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Duality constraints and the baryon spectrum

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It is shown that the exact duality constraints on the baryons, when taken with the baryon spectrum of the harmonic-oscillator quark model, allow $SU(6)_W$-symmetric and crossing-invariant solutions for a wide class of reactions involving the pseudoscalar-meson–baryon scattering. A simple prescription is given for the construction of these solutions. Implications of the solutions on the signs of the resonant amplitudes in SU(3) inelastic processes such as $\pi N \to \pi\Delta$ are deduced and compared with the higher-symmetry predictions.

I. INTRODUCTION

It has been known for some time that duality, when combined with the SU(3) symmetry and the absence of exotic particles, leads to exchange degeneracies among the trajectories in meson-baryon scattering amplitudes.[1] The patterns of the exchange-degenerate trajectories and their coupling strengths are determined by solving the duality constraint equations.

The self-consistency of such a scheme has been a subject of much debate in the past. It was found quite early[2,3] that the exact duality constraints on meson-baryon scattering amplitudes imply the existence of nonsymmetric baryonic states as $\underline{70}$ with $L=0$ (L is the total quark orbital angular momentum). Empirical nonexistence of such states has been considered as a serious difficulty of the theories. A proposed solution to this difficulty was the "broken duality" approach[4,5] which rejects the duality constraints coming from the $MM \to \bar{b}b$ channels. Such a proposal does not seem to receive good experimental support, however, since there has been yet no convincing empirical evidence in meson-baryon scattering which favors the duality constraints coming from the u channels rather than those arising from the t channels.[6]

Another outstanding difficulty of the baryon dual theories has been that concerned with the s, u crossing properties of the baryon spectrum.[7] Since s and u channels are formally the same in meson-baryon scattering, we have not only to require forbidding exotic states in all channels but also to ensure the crossing invariance of the baryon spectrum. Until now several attempts[8,9] have been made to accommodate these requirements into constraint solutions; however, as yet no satisfactory solutions of the baryon exchange degeneracy seem to exist which fulfil the requirement of s, u crossing invariance. As will be made clear in the following, these points are also intimately related to the problems of the t-channel vector-tensor couplings into baryon and antibaryon, which are not determined and left arbitrary in the usual approach of duality.[10]

As regards the difficulty of predicting unobserved states mentioned above, a new type of possible solution has recently been proposed by Rosner,[11] who pointed out that the exact duality constraints on the meson-baryon scattering amplitude can be made consistent with the observed spectrum of the hadrons if we allow the momentum-transfer dependence for the ratios of the residues of the baryon Regge trajectories. This observation originates from the work of Mandelstam,[12] who noted the difference in the relative weights of the symmetric ($\underline{56}$) and mixed-symmetric ($\underline{70}$) representations in s, t and s, u dual meson-baryon amplitudes which come from different topologies of s, t and s, u duality diagrams. Indeed, we find the following combinations of the symmetric and the mixed-symmetric representations for (s, t) and (s, u) dual amplitudes:

$$\begin{aligned} (s,t) &= f(L)(15[56]_s + 16[70]_s), \\ (s,u) &= g(L)(15[56]_s - 8[70]_s). \end{aligned} \tag{1}$$

Here $f(L)$ and $g(L)$ denote the L dependence of the s-channel baryon Regge trajectories in s, t and s, u dual amplitudes. The sum and difference of these two terms give the baryon spectrum at L = even and odd integral values, respectively. Thus if we allow $f(L) \neq g(L)$ and assume

$$\begin{aligned} f(0):g(0) &= 1:2, \\ f(1):g(1) &= 1:1, \end{aligned} \tag{2}$$

then we can eliminate from our spectrum the unobserved states $\underline{70}$ $L=0$ and $\underline{56}$ $L=1$ and we end up with the baryon spectrum predicted by the harmonic-oscillator quark model,[13-15]

$$\begin{array}{lllll} L=0 & L=1 & L=2 & L=3 & \cdots \\ \underline{56} & \underline{70} & \underline{56},\underline{70} & \underline{70},\underline{56} & \cdots . \end{array} \quad (3)$$

This possibility of the harmonic-oscillator spectrum providing an acceptable solution to the exact duality constraints on meson-baryon scattering was conjectured quite early[16]; however, interest in this possibility has only recently been revived by Ref. 11.

