"Power" vs "radius" models of form factors: Decay characteristics

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We present a detailed study of the relative performance of two classes of phenomenological form factors, the "power" form factor (PFF) and the "radius" form factor (RFF), in the understanding of pseudoscalar decays of a large number of experimentally known baryon and meson resonances in both $(L \pm 1)$ waves. The PFF which turns out to work best is essentially characterized by the division of each 4-momentum (in the multiple momentum couplings) by the c.m. energy of the daughter baryon and daughter meson for the $(L \pm 1)$ waves, respectively. For the RFF we use two versions, one employed by the Particle Data Group and another proposed by us to incorporate certain features of the Harari radius model. Both sets provide a consistent pattern for $(L + 1)$-wave decays compatible with the universality of reduced coupling constants for Regge and exchange-degenerate partners. For the $(L - 1)$-wave decays, the PFF has an edge over the RFF because of its flexibility to incorporate the Gell-Mann, Oakes, and Renner effect. It is also pointed out that reaction processes as well as electromagnetic mass differences (considered in separate papers) can discriminate between the two forms, considerably favoring PFF over the RFF.

I. INTRODUCTION

The language of supermultiplet form factors within the framework of general principles such as partial symmetry and Regge universality [and hopefully exchange degeneracy (EXD)] at the level of the reduced coupling constant provides a powerful though phenomenological tool for the study of two-body reactions. Such an approach has been suggested by the authors and collaborators[1] over the past few years. Using as a specific model, wherein the classification and geometrical coefficients of various hadronic resonances are generated by a convenient relativistic version of $SU(6)\times O(3)$, a class of form factors characterized by a power law has been advocated[2,3] and applied to several two-body reactions.[4-6] This class of form factors for $(L\pm 1)$ waves of pseudoscalar meson decays with parameters fixed through certain sets of reliable decay modes has been continuously sharpened (and successively denoted as I, II, III) over the years. This class will be called the "power" model in this paper.

For the $(L+1)$ waves, the structure which gives the best performance among this class (i.e., III), and which has been tailored to the requirements of the off-shell extension so as to be applicable to processes such as electroproduction of resonances[5] and photoproduction of (π, η),[6] behaves essentially as E_k^{-L-1}, where E_k is the energy of the daughter baryon in the rest frame of the parent hadron. Regarding the $(L-1)$ waves, a corresponding degree of clarity has been lacking due to paucity of data at the decay level, and the passive role played by $(L-1)$-wave couplings in most two-body reactions. However, it is fair to suppose that any structure for the $(L-1)$-wave form factor must incorporate the Gell-Mann, Oakes, and Renner (GOR) effect[7] [which essentially amounts to a multiplication by the factor $(M_L - m)$ on the mass shell]. It also goes without saying that the parametrization for $(L-1)$ waves must be dominated by the "recoil" term[8] or its essentially equivalent version of L-broken $SU(6)_W$ suggested by Rosner[9] and collaborators. For a detailed and comparative discussion of the subject as well as of the GOR effect[7] in this context, we refer the interested reader to the recent review article by one of the authors.[1]

As for pure meson couplings, again the general paucity of data has prevented any firm conclusions from being drawn on their form factors so far.

Another class of form factors which has been studied by various authors[10] (including the present ones[11]) arises out of the concept of a fixed radius of interaction. One version of this "radius" model, advocated by the Particle Data Group[12] on the lines of a barrier penetration factor, has acquired a fair degree of acceptance in the phenomenological analysis of the data.[13] A somewhat different version for the $(L+1)$-wave form factor, suggested by the present authors,[11] is characterized by the factor $j_{L+1}(kR)$ which also appears as a nonrelativistic limit of an $O(3,1)$ model.[14] The structure of the $(L-1)$-wave form factor is less well understood in this case also.

