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Lepton number as the fourth "color"

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Universal strong, weak, and electromagnetic interactions of leptons and hadrons are generated by gauging a non-Abelian renormalizable anomaly-free subgroup of the fundamental symmetry structure $SU(4)_L \times SU(4)_R \times SU(4')$, which unites three quartets of "colored" baryonic quarks and the quartet of known leptons into 16-folds of chiral fermionic multiplets, with lepton number treated as the fourth "color" quantum number. Experimental consequences of this scheme are discussed. These include (1) the emergence and effects of exotic gauge mesons carrying both baryonic as well as leptonic quantum numbers, particularly in semileptonic processes, (2) the manifestation of anomalous strong interactions among leptonic and semileptonic processes at high energies, (3) the independent possibility of baryon-lepton number violation in quark and proton decays, and (4) the occurrence of $(V+A)$ weak-current effects.

I. INTRODUCTION

In two recent notes[1,2] we proposed grouping baryonic quarks ($B=1$) and leptons ($L=1$) together as members of the same fermionic multiplet ($F=B+L=1$) and generating weak and electromagnetic as well as strong interactions through a gauging of the symmetry group of this multiplet. In the first place, this postulate of a common fermionic multiplet for all fundamental matter guarantees that in any model of weak interactions, the same[3] $[\frac{1}{2}(1+\gamma_5)]$ helicity projection manifests itself for leptons (as contrasted to antileptons) as is manifested for baryonic quarks. In the second place, the gauging of the symmetry group of matter ensures that all interactions, weak and electromagnetic as well as strong, are universal with respect to baryons and leptons. While the detailed dynamical model of gauge interactions clearly depends on the precise symmetry group one may choose for the fermions (quarks + leptons), it must be emphasized that all such models share the following three characteristics:

(1) Among the gauge particles, there must exist *exotic* particles (X particles) carrying both baryonic as well as leptonic quantum numbers. In the lowest orders of perturbation theory such particles would mediate semileptonic interactions only.

(2) If all allowed gauge degrees of freedom are realized through appropriate gauge bosons, the universality of gauge interactions implies that leptonic and semileptonic interactions must eventually become strong. The asymmetric response of leptons and baryons to strong interactions at presently attained energies would then be interpreted as a "low"-energy phenomenon.[4]

(3) If appropriate spontaneous symmetry-breaking is postulated, there is the (logically independent) possibility of baryonic quarks transforming into leptons, with a violation of baryon and lepton number conservation (though the fermion number $F=B+L$ is still conserved).

In this paper, we wish to concentrate on one class of fermionic models for quarks and leptons. This class was briefly motivated in II; here we shall be concerned with the experimental consequences. However, we wish to emphasize once again that the notion that all fundamental matter is of one variety and that this lepton-baryon unification leads to the three general consequences enumerated above is something which lies at a level much deeper than the particular models discussed in this paper, which may or may not need modifications as new experimental facts emerge, and it is this unification which we principally wish to stress.

II. THE "BASIC" MODEL AND ITS VARIANTS

The central assumption of the "basic" model we propose is that quarks carry four "colors:" Three of these (a, b, and c in our notation; red, blue, and white in the more familiar terminology) represent baryonic matter ($B=1$), and the *fourth* (d or lilac) represents lepton number L.[5] The unification of baryonic and leptonic matter arises by extending the gauge symmetry $SU(3')$ of the three colors[1] (a, b, c) to $SU(4')$ of the four colors (a, b, c, d). We shall assume that the fifteen (1^-) gauge mesons corresponding to $SU(4')$ generate strong interactions with $f^2/4\pi \simeq 1-10$.

Accepting that (spin-$\frac{1}{2}$) quarks form quartets with four *valency* quantum numbers ($I_3 = \pm\frac{1}{2}$, strangeness S, and charm C), with an underlying group structure $SU(4)_L \times SU(4)_R$, the full *global* structure we are postulating [and one which contains the classification symmetry[6] $SU(3) \times SU(3')$ of hadrons] corresponds to

$$G = SU(4)_L \times SU(4)_R \times SU(4') \,. \qquad (1)$$

This symmetry is mathematically realized by a composite structure,[7]

$$\psi_{L,R} = \begin{bmatrix} \mathcal{P} \\ \mathcal{N} \\ \lambda \\ \chi \end{bmatrix}_{L,R} \otimes (a, b, c, d) \,,$$

where the (spin-$\frac{1}{2}$) column ($\mathcal{P}, \mathcal{N}, \lambda, \chi$) indicates valency and the (spin-zero) row (a, b, c, d) indicates color degrees of freedom. A *physical* realization of this structure is provided by the following two 16-fold fermions.

A. Fermions

$$\Psi_{L,R} = \begin{bmatrix} \mathcal{P}_a & \mathcal{P}_b & \mathcal{P}_c & \mathcal{P}_d = \nu \\ \mathcal{N}_a & \mathcal{N}_b & \mathcal{N}_c & \mathcal{N}_d = e^- \\ \lambda_a & \lambda_b & \lambda_c & \lambda_d = \mu^- \\ \chi_a & \chi_b & \chi_c & \chi_d = \nu' \end{bmatrix}_{L,R} = [\Psi^i_\alpha]_{L,R} \,.$$

Their transformation properties are

$$\Psi_L = (4, 1, \bar{4})_G \,,$$

$$\Psi_R = (1, 4, \bar{4})_G \,,$$

$$i = \mathcal{P}, \mathcal{N}, \lambda, \chi \,,$$

$$\alpha = a, b, c, d \,.$$

These multiplets contain twelve baryonic quarks, together with a lepton quartet which we have identified with the *known* leptons.[8]

B. Gauge mesons in the "basic" model

The maximal[9] anomaly-free (renormalizable) subgroup of the valency group $SU(4)_L \times SU(4)_R$ is $Sp(4)_L \times Sp(4)_R$, for which each quark (or lepton) quartet transforms as a 4-component internal-symmetry spinor. Likewise (without a doubling of quarks and leptons), the maximal anomaly-free strong gauge group, which contains the strong $SU(3')$ as a subgroup, is $SU(4')_{L+R}$. Accepting the principle that a symmetry group is manifested only through the dynamical interactions of the theory, we should gauge

$$Sp(4)_L \times Sp(4)_R \times SU(4')_{L+R} \,,$$

yielding a total of $10 + 10 + 15 = 35$ gauge fields. However, most of the essential features of the model are retained, insofar as its physical predictions are concerned, if we simplify our considerations and choose to work with the smaller local subgroup

$$\mathcal{G} = [SU(2)_L^{\mathrm{I+II}}] \times [SU(2)_R^{\mathrm{I+II}}] \times SU(4')_{L+R} \,, \qquad (2)$$

for which Ψ_L and Ψ_R transform as $(2+2, 1, \bar{4})$ and $(1, 2+2, \bar{4})$, respectively. In the sequel we shall do this. The groups $SU(2)_L^{\mathrm{I}}$ and $SU(2)_L^{\mathrm{II}}$ act on the $(\mathcal{P}, \mathcal{N})_L$ and $(\lambda, \chi)_L$ indices, respectively (or rather, on the corresponding Cabibbo-rotated fields; see Sec. IV), while $SU(2)_L^{\mathrm{I+II}}$ is their diagonal sum.

Before we discuss the structure of the local gauges, let us list some of the general features of the proposed gauge scheme:

(1) In contrast to the scheme proposed in I [where only the subgroup $SU(3') \times U(1')$ of $SU(4')$ was gauged], the present scheme treats leptons and baryons universally even so far as strong gauge couplings are concerned. As will be seen in Sec. III, the presently observed differences between leptons and baryons in this regard will be attributed (through a mechanism of spontaneous symmetry breaking) to a heavy mass of those strong gauge mesons, which interact with the leptons. The advantages[10] of the restricted gauge scheme proposed in I, in respect of effective strong interactions generated through the mediation of a relatively light $SU(3')$ octet, are of course preserved in the present scheme.

(2) If the bare coupling constants $g_L^{(0)}$ and $g_R^{(0)}$ are equal, the Lagrangian would exhibit *complete symmetry* between left and right helicities insofar as fermion-gauge-meson interactions are concerned. The observed left-right asymmetry (i.e., parity violation) at low and medium energies may thus be ascribed to heavier masses of the "right-hand" weak gauge mesons compared to the "left-hand" ones, introduced via spontaneous symmetry breaking.

(3) An advantage of gauging the full $SU(4')$ *and* the right-hand gauges[11] (in contrast to the restricted scheme of I) is that it is possible to generate electromagnetism without ever introducing an Abelian $U(1)$-gauge group for this purpose. The elimination of an Abelian quantum-number contribution to electric charge is most desirable in understanding why electric charge is so quantized. Furthermore, the absence of $U(1)$ may have importance in securing "asymptotic freedom" for the complete theory, including electromagnetism.

Below we list the set of 21 $(=3+3+15)$ gauge particles corresponding to the "basic" model with the local gauge group $\mathcal{G}$. These are

$$W_L = (\underline{3}, 1, 1) = \begin{bmatrix} \tau\cdot W_L & 0 \\ 0 & \tau_1(\tau\cdot W_L)\tau_1 \end{bmatrix}, \qquad \text{coupling } \frac{g_L{}^2}{4\pi} \simeq \alpha$$

$$W_R = (1, \underline{3}, 1) = \begin{bmatrix} \tau\cdot W_R & 0 \\ 0 & \tau_1(\tau\cdot W_R)\tau_1 \end{bmatrix}, \qquad \text{coupling } \frac{g_R{}^2}{4\pi} \simeq \alpha \tag{3}$$

$$V = (1, 1, \underline{15}) = \frac{1}{\sqrt{2}} \begin{bmatrix} V(8) - \dfrac{S^0\times 1}{\sqrt{12}} & X \\ \bar{X} & (\frac{3}{4})^{1/2} S^0 \end{bmatrix}, \qquad \text{coupling } \frac{f^2}{4\pi} \simeq 1 \sim 10 .$$

$V(8)$ in the 3×3 matrix block for V denotes the SU(3′) color octet of gauge mesons consisting of V_ρ, V_{K^*}, and V_8. X is an exotic ($B=+1$, $L=-1$) SU(3′) triplet[12] with members $(X^0, X^-, X^{-\prime})$, and S^0 is an SU(3′) singlet.