In this note we investigate this possibility in detail and show that in this case there indeed exist consistent and factorized solutions of the baryon exchange degeneracy for a wide class of reactions. In all our solutions the meson and baryon exchange-degenerate trajectories appear as $U(6)_W$ multiplets; also, the ratios of their coupling strengths in each partial-wave amplitude possess the values prescribed by $SU(6)_W$. All of our solutions fulfil the requirement of the s, u crossing invariance of the baryon spectrum. The crossing invariance will be realized by making specific combinations of the quark-spin doublet and quartet baryon trajectories in the direct channel corresponding to definite W-spin exchanges in the t channel. Our solutions include all the reactions possible with the external mesons and baryons belonging to $\underline{35}$ and $\underline{56}$ of $SU(6)_W$ representations. Solutions for the processes $PB \to P'B'$, $PB \to P'D'$, and $PD \to P'D'$ (P, B, and D denote the pseudoscalar-meson octet, baryon octet, and baryon decimet, respectively) and those previously obtained by Rosner[11] (quark-spin quartets) and by the present author[17] (quark-spin doublets).

We first derive our solutions by directly solving the duality constraint equations in Secs. II and III, and then give a simple prescription to obtain these solutions without recourse to the SU(3) crossing matrices in Sec. IV. The implications of these solutions on the amplitude signs in SU(3) inelastic processes are deduced and compared with the higher-symmetry predictions in Sec. V. Finally, Sec. VI is devoted to summary and discussions. Technical details about the way of solving the constraint equations are given in the Appendix.

Throughout this paper we confine ourselves to the discussions on meson-baryon scattering and we do not consider the difficulties of exact duality associated with baryon-antibaryon scattering. These problems are out of the scope of the present article.

II. SOLUTION FOR PSEUDOSCALAR-MESON-BARYON SCATTERING

In this section let us consider the case of the pseudoscalar-meson–baryon scattering. The duality constraint equations are given by

$$\sum_i (X_{ts})_{ji}\,\beta_s^i = 0 \quad \text{for } j=\underline{10},\ \underline{10}^*,\ \text{and } \underline{27},$$

and (4)

$$\sum_i (X_{us})_{ji}\,\tau^i\beta_s^i = 0 \quad \text{for } j=\underline{10}^* \text{ and } \underline{27},$$

for this process. Here X_{ts} and X_{us} are the s, t and s, u SU(3) crossing matrices for the case $\underline{8}+\underline{8} \to \underline{8}+\underline{8}$. β_s^i and τ^i are the residues and signatures of the s-channel Regge trajectories, and the summation i runs over different SU(3) representations. We shall use the normalization of the octets as

$$\begin{aligned}[8_{F/D',F/D}]_s = {} & \tfrac{20}{3}D'D[8_{ss}]_s \\ & - 4\sqrt{5}\,(F'D[8_{as}]_s + D'F[8_{sa}]_s) \\ & + 12F'F[8_{aa}]_s ,\end{aligned}$$

where $F+D=F'+D'=1$ and suffices to the octets are the F/D values at each vertex. Labels of the SU(3) states are put into square brackets, and their coupling strengths outside.

We consider in the following the case of the quark-doublet and quark-quartet trajectories which are separately exchange-degenerate in the baryon spectrum of the harmonic-oscillator quark model.

$$\begin{array}{ccccc} L=0 & L=1 & \multicolumn{2}{c}{L=2} & \cdots \\ \underline{56} & \underline{70} & \underline{56} & \underline{70} & \cdots \\ (10,4) & (8,4) & (10,4) & (8,4) & \\ (8,2) & (10,2) & (8,2) & (10,2) & \cdots \\ & (8,2) & & (8,2) & \\ & (1,2) & & (1,2) & \end{array} \quad (3')$$

As will be shown immediately below, such an exchange-degeneracy scheme applying separately to each quark spin is indispensable to obtain the crossing-invariant results.

Then, by making use of the SU(3) crossing matrices[18] and solving the duality constraint equations for each quark spin, we find the following solutions for the pseudoscalar-meson–baryon scattering.[11,17] Details about the way of obtaining them are given in the Appendix.