The purpose of the present paper is to provide the main results of a thorough reassessment of the "power" and "radius" form factors for both baryons and mesons in relation to the decay data of almost all the resonances listed in the Particle Data Group tables.[12]

A. Power form factors (PFF)

For the "power" form factor for $(L+1)$-wave decays we take the structure

$$f_L^{(+)}(P)=E_k^{-L-1}S_F^{L+1}g_L^{(+)}(P)\,,\qquad E_k=(2M_L)^{-1}(M_L^2+m^2-\mu^2)\,, \tag{1.1}$$

with $S_F=1.16$. This structure has already been used in Ref. 5. For the $(L-1)$ wave, our present studies indicate that the GOR effect cannot be successfully incorporated in a structure behaving as E_k^{-L}. On the other hand, a slight modification of the earliest version (I),[2] so as to conform to the power indices $L+1$, L for $(L\pm1)$ waves, respectively, according to the pattern of III would be ω_k^{-L}, where ω_k is the energy of the quantum of mass μ. This structure differs from the earliest version (I) only in having the replacement ω_k^{-L} for ω_k^{-L+1}, the latter being too inadequate for the data. On the other hand, for applications involving extrapolation in the mass of the quantum (μ^2 to $-k^2$), ω_k is unsuitable as discussed in Ref. 5. We meet this problem through the modification

$$\omega_k^{-1}=2M_L(M_L^2-m^2+\mu^2)^{-1} \to \chi_k^{-1}[2M_L(M_L^2-m^2-\mu^2)^{-1}]\,.$$

χ_k may be interpreted as the c.m. energy of a particle of fictitious mass $(m\mu/M_L)$. Our final ansatz for the $(L-1)$-wave form factor, in conformity with "recoil"-effect dominance,[8] is given by

$$f_L^{(-)}(P)=\chi_k^{-L}g_L^{(-)}(P)\,. \tag{1.2}$$

We emphasize at this point that the marginal changes suggested in the PFF are unlikely to affect the good features of the results of their application to two-body reactions.

B. Radius form factors (RFF)

The radius form factor for the $(L+1)$ wave, proposed in Ref. 11, is

$$f_L^{(+)}(R)=k^{-L-1}j_{L+1}(kR)p_l^{1/2}\left(\frac{2l+1}{l+1}\right)^{1/2}g_L^{(+)}(R)\,. \tag{1.3}$$

An analogous proposal for the radius model in the $(L-1)$ wave on the lines of (1.1) and (1.2), again in conformity with dominance of the recoil effect,[8] is

$$f_L^{(-)}(R)=k^{-L}j_L(kR)p_l^{1/2}\left(\frac{2l+1}{l+1}\right)^{1/2}g_L^{(-)}(R)\,. \tag{1.4}$$

As to the values of R in (1.3) and (1.4), there is no intrinsic reason for them to be equal. While for (1.3) R was found to be of the order of 1 F, the corresponding value which seems to give a better measure of the fit to the data is $R\sim0.6$ F (see further below).

The above structures of RFF may be compared with those used by the Particle Data Group[12] which are given effectively by the replacement $j_L(kR)\to v_L(kR)$, where

$$(kR)^{-2}v_l^{-2}(kR)=j_l^2(kR)+n_l^2(kR)\,. \tag{1.5}$$

C. Mixing effects

Whereas the $(L+1)$-wave modes for the baryons present a relatively clean picture, the study of $(L-1)$-wave modes is beset with the problems of mixing effects. We believe that in the absence of any reliable theory of mixing, even at the phenomenological level, and the present paucity of experimental data for the $(L-1)$-wave decays, a simultaneous study of the structure of form factors as well as mixing effects is unfeasible. So we merely consider the cases where well-known "guesses"[12,15] on mixing angles are available, and that too for illustrative purposes only.