Defining $\nabla_\mu \Psi = \partial_\mu \Psi + igW\Psi_\mu - if\Psi V_\mu$, the Fermi Lagrangian is given by

$$-\mathrm{Tr}[\bar{\Psi}_L(\gamma_\mu\nabla_\mu)_L\Psi_L + (L \to R)] . \tag{4}$$

Thus

$$L_{\text{int}} = g_L \sum_{\alpha=a,b,c,d} (\bar{\mathcal{P}}_\alpha \bar{\mathfrak{N}}_\alpha \bar{\lambda}_\alpha \bar{\chi}_\alpha)_L (W_L)_\mu \gamma_\mu [\tfrac{1}{2}(1+\gamma_5)] \begin{bmatrix} \mathcal{P}_\alpha \\ \mathfrak{N}_\alpha \\ \lambda_\alpha \\ \chi_\alpha \end{bmatrix}_L + g_R \sum_{\alpha=a,b,c,d} (\bar{\mathcal{P}}_\alpha \bar{\mathfrak{N}}_\alpha \bar{\lambda}_\alpha \bar{\chi}_\alpha)_R (W_R)_\mu \gamma_\mu [\tfrac{1}{2}(1-\gamma_5)] \begin{bmatrix} \mathcal{P}_\alpha \\ \mathfrak{N}_\alpha \\ \lambda_\alpha \\ \chi_\alpha \end{bmatrix}_R$$

$$+ f \sum_{i=\mathcal{P},\mathfrak{N},\lambda,\chi} (\bar{\psi}^i_a \bar{\psi}^i_b \bar{\psi}^i_c \bar{\psi}^i_d)_{L+R} V_\mu \gamma_\mu \begin{bmatrix} \psi^i_a \\ \psi^i_b \\ \psi^i_c \\ \psi^i_d \end{bmatrix}_{L+R} . \tag{5}$$

The complete Lagrangian (after Cabibbo rotations) is exhibited in (16a) and (16c) of Secs. IV B and IV C.

C. The photon

To identify the photon field we must fix on a charge formula for the fermionic multiplet. It is easy to show that the postulate that the known baryons are three baryonic quark composites and have $F_3' = F_8' = 0$ leaves us with just the following choice for the charge operator[13] Q:

$$Q = I_{3L}^{\mathrm{I+II}} + I_{3R}^{\mathrm{I+II}} + \left[\alpha F_3' + \frac{\beta}{\sqrt{3}} F_8' - (\tfrac{2}{3})^{1/2} F_{15}'\right] . \tag{6}$$

Here $(I_3^{\mathrm{I+II}})_{L,R}$ denote the diagonal generators of $\mathrm{SU}(2)^{\mathrm{I+II}}_{L,R}$, while F_3', F_8', and F_{15}' are the diagonal generators of $\mathrm{SU}(4')_{L+R}$. The coefficients α and β of F_3' and F_8' are arbitrary. This results in the following charge assignments for the fermionic multiplets $\Psi_{L,R}$:

$$[Q(\alpha,\beta)] = \begin{bmatrix} \frac{2}{3} - \frac{1}{2}\alpha - \frac{1}{6}\beta & \frac{2}{3} + \frac{1}{2}\alpha - \frac{1}{6}\beta & \frac{2}{3} + \frac{1}{3}\beta & 0 \\ -\frac{1}{3} - \frac{1}{2}\alpha - \frac{1}{6}\beta & -\frac{1}{3} + \frac{1}{2}\alpha - \frac{1}{6}\beta & -\frac{1}{3} + \frac{1}{3}\beta & -1 \\ -\frac{1}{3} - \frac{1}{2}\alpha - \frac{1}{6}\beta & -\frac{1}{3} + \frac{1}{2}\alpha - \frac{1}{6}\beta & -\frac{1}{3} + \frac{1}{3}\beta & -1 \\ \frac{2}{3} - \frac{1}{2}\alpha - \frac{1}{6}\beta & \frac{2}{3} + \frac{1}{2}\alpha - \frac{1}{6}\beta & \frac{2}{3} + \frac{1}{3}\beta & 0 \end{bmatrix} . \tag{7}$$

Note that the baryonic quarks (in the first three columns) may be assigned a wide variety of charges, but leptons associated with the fourth color possess the *unique assignment* of charges[3] (0, −1, −1, 0). In the sequel we shall consider two special choices for α and β:

the integer-charge model: $\alpha = \beta = +1$, (8)

the fractional-charge model: $\alpha = \beta = 0$, (9)

which serve to bring out the main contrasting features of different submodels within our scheme. Corresponding to the charge formula (6), the photon field will be made up of appropriate pieces from W_{3L}, W_{3R}, $V(8)$, and S^0 (see Sec. IV). [Note that SU(4) and SU(4′) contribute symmetrically to

Q for the integer-charge model, which we concentrate on, in the main, unless otherwise stated.]

D. Variants to the "basic" model

If electron number L_e and muon number L_μ correspond to *distinct* colors, the following simple variants may be considered:

(a) The "economical" model. Take as basic fermions the four 8-folds:

$$(\Psi_e)_{L,R} = \begin{pmatrix} \mathcal{P}_a & \mathcal{P}_b & \mathcal{P}_c & \nu \\ \mathfrak{N}_a & \mathfrak{N}_b & \mathfrak{N}_c & e^- \end{pmatrix}_{L,R},$$

$$(\Psi_\mu)_{L,R} = \begin{pmatrix} \lambda_a & \lambda_b & \lambda_c & \mu^- \\ \chi_a & \chi_b & \chi_c & \nu' \end{pmatrix}_{L,R},$$

with the symmetry group

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4')_e \times \mathrm{SU}(4')_\mu .$$

The number of fermions is the same as in the "basic" model; however, the number of gauge bosons has increased to $3+3+15+15=36$. The physical SU(3′) may now be identified with the diagonal sum of $\mathrm{SU}(3')_e$ and $\mathrm{SU}(3')_\mu$, whose emergence will require a more elaborate Higgs-Kibble set of scalars than are needed for the "basic" model (see Sec. IV).

(b) The "prodigal" model. A model similar in structure to the "basic" model, although more prodigal in quarks and leptons needed, could be constructed with the following basic fermions:

$$(\Psi_e)_{L,R} = \begin{bmatrix} \mathcal{P}_a & \mathcal{P}_b & \mathcal{P}_c & E^0 \\ \mathfrak{N}_a & \mathfrak{N}_b & \mathfrak{N}_c & E^- \\ \lambda_a & \lambda_b & \lambda_c & e^- \\ \chi_a & \chi_b & \chi_c & \nu \end{bmatrix}_{L,R},$$

$$(\Psi_\mu)_{L,R} = \begin{bmatrix} \mathcal{P}'_a & \mathcal{P}'_b & \mathcal{P}'_c & M^0 \\ \mathfrak{N}'_a & \mathfrak{N}'_b & \mathfrak{N}'_c & M^- \\ \lambda'_a & \lambda'_b & \lambda'_c & \mu^- \\ \chi'_a & \chi'_b & \chi'_c & \nu' \end{bmatrix}_{L,R}.$$

Here E^0, E^-, M^0, M^- are new heavy leptons and the primed particles are new quarks.[14] Notice that for this model both neutrinos (ν' as well as ν) can be "charmed" so that they may be coupled to charmed quarks χ and χ' through X. (This will have implications for the limits on masses of X particles; see Sec. III.)

(c) The five-color model. One may take as the basic set of fermions a 20-fold Ψ with the symmetry group, $\mathrm{SU}(4)_L \times \mathrm{SU}(4)_R \times \mathrm{SU}(5')$, where

$$\Psi_{L,R} = \begin{bmatrix} \mathcal{P}_a & \mathcal{P}_b & \mathcal{P}_c & E^0 & M^0 \\ \mathfrak{N}_a & \mathfrak{N}_b & \mathfrak{N}_c & E^- & M^- \\ \lambda_a & \lambda_b & \lambda_c & e^- & \mu^- \\ \chi_a & \chi_b & \chi_c & \nu & \nu' \end{bmatrix}_{L,R}.$$

Once again, we can assign "charm" both to ν and ν'. [As will be seen in Sec. III, what chiefly distinguishes all these variants from the "basic" model is the forbiddenness of the transition $K^0 \to e^- + \mu^+$. This transition is allowed in the "basic" model through the mediation of the exotic X's. As a consequence of this, while for the "basic" model the X's must be superheavy ($m_X > 10^4$–10^5 GeV), they need not be much more massive than 10^2–10^3 GeV for the variants.]

From a pure theoretical point of view, none of these "variants" is as attractive as the "basic model." However, at this state of experimental uncertainty, we do not wish to prejudice the issue of a final choice.

III. LIMITS ON GAUGE-MESON MASSES

All models discussed above give rise to exotic strong interactions. In order to account for their absence in the present energy domain, some of the gauge mesons must be heavy or superheavy. Such interactions are generated by three sets of gauge bosons in the scheme:

(1) the exotic vector X triplet ($X^0, X^-, X^{-\prime}$), whose interactions in the "basic" model read [see (5) and (16a)]

$$\begin{aligned} f[X^0(\bar\nu\mathcal{P}_a + \bar e\mathfrak{N}_a + \bar\mu\lambda_a + \bar\nu'\chi_a) \\ + X^-(\bar\nu\mathcal{P}_b + \bar e\mathfrak{N}_b + \bar\mu\lambda_b + \bar\nu'\chi_b) \\ + X^{-\prime}(\bar\nu\mathcal{P}_c + \bar e\mathfrak{N}_c + \bar\mu\lambda_c + \bar\nu'\chi_c) + \mathrm{H.c.}] ; \quad (10) \end{aligned}$$

(2) the exotic S^0 meson, whose coupling is given by

$$\frac{-f}{\sqrt{24}}\Big[\sum_{\alpha=a,b,c}(\bar{\mathcal{P}}_\alpha\mathcal{P}_\alpha + \bar{\mathfrak{N}}_\alpha\mathfrak{N}_\alpha + \bar\lambda_\alpha\lambda_\alpha + \bar\chi_\alpha\chi_\alpha) - 3(\bar\nu\nu + \bar e e + \bar\mu\mu + \bar\nu'\nu')\Big]S^0; \quad (11)$$

(3) the right-hand gauge mesons W_R, which lead to weak $(V+A)$ interactions.