The solution for quark-spin doublets is

$$\begin{aligned}(s,t) &= f(L)([8_{2/3}]_s + \tfrac{216}{25}[1]_s \\ &\qquad + \tfrac{16}{25}[8_{5/3}]_s + \tfrac{12}{25}[10]_s), \\ (s,u) &= g(L)([8_{2/3}]_s - \tfrac{108}{25}[1]_s \\ &\qquad - \tfrac{8}{25}[8_{5/3}]_s - \tfrac{6}{25}[10]_s) .\end{aligned} \quad (5)$$

The solution for quark-spin quartets is

$$(s,t)=f(L)(\tfrac{48}{25}[10]_s+\tfrac{2}{25}[8_{-1/3}]_s),$$
$$(s,u)=g(L)(\tfrac{48}{25}[10]_s-\tfrac{1}{25}[8_{-1/3}]_s). \tag{6}$$

Here $f(L)$ and $g(L)$ are the same as those which appeared in Eq. (1) and we have chosen the overall normalization such that the coefficient of $[8_{2/3}]_s$ is unity.

Trajectories $\underline{8}_{2/3}$ (octet with $F/D=\frac{2}{3}$) with S (quark spin) $=\frac{1}{2}$ and $\underline{10}$ with $S=\frac{3}{2}$ are identified as those belonging to $\underline{56}$, and $\underline{1}$, $\underline{8}_{5/3}$, and $\underline{10}$ with $S=\frac{1}{2}$, and $\underline{8}_{-1/3}$ with $S=\frac{3}{2}$ belonging to $\underline{70}$. Then, in the above solutions the relative weights of the $\underline{56}$ and $\underline{70}$ representations differ by a factor -2 in s,t and s,u terms just as in Eq. (1). Hence by assuming Eq. (2) we can eliminate the unobserved states $\underline{1}$, $\underline{8}_{5/3}$, and $\underline{10}$ with $S=\frac{1}{2}$ and $\underline{8}_{-1/3}$ with $S=\frac{3}{2}$ at $L=0$ and also the states $\underline{8}_{2/3}$ with $S=\frac{1}{2}$ and $\underline{10}$ with $S=\frac{3}{2}$ at $L=1$. Then we are able to obtain the spectrum of the harmonic-oscillator quark model.

In the above solutions (5) and (6) we have fixed the relative normalization between the quark-spin doublets and quartets in such a way that they fulfil the requirement of the s,u crossing invariance of their baryon spectrum.

These points may be illustrated as follows. Since the exchange of the quark spin $\frac{1}{2}$ or $\frac{3}{2}$ in the s channel corresponds to the exchanges of the quark spin (W spin) 0 and 1 in the t channel, if we make an appropriate linear combination of the $W_s=\frac{1}{2}$ and $W_s=\frac{3}{2}$ solutions to single out the $W_t=0$ or $W_t=1$ exchange in the t channel, then the baryon spectrum should close under s,u crossing in this combination. By making use of the SU(2) crossing matrix we find the following combinations corresponding to definite W spins in the t channel (for the definitions of the W spin, see Ref. 19):

$$(W_t=0)=-\tfrac{1}{3}\sqrt{6}\,(W_s=\tfrac{1}{2})-\tfrac{2}{3}\sqrt{6}\,(W_s=\tfrac{3}{2}),$$
$$(W_t=1)=-\tfrac{2}{3}(W_s=\tfrac{1}{2})+\tfrac{2}{3}(W_s=\tfrac{3}{2}). \tag{7}$$

The s,u terms of these equations must cross into themselves when multiplied by the s,u SU(3) crossing matrix. By substituting our solutions (5) and (6) into Eq. (7) one can check that this is indeed the case as follows.

(i) $W_t=0$:

$$\begin{aligned}-\tfrac{1}{2}\sqrt{6}\,(s,u)&=g(L)(-\tfrac{108}{25}[1]_s+[8_{2/3}]_s-\tfrac{8}{25}[8_{5/3}]_s-\tfrac{2}{25}[8_{-1/3}]_s+\tfrac{18}{5}[10]_s),\\&=g(L)(-\tfrac{108}{25}[1]_u+[8_{2/3}]_u-\tfrac{8}{25}[8_{5/3}]_u-\tfrac{2}{25}[8_{-1/3}]_u+\tfrac{18}{5}[10]_u).\end{aligned} \tag{8}$$

