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# Effect of neutral weak current in the decay $\Sigma^0 \to \Lambda^0 + e^+ + e^-$ in Weinberg's model

H. S. Mani and H. S. Sharatchandra
Department of Physics, Indian Institute of Technology, Kanpur 208016, India
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Weinberg's model of weak and electromagnetic interaction predicts a longitudinal polarization of $\Lambda^0$ in the decay $\Sigma^0 \to \Lambda^0 + e^+ + e^-$ which can be calculated uniquely, in terms of the Weinberg angle. The polarization of $\Lambda^0$ is in general of the order of $10^{-5}$. However, for small $|\vec{p}_\Lambda|$ ($\Lambda^0$ momentum)$\simeq 10^{-2}$ MeV/$c$, the polarization is large and is $\simeq 0.1\%$ for the Weinberg angle $\theta_W \simeq 35°$. The width of the peak, where the polarization is large, is narrow and is in the region where the number of events expected is small.

## I. INTRODUCTION

Models[1] for weak interactions using gauge theories predict a neutral weak current in addition to charged ones and the electromagnetic current. Experiments[2] stimulated by this model do appear to indicate the existence of a neutral current. For the weak-interaction theory, establishing such a weak neutral current is of great importance. The experiments on the neutrino scattering on hadrons have been theoretically analyzed on the basis of the parton model, and one finds that the data are consistent with the model of Weinberg,[1] with the Weinberg angle around 35°.

We have investigated the effect of neutral weak currents in the decay of $\Sigma^0 \to \Lambda^0 + e^+ + e^-$ using the Weinberg model. The decay is mainly due to electromagnetic interactions, and the effect of neutral currents can be identified by looking for the parity-violating effects.[4] A simple order-of-magnitude estimate for the ratio

$$\left|\frac{\text{weak amplitude}}{\text{em amplitude}}\right|$$

for $\Sigma^0 \to \Lambda^0 + e^+ + e^-$ can be obtained[4] and is $\simeq 10^{-3}$. We have calculated the polarization of $\Lambda^0$ as a function of its momentum $|\vec{p}_\Lambda|$ for the case of unpolarized $\Sigma^0$. It turns out that, to a great accuracy, the polarization is predicted uniquely in terms of the Weinberg angle. In general the polarization is small and is of the order of $10^{-5}$. Also we find that for small values of $|\vec{p}_\Lambda|$ the polarization increases and the maximum occurs at $|\vec{p}_\Lambda| \simeq 10^{-2}$ MeV/$c$ where the polarization is 0.1%. The details of the calculation are given in Sec. II. In Sec. III we make a few remarks about the calculations.

## II. DETAILS OF THE CALCULATION

In lowest order the electromagnetic amplitude for the decay of $\Sigma^0 \to \Lambda^0 + e^+ + e^-$ is

$$\mathfrak{M}_{\text{em}} = (-i)^2(e)(-e)\langle \Lambda^0 | j_\mu^{\text{em}} | \Sigma^0 \rangle \left(\frac{-i}{q^2}\right) \times \bar{u}(p_1)\gamma^\mu v(p_2) . \qquad (1)$$

The lowest-order contributions to the neutral weak current are

$$\mathfrak{M}_W = (-i)^2 \frac{(g^2+g'^2)^{1/2}}{2} \langle \Lambda^0 | \mathcal{J}_\mu^W | \Sigma^0 \rangle \left(\frac{-i}{-m_Z{}^2}\right) \times \frac{(g^2+g'^2)^{1/2}}{4} \bar{u}(p_1)\gamma^\mu (C_V + C_A\gamma_5) v(p_2). \qquad (2)$$

The Weinberg model[1] predicts a neutral weak current,

$$\mathcal{J}_\mu^Z = 4[\cos^2\theta_W \bar{L}\gamma_\mu \tfrac{1}{2}\tau_3 L + \sin^2\theta_W(\tfrac{1}{2}\bar{L}\gamma_\mu L + \bar{R}\gamma_\mu R)] , \qquad (3)$$

where $\theta_W$ is the Weinberg angle, $L$ is the SU(2) doublet $\frac{1}{2}(1-\gamma_5)\binom{\nu_e}{e}$, and $R$ is the SU(2) singlet $\frac{1}{2}(1+\gamma_5)e$. This gives for the electron part