The following sequences of masses will suppress reactions arising from sources (1), (2), and (3) to the presently observed extent, both in tree and (one can show) also for the loop diagrams (see Sec. IV D for an example of the operation of the suppression mechanism for loop diagrams[15]).

(i) The X couplings contribute to $\eta^0, \pi^0 \to e^+e^-$, $\mu^+\mu^-$, and (in the "basic" model *only*) to $K^0 \to e^- + \mu^+$, $\bar K^0 \to e^+ + \mu^-$. Since the observed amplitude for $K_L \to \mu^+ + \mu^-$ is of the order of $G_F\alpha^2$ and no events of the variety $K_L \to \mu^\pm + e^\mp$ have yet been observed, there is a lower limit on the mass of X in the "basic" model given by $f^2/m_X^2 < G_F \alpha^2$. For $f^2/4\pi \approx 1$, this implies that X must be superheavy ($m_X > 3\times 10^4$ GeV). For variants to the "basic" model, where $K \to e^- + \mu^+$ is forbidden, X need not be much more massive than $m_X^2 \approx G_F^{-1} f^2$, the severest lower limit on m_X coming from nuclear β decay and the ν_e hadronic interaction in the "economical" model. (Note that the ν_μ hadronic interactions through the mediation of X particles are suppressed in the "economical" model since known hadrons are basically charmless and ν_μ carries charm. In the "prodigal" model, both ν_e and ν_μ are charmed so that the lower limit restrictions on m_X are even less severe.)

In the following section, we exhibit the scheme for generating masses of the gauge bosons for the "basic" model only. It is worth remarking, however, that a nonsuperheavy X ($m_X \simeq 100$ GeV in the "prodigal" model) will influence $e^-e^+ \to$ hadrons at present center-of-mass energies $\simeq 5$ GeV and may provide an explanation[16] of the recently observed near constancy of the annihilation cross section over a wide range of energies.

(ii) The S^0 coupling leads to order f^2 interactions of neutrinos with hadrons and leptons. In order that the effective strength of such interactions is less than or of order G_{Fermi} at low energies, we expect $(f^2/m_{S^0}{}^2) \lesssim G_F$.

(iii) From the presently observed helicities and other weak-interaction experiments it appears that the $V+A$ amplitudes are at most of order 10% of $V-A$ amplitudes, from which we may conclude (*if* $g_L \simeq g_R$) that $m_{W_R^\pm} \gtrsim 3 m_{W_L^\pm}$.

In addition to the restrictions on the masses of the exotic gauge bosons, there are constraints on the masses of $W_L^\pm$ and the color octet $V(8)$, due to the fact that they should mediate the known $V-A$ interactions and effective strong interactions (between baryonic quarks), respectively. From this we expect that

$$(m_{W_L^\pm})^2 \gtrsim G_F^{-1}\alpha,$$
$$m(V(8)) \simeq 3\text{–}10 \text{ GeV}. \tag{12}$$

We present a summary of expected masses for gauge particles in Table I.

IV. SPONTANEOUS-SYMMETRY-BREAKING MECHANISM

A. Higgs-Kibble particles

In order to generate the postulated sequences of gauge masses (as well as Fermi masses)—and even more important, in order to motivate the broken symmetries observed in nature [i.e., global SU(3), or rather $\mathrm{SU}_I(2)\times \mathrm{U}_Y(1)$ when $g=0$, and the Cabibbo rotation when $g\neq 0$]—one is obliged, until a new renormalizable mechanism is invented, to implement spontaneous symmetry breaking through the expectation-value mechanism of Higgs-Kibble scalar multiplets. (We expect the situation will change with the advent of new ingredients, which may eliminate the need for such scalars, except as a means for bookkeeping in the orderly emergence of the symmetry-breaking pattern.)

At the present state of the art, there is a considerable degree of flexibility in the choice of basic Higgs-Kibble multiplets. However, it is good to reemphasize that once these multiplets are chosen and their general invariant renormalizable (cubic or quartic) interaction potential written down, the pattern of (lowest-order) symmetry breaking which emerges on minimizing this potential is (as a rule) fairly restrictive. This pattern may, of course, get drastically modified through the (radiatively generated) higher-order terms in the effective potential, as shown by Coleman and Weinberg.[17] However, as a first orientation the demand that this particular lowest-order pattern correspond fairly to the physically observed pattern of broken symmetries, or at least to a set of natural symmetries, radiative deviations from which are in principle calculable, may make some choices of basic Higgs-Kibble multiplets more desirable than others.

Be that as it may, a simple choice, capable of satisfying the restrictions on the gauge meson masses for the "basic" model discussed in Sec. III, is provided by a set of three 16-fold complex multiplets, with the cyclic transformation properties:

TABLE I. Summary of expected masses for gauge particles.

Particle	Coupling	Expected (mass)^2
$V(8)$	$\frac{f^2}{4\pi} \simeq 1\text{–}10$	$\simeq \alpha^2 G_F^{-1}$
W_L	$\frac{g_L^2}{4\pi} \simeq \alpha = \frac{1}{137}$	$\gtrsim 4\pi\alpha G_F^{-1}$
W_R	$\frac{g_R^2}{4\pi} \simeq \alpha = \frac{1}{137}$	$> (3 m_{W_L^\pm})^2$
S^0	$\frac{f^2}{4\pi} \simeq 1\text{–}10$	$> f^2 G_F^{-1}$
X	$\frac{f^2}{4\pi} \simeq 1\text{–}10$	superheavy $> \alpha^{-2} G_F^{-1}$ "basic" model
		heavy $> f^2 G_F^{-1}$ "economical" model
		heavy $\lesssim f^2 \alpha G_F^{-1}$ "prodigal" model

$$A=(4,\bar{4},1)_G \to U_L A U_R^{-1},$$
$$B=(1,4,\bar{4})_G \to U_R B V^{-1}, \tag{13}$$
$$C=(\bar{4},1,4)_G \to V C U_L^{-1},$$

where U_L, U_R, V refer to the three global groups $\mathrm{U}(4)_L$, $\mathrm{U}(4)_R$, and $\mathrm{U}(4')_{L+R}$, respectively.

The most general renormalizable quadratic and quartic potential[18] for the three multiplets $V(A,B,C)$ invariant under $\mathrm{U}(4)_L\times \mathrm{U}(4)_R\times \mathrm{U}(4')$ contains twelve parameters [fifteen if the global group is specialized to $\mathrm{SU}(4)_L\times\mathrm{SU}(4)_R\times\mathrm{SU}(4')_{L+R}$] besides the three mass parameters for A, B, and C, provided we impose on the theory the discrete symmetry $A\to -A$, $B\to -B$, $C\to -C$, and $\psi\to\gamma_5\psi$. One can now show that this 12-parameter potential possesses a minimum,[19] provided that $\langle A\rangle$, $\langle B\rangle$, $\langle C\rangle$ are of the form

$$\langle A\rangle = R(\theta_L)R(\phi_L)\langle A_D\rangle R^{-1}(\theta_R)R^{-1}(\phi_R),$$
$$\langle B\rangle = R(\theta_R)R(\phi_R)\langle B_D\rangle, \tag{14}$$
$$\langle C\rangle = \langle C_D\rangle R^{-1}(\theta_L)R^{-1}(\phi_L)$$

Here $R(\theta)$ and $R(\phi)$ are "Cabibbo rotations" of angles θ and ϕ in the $(\mathfrak{N},\lambda)$ and $(\mathcal{P},\chi)$ spaces, respectively, i.e.,

$$R(\theta)=\begin{bmatrix}1&0&0&0\\0&\cos\theta&\sin\theta&0\\0&-\sin\theta&\cos\theta&0\\0&0&0&1\end{bmatrix}, \text{ etc.}$$

while $\langle A_D\rangle$, $\langle B_D\rangle$, and $\langle C_D\rangle$ are diagonal and of the form

$$\langle A_D\rangle=\begin{bmatrix}a_1&&&\\&a_1&&\\&&a_1&\\&&&a_4\end{bmatrix},$$
$$\langle B_D\rangle=\begin{bmatrix}0&&&\\&0&&\\&&0&\\&&&b_4\end{bmatrix}, \tag{15}$$
$$\langle C_D\rangle=\begin{bmatrix}c_1&&&\\&c_1&&\\&&c_1&\\&&&c_4\end{bmatrix}$$

The four angles $\theta_{L,R}$ and $\phi_{L,R}$ are arbitrary at this stage, but the five parameters a_1, a_4, b_4, c_1, c_4 are fully determined in terms of the fifteen parameters of $V(A,B,C)$. Note the remarkable emergence of a global U(3) as the residual symmetry at this stage. [For the 15-parameter potential invariant for $\mathrm{SU}(4)_L\times\mathrm{SU}(4)_R\times\mathrm{SU}(4')$, an extremum exists with

$$A_D=\begin{bmatrix}a_1&&&\\&a_1&&\\&&a_3&\\&&&a_4\end{bmatrix},$$

where $a_1\neq a_3$, so that there is a possibility of residual symmetry being in fact $\mathrm{SU}(2)\times\mathrm{U}(1)$.]

B. The Lagrangian of the model

Consider now the Lagrangian for the "basic" model

$$-\mathcal{L}=\mathrm{Tr}\Big[\sum_{L,R}(\bar{\Psi}\nabla\!\!\!/\,\Psi)+\sum_{A,B,C}|\nabla A|^2+\sum_{V,W_L,W_R}|\nabla V|^2 + V(A,B,C)+\mu\bar{\Psi}_L A\Psi_R+\mathrm{H.c.}\Big], \tag{16a}$$

where

$$\nabla\!\!\!/\,\Psi_{L,R}=\gamma_\mu(\partial_\mu\Psi+igW_\mu\Psi-if\Psi V_\mu)|_{L,R},$$
$$\nabla A=\partial A+ig_L W_L A-ig_R A W_R,$$
$$\nabla B=\partial B+ig_R W_R B-ifBV,$$
$$\nabla C=\partial C+ifVC-ig_L C W_L.$$

Barring the W-containing terms ($g_L=g_R=0$), this Lagrangian is invariant for the full symmetry $G=\mathrm{SU}(4)_L\times\mathrm{SU}(4)_R\times\mathrm{SU}(4')_{L+R}$.