(ii) $W_t=1$:

$$\begin{aligned}-\tfrac{3}{2}(s,u)&=g(L)(-\tfrac{108}{25}[1]_s+[8_{2/3}]_s-\tfrac{8}{25}[8_{5/3}]_s+\tfrac{1}{25}[8_{-1/3}]_s-\tfrac{54}{25}[10]_s),\\&=g(L)(\tfrac{108}{25}[1]_u-[8_{2/3}]_u+\tfrac{8}{25}[8_{5/3}]_u-\tfrac{1}{25}[8_{-1/3}]_u+\tfrac{54}{25}[10]_u).\end{aligned} \tag{9}$$

Structures of their t-channel couplings are given by the following expressions (subscripts to the octet amplitudes in the t-channel denote the couplings into meson-meson channel).

(i) $W_t=0$:

$$\begin{aligned}-\tfrac{1}{2}\sqrt{6}\,(s,t)&=f(L)(\tfrac{216}{25}[1]_s+[8_{2/3}]_s+\tfrac{16}{25}[8_{5/3}]_s+\tfrac{4}{25}[8_{-1/3}]_s+\tfrac{108}{25}[10]_s),\\&=f(L)(\tfrac{432}{25}[1]_t+\tfrac{54}{25}\sqrt{5}\,[8_{as}]_t+\tfrac{162}{25}[8_{aa}]_t),\end{aligned} \tag{10}$$

and

(ii) $W_t=1$:

$$\begin{aligned}-\tfrac{3}{2}(s,t)&=f(L)(\tfrac{216}{25}[1]_s+[8_{2/3}]_s+\tfrac{16}{25}[8_{5/3}]_s-\tfrac{2}{25}[8_{-1/3}]_s-\tfrac{36}{25}[10]_s),\\&=f(L)(\tfrac{144}{25}[1]_t+\tfrac{36}{25}\sqrt{5}\,[8_{as}]_t+\tfrac{18}{5}[8_{ss}]_t+\tfrac{54}{25}\sqrt{5}\,[8_{sa}]_t+\tfrac{108}{25}[8_{aa}]_t).\end{aligned} \tag{11}$$

Here the trajectories $\underline{1}$ and $\underline{8}$ with symmetric couplings at the meson-meson vertices are identified as those belonging to $\underline{36}$ with $L=$ odd, and the trajectory $\underline{8}$ with antisymmetric couplings as belonging to $\underline{36}$ with $L=$ even. Then Eqs. (10) and (11) correspond to the electric and magnetic couplings of the vector- and tensor-meson exchanges at the baryon and antibaryon vertices. Their F/D ratios are given by $F/D=\infty$ and $\frac{2}{3}$, respectively.

Hence, by requiring the crossing invariance of the baryon spectrum we are at the same time able to determine the relative strengths of the quark doublets and quartets in the s channel and the SU(3) structures of the vector-tensor exchanges in the t channel. Note that if we had restricted ourselves to only requiring the absence of exotics in all channels, the $W_s=\frac{1}{2}$ and $\frac{3}{2}$ solutions would have never been related to each other and hence their relative strengths or the t-channel couplings would not have been determined. In these respects, our approach of duality supplemented by the crossing invariance has a much stronger predictive power than the usual approaches.

Next, by making the sum and difference of Eqs. (5) and (6), we find that the residues of the baryon Regge trajectories are given by

$$\left.\begin{aligned}\underline{8}_{2/3}&=u_\pm(L),\\ \underline{10}&=\tfrac{48}{25}u_\pm(L)\end{aligned}\right\}\ \text{belonging to } \underline{56} \tag{12}$$

and

$$\left.\begin{aligned}\underline{1}&=\tfrac{81}{10}v_\pm(L),\quad \underline{8}_{5/3}=\tfrac{3}{5}v_\pm(L),\\ \underline{10}&=\tfrac{9}{20}v_\pm(L),\quad \underline{8}_{-1/3}=\tfrac{3}{40}v_\pm(L)\end{aligned}\right\}\ \text{belonging to } \underline{70}. \tag{13}$$

Here $u_\pm(L)$ and $v_\pm(L)$ are defined by

$$\begin{aligned}u_\pm(L)&=\tfrac{1}{2}[f(L)\pm g(L)],\\ v_\pm(L)&=\tfrac{8}{15}[f(L)\mp\tfrac{1}{2}g(L)],\end{aligned} \tag{14}$$

and characterize the L dependence of the baryon Regge residues for $\underline{56}$ and $\underline{70}$ multiplets. The signs + and − refer to even and odd L values, respectively. They satisfy $u_-(1)=0$ and $v_+(0)=0$.