$$\mathcal{J}_\mu^Z = \bar{e}\gamma_\mu[(4\sin^2\theta_W - 1) + \gamma_5] e .$$

Thus the vector and axial-vector constants are

$$C_V = -1 + 4\sin^2\theta_W, \quad C_A = +1. \qquad (4)$$

The matrix element of the em current is

$$\langle \Lambda^0 | \mathcal{J}_\mu^{\text{em}} | \Sigma^0 \rangle = + \frac{\mu_{\Sigma\Lambda}}{2m_N} \bar{u}(p_\Lambda) i\sigma_{\mu\nu} q^\nu u(p_\Sigma) , \qquad (5)$$

where $\mu_{\Sigma\Lambda}$ is the transition magnetic moment and $m_N$ is the nucleon mass; $q = p_1 - p_2$. $\mu_{\Sigma\Lambda}$ is to be fixed by the $\Sigma^0 \to \Lambda^0 + \gamma$ decay, whose width is not definitely known. Hence we take recourse to the SU(3) value,

$$\mu_{\Sigma\Lambda} = -\tfrac{1}{2}\sqrt{3}\,\mu_n = +\tfrac{1}{2}\sqrt{3}\,\frac{1.91}{2m_p} , \qquad (6)$$

where $\mu_n$ is the neutron magnetic moment. [See remark (3) in Sec. III.]

The hadron part of the neutral weak current can be written in a model-independent way[5] as

$$\mathcal{J}^W_\mu = \tfrac{1}{2} A^3_\mu + (1 - 2\sin^2\theta_W) V^3_\mu - 2\sin^2\theta_W S_\mu\,, \tag{7}$$

where $A^3_\mu$ and $V^3_\mu$ are isovectors, whereas $S_\mu$ is an isoscalar. $S_\mu$, having $I=0$, cannot contribute to the $\Sigma^0 - \Lambda^0$ matrix element. Actually it turns out that only $A^3_\mu$ gives a major contribution to the decay.

$A^3_\mu$ is related to the charged axial-vector current by an isospin transformation

$$\begin{aligned}\langle \Lambda^0 | A^3_\mu | \Sigma^0 \rangle &= \frac{1}{\sqrt{2}} \langle \Lambda^0 | A^-_\mu | \Sigma^+ \rangle \\ &= + \bar{u}(p_\Lambda)\gamma_\mu \gamma_5 u(p_\Sigma) \times \frac{1}{\sqrt{3}} D\,,\end{aligned} \tag{8a}$$

where $D = 0.79$ from Cabibbo phenomenology.[6]

The vector-current matrix element is

$$\langle \Lambda^0 | V^3_\mu | \Sigma^0 \rangle = + \frac{\mu_{\Sigma\Lambda}}{2m_N} \bar{u}(p_\Lambda) i\sigma_{\mu\nu} q^\nu u(p_\Sigma)\,. \tag{8b}$$

The unknown parameters $g$ and $m_Z$ can be related to the weak-interaction coupling constant:

$$\frac{-iG}{\sqrt{2}} = \left(\frac{g}{2\sqrt{2}}\right)^2 \left(\frac{-i}{-M_W^2}\right)(-i)^2$$

and

$$\frac{g^2}{M_W^2} = \frac{g^2 + g'^2}{M_Z^2}\,. \tag{9}$$

With these signs, the amplitudes are

$$\mathfrak{M}_{\rm em} = \frac{e^2}{q^2} \frac{\sqrt{3}}{2} \times \frac{1.91}{2m_p} \bar{u}(p_\Lambda)\sigma_{\mu\nu} q^\nu u(p_\Sigma) \bar{u}(p_1)\gamma^\mu v(p_2), \tag{10}$$

$$\begin{aligned}\mathfrak{M}_W = &- \frac{iGd}{\sqrt{6}} \bar{u}(p_\Lambda)\gamma_\mu\gamma_5 u(p_\Sigma) \\ &\times \bar{u}(p_1)\gamma^\mu (C_V + C_A \gamma_5) v(p_2)\,.\end{aligned} \tag{11}$$

We can now evaluate the various effects due to the presence of weak interactions. We have chosen to evaluate the longitudinal polarization $P(q)$ of $\Lambda^0$, summing over $\Sigma^0$ polarization. We believe that this is the simplest effect to observe experimentally.

For the polarization calculation, we write in a self-evident notation

$$\mathfrak{M}_W = (v^h_\mu + a^h_\mu)(v^{l\mu} + a^{l\mu})$$

and

$$\mathfrak{M}_{\rm em} = V^h_\mu V^{l\mu}\,.$$

In $|\mathfrak{M}_{\rm total}|^2$, the main contribution comes from the em part $|V^h_\mu V^{l\mu}|^2$. But this goes to zero like $|\vec{p}_\Lambda|^2$ for small $\Lambda^0$ momenta. Hence we would like to add the dominant contribution coming from weak interactions, which remains nonzero in this limit.