For $g_L\neq0$, $g_R\neq0$, invariance holds for the *local* subgroup $\mathcal{G}=\mathrm{SU}(2)_L^{\mathrm{I+II}}\times\mathrm{SU}(2)_R^{\mathrm{I+II}}\times\mathrm{SU}(4')_{L+R}$, with Ψ_L, Ψ_R, A, B, and C transforming[20] as $(2+2,1,\bar{4})$, $(1,2+2,\bar{4})$, $(2+2,2+2,1)$, $(1,2+2,\bar{4})$, and $(2+2,1,4)$ representations of $\mathcal{G}$. The only new feature, so far as invariance for $\mathcal{G}$ is concerned, is that in addition to the terms included in $V(A,B,C)$ and $\mu\,\mathrm{Tr}[\bar{\Psi}_L A\Psi_R]$, one could now write a host of new renormalizable couplings among these fields:

$$-\delta\mathcal{L}=\lambda\delta V(A,B,C) + \sum_i \mu_{ij}\,\mathrm{Tr}[\bar{\Psi}_L\Gamma_i A\Gamma_j\Psi_R]+\mathrm{H.c.}, \tag{16b}$$

where the Γ_i's are numerical matrices. Such terms are invariant for the subgroup $\mathcal{G}$ but not for G, and act as a perturbation $\delta\mathcal{L}$ to $\mathcal{L}$.

If the local subgroup $\mathcal{G}$ we are dealing with were $\mathrm{Sp}(4)_L\times\mathrm{Sp}(4)_R\times\mathrm{SU}(4')$ [or even $(\mathrm{SU}(2)^{\mathrm{I}}\times\mathrm{SU}(2)^{\mathrm{II}})_L\times(\mathrm{SU}(2)^{\mathrm{I}}\times\mathrm{SU}(2)^{\mathrm{II}})_R\times\mathrm{SU}(4')$], one may prove an important result about the minimization of $V+\lambda\delta V$. This states that the minimization of $V+\lambda\delta V$ leads to solutions for $\langle A\rangle$, $\langle B\rangle$, $\langle C\rangle$, which in the limit $\lambda\to0$ reduce to the unperturbed solutions given (for example) by (15). We have verified the result by examining the detailed structure of the "perturbation" term $\lambda\delta V(A,B,C)$. We conjecture that the same result holds for the local subgroup of interest here, viz. $SU(2)_L^{I+II} \times SU(2)_R^{I+II} \times SU(4')$. If true, this would imply that the (Cabibbo) angles θ, ϕ, etc., as well as departures from the residual global symmetry SU(3), are noncatastrophic functions of λ (and of the radiative corrections to λ, of order g^2, $g^2 f^2$, λf^2, etc.). This has the consequence that by ignoring $\delta\mathcal{L}$ in the first instance [for small or zero renormalized λ, and with the neglect of $O(g^2)$ radiative corrections], we are not running the risk of losing out in unexpected physics so far as the pattern of symmetry breaking is concerned. In view of this we shall henceforth drop the $\delta\mathcal{L}$ terms and work with $\mathcal{L}$. This implies that we expect all further breakdown of symmetries to be radiative in origin.

C. The mass matrix

Returning to the Lagrangian $\mathcal{L}$, let us study the mass terms, obtained by replacing A, B, C by $\langle A\rangle$, $\langle B\rangle$, $\langle C\rangle$. It is convenient to define the physical Fermi fields which diagonalize the Fermi mass matrix through the relation

$$\Psi = R(\theta)R(\phi)\Psi_D .$$

In terms of Ψ_D, and the fields A_D, B_D, C_D [defined through relations similar to (14)], we can write $\mathcal{L}$ in the form

$$-\mathcal{L} = \mathrm{Tr}\Big[\sum_{L,R}(\overline{\Psi}_D \nabla_D \Psi_D) + \sum_{A,B,C} |\nabla_D A_D|^2 + \sum_{V,W_L,W_R} |\nabla V|^2 + V(A_D, B_D, C_D) + \mu(\overline{\Psi}_L A \Psi_R)_D + \mathrm{H.c.}\Big]. \qquad (16c)$$

Here

$$\nabla_D \Psi_D = \partial\Psi_D + igW(\theta,\phi)\Psi_D - if\Psi_D V ;$$

$$\nabla_D A_D = \partial A_D + ig_L W_L(\theta,\phi)A_D - ig_R A_D W_R(\theta,\phi),$$

etc.

with

$$W(\theta,\phi) = R^{-1}(\theta)R^{-1}(\phi)WR(\theta)R(\phi) .$$

We wish now to consider the gauge-meson sector of the mass matrix, including the mixing terms, in more detail. For a first orientation, take $\theta_L = \phi_L = \phi_R = \theta_R = 0$. This leads [with the usual replacement of A by $(A + \langle A\rangle)$ in the $|\nabla_\mu A|^2$ term, and similarly for the B and C fields] to the following expression for the gauge-meson mass terms:

$$\begin{aligned} L^I_{\text{mass}} = {} & \tfrac{1}{4} g_L{}^2(a^2+c^2)[2W_L^+W_L^- + (W_L^3)^2] + \tfrac{1}{4} g_R{}^2(a^2+b_4{}^2)[2W_R^+W_R^- + (W_R^3)^2] \\ & - \tfrac{1}{2} g_L g_R (a^2)(W_L^3 W_R^3) + \tfrac{1}{2} g_L g_R (a_1{}^2 + a_1 a_4)(W_L^+W_R^- + W_L^-W_R^+) + \tfrac{1}{2} f^2 c_1{}^2 (V_3{}^2 + V_8{}^2 + 2V_\rho^+V_\rho^- + 2V_{K*}^+V_{K*}^- + 2V_{K*}^0\overline{V}_{K*}^0) \\ & + \tfrac{1}{8} f^2(c_1{}^2 + 3c_4{}^2 + 3b_4{}^2)(S^0)^2 + \tfrac{1}{2} f^2(c_1{}^2 + c_4{}^2 + b_4{}^2)(X^0\overline{X}^0 + X^-X^+ + X^{-\prime}X^{+\prime}) - fg_L W_L^3\left[c_1{}^2\left(V_3 + \frac{V_8}{\sqrt{3}} - \frac{S^0}{\sqrt{24}}\right) + \frac{3c_4{}^2}{\sqrt{24}} S^0\right] \\ & + fg_R \frac{3b_4{}^2}{\sqrt{24}}(W_R^3 S^0) + fg_L(c_1{}^2)(W_L^+V_\rho^- + W_L^-V_\rho^+) + fg_L(c_1c_4)(W_L^+X^{-\prime} + W_L^-X^{+\prime}), \end{aligned} \qquad (17)$$

where

$$a^2 \equiv 3a_1{}^2 + a_4{}^2 ,$$

and

$$c^2 = 3c_1{}^2 + c_4{}^2 .$$

These give rise to the following masses for the gauge bosons (note that for *this* purpose one may safely ignore the mixing terms such as $W_R^+V_\rho^-$, $W_R^+X^{-\prime}$, and even $W_L^+W_R^-$, a fact which is better justified *a posteriori*; of course, we do not neglect the important physical consequences of the mixing terms):

$$m_{W_L^\pm} \simeq \tfrac{1}{2} g_L (a^2+c^2)^{1/2} ,$$

$$m_{W_R^\pm} \simeq \tfrac{1}{2} g_R (a^2 + b_4{}^2)^{1/2} ,$$

$$m(V(8)) \simeq \frac{fc_1}{\sqrt{2}} ,$$

$$m(X^0, X^-, X^{-\prime}) \simeq \frac{f(c_1{}^2 + c_4{}^2 + b_4{}^2)^{1/2}}{\sqrt{2}} , \qquad (18)$$

$$m_{S^0} \simeq \frac{f}{2\sqrt{2}}(c_1{}^2 + 3c_4{}^2 + 3b_4{}^2)^{1/2} ,$$

$$m_{A^0} = 0 ,$$

$$m_{Z^0} \simeq \tfrac{1}{2}(g_L{}^2 + g_R{}^2)^{1/2} a .$$

Here A^0 denotes the photon and Z^0 a neutral eigenstate, whose complexion is exhibited later. From these expressions one may infer that the restrictions on the gauge-meson masses for the "basic" model outlined in Sec. III are satisfied if we assume that there are basically *three essentially different scales of masses* (vacuum expectation values), characterized by

$$\begin{aligned} c &\simeq (c_1, c_4) \lesssim 1\ \text{GeV} , \\ a &\simeq (a_1, a_4) \approx 300\ \text{GeV} , \\ b &= b_4 \approx 10^4\text{–}10^5\ \text{GeV} . \end{aligned} \qquad (19)$$

The above pattern of vacuum expectation values has the consequence that members of the SU(3′) octet of gauge mesons $V(8)$ possess nearly equal masses of few GeV; $W_L^\pm$ and Z^0 have masses of order 100 GeV; *while all*[21] *the exotic gauge mesons* ($X^0, X^-, X^{-\prime}, S^0$, and $W_R^\pm$) acquire (heavy or) superheavy masses because of the *single parameter* b_4 being large. Looked upon from this point of view, baryon-lepton asymmetry and left-right asymmetry in the low-energy domain is due to this new scale of mass b_4. [Note that *a priori* the roles of b_4 and c_4 are interchangeable for most purposes except for baryon-lepton number violation. One may remark that for the "prodigal" model (which we have not treated in any detail here) b_4 could be as small as 10^2–10^3 GeV, while[19] $c_4 \approx c_1\alpha^2$.]