On the other hand, the residues of the meson Regge trajectories are given by

$$\left.\begin{aligned}\underline{8}_\infty&=1\quad\text{for } W=0,\\ \underline{8}_{2/3}&=\tfrac{5}{9}\sqrt{6}\quad\text{for } W=1\end{aligned}\right\}\ \text{belonging to } \underline{36}\ L=\text{even}, \tag{15}$$

$$\left.\begin{aligned}&\underline{1}=32,\ \underline{8}_\infty=1\quad\text{for } W=0,\\ &\underline{1}=\tfrac{32}{9}\sqrt{6},\ \underline{8}_{2/3}=\tfrac{5}{9}\sqrt{6}\quad\text{for } W=1\end{aligned}\right\}\ \text{belonging to } \underline{36}\ L=\text{odd}. \tag{16}$$

Here we have normalized $\underline{8}(W=0,\ L=\text{even})=1$. Note that there is no momentum-transfer dependence in the ratios of the residues of the meson Regge trajectories.

The above solutions (12)–(16) possess striking consistency with $SU(6)_W$ as follows.

(i) The meson and baryon exchange-degenerate trajectories appear as $U(6)_W$ multiplets.

(ii) The octet F/D ratio of the baryons at the pseudoscalar-meson–baryon vertices are predicted to be $\frac{2}{3}$ for quark doublets belonging to $\underline{56}$, $\frac{5}{3}$ for quark doublets belonging to $\underline{70}$, and $-\frac{1}{3}$ for the quark quartets of $\underline{70}$. Also, the F/D ratio of the meson octet at the baryon-antibaryon vertices is predicted to be ∞ for $W=0$ and $\frac{2}{3}$ for $W=1$. These values agree with the $SU(6)_W$ symmetry.

(iii) The ratio of the strengths of the baryons $\underline{8}_{2/3}$ and $\underline{10}$ belonging to $\underline{56}$ is predicted to be $1:\frac{48}{25}$ for their couplings into PB. Also the ratios of $\underline{1}$, $\underline{8}_{5/3}$, $\underline{10}$, and $\underline{8}_{-1/3}$ belonging to $\underline{70}$ are predicted to be $1:\frac{2}{27}:\frac{1}{18}:\frac{1}{108}$. These values are the one predicted by the invariance under $SU(6)_W$ [for the $SU(6)_W$ Clebsch-Gordan coefficients, see Ref. 20].

(iv) The ratios of the mesons $\underline{8}(W=0)$, $\underline{1}(W=1)$, and $\underline{8}(W=1)$ belonging to $\underline{35}$ are predicted to be $1:\frac{32}{9}\sqrt{6}:\frac{5}{9}\sqrt{6}$ for their couplings into $PP\to\bar{B}B$. These values are also in accord with $SU(6)_W$.

We find nowhere in our solutions predictions which disagree with $SU(6)_W$. Such was certainly not the case in the broken-duality approach.[4,5] For instance, the F/D ratios of the quark doublets belonging to $\underline{56}$ and $\underline{70}$ were both predicted to be unity, in disagreement with $SU(6)_W$.

Thus we have succeeded in constructing a crossing-invariant and $SU(6)_W$-symmetric theory of the baryon exchange degeneracy for the pseudoscalar-meson–baryon scattering with the baryon spectrum of the harmonic-oscillator quark model. We shall show in the following that such solutions exist also for a wide class of reactions with the external mesons and baryons belonging to $\underline{35}$ and $\underline{56}$ representations. Since the search for the existence of such baryon duality solutions seems to have been one of the main theme in the theory of baryon exchange degeneracy, their successful construction will provide a considerable step forward in our understanding of the duality constraints on meson-baryon scattering amplitudes.