$V^h_\mu v^{h}_\nu{}^*$ and $v^h_\mu v^{h}_\nu{}^*$ go to zero like $|\vec{p}_\Lambda|^2$ and are of much smaller magnitude compared with $V^h_\mu V^{h\mu}$. $V^l_\mu a^{l}_\nu{}^*$ and $v^l_\mu a^{l}_\nu{}^*$ terms vanish on integration over the lepton momenta. The term $V^h_\mu a^{h}_\nu{}^* V^{l\mu} v^{l\nu *}$ is zero when the hadron trace is calculated. Thus, the only term to be retained is

$$a^h_\mu a^{h}_\nu{}^* (v^{l\mu} v^{l\nu *} + a^{l\mu} a^{l\nu *})\,.$$

For the parity-violation effects, the dominant contribution comes from the interference term $V^h_\mu a^{h}_\nu{}^* V^{l\mu} v^{l\nu *}$. This goes to zero like $|\vec{p}_\Lambda|$. No other term gives a nonzero contribution in this limit.

Now, after a straightforward calculation, we find

$$P(q) = + C_V \frac{\sqrt{2}GD}{3\pi\alpha \times 1.91} m_N m_\Lambda (m_\Lambda + 3m_\Sigma) \frac{s_\Lambda \cdot p_\Sigma}{\{3m_\Lambda m_\Sigma + p_\Sigma \cdot p_\Lambda - 4p_\Sigma \cdot q p_\Lambda \cdot q/q^2\} + [\text{a correction term}]}\,. \tag{12}$$

The term in curly brackets in the denominator of the right-hand side comes from the dominant electromagnetic contribution. The correction term is the weak contribution, and is of importance only when the $\Lambda^0$ momentum is as low as a few tens of keV/$c$. The em contribution rapidly tends to zero as the momentum transfer (to $\Lambda^0$) becomes smaller. The weak correction makes the denominator nonzero, in the limit of zero momentum transfer.

For $|\vec{p}_\Lambda|$ larger than ~0.1 MeV/$c$ we have, as an excellent approximation,

$$P(q) = - C_V \frac{\sqrt{2}GD}{3\pi\alpha \times 1.91} m_N m_\Sigma \frac{(m_\Lambda + 3m_\Sigma)|\vec{p}_\Lambda|}{m_\Sigma(3m_\Lambda + E_\Lambda) - 4\Delta m_\Sigma (m_\Sigma E_\Lambda - m_\Lambda^2)/(m_\Sigma^2 + m_\Lambda^2 - 2m_\Sigma E_\Lambda)}\,. \tag{13}$$

Figure 1 shows a plot of this.

For $\Lambda^0$ momentum less than 0.1 MeV/$c$, we write

$$\frac{P(q)}{P(|\vec{p}_\Lambda| = 10\,\text{MeV}/c)} = \frac{|\vec{p}_\Lambda| \times 10}{|\vec{p}_\Lambda|^2 + K^2}\,, \tag{14}$$

where

$$K^2 = (|C_V|^2 + |C_A|^2) \frac{2d^2}{3\pi^2 \times (1.91)^2} \frac{\Delta^4}{8m_\Lambda m_\Sigma + 3\Delta^2} \times \frac{G^2 m_p^4}{\alpha^2} \left(\frac{m_\Lambda}{m_p}\right)^2,$$

$$\Delta = m_\Sigma - m_\Lambda\,.$$

A plot of this is shown in Fig. 2.

## III. DISCUSSION

We would like to make the following remarks:

(1) The polarization becomes large for wide-angle $e^+$-$e^-$ pairs. This is essentially because the em cross section decreases rapidly. However, the weak-interaction effects give a nonzero contribution in the limit of zero momentum transfer to $\Lambda^0$. This has the effect of producing a sharp peak in the polarization curve, and making polarization vanish at zero momentum transfer (see Fig. 2).

(2) We need not include the higher order em interaction effects because we are only looking for the parity-violating effects, which come from the interference between the weak and the em terms.

(3) Note that the SU(3) values for $\mu_{\Sigma\Lambda}$ have been used. The experimental value of $\mu_{\Sigma\Lambda}$ is not known. It may be that it is an order of magnitude less. This would be consistent with the experimental limit for the decay widths of $\Sigma^0 \rightarrow \Lambda^0\gamma$ decay. In such a fortunate but unlikely event, the polarization would increase by an order of magnitude.