It is straightforward to diagonalize the vector-meson mass matrix. For the sake of facilitating further discussions, we show below the composition of only the neutral "diagonal" fields. These are obtained by consistently neglecting terms of order (c^2/a^2), (c^2/b^2), and $(g_{L,R}^2/f^2)$, whenever such terms are of no physical significance[22]:

$$\begin{aligned}
\frac{A}{e} &= \left(\frac{W_L^3}{g_L} + \frac{W_R^3}{g_R}\right) + \frac{(V_3 + V_8/\sqrt{3} - (\frac{2}{3})^{1/2} S^0)}{f}, \quad m_A = 0 \\
Z^0 &= \frac{f(g_R W_R^3 - g_L W_L^3) - (\frac{2}{3})^{1/2} g_R^2 S^0 + O(g^2/f^2, c^2/a^2)}{[f^2(g_R^2 + g_L^2) + \frac{2}{3} g_R^4]^{1/2}}, \quad m_{Z^0} \simeq \tfrac{1}{2}(g_L^2 + g_R^2)^{1/2} a \\
\tilde{S}^0 &= \frac{f S^0 + (\frac{2}{3})^{1/2} g_R W_R^3 + O(g^2/f^2, c^2/a^2)}{[f^2 + \frac{2}{3} g_R^2]^{1/2}}, \quad m_{S^0} \simeq (\tfrac{3}{8})^{1/2} f b_4 \\
\tilde{U}^0 &= \frac{g_R W_L^3 + g_L(1+\Delta) W_R^3 - \sqrt{3}\,\bar{f} U^0 - (\frac{2}{3})^{1/2}(g_L g_R/f)(1 + \Delta + \frac{4}{3} c_1^2/b_4^2) S^0}{[g_R^2 + g_L^2 + 3\bar{f}^2 + \frac{2}{3}(g_L g_R/f)^2]^{1/2}}, \quad m_{\tilde{U}^0} \simeq \frac{f c_1}{\sqrt{2}}
\end{aligned} \tag{20}$$

and

$$V^0 = \tfrac{1}{2}(V_3 - \sqrt{3}\, V_8), \quad m_{V^0} = \frac{f c_1}{\sqrt{2}},$$

where[23]

$$\begin{aligned}
\frac{1}{e^2} &= \frac{1}{g_L^2} + \frac{1}{g_R^2} + \frac{2}{f^2}, \\
U^0 &= \tfrac{1}{2}(\sqrt{3}\, V_3 + V_8), \\
\Delta &= 2(f^2/g^2)(c_1^2/a^2), \\
\bar{f} &= \frac{f(g_R^2 + g_L^2)}{2 g_R g_L}.
\end{aligned} \tag{21}$$

D. Color-valency mixing

Note the important circumstance that as a consequence of gauging the B and C multiplets, color and and valency mix,[24] and in particular the exotic X's mix with the W's, leading to a nonconservation of baryon (and lepton) numbers. This mixing term in the mass matrix equals $f g_L c_1 c_4 (W_L^\dagger X^{-\prime} + \text{H.c.})$. Note the following features of this term:

(a) The strength of baryon-lepton number (B-L) violating interaction is directly proportional to c_4, with exact conservation[25] obtaining for $c_4 = 0$.

(b) The W-X mixing term responsible for baryon violation gives rise to the effective propagator

$$\langle WX \rangle = \frac{f g c_1 c_4}{(k^2 + m_X^2)(k^2 + m_W^2)}$$

in momentum space. This propagator is highly convergent so that no infinities are ever encountered in closed-loop calculations involving W-X mixing. [Using standard arguments, i.e., diagonalization of the W-X mass matrix, it is easy to see that the effective coupling strength for the baryon-lepton number violation is $\simeq (f^2 g^2 c_1 c_4)/m_X^2$ so far as closed-loop contributions are concerned.]

E. Gauge-meson masses for the fractional-charge model

In this case, it is worth noting that simple representations of Higgs-Kibble multiplets (for example, B and C) *cannot* be utilized to give masses[26] to the $V(8)$ octet of gauge mesons, though they can furnish masses for the X's and S^0. This is because the appropriate entries in the multiplet, capable of giving masses to $V(8)$, carry electric charge (if $\alpha = \beta = 0$) and therefore must possess zero vacuum expectation values. One can, however, introduce higher *reducible* multiplets such as $(1, 1, 4\times4\times4)$ if one wishes to give masses to the octet SU(3′) gauge mesons. [Note that these restrictions do not apply if (unlike the case for the present scheme) the electric charge contain contributions from an Abelian U(1) gauge.] Note also that the X's in this scheme are fractionally charged so that the X's and W's can never mix. In other words, in a model of the type described above, baryon-lepton number conservation is a consequence of the twin postulates of fermion number and electric charge conservation.

V. FERMION MASSES

A. The fermion mass term

The fermion mass term

$$\mu \operatorname{Tr}[(\overline{\Psi}_L \langle A\rangle \Psi_R)_D + \text{H.c.}],$$

with

$$\langle A_D\rangle = \begin{bmatrix} a_1 & & & \\ & a_1 & & \\ & & a_3 & \\ & & & a_4 \end{bmatrix}$$

cannot provide a distinction between fermions of different colors. Thus baryonic quarks and leptons in the same row of Ψ possess the same mass (in particular $e \leftrightarrow \mathfrak{N}$, $\mu \leftrightarrow \lambda$). The situation is not remedied by the possibility [see (16b)] of adding Yukawa couplings of the type

$$\mu_{ij} \operatorname{Tr}(\overline{\Psi}_L \Gamma_i A \Gamma_j \Psi_R) .$$

Such couplings [which in effect treat A as constituted of four independent submultiplets[20] (2, 2, 1) of $SU(2)_L \times SU(2)_R \times SU(4')$] may assign different masses to $\mathcal{P}$, $\mathfrak{N}$, λ, and χ within the same column. However, the SU(4′)-singlet character of the submultiplets means that there still would be no color distinction.

Such a color distinction could arise if we were willing to introduce an SU(4′)-nonsinglet scalar multiplet such as $A' = (4, \bar{4}, 15)_G$ [or a smaller multiplet $(2, 2, 15)_{\mathcal{G}}$], which is capable of having gauge-invariant Yukawa coupling with fermions. This would have the consequence of assigning one mass (call it a') to the three baryonic quarks in a given row and $-3a'$ to the lepton in the same row of Ψ. Thus the multiplets A and A' could collaborate[27] to make the baryonic quarks consistently heavy and the leptons light. We have, however, avoided[27] introducing a multiplet such as A'. Apart from the undesirability of proliferating Higgs-Kibble multiplets, we wish to retain the SU(4′) color group as a "natural" symmetry in the sense that deviations from it (including baryon-lepton number violation) should eventually be computable.

Even without an A', there exists a definite mechanism in the scheme which could boost the masses of the baryonic quarks without boosting the masses of the leptons. In the strong-interaction sector the baryonic quarks are coupled to the light gauge mesons $V(8)$ as well as to the superheavy ones, whereas the leptons are coupled only to the superheavy or heavy gauge mesons (X's and S^0). Thus, the former may get most of their mass through self-energy contributions involving the *light* $V(8)$ exchanges—something not available to the leptons. Note that the mass difference $(m_q - m_l)$ is *computable* in the scheme, with SU(4′) as a natural symmetry.[28] Of course, second-order perturbation in f is not reliable; it is amusing, however, that one gets[15] the right sign and roughly the correct order of magnitude:

$$\frac{m_q - m_l}{m} \simeq \frac{3}{4\pi}\left(\frac{f^2}{4\pi}\right) \ln\left(\frac{m_X^{\ 2}}{m_{V(8)}^{\ 2}}\right) , \tag{22}$$

which, using renormalization-group ideas, may possibly be an approximation to $\simeq (m_X^{\ 2}/m_{V(8)}^{\ 2})^{3f^2/(4\pi)^2} - 1$. Here m is the zeroth-order common mass of quarks and leptons which may be $\approx m_l$. The baryon-lepton mass difference may quite possibly have its origin in the large magnitude for m_X.

B. The massless neutrinos

In the theory developed so far, the neutrinos ν and ν' are 4-component objects, and even if one could arrange zero bare masses for them (by introducing the multiplet A', for example), nothing can stop their acquiring mass through radiative corrections (e.g., through the γ_5-noninvariant vector interaction[29] $\bar{\nu}\gamma_\mu \mathcal{P} X_\mu$). If the physical neutrinos $(\nu_e)_L$ and $(\nu_\mu)_L$ are indeed (2-component) *massless* objects, the model is presented with a dilemma of massive neutrinos.

Below we suggest a mechanism to resolve this dilemma. The idea is this: A physical spin-$\frac{1}{2}$ particle is massless only if it can be described by a 2-component spinor. To implement the 2-component nature of the physical neutrino, postulate in the model two additional 2-component fermions, ζ_L^e and ζ_L^μ, which are singlets (1, 1, 1) of the *local* group $\mathcal{G}$, and thus possess no gauge interaction. The *only* renormalizable invariant interaction they can possess is of the Yukawa type:

$$h\bar{\zeta}_L^\mu \operatorname{Tr}[\bar{B}\Psi_R] + h'\bar{\zeta}_L^e \operatorname{Tr}[\bar{B}\Gamma\Psi_R + \text{H.c.}] , \tag{23}$$

where the matrix

$$\Gamma = \begin{pmatrix} & \tau_1 \\ \tau_1 & \end{pmatrix}$$

commutes with the generators of $\mathcal{G}$.

Consider now the mass matrix for the complex ζ_L^μ, ν_L', ν_R'. Assuming $\theta_{L,R} = \phi_{L,R} = 0$, for simplicity of discussion and diagonalizing the relevant terms:

$$hb_4 \bar{\zeta}_L^\mu \nu_R' + \mu a_4 \bar{\nu}_L' \nu_R' + \text{H.c.} ,$$

we immediately see that among the 6-component complex (ν_L', ν_R', and ζ_L^μ) there is one massless

2-component left-handed particle which we identify with the physical ν^{μ}:

$$\nu^{\mu}_{\text{physical}} = \frac{\mu a_4 \zeta^{\mu}_L - h b_4 \nu'_L}{(\mu^2 a_4{}^2 + b_4{}^2 h^2)^{1/2}} , \tag{24}$$

plus a massive 4-component fermion. Likewise for the complex $(\nu_L, \nu_R, \zeta^e_L)$. (The restrictions imposed on the parameters h, h' and the angles θ, ϕ by the demands of the μ-e universality can easily be worked out and will not be exhibited here.)