From the theoretical point of view, we consider that our solutions surpass those of the broken-duality approach in the following respects. They consist in (i) the impartial treatment of the duality constraints arising from t and u channels, (ii) the strong consistency with the higher symmetry predictions, and (iii) the closure of the baryon spectrum under s, u crossing. From the experimental point of view, our approach has the apparent advantage of taking over almost all of the empirical successes of the theories of $SU(6)_W$ or the naive quark model. At the same time it does not share the empirically unsuccessful predictions of the exact $SU(6)_W$ on the relations between different relative orbital angular momenta for a single resonance decay (these points will be explained in Sec. V). In this way our approach provides a consistent and empirically successful basis for a baryon bootstrap where the constituent-quark description of the hadrons merges quite naturally with the restrictions arising from duality.

III. SOLUTIONS FOR $\underline{35}+\underline{56}\to\underline{35}+\underline{56}$

Next let us enumerate our solutions for the processes possible with the external mesons and baryons belonging to $\underline{35}$ and $\underline{56}$ representations. In terms of $SU(6)_W$ the above solution for $PB\to P'B'$ is identified as the one for $(8,3)+(8,2)\to(8,3)+(8,2)$. Since the duality constraints do not depend on the "magnetic quantum numbers" W_z, the same solution should apply also to $PB\to V_\pm B'$ and $V_\pm B\to V'_\pm B'$ ($V_\pm$ stands for a transversely polarized vector meson). Then, by making use of the naive vector-meson dominance we reproduce the well-known $SU(6)_W$ results on the electromagnetic couplings of the baryons: $\mu_p/\mu_n=-\frac{3}{2}$, $\mu^*/\mu_p=\frac{2}{3}\sqrt{2}$, pure $M1$ transition for Δ production, the Moorhouse selection[21] for the couplings of the $\underline{70}$ $S=\frac{3}{2}$ baryons, etc. We call this the solution for the process 1 $(8,3)+(8,2)\to(8,3)+(8,2)$ hereafter.

Other solutions are also obtained by making use of SU(3) crossing matrices and by directly solving the duality constraint equations like Eq. (4).

Our solutions are enumerated as follows.

(i) Solution for the process 1, $(8,3)+(8,2)\to(8,3)+(8,2)$.

$W_s=\frac{1}{2}$:

$$\begin{aligned}(s,t)&=[8_{2/3}]_s+\tfrac{216}{25}[1]_s+\tfrac{16}{25}[8_{5/3}]_s+\tfrac{12}{25}[10]_s\,,\\(s,u)&=[8_{2/3}]_s-\tfrac{108}{25}[1]_s-\tfrac{8}{25}[8_{5/3}]_s-\tfrac{6}{25}[10]_s\,,\end{aligned}\tag{17}$$

$W_s=\frac{3}{2}$:

$$\begin{aligned}(s,t)&=\tfrac{48}{25}[10]_s+\tfrac{2}{25}[8_{-1/3}]_s\,,\\(s,u)&=\tfrac{48}{25}[10]_s-\tfrac{1}{25}[8_{-1/3}]_s\,.\end{aligned}\tag{18}$$

(ii) Solution for the process 2, $(8,3)+(8,2)\to(8,3)+(10,4)$.

$W_s=\frac{1}{2}$:

$$\begin{aligned}(s,t)&=[8_{2/3}]_s-\tfrac{4}{5}[8_{5/3}]_s+\tfrac{4}{25}\sqrt{30}\,[10]_s\,,\\(s,u)&=[8_{2/3}]_s+\tfrac{2}{5}[8_{5/3}]_s-\tfrac{2}{25}\sqrt{30}\,[10]_s\,,\end{aligned}\tag{19}$$

$W_s=\frac{3}{2}$:

$$\begin{aligned}(s,t)&=\tfrac{4}{5}\sqrt{3}\,[10]_s+\tfrac{1}{10}\sqrt{10}\,[8_{-1/3}]_s\,,\\(s,u)&=\tfrac{4}{5}\sqrt{3}\,[10]_s-\tfrac{1}{20}\sqrt{10}\,[8_{-1/3}]_s\,.\end{aligned}\tag{20}$$

(iii) Solution for the process 3, $(8,3)+(10,4)\to(8,3)+(10,4)$.