D. Meson form factors

For the mesonic decays, we insist, as usual, on the principle of similarity in the form factors for mesons and baryons.[1,3] Thus, Eqs. (1.1) to (1.4) also define the form factors for mesons, except for an extra normalization factor $2M_L$ and perhaps also a change in the values of the reduced coupling constants. For the details of the argument related to the choice of $2M_L$, see Ref. 1. The role of the VW (Van Royen and Weisskopf) factor,[1,16] which has been used on several occasions, has been excluded altogether from the present investigation.

In Sec. II, we present the results for baryons. The profusion of data for the $(L+1)$-wave applications makes it possible for us to display these results in the form of graphs which clearly bring out the L independence of the reduced coupling constant. For the $(L-1)$-wave decays, which contain a few mixed modes as well, the results have been presented in the form of two tables, one for the unmixed and the other for the mixed cases. Section III is devoted to the mesonic decays. Finally, Sec. IV is a summary of the comparison between the PFF and RFF with the long-range objective of finding applications to some two-body reactions, where the (more important) off-shell structure of the form factors is manifested.

II. BARYON DECAYS

A. $(L+1)$ waves

Since there are a sufficiently large number of $(L+1)$-wave modes, we prefer to present the results in the form of a plot of reduced coupling constant ($g_B{}^2/4\pi$) as deduced from the data on baryonic decays against L (Fig. 1). This type of presentation, which involves a vertical (tower) dependence of the form factor on L or J, is more meaningful than a "horizontal" study of decay modes from within the same supermultiplet, from the point of view of detecting evidence of Regge universality, and exchange degeneracy (EXD) at the level of the reduced coupling constant. Such an analysis was presented for the RFF version in Ref. 11. In this paper we systematize such studies to include other decay modes, especially $\Sigma\pi$ and $\Lambda\pi$, as well as a corresponding analysis with the PFF version.

With the PFF [Fig. 1(a)], the universality requirements (both Regge and EXD) seem to be well satisfied for the $N\pi$ and $N\bar{K}$ modes of the towers $N_{\alpha,\gamma}$, Δ_δ, $\Lambda_{\alpha,\gamma}$, and $\Sigma_{\delta,\beta}$. For the $\Lambda(1520)$ and its Regge recurrences, the standard mixing angle of 20° between the Λ_1^d and Λ_8^d states[13,17] has been used. Even the $\Lambda\pi$ and $\Sigma\pi$ modes of these towers seem to fit in with such a pattern, which indicates a universal reduced coupling constant ($g_B{}^2/4\pi$) confined to the relatively narrow band 1.0 to 2.0 except for a few stray cases such as $\Lambda(1520)\to\Sigma\pi$ and $\Sigma(1765)\to\Lambda\pi$. Figure 1(a) also includes the decays of the first members of the trajectories N_B, Δ_α, Δ_γ, Λ_γ, Λ_β, Σ_γ, Ξ_δ^*. These also help confirm the "narrow band" picture obtained above, except for the isolated case of $N(1670)\to N\pi$. Unfortunately the higher-L counterparts of these first members are not yet available for comparison. From these studies we estimate an average value of $[g_B^{(+)}(P)]^2/4\pi = 1.5\pm0.2$.

For the RFF $[j_l(kR)]$, the results for g_B [Fig. 1(b)] seem to be comparable to those of the PFF, with most of the cases again lying within a narrow band. Some of the modes, e.g., the island of three Λ's [Λ(1520), Λ(1690), and Λ(1830)] show much less scatter with the RFF than with the PFF. On the other hand, a few cases show the opposite trend, notably $\Delta(1670)\to N\pi$, which is wide off the mark. The mean value of $[g_B^{(+)}(R)]^2/4\pi$ is about 0.18 ± 0.03.