Since the entire set of couplings of ζ^{μ} and ζ^e are contained in (23), these fields may be integrated out for processes not involving them as incoming or outgoing particles, leaving us with an effective interaction

$$\mathcal{L}_{\text{eff}} \sim h^2 \text{Tr}(\bar{B}\Psi_R)\frac{1}{\not{p}}\text{Tr}(\bar{\Psi}_R B) + (h')^2 \text{Tr}(\bar{B}\Gamma\Psi_R)\frac{1}{\not{p}}\text{Tr}(\bar{\Psi}_R\Gamma B) . \tag{25}$$

It is amusing that $\mathcal{L}_{\text{eff}}$ does not contribute to processes involving the physical (left-handed) massless neutrinos, although it would contribute (with strength $\approx h^2$) to semileptonic processes like $e^+ + e^- \to$ hadrons, its characteristic feature being its long-range character (on account of the factor $1/\not{p}$) and the appearance of right-polarized particles Ψ_R.

Before closing, we remark that if radiative corrections to $\langle B\rangle$ are taken into account, such that after including these $\langle B\rangle$ has the most general form (consistent with conservation of charge)

$$\langle B\rangle = \begin{bmatrix} b_1 & & & f_1 \\ & b_2 & f_2 & \\ & f_3 & b_3 & \\ f_4 & & & b_4 \end{bmatrix} , \tag{26}$$

then the interaction term (23) will read

$$h\bar{\zeta}^{\mu}_L(b_1\mathcal{P}_a + b_2\mathfrak{N}_b + b_3\lambda_c + b_4\nu')_R + \text{terms containing } \zeta^e_L \text{ and } f\text{'s} . \tag{27}$$

Clearly, (27) will contribute further to a mixing of different colors and valencies, this mixing being proportional to the (small) parameter h. Conversely, the masslessness of the physical neutrinos [which led to the introduction of the interaction (23) in the first place and the necessity for $\langle B\rangle \neq 0$] may possibly rank as the deeper primary reason behind the mixing of colors and valencies and thus for the violation of *internal* symmetries observed in nature.

VI. EXPERIMENTAL CONSEQUENCES

In this section we list some of the experimental consequences of our scheme.

A. The color octet of vector gluons $V(8)$

For energies sufficiently above threshold, one may expect to produce these particles (expected masses ≈ 3–10 GeV) in *pairs* in normal hadronic collisions with reasonable cross sections. Depending on the nature of conservation of color quantum numbers, one or more of these particles may be semistable. The whole octet is electrically neutral for the fractionally charged quark model ($\alpha = \beta = 0$), while some of its members carry unit charge for the integer-charge quark model.

B. The color components of the photon (U^0)

If the photon contains a color octet component U^0 (i.e., quarks carry integer charges), the object "U^0" can be produced *singly* in photon-induced reactions such as

$$\begin{aligned} &\gamma + p \to p + \text{“}U^0\text{”} , \\ &e^+ + e^- \to \text{“}U^0\text{”} . \end{aligned} \tag{28}$$

We expect that the production cross sections for "U^0"[30] in either reaction should be comparable to that of ρ^0 at appropriate energies. The production of "U^0" in (e^-e^+) annihilation should exhibit a resonant structure similar to that of ρ^0 except that its expected total width is uncertain. If there exist color octet states[31] C' lighter than "U^0," this object would decay strongly to $(C'+\text{hadron})$ or $(C'+C')$; otherwise its primary decay mode would be $(\pi^0+\gamma)$, with secondary decays to (e^+e^-) and $(\mu^+\mu^-)$. A reasonable expectation would be $\Gamma_{(\pi^0+\gamma)} \Gamma_{\text{lepton pair}} \simeq 1:(10^{-3}\text{–}10^{-4})$. To summarize, a search for U^0 using (1) *missing-mass measurements*, (2) $(\mu^+\mu^-)$ production in $(\gamma+p)$ reactions, and (3) (e^+e^-)-annihilation experiments, may offer a direct means of establishing whether the photon contains "color" pieces. If it does, this would favor the integer-charge quark model in contrast to the model with fractional charges. In (e^-e^+) annihilation, apart from single production of "U^0"'s, one should also expect *pair production*[32] of the charged members of the spin-1 color octet mesons (in the integer-charge quark model) above the necessary threshold. It is worth emphasizing that in accordance with the light-cone or parton-model ideas, one may expect the ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ to settle to a value $\sum Q_i{}^2 = 6$ (plus possible contributions from the charged spin-1 gluons) for the integer-charge model and to a value $\frac{10}{3}$ for the fractional-charge model. Also for the integer-charge case, due to the charged spin-1 gluon contributions in a parton model, one should expect $\sigma_{\text{longitudinal}}/\sigma_{\text{transverse}}$ to remain nonvanishing in inelastic electron-nucleon scattering.

C. Neutral neutrino-current processes

$$\nu_\mu + e \to \nu_\mu + e\,,$$
$$\nu_e + e \to \nu_e + e,\ \text{etc. (leptonic)},$$
and
$$\nu + p \to (\nu + \text{hadrons}),\ \text{etc. (semileptonic)}\,. \tag{29}$$

(a) Z^0 exchange. Our Z^0 is almost identical to the "Z^0" of the simple $SU(2)_L \times U(1)$ gauge theories[33] insofar as *leptonic* currents are concerned. One may in fact make the correspondence[34] $(g_L, g_R) \leftrightarrow (g, g')$, where g and g' denote the coupling constants associated with the gauge groups $SU(2)_L$ and $U(1)$ respectively. The "Z^0"'s in the two schemes differ in their coupling to hadronic currents since the U^0 term in our photon does not have a counterpart in our Z^0. The differences become significant when one starts producing $SU(3')$ nonsinglet hadrons. This is because the U^0 current is $SU(3')$ octet.

(b) U^0 exchange. It is important to note that $\tilde{U}^0$ contains (W_L^3, W_R^3, and S^0) essentially in the combination

$$[g_R W_L^3 + g_L W_R^3 - (\tfrac{2}{3})^{1/2}(g_L g_R/f) S^0]\ .$$

This same combination also enters into the photon and is *exactly* decoupled from $(\bar{\nu}\nu)$ current. Thus $\tilde{U}^0$ contributes to leptonic processes *only through correction terms* of order Δ [see Eq. (20)], whose net effect towards leptonic amplitudes can be seen to be of order[35] $(g^2/f^2)^2(\Delta/C^2) \simeq (10^{-6}) \times$ (strong amplitude).

For semileptonic processes there is an additional contribution due to interference of U^0 and S^0 currents which leads to an amplitude of order $(g^2/f^2) \times (\Delta^2/C^2)$. Because the U^0 current is a color octet this will become significant once we are above threshold for production of color nonsinglet states.

(c) S^0 exchange. This directly contributes to both leptonic and semileptonic processes. If it is not superheavy,[21] but instead has a mass $\approx (f/g) M_{W_L^\pm}$, its contribution would be of order G_{Fermi}. There is thus a distinct possibility in our scheme of a *new variety of contribution,* which does not exist in the simple $SU(2)_L \times U(1)$ theory. Since the hadronic current, which is coupled to S^0, is a singlet under *both* $SU(3)$ and $SU(3')$ its contribution to semileptonic processes involving low-lying hadrons is not suppressed by $SU(3')$ selection rules.

To summarize, a study of the cross sections of neutral neutrino-current leptonic *as well as* semileptonic processes can in the first place determine whether departures from the simple $SU(2)_L \times U(1)$-theory[33] predictions are warranted; secondly (since S^0 current is pure *isoscalar*), a study of the *isospin structure* of the hadronic current for semileptonic processes would help determine whether S^0 contribution is significantly present. We should stress that the introduction of S^0 is a consequence of our gauging of the full $SU(4')$ group.

D. Right-handed currents

The scheme explicitly uses $V-A$ as well as $V+A$ currents. If the W_R are not superheavy (i.e, $M_{W_R^\pm} \sim 3M_{W_L^\pm}$), one may expect $(V+A)$ amplitudes at around 10% level of the $(V-A)$ amplitudes. Observable parameters involving helicity and correlation measurements in leptonic and semileptonic weak decays, however, depend upon squares of $(V-A)$ and $(V+A)$ amplitudes without an interference term to the extent that one may neglect the masses of the neutrinos (if they are light-mass 4-component objects).

E. Anomalous interactions of e and μ

Regarding contributions from gauge mesons to such anomalous interactions, the only relevant exchange in the "basic" model is $\tilde{U}^0$, since all other gauge mesons coupled to e and μ (i.e., Z^0, S^0, and the X's) are much too heavy. One may verify [using Eq. (20)] that the $\tilde{U}^0$-exchange contribution to (ee) and $(\mu\mu)$ scattering is of order $(g^2/f^2) \times (k^2/m_U{}^2)$, compared to the one-photon-exchange contribution to the same process, where k^2 denotes (momentum transfer)^2; this is too small to be observable at present. By the same token, the contributions of these additional interactions to anomalous magnetic moments of the electron and the muon appear to be too small to be relevant for a present comparison of theory versus experiment. $\tilde{U}^0$ exchange would, however, contribute significantly in inelastic ep scattering once we are above the threshold for production of $SU(3')$ nonsinglet states. Also, so far as the variants to the "basic" model are concerned, the effects of nonsuperheavy X's could be significant for semileptonic processes[16] and particularly for $e^+ + e^- \to$ hadrons.

F. Parity violation

Parity and strangeness violations in hadronic processes arising from radiative corrections with W_L loops are computable and of order G_{Fermi} (similar to the situation for the gauge model of paper I and for exactly the same reasons)[36]; the contributions of W_R loops are suppressed by the heavier W_R mass. Furthermore, it is easy to show that no large parity and strangeness violations occur in tree diagrams involving the "diagonalized" gauge-meson fields. The one field which might have caused concern is $\tilde{U}^0$, since it is "light." However, $\tilde{U}^0$ contains W_L^3 and W_R^3 essentially in the combination $(g_R W_L^3 + g_L W_R^3)$, which is coupled to pure vector strangeness-conserving current. Thus parity violation due to $\tilde{U}^0$ exchange can arise only through a term of order Δ in the coefficient of W_R^3. Such a term interfering with the U^0 term in $\tilde{U}^0$ leads to nonleptonic amplitudes of order $(g^2/f^2)\Delta \lesssim 10^{-4}$ (compared to normal hadronic amplitudes) and, when interfering with the S^0 term, leads to amplitudes $\simeq (g^2/f^2)\Delta^2 \lesssim 10^{-5}$. The fact that for the U^0 case we are dealing with an SU(3′)-octet transition operator means that there is a further suppression factor for processes involving low-lying hadrons.

