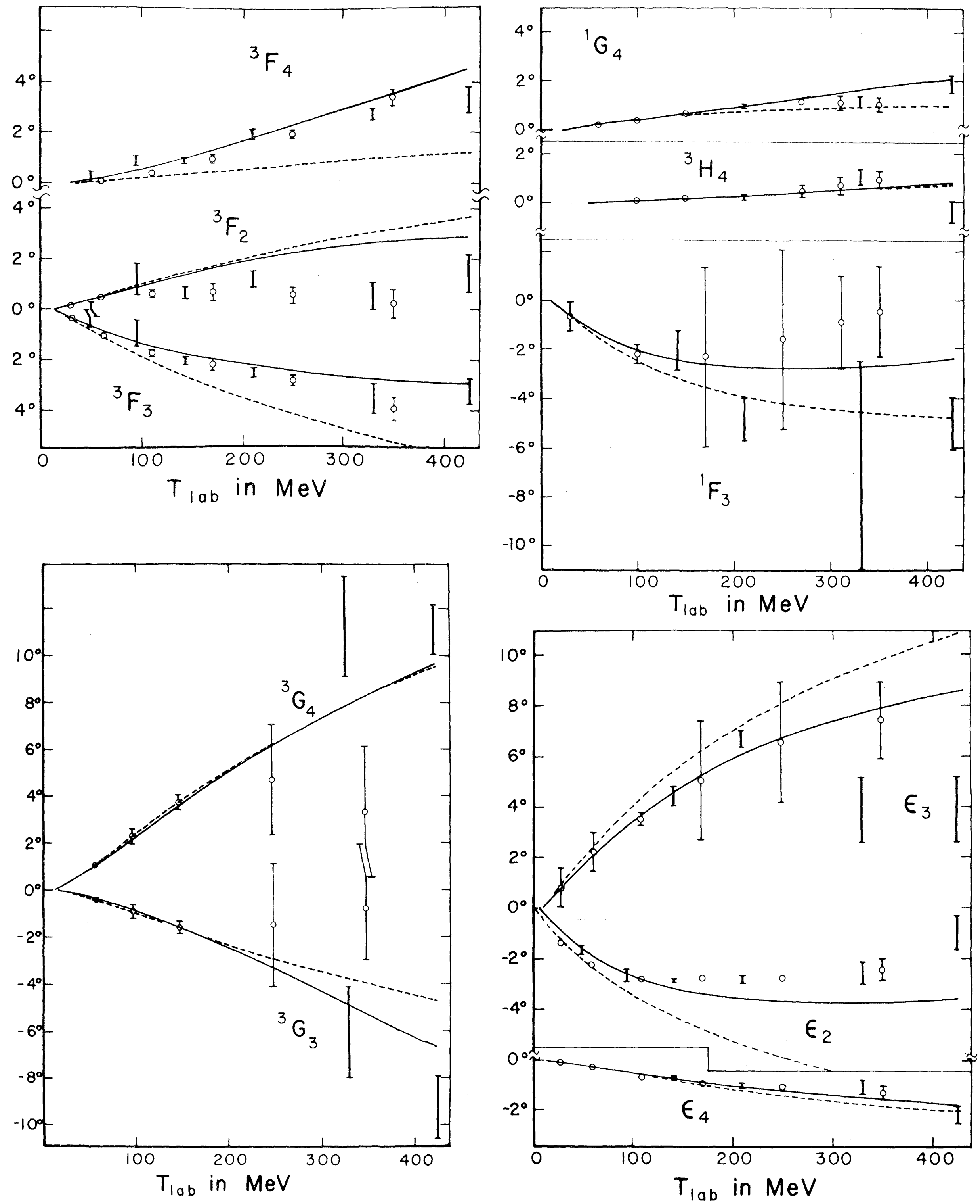

FIG. 2. Nucleon-nucleon bar phase shifts calculated by using MPA's (solid line) and OPE (dashed line). The heavy error lines depict experimental phase shifts found by the Livermore group (Ref. 6), while the circles and thin error lines correspond to an energy-dependent solution of the Yale group (Ref. 7). The phase shifts in degrees are given as a function of the laboratory kinetic energy T_{lab} in MeV.

the coupling constant. Here δ_i and δ_{ei} are the theoretical and experimental phase shifts, respectively, and Δ_i are the phase-shift errors.

III. RESULTS AND DISCUSSION

In Fig. 1 the χ^2 dependence on $g^2/4\pi$ is displayed for calculations where the average pion mass $m_\pi = 138$ MeV and average nucleon mass $M_N = 938.5$ MeV were used. The calculations were performed for both MPA's and KMU. The results, for the selected phase shifts and energy regions, are quite similar for both methods. A rather deep minimum was obtained which led to $g^2/4\pi = 13.20 \pm 0.14$ for the MPA ($\chi^2 = 89$) and $g^2/4\pi = 13.30 \pm 0.15$ for the KMU ($\chi^2 = 93$) using 44 experimental data. For comparison, the same calculations were performed with OPE (in the KMU) only. The χ^2 minimum was at $\chi^2 = 916$ for $g^2/4\pi = 15.31 \pm 0.24$. Thus, the inclusion of the TPE reduced the χ^2 minimum by a factor of about 10. This is a very significant result which gives us more confidence in the use of the pseudoscalar interaction. The phase shifts obtained using the MPA's for states with orbital momentum $L \geq 2$ are displayed in Fig. 2 (see Refs. 6 and 7). For comparison the OPE phase shifts are also given. Here the OPE calculations were done using the KMU and $g^2/4\pi = 14.43$, $m_\pi = 135$ MeV as in Ref. 6.