Finally, for the RFF with $v_l(kR)$ in place of $j_l(kR)$, the Δ modes, which represent an important ingredient in the study of tower structures,

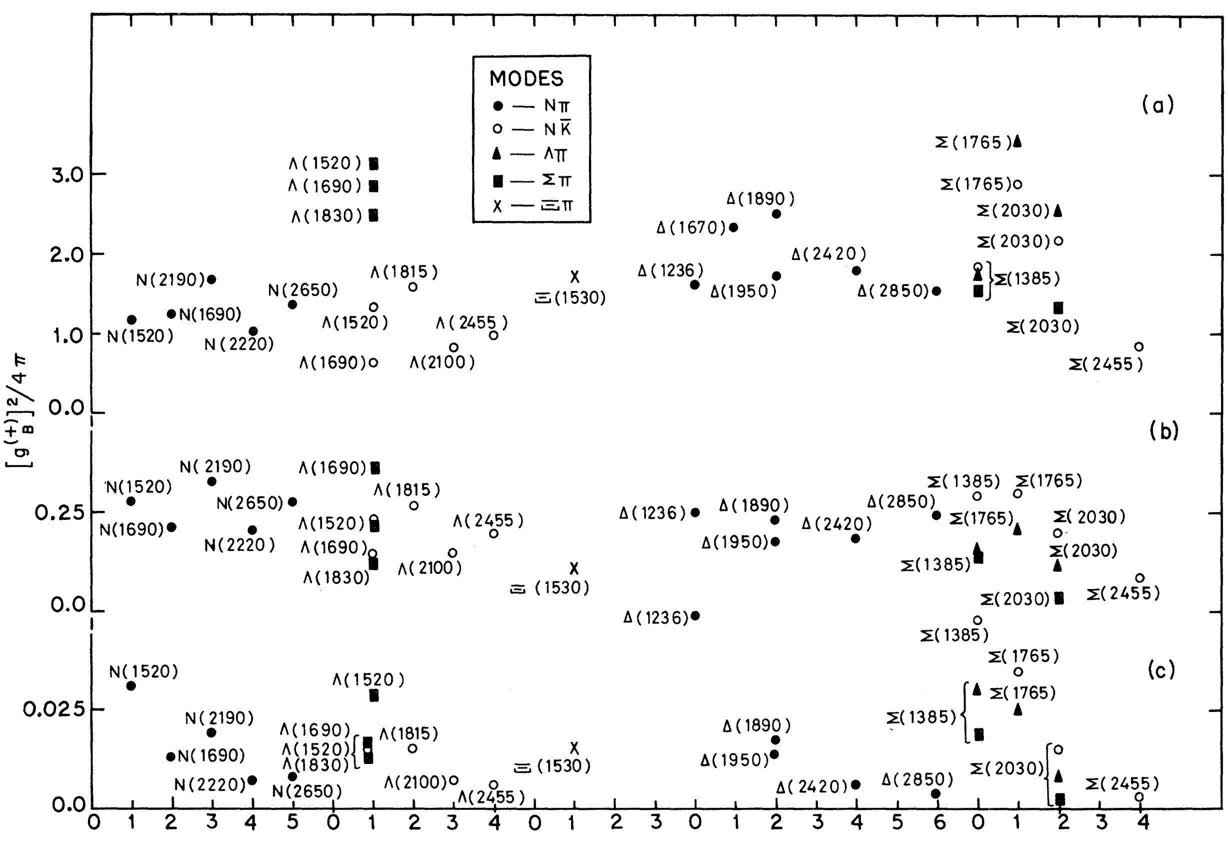

FIG. 1. Plot of the $(L+1)$-wave reduced coupling constant $[g_B^{(+)}]^2/4\pi$ against L for the baryon resonances, for the various form factors (a) PFF, (b) $j_l(kR)$ version of RFF, and (c) $v_l(kR)$ version of RFF.

are rather badly scattered [Fig. 1(c)]. This discrepancy is not removed even with the inclusion of an additional scale factor. Excluding the Δ modes, however, the overall results do not seem to differentiate much between this form and the other two.

On the whole, the $(L+1)$-wave decays do not seem to provide much of a discrimination between the PFF and the RFF.