(4) $C_V$ can lie between $-\frac{1}{2}$ and $\frac{3}{2}$. If we use the values obtained by Palmer,[3] $C_V = 0.34 \pm 0.16$.

(5) The coupling of hadrons and leptons with the heavy scalar mesons of the Weinberg theory need not be taken into account: The coupling is $\sim(m_e/2\lambda)\bar{R}\chi L$. This gives rise to a matrix element with a coefficient $m_e/\lambda m_\phi^2$. This term must be compared with a similar matrix element with a coefficient $\sim(g^2/m_Z^2) = 1/2\lambda^2$. We can neglect the scalar coupling if

$$\frac{m_e}{\lambda m_\phi^2} \ll \frac{1}{\lambda^2}, \text{ i.e., } \frac{m_e \lambda}{m_\phi^2} \ll 1 .$$

Now $\lambda = 1/\sqrt{G}$. Hence

$$\frac{m_e\lambda}{m_\phi^2} = \frac{m_e}{m_\phi^2}\frac{m_p}{(Gm_p^2)^{1/2}} \simeq \frac{m_e}{m_\phi^2}\frac{m_p}{(10^{-5})^{1/2}} \simeq O\left(\frac{m_p^2}{m_\phi^2}\right) .$$

If the scalar meson of Weinberg's theory is heavier than the proton the scalar coupling is of negligible importance.

Even though the polarization obtained is small by the present experimental standards in particle physics, such small polarizations have been measured in nuclear physics.[7] An attempt to measure this is warranted, as the prediction from the Weinberg theory is unique.

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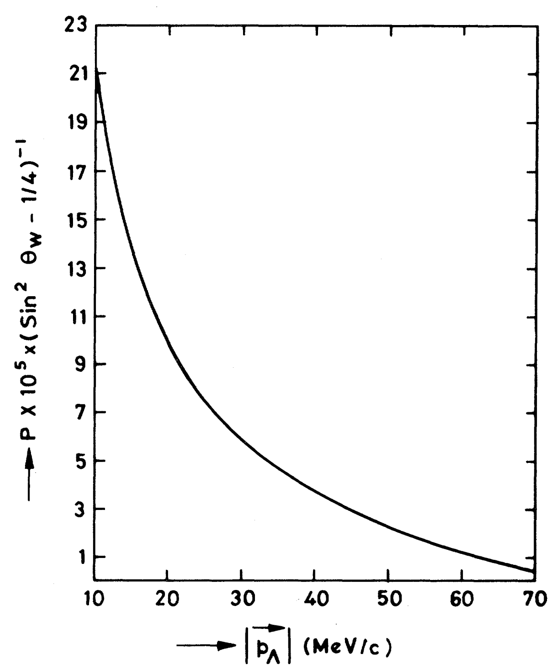


FIG. 1. A plot of $\Lambda^0$ polarization vs its momentum in the range 10–70 MeV/$c$.

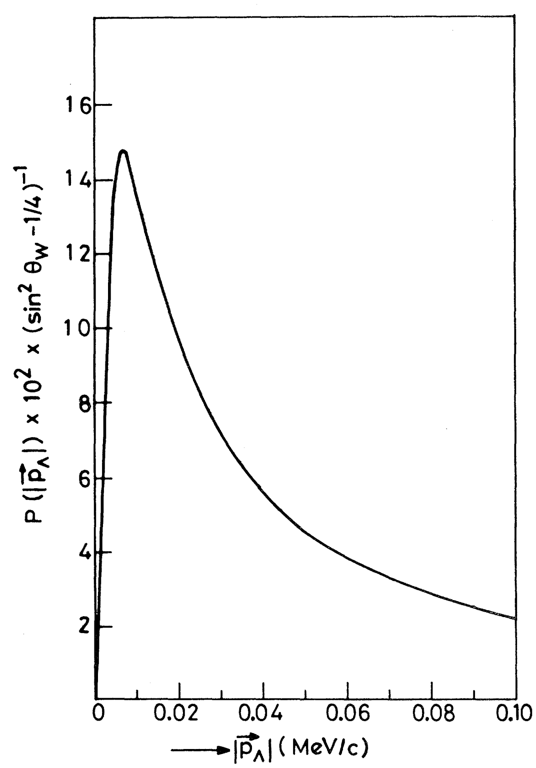


FIG. 2. A plot of $\Lambda^0$ polarization vs its momentum in the range 0–0.1 MeV/$c$.

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