G. Violation of baryon and lepton numbers

The present gauge scheme can lead—through a spontaneous-symmetry-breaking mechanism—to a violation of baryon and lepton numbers in the integer-charge quark model. This would lead to quarks disintegrating into leptons. Even with a reasonably large strength for $q \to l + l + \bar{l}$ decays $(G_q \simeq 10^{-9} m_p{}^2)$, there is, however, no conflict[2] with the extraordinary stability of the proton, since the latter is a three-quark composite $(B=3)$ and its triple B decay $(|\Delta B| = 3)$ can occur only in order G_q^3 or higher (depending on additional selection rules). This is assuming that both the quark and the diquark systems are more massive than the proton, or even, alternatively, that there is some field-theoretic mechanism of quark imprisonment, which does not permit hadronic quarks to materialize as physical particles. [In Amati's graphic phrase, in this model, "for a quark imprisoned within a nucleon, the price of liberty is death" (Aix-En-Provence Conference, 1973).]

If G_q is deduced from the proton's lifetime, there are uncertainties in the determination of its precise value. These arise from the fact that the proton decay may be subject to additional selection rules due to SU(3′) quantum numbers (see Ref. 2) and also from the details of the wave function of the proton considered as a three-quark composite. Thus from a purely phenomenological point of view, one may search for (integer-charge) quark decays (for example, in process $q \to l + \pi$ or $q \to l + l + \bar{l}$) with lifetimes ranging from 10^{-10} to 10^{-6} sec on the one hand and very much shorter than 10^{-10} sec on the other. In any case, the highly *energetic* lepton (or leptons) in the decay products should provide a characteristic signature for quark decay. Furthermore, in the decay $q \to l + l + \bar{l}$, one pair of leptons $(l\bar{l})$ coming from virtual W decay would carry longitudinal polarization exhibiting parity violation.

As far as proton decay is concerned, one must emphasize that it is essential to search for *multiparticle* decays of the proton. This is because the minimum number of particles—with fermion number conservation—into which the proton (as a three-quark composite) can decay is three neutrinos plus a pion. Thus we may expect the following decay modes:

$$\begin{aligned} p &\to 3\nu + \pi^+ \\ &\to 4\nu + e^+ \text{ or } 4\nu + \mu^+ \\ &\to 4\nu + \mu^+ + e^+ + e^-, \text{ etc.} \end{aligned} \tag{30}$$

The crucial point is that no two- or three-body decays are allowed.[37]

To conclude, the model provides a number of intriguing experimental possibilities. In particular, we urge a search for (1) color (either gauge-vector bosons or color nonsinglet states) in photon-induced reactions, (2) large isoscalar component in neutrino-induced neutral-current processes, (3) possible anomalous interactions of e and μ, specially at energies above the threshold for color production (for example in *ep* reactions), (4) right-handed $(V+A)$ weak interactions, (5) muon-electron number-violating transitions such as $K \to \mu + e$ decays—this is relevant for the "basic" model only, (6) "nonsuperheavy" exotic X-meson effects in semileptonic processes in general, and for $e^+ + e^- \to$ hadrons in particular (relevant for models other than the "basic"), and finally (7) baryon-lepton number nonconservation in quark and proton decays.

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[1]J. C. Pati and Abdus Salam, Phys. Rev. D 8, 1240 (1973), to be referred to as I.

[2]J. C. Pati and Abdus Salam, Phys. Rev. Lett. 31, 661 (1973), to be referred to as II.

[3]If baryons and leptons belong to distinct representations of some underlying symmetry group, there is no reason why weak interactions might not have picked $V-A$ for *baryons* and *antileptons* $(\bar{\nu}_e, e^+, \mu^+, \bar{\nu}_\mu)$. Since charges of leptons within the same fermionic multiplet as the quarks get *fixed* due to our assumption that they are members of a common fermionic multiplet (see Sec. II), the statement that the same $\frac{1}{2}(1+\gamma_5)$ projection occurs for the two types of matter is *not* a matter of mere terminology of what one may have chosen to call leptons.

[4]The situation here is qualitatively the same as the one encountered in the unification of weak and electromagnetic interactions, with ν_μ and μ^-, for example, sharing universal gauge interactions. The fact that ν_μ exhibits *only* weak interactions (and no interaction of electromagnetic strength) at presently available energies is attributed to the massiveness of W and Z^0.

[5]By L we mean $(L_e + L_\mu)$, where L_e and L_μ are additive electron and muon numbers. It is conceivable that L_e and L_μ need to be further distinguished and correspond to distinct colors. This would give variants of the "basic" model which are briefly considered in Sec. II D.

[6]Whether the observed SU(3) is the diagonal sum of SU(3) × SU(3′) or the SU(3) within our SU(4) may be left open until one begins to observe SU(3′) nonsinglet states. Either possibility can be accommodated into our scheme. Note that the notation of SU(4′) × SU(4″) of I is replaced by SU(4) × SU(4′) in this paper.

[7]In many ways it is attractive to consider the mathematical objects $F = (\mathcal{P}, \mathfrak{N}, \lambda, \chi)$ and $B = (a, b, c, d)$ as fundamental fields and the 16-fold Ψ's to be composite. [For the "five-color" model of Sec. II D (a, b, c, d) are replaced by five colors (a, b, c, d_e, d_μ).] This may provide an answer to why the relevant global symmetry group underlying our scheme is $SU(4)_L \times SU(4)_R \times SU(4')_{L+R}$, and not SU(16) or $SU(16)_L \times SU(16)_R$. It may also provide a reason for the circumstance that one set of interactions (the weak) is chiral in nature, while the other (the strong) is vector; the difference may lie in the spins ($\frac{1}{2}$ and zero) of F and B. In this picture, a binding between F and B {which need not be strong and may be $\approx \alpha$ [see H. Fritzsch and P. Minkowski, Caltech Report No. 68-421 (unpublished)]} would have to arise through some additional mechanism. We propose to pursue the consistency of this idea elsewhere (in particular, the consequences of treating the known leptons as composites). J. D. Bjorken and, independently, O. W. Greenberg (private communications) have been interested in related ideas (specially with regard to the 3-triplet model) from different motivations.

[8]Here ν and ν' are 4-component objects. Later we shall introduce two left-handed singlets ζ_L^e and ζ_L^μ to ensure the emergence of 2-component massless neutrinos ν_L^e and ν_L^μ. The "basic" model, in its final form, thus may contain a total of 16 + 16 + 2 = 34 2-component fields, if neutrinos are massless.

[9]We are indebted to Professor S. Glashow for pointing this out to us.

[10]We have in mind questions such as (i) calculability and strength of parity and strangeness violations [see S. Weinberg, Phys. Rev. Lett. 31, 494 (1973); R. N. Mohapatra, J. C. Pati, and P. Vinciarelli, Phys. Rev. D 8, 3652 (1973)], (ii) classification symmetry of hadrons being limited to SU(3) × SU(3′) and no higher [see J. C. Pati and Adbus Salam, ICTP, Trieste, Internal Report No. IC/73/81 (unpublished)], (iii) saturation properties of hadrons treated as bound states of qqq and $q\bar{q}$ [see M. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965); O. W. Greenberg and D. Zwanziger, Phys. Rev. 150, 1177 (1966); and H. Lipkin, SLAC report, 1973 (unpublished)], and (iv) asymptotic freedom for strong interactions [D. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); and D. Politzer, *ibid.* 30, 1346 (1973)]. {Whether asymptotic freedom is lost by the introduction of Higgs-Kibble scalars in a model like ours based on a semisimple group like $SU(4)_L \times SU(4)_R \times SU(4')$ is discussed in a forthcoming paper by R. Delbourgo and Abdus Salam [Imperial College report (unpublished)]. At any rate this loss is unlikely to be serious if Higgs-Kibble scalars are composites.}

[11]Note that if we did not insist on the full SU(4′) gauging, we could still obtain an elegant left-right symmetric anomaly-free gauge scheme given by $SU(2)_L^{I+II} \times SU(2)_R^{I+II} \times SU(3')_{L+R} \times U(1)_{L+R}$, where U(1) is given by the charm generator $(\frac{2}{3})^{1/2}(F'_{15})_{L+R}$ of SU(4′). This scheme will have all the consequences of the present extended scheme except for the interactions mediated by the X particles. [The U(1)-gauge particle will correspond to S^0 of the present scheme, though its coupling strength would not be related to that of the SU(3′) octet.] In this restricted scheme, it may be possible to preserve masslessness of neutrinos without invoking ζ's (footnote 8). This is because $\nu_L^{e,\mu}$ and $\nu_R^{e,\mu}$ interactions (mediated through W_L, W_R and S^0 gauge mesons) are γ_5-invariant. (One must still examine whether the leftover Higgs-Kibble meson interactions can be arranged to preserve the said γ_5 invariance. We thank Dr. R. N. Mohapatra for pointing this out to us.)

[12]The charges of the X triplet $(0, -1, -1)$ correspond to the integer-charge model. For the fractional-charge model these would be $(-\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3})$.

[13]Note that $(I_{3L}^{I+II} + I_{3R}^{I+II})$ equals $(F_3 + F_8/\sqrt{3} - (\frac{2}{3})^{1/2} F_{15})_{L+R}$, where F_3, F_8, and F_{15} are the diagonal generators of SU(4) defined explicitly in I.

[14]The symmetry group is $SU(4)_L \times SU(4)_R \times SU(4')$. It could also be $SU(4)_L \times SU(4)_R \times SU(4)'_e \times SU(4)'_\mu$, in which case the μ-e distinction is emphasized even more.

[15]The problems of calculability (and renormalizability) of the relevant radiative corrections in the present model have been considered by Dr. D. Ross [Imperial College report (unpublished)].