B. $(L-1)$ wave

Unlike the $(L+1)$-wave modes, lack of adequate data prevents a trajectory-wise analysis of $(L-1)$-wave decays. The data that are available correspond to $L \leq 2$; we have therefore presented the results in a tabular form, separately for the unmixed (Table I) and mixed (Table II) cases, with certain phenomenologically accepted mixing angles for the latter.

As remarked in Sec. I, we have devised a variant of the earliest version (I) of the form factor for the $(L-1)$ wave to accommodate the GOR effect and this is incorporated in Eq. (1.2). Considering first the unmixed cases (Table I), of nine such cases, three involve $1^- \to 0^+$ transitions and the rest are $2^+ \to 0^+$ transitions. These include the important $\Delta\pi$ modes which are mostly dominated by the $(L-1)$ wave (because of the recoil effect[1]), except perhaps for $\Delta(1890) \to \Delta\pi$.[18] As seen from Table I, the $2^+ \to 0^+$ transitions give an almost consistent value of $[g_B^{(-)}(P)]^2/4\pi$ of the order of 0.015. For the $1^- \to 0^+$ transition, there are two anomalous cases, $N(1520) \to \Delta\pi$ and $\Delta(1650) \to N\pi$. We have also included a column with $\chi_k \to E_k$, with the earlier recipe $(E_k{}^{-L})$ for the $(L-1)$ modes.[5] The possibility of an L-independent $g_B^{(-)}$ [as in the $(L+1)$ modes], even for the radius form factor, is ruled out with $R \sim 1$ F. As observed in Sec. I, we have explored the effect of varying R in this regard. Table I shows that at $R \sim 0.6$ F the values of $g_2^{(-)}(R)$ and $g_1^{(-)}(R)$ almost overlap each other compared to their relative scatter for $R = 1.0$ F and 0.4 F. Thus at $R = 0.6$ F it is meaningful to speak of an L-independent reduced coupling constant $[g_B^{(-)}(R)]^2/4\pi$, which works out at about 0.15. The "bad" mode of $\Delta(1650) \to N\pi$ is seen to persist here as well (cf. Rosner[19]).

As to the decays of certain mixed states, the lack of a viable theory of mixing greatly limits our efforts. We confine ourselves to some "phenomenologically accepted" mixing angles (Table II) proposed by various authors. Within these bounds we still seem to find a few interesting items, notably some evidence against a mixing[15] angle of 63°. Rather a small mixing angle (~4°) for the $\underline{8}_q$ and $\underline{8}_d$ states would seem to reopen the question of quartet assignment to $N(1535)$ as first suggested in Ref. 8. Again for $\Lambda(1405)$ a pure singlet assignment seems to be favored over a mixing angle of 8° with $\Lambda(1670)$.[12]

We note in passing the important case of $\Lambda(1405) \to \Sigma\pi$ decay (which is well known to be the only mode for the state) as a possible source of discrimination between the PFF and RFF versions. As is clear from Table II the new $(L-1)$ PFF is found to bridge the gap between the earlier anomalous value of $g_B^{(-)}(p)$ for this mode vis-a-vis other values. On the other hand, a marked disparity between this case and others persist for the RFF. Thus the $(L-1)$-wave decays (within the framework of the GOR effect) seem to offer some means of favoring the PFF over the RFF.

Before concluding the subsection we would like to comment on the performance of the $v_l(kR)$ for the $(L-1)$ decays. While the unmixed modes (Table I) do not provide much discrimination be-

TABLE I. Comparison of the $(L-1)$-wave reduced coupling constant for the unmixed baryon states for the power form factor (both E_k and χ_k) and the radius form factor (both j_l and v_l), the latter for three different radii at 0.4 F, 0.6 F, and 1.0 F.