The best values of $g^2/4\pi$ are quite sensitive to the change of the pion mass m_π. This is displayed in Fig. 3, where the best values of $g^2/4\pi$ are given for different values of m_π. The shaded area includes the statistical error. This sensitivity of the best value of $g^2/4\pi$ on the average pion mass m_π should be taken into account. If we consider the values of $g^2/4\pi$ for m_π between the masses of π^0 and π^+ (m_{π^0} and m_{π^+}) we obtain, using the results of Fig. 3, the following values:

$$g^2/4\pi = 13.1 \pm 0.4 \quad \text{for MPA},$$
$$g^2/4\pi = 13.2 \pm 0.4 \quad \text{for KMU},$$
$$g^2/4\pi = 15.3 \pm 0.5 \quad \text{for OPE}.$$

In Fig. 4 the minimal χ^2, for the best values of $g^2/4\pi$, is given for different values of m_π. A minimum is obtained for $m_\pi = 145$ MeV using the MPA's, and $m_\pi = 149$ MeV for KMU, which are slightly higher than the physical masses of the pions. As one can see, the results with the MPA's are somewhat better than the results obtained by using the KMU. Therefore this analysis favors the result obtained by using the MPA's, $g^2/4\pi = 13.1 \pm 0.4$.

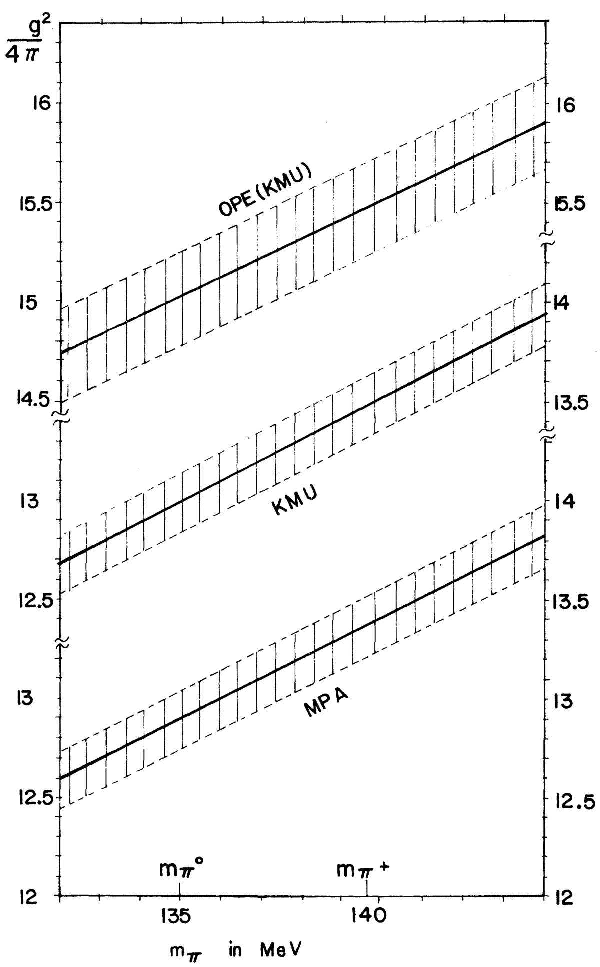

FIG. 3. The best values of $g^2/2\pi$ determined by searching, versus the average pion mass m_π. The shaded area corresponds to the statistical error.

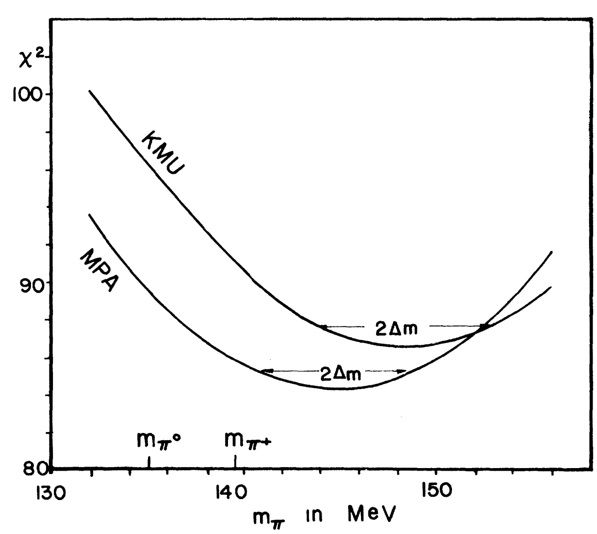

FIG. 4. The minimal χ^2 for the best values of $g^2/4\pi$ versus the average pion mass m_π.

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