[16]This suggestion (first made at the Irvine Conference on Weak Interactions, 1973) is the subject of a separate paper [J. C. Pati and Abdus Salam, Phys. Rev. Lett. 32, 1083 (1974)]. *Note added in proof.* However, it should be stressed that in the gauge-theory context, we find it difficult to find a suitable Higgs-Kibble system, which in zeroth order can assign light mass of order 100 BeV to x's and yet appropriately heavy mass to s.

[17]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[18]The general 15-parameter potential $V(A,B,C)$ contains mass terms like $\mathrm{Tr}[AA^\dagger]$ and terms of the type $(\mathrm{Tr}[AA^\dagger])(\mathrm{Tr}[BB^\dagger])$, $\mathrm{Tr}[CAA^\dagger C^\dagger]$, etc., and is invariant for $U(4)_L \times U(4)_R \times U(4')$. When one specializes to $SU(4)_L \times SU(4)_R \times SU(4')$, three additional terms appear. These are proportional to $\det A$, $\det B$, $\det C$.

[19]We are indebted to Dr. Ling-Fong Li and Dr. M. A. Rashid for kindly showing that the solution (15) is not just an extremum of the (12 + 3)-parameter potential $V(A, B, C)$ (with three mass and twelve interaction terms) but represents a possible minimum for a specified sequence of signs of these parameters. A simple illustrative example for the sequence of signs [see Ling-Fong Li, Phys. Rev. D 9, 1723 (1973)] is provided by the (6 + 3 parameter) potential. Take

$$V(A, B, C) = \sum_{A,B,C,\alpha,\beta,\gamma} \{\alpha_1 \mathrm{Tr}[AA^\dagger] + \alpha_2 \mathrm{Tr}[(AA^\dagger)^2] + \alpha_3 \mathrm{Tr}[AA^\dagger AA^\dagger]\},$$

where (besides $\alpha_1, \beta_1, \gamma_1$) β_3 is negative, while all other parameters are positive. Arrange $\gamma_1 : \alpha_1 : \beta_1$ (the mass parameters for C, A, and B) to be in the ratios $1 : \alpha^{-2} : \alpha^{-4}$, while all other parameters are of magnitude $f^2/4\pi \approx 1$. One finds that (c_1, c_4), (a_1, a_4), and (b_4) are in the ratios $1 : \alpha^{-1} : \alpha^{-2}$ ($\alpha = \frac{1}{137}$). If $V(A, B, C)$ contains terms proportional to det A, det B, and det C [corresponding to the symmetry $SU(4)_L \times SU(4)_R \times SU(4')$ rather than $U(4)_L \times U(4)_R \times U(4')$], an extremum exists with

$$\langle A_D \rangle = \begin{bmatrix} a_1 & & & \\ & a_1 & & \\ & & a_3 & \\ & & & a_4 \end{bmatrix},$$

i.e., the solution exhibits a residual symmetry SU(2) $\times$ U(1). We have not examined whether this solution represents a true minimum.

[20]One might have been tempted to replace the multiplet $A = (4, \bar{4}, 1)$ of G by a more economical submultiplet $\tilde{A} = (2, 2, 1)$ of $\mathcal{G}$ with the (apparently allowed) pattern

$$\langle \tilde{A} \rangle = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix},$$

together with $\langle B \rangle$ and $\langle C \rangle$ as in (15). So far as W_L, W_R, and V masses are concerned, one might then have the desired sequence of masses (except that $W_L^+ W_R^-$ mixing would be absent). However, we have preferred to use the larger multiplet A, since the global SU(3) then has a chance to emerge more naturally.

[21]By adding a new multiplet $D = (1, 1, \underline{15})$, it should be possible to make the X's superheavy without affecting the mass of S^0. Likewise, a multiplet $E = (1, \underline{3}, 1)$ would affect only the mass of $W_R^\pm$. With both these multiplets present, b_4 need be no larger than a_4, and S^0 may have a mass as light as $(\frac{3}{8})^{1/2} f a \simeq (f/g) m_{W_L^\pm}$. The introduction of such multiplets does not disturb the pattern of solutions $\langle A_D \rangle$, $\langle B_D \rangle$, $\langle C_D \rangle$ shown in the text.

[22]We have retained seemingly insignificant terms of order $\Delta(g^2/f^2)$ and (c^2/b^2) in $\tilde{U}^0$ partly because the Δ terms may have observable consequences in parity violation in nuclear transitions and partly because without these terms $\tilde{U}^0$ will not appear to be anywhere near an eigenstate of the mass matrix.

[23]Note that the physical particles [with broken SU(3')] may not be $\tilde{U}^0$ and V^0, but rather their linear combinations. This will be the case if the mass matrix assigned unequal to V_3 and V_8 before diagonalization.

[24]If we had not set the θ's and ϕ's equal to zero, all color and valency components would have mixed, and not just the strange exotic X' with W_L. An attractive choice which breaks all color symmetries is $|\theta_L - \theta_R| = 90°$.

[25]In the present scheme (with both B and C present), c_4 can be as small as we wish since the X's can be superheavy through b_4. (This is in contrast to the limited multiplet scheme exhibited in II, where only either B or C was introduced.)

[26]This contrasts with the view of D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973), and S. Weinberg, Phys. Rev. Lett. 31, 494 (1973), who have suggested from independent considerations that the $V(8)$ octet consists of massless particles and the SU(3') color symmetry is exact; the consequent infrared infinities have been welcomed by these authors with the hope that they may prevent emission of quarks and the $V(8)$ gluons.

[27]Such a collaboration may perhaps receive a "natural" interpretation within a U(4') structure if A and A' correspond to $(\bar{\psi}\psi)$ composites. In this case A' may play its legitimate role without spoiling SU(4') natural symmetry and the postulate of $\langle \bar{\psi}\psi \rangle \neq 0$ (with A and A' collaborating) would provide perhaps a more attractive way of building in fermion masses.

[28]We use the words "natural symmetry" in the sense of S. Weinberg, Phys. Rev. Lett. 29, 338 (1972), and H. Georgi and S. L. Glashow, Phys. Rev. D 6, 2977 (1972). The problems of calculability (and renormalizability) of radiative corrections as well as of possible pseudo-Goldstone bosons in the present model have been considered in detail by Dr. D. Ross (see Ref. 15).

[29]Of course, if one could leave $\mathcal{P}_{a,b,c}$ quarks massless [a possibility which is attractive for chiral $SU(2)_L \times SU(2)_R$ symmetry], the $\bar{\nu}\gamma_\mu \mathcal{P} X_\mu$ interaction would be γ_5-invariant. This, however, may not apply to ν' interactions, since the companion χ quarks are presumably massive. We should remark that light quark masses (*analogous* to light lepton masses) would be an interesting possibility, if the idea of "hiding" quarks becomes feasible.

[30]By "U^0" we mean $\tilde{U}^0$ or linear combinations of U^0 and V^0 depending upon the complexion of the physical particles (see footnote 20).

[31]The possible occurrence of such states has been pointed out by M. Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965), and has been considered subsequently by many authors.

[32]The contribution of these spin-1 gluons (which should possess an intrinsic magnetic moment) to the electromagnetic current may play a role either directly (corresponding to the production of these gluons in *pairs*) or indirectly (via the light-cone picture) in providing an explanation for the recently observed near "constancy" of ($e^+e^- \to$ hadrons) over a large range of s = center-of-mass (energy)^2. The gluon pair could contribute a term growing with s in the region of interest, which might compensate for the expected $(1/s)$ fall-off of other contributions.

[33]By simple $SU(2)_L \times U(1)$ theory, we mean the gauge theory of the type suggested by Abdus Salam and J. C. Ward, Phys. Lett. 13, 168 (1964); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967), and Abdus Salam, in *Elementary Particle Physics*, edited by N. Svartholm (Almqvist and Wiksells, Stockholm, 1968), p. 367, together with extension to hadrons as given by S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[34]With this correspondence, the predictions of the two schemes coincide for leptonic processes provided the contributions from other exchanges (in particular S^0) are negligible. In this case, one may describe the neutral-current processes by one gauge angle ξ (see Ref. 33) given by $\sin^2\xi = g_R^2/(g_L^2 + g_R^2)$. If $g_L \simeq g_R$, we expect $\sin^2\xi$ to be nearly 0.5. In line with our suggestion in Sec. II that the starting Fermi Lagrangian may be completely left-right symmetric with equality of *bare* coupling constants $g_L^{(0)}$ and $g_R^{(0)}$, the ratio of the renormalized coupling constants (g_L/g_R) would

differ from unity by calculable finite radiative corrections of order $\alpha \ln(m_{W_R}{}^2/m_{W_L}{}^2)$ to the extent that spontaneous symmetry breaking introduces different masses for W_L and W_R (for example). Thus a study of neutral-current processes and $(V+A)$ interactions could help determine whether there is a complete left-right symmetry $(L \longleftrightarrow R)$ in the basic Fermi Lagrangian.

[35]Note that strong-interaction quark-quark scattering amplitudes arising due to $V(8)$ exchange are of order $(f^2/f^2c^2) \simeq (1/c^2)$.

[36]S. Weinberg, Phys. Rev. Lett. 31, 494 (1973); R. N. Mohapatra, J. C. Pati, and P. Vinciarelli, Phys. Rev. D 8, 3652 (1973).

[37]In the classic experiments of H. S. Gurr, W. R. Kropp, F. Reines, and B. S. Meyer [Phys. Rev. 158, 1321 (1967)], attention was concentrated on relatively high-energy charged secondaries, so that possibly the two-body decays of the proton were the ones which received more emphasis. Such decays are forbidden in the model presented in this paper. One may wonder whether some of the unidentified low-energy charged secondaries could have come from four- (or more-) particle decays of the proton. In a recent paper, F. Reines and M. F. Crouch [Phys. Rev. Lett. 32, 493 (1974)] report five μ events which may possibly have resulted from proton decays in their detectors. In the authors' view, so far as this experiment is concerned, "it seems prudent to interpret the signal so as to yield a lower limit on nucleon lifetime" of 2×10^{30} years. We are indebted to Professor F. Reines, Professor H. S. Gurr, and Professor Z. Zichichi for numerous kind discussions.