		Power FF		Radius FF					
					$j_L(kR)$			$v_L(kR)$	
L^P	Decay mode	$\sim E_k{}^{-L}$	$\sim \chi_k{}^{-L}$	0.4 F	0.6 F	1.0 F	0.4 F	0.6 F	1.0 F
1^-	$N(1520) \to \Delta\pi$	0.148	0.006	0.200	0.100	0.060	0.026	0.014	0.012
	$\Delta(1650) \to N\pi$	0.764	0.126	1.188	0.725	0.819	0.227	0.170	0.141
	$\Delta(1670) \to \Delta\pi$	0.119	0.010	0.149	0.075	0.042	0.023	0.014	0.010
2^+	$N(1860) \to N\pi$	0.170	0.020	0.369	0.104	0.047	0.023	0.010	0.006
	$N(1860) \to \Lambda K$	0.397	0.021	0.910	0.201	0.043	0.043	0.012	0.004
	$N(1860) \to N\eta$	0.155	0.015	0.398	0.101	0.027	0.021	0.007	0.003
	$\Lambda(1815) \to \Sigma^*\pi$	0.594	0.006	1.052	0.229	0.041	0.047	0.012	0.003
	$N(1690) \to \Delta\pi$	0.413	0.010	1.015	0.222	0.080	0.046	0.012	0.007
	$\Delta(1890) \to \Delta\pi$	0.380	0.016	0.690	0.167	0.043	0.035	0.012	0.005

TABLE II. Comparison of the $(L-1)$-wave reduced coupling constant for the mixed baryon states for the power form factor (both E_k and χ_k) and the radius form factor (both j_l and v_l), the latter for three different radii at 0.4 F, 0.6 F, and 1.0 F. The mixing angles used have been mentioned in the text in Sec. II.

		Power FF		Radius FF					
	Mixing				$j_L(kR)$			$v_L(kR)$	
Decay mode	angle	$\sim E_k^{-L}$	$\sim\chi_k^{-L}$	0.4 F	0.6 F	1.0 F	0.4 F	0.6 F	1.0 F
$N(1535)\to N\pi$	63°	0.082	0.012	0.137	0.077	0.061	0.024	0.017	0.013
$\to N\eta$		2.347	0.258	4.071	1.871	0.745	0.499	0.254	0.129
$N(1700)\to N\pi$		34.111	5.953	51.418	32.750	44.272	10.184	7.772	6.537
$\to \Lambda K$		0.661	0.063	0.879	0.416	0.185	0.116	0.065	0.038
$\to N\eta$		0.130	0.020	0.191	0.005	0.002	0.028	0.001	0.001
$N(1535)\to N\pi$	4°	0.241	0.034	0.404	0.225	0.178	0.070	0.050	0.039
$\to N\eta$		0.199	0.022	0.345	0.159	0.064	0.042	0.022	0.011
$N(1700)\to N\pi$		0.318	0.056	0.479	0.305	0.413	0.095	0.073	0.061
$\to \Lambda K$		0.137	0.013	0.182	0.100	0.038	0.024	0.013	0.008
$\to N\eta$		0.013	0.002	0.018	0.100	0.005	0.003	0.002	0.001
$\Lambda(1405)\to \Sigma\pi$	0°	1.332	0.041	2.010	0.912	0.351	0.238	0.116	0.053
$\Lambda(1670)\to \Sigma\pi$		0.035	0.003	0.141	0.023	0.014	0.016	0.005	0.003
$\to \Lambda\eta$		0.110	0.010	0.044	0.089	0.025	0.008	0.008	0.003
$\Lambda(1405)\to \Sigma\pi$	8°	1.735	0.032	2.617	1.187	0.456	0.309	0.151	0.067
$\Lambda(1670)\to \Sigma\pi$		0.041	0.004	0.172	0.028	0.017	0.019	0.006	0.004
$\to \Lambda\eta$		0.127	0.011	0.053	0.100	0.028	0.009	0.009	0.003

tween j_l and v_l, the "mixed" cases (Table II) seem to favor the former over the latter. Both these forms are equally unable to accommodate the GOR factor, unlike the PFF.

III. MESON DECAYS

The analysis of data for meson decays on similar lines is necessarily much less systematic than for the baryonic case because of the availability of much fewer data (mainly $L \leq 1$). Indeed we take a rather passive point of view in this regard; namely, we tentatively accept the success of the PFF and RFF [Eqs. (1.1)–(1.4)] and expect it to be generally applicable to the mesonic case on grounds of similarity of form factors. Further, because of the restriction mainly to $L \lesssim 1$, in this case the only meaningful comparison of the data with respect to the L variable must be of a "horizontal" rather than of a "vertical" nature. Before showing the data (in Table III) we must specify our radiation quantum convention. Unfortunately, there is no specific guideline in this regard, so the following convention must be regarded as largely arbitrary:

Generally the *lighter* meson is the *radiation quantum except when* one of them happens to be a vector meson (in which case the latter must be taken as the radiation quantum). This qualification is necessitated by the fact that only certain V mesons can couple to the electromagnetic field via vector-meson dominance (VMD), and the inclusion of K^* in this list is mainly for the sake of completeness.

As to the relativistic normalization factor, which again has been described in detail elsewhere,[1,16] the only choice consistent with energy conservation in the zero-frequency limit is the factor $(4M_L{}^2)^{1/2}$ (M_L = mass of decaying meson), except for the case of equal-mass decays (e.g., $\pi\pi$, $K\bar{K}$), where for some inexplicable reason, which we

TABLE III. List of the $(L+1)$-wave reduced coupling constant $g_M^{(+)2}/4\pi$ for the various meson resonances using the same form factors as for the baryon case with all three form factors.

			RFF at R = 1 F	
L	Decay mode	PFF	$j_l(kR)$	$v_l(kR)$
0	$\rho\to\pi\pi$	0.249	0.352	0.084
	$\phi\to K^+K^-$	0.289	0.130	0.019
	$K^*\to K\pi$	0.260	0.131	0.029
1	$f\to\pi\pi$	0.157	0.224	0.029
	$A_2\to\pi\rho$	0.059	0.156	0.015
	$A_2\to\eta\pi$	0.107	0.048	0.006
	$f'\to K\bar{K}$	0.301	0.166	0.021
	$K_v\to K\pi$	0.169	0.100	0.013
	$K_v\to K\rho$	0.253	0.158	0.011
	$K_v\to K\omega$	0.449	0.291	0.020
	$K_v\to\pi K^*$	0.057	0.158	0.015
2	$g\to\pi\pi$	0.154	0.203	0.018

cannot logically defend, we prefer the factor[1] $(4M_L E_k)^{1/2}$ which was first suggested by Becchi and Morpurgo.[20] Table III summarizes our results for the PFF and two RFF's [Eqs. (1.1)–(1.4)]. It is rather remarkable that the reduced coupling constants deduced from as many as twelve pieces of decay data involving three different values of L ($L=0$, 1, and 2) turn out to be quite consistent with one another, within fairly narrow bands for both the PFF and the RFF. The only serious anomalies are the $A_2 \to \rho\pi$ and $K_V \to \pi K^*$ modes (for the PFF), and even this seems to disappear when the RFF (with symmetrical dependence on ρ and π masses) is used.

For the $(L-1)$ wave, perhaps the only meaningful data are represented by $A_1 \to \pi\rho$ and $B \to \pi\omega$, both of which may be assumed to be dominated by S-wave couplings. Assuming the same form factor as for baryons, viz. Eqs. (1.1)–(1.4), the reduced coupling constants may be estimated from these modes and these are listed in Table IV.

TABLE IV. The value of the $(L-1)$-wave reduced coupling constant for the meson decays $A_1 \to \pi\rho$ and $B \to \pi\omega$ for the three form factors. The radius used here is the same as in the baryon case, namely $R = 0.6$ F.

Decay mode	PFF	RFF at $R=0.6$ F $j_L(kR)$	$v_L(kR)$
$A_1 \to \pi\rho$	0.016	0.043	0.007
$B \to \pi\omega$	0.012	0.028	0.005

IV. SUMMARY AND CONCLUSIONS

We have presented the results of a comparative assessment of the PFF vs RFF via the decay modes of a fairly large number of baryon and meson resonances. Such a comparison at the level of the decay data, which has unfortunately been a rather old-fashioned practice, must, however, be regarded as a first step towards a more meaningful comparison of phenomenological form factors with experiment by considering reaction processes which require extrapolation in the masses.

One of the objectives of the present comparison has been to explore the existence of approximate equality among reduced coupling constants for the successive Regge recurrences as well as EXD partners. For the PFF this objective is largely fulfilled with the form E_k^{-L-1} for the $(L+1)$ wave [Eq. (1.1)], which has found considerable support via several off-mass-shell applications [charge-exchange (CEX) πN scattering,[5] electroproduction,[8] etc.]. However, for the $(L-1)$ wave we have made an important departure from our earlier recipe through the replacement $E_k \to \chi_k$, which seems to be strongly indicated by the data if the GOR effect is to be included. For the RFF we have two versions, one used by the Particle Data Group and the other proposed recently to incorporate certain desirable features of the Harari radius model. We have also considered the possibility of different radii for the $(L\pm1)$ waves.

For completeness we have also included a corresponding comparison between the PFF and RFF for meson decays, using for this purpose the (generally accepted by unproved) principle of similarity between meson and baryon form factors (except for the "quantum" conventions as well as relativistic boson normalization which have been set out in Sec. III).

The results of our analysis indicate that the expectations of a universal reduced coupling constant within a fairly narrow band for both Regge and EXD partners is largely fulfilled, separately for the PFF and the RFF. This is particularly true for the $(L+1)$ wave, where there are plenty of data (both "vertical" and "horizontal") for the baryons and adequate figures (mostly "horizontal" and a few "vertical") for the mesons. Even for the $(L-1)$-wave baryon decays for which the data are confined to $L \leq 2$, a universal reduced coupling constant is seen to be a distinct possibility at least with the PFF.

For the RFF, its lack of compatibility with the GOR effect, as manifested through the important $\Lambda(1405) \to \Sigma\pi$ case, makes it difficult to fulfill the requirements of universality to the same extent. Another interesting result of this study is that the estimated radius for the $(L-1)$-wave RFF is appreciably different (0.6 F) from its more standard value (1.0 F) in the $(L+1)$ case. [For the $(L-1)$-wave meson decays there are hardly any data for comparison between the PFF and RFF.]

The decay data unfortunately do not seem to indicate any preference between the PFF and the RFF. The quality of fits to the decay data for both mesons and baryons is by and large comparable for these two versions, and there seem to be few discriminating criteria except for the GOR effect which favors the PFF somewhat. Finally, the overall performance of the v_l is somewhat weaker than that of the j_l.

One would hope that a more promising approach to possible discrimination between the PFF and RFF might lie in their respective effects on two-body processes requiring extension off the mass shell. While the details of such a study, which indeed seem to fulfill these expectations, will be presented separately, it is worthwhile to mention some of those features which are well borne out by these calculations. In general, the RFF underestimates the cross sections considerably because of an early onset, compared to the PFF, of convergence with J.

The latter is found to give good overlap with the data on several processes [hypercharge exchange (HCEX) reactions, electroproduction, etc.], while the former falls rather short of these data. Thus one would expect poorer performance of RFF where a long tail of resonance contributions is required. One such example is provided by the electromagnetic mass patterns of certain hadrons. Indeed, the results of fairly systematic calculations of n-p and K^+-K^0 mass differences brings out this difference between PFF and RFF rather clearly. The slower convergence of the PFF plays an important role in providing a semiquantitative understanding of these mass differences, while the faster convergence of the RFF is largely responsible for its failure to account for them. The details of the results on electromagnetic mass differences will be reported elsewhere.

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