$W_s=\frac{1}{2}$:

$$\begin{aligned}(s,t)&=[8_{2/3}]_s+[8_{5/3}]_s+\tfrac{8}{5}[10]_s\,,\\(s,u)&=[8_{2/3}]_s-\tfrac{1}{2}[8_{5/3}]_s-\tfrac{4}{5}[10]_s\,,\end{aligned}\tag{21}$$

$W_s=\frac{3}{2}$:

$$\begin{aligned}(s,t)&=[10]_s+\tfrac{5}{4}[8_{-1/3}]_s\,,\\(s,u)&=[10]_s-\tfrac{5}{8}[8_{-1/3}]_s\,.\end{aligned}\tag{22}$$

(iv) Solution for the process 4, $(8,1)+(8,2)\to(8,1)+(8,2)$.

$W_s=\frac{1}{2}$:

$$\begin{aligned}(s,t)&=[8_\infty]_s+24[1]_s+4[8_{1/3}]_s+12[10]_s\\&=48[1]_t+6\sqrt{5}\,[8_{as}]_t+18[8_{aa}]_t\,,\\(s,u)&=[8_\infty]_s-12[1]_s-2[8_{1/3}]_s-6[10]_s\\&=[8_\infty]_u-12[1]_u-2[8_{1/3}]_u-6[10]_u\,.\end{aligned}\tag{23}$$

(v) Solution for the process 5, $(8,1)+(8,2)\to(8,3)+(8,2)$.

$W_s=\frac{1}{2}$:

$$\begin{aligned}(s,t)&=[8_{2/3,\infty}]_s+\tfrac{72}{5}[1]_s+\tfrac{8}{5}[8_{5/3,\,1/3}]_s\\&\quad-\tfrac{12}{5}[10]_s\\&=\tfrac{48}{5}[1]_t+6[8_{ss}]_t+\tfrac{12}{5}\sqrt{5}\,[8_{as}]_t\\&\quad+\tfrac{18}{5}\sqrt{5}\,[8_{sa}]_t+\tfrac{36}{5}[8_{aa}]_t\,,\\(s,u)&=[8_{2/3,\infty}]_s-\tfrac{36}{5}[1]_s-\tfrac{4}{5}[8_{5/3,\,1/3}]_s\\&\quad+\tfrac{6}{5}[10]_s\\&=[8_{2/3,\infty}]_u-\tfrac{36}{5}[1]_u-\tfrac{4}{5}[8_{5/3,\,1/3}]_u\\&\quad+\tfrac{6}{5}[10]_u\,.\end{aligned}\tag{24}$$

(vi) Solution for the process 6, $(8,1)+(8,2)\to(8,3)+(10,4)$.

$W_s=\frac{1}{2}$:

$$\begin{aligned}(s,t)&=[8_\infty]_s-2[8_{1/3}]_s-\tfrac{4}{5}\sqrt{30}\,[10]_s\\&=-\sqrt{15}\,[8_s]_t-3\sqrt{3}\,[8_a]_t\,,\\(s,u)&=[8_\infty]_s+[8_{1/3}]_s+\tfrac{2}{5}\sqrt{30}\,[10]_s\\&=-\tfrac{4}{5}\sqrt{30}\,[10]_u+\tfrac{1}{2}[8_{-1/3}]_u\,.\end{aligned}\tag{25}$$

(vii) Solution for the process 7, $(8,3)+(8,2)\to(8,1)+(10,4)$.

$W_s=\frac{3}{2}$:

$$\begin{aligned}(s,t)&=\tfrac{4}{5}\sqrt{30}\,[10]_s+[8_{-1/3}]_s\\&=\sqrt{15}\,[8_s]_t+3\sqrt{3}\,[8_a]_t\,,\\(s,u)&=\tfrac{4}{5}\sqrt{30}\,[10]_s-\tfrac{1}{2}[8_{-1/3}]_s\\&=-[8_\infty]_u-[8_{1/3}]_u-\tfrac{2}{5}\sqrt{30}\,[10]_u\,.\end{aligned}\tag{26}$$

(viii) Solution for the process 8, $(8,1)+(10,4)\to(8,1)+(10,4)$.

$W_s=\frac{3}{2}$:

$$\begin{aligned}(s,t)&=[10]_s+\tfrac{5}{4}[8]_s\\&=\sqrt{5}[1]_t+\tfrac{5}{8}\sqrt{2}\,[8_s]_t+\tfrac{3}{8}\sqrt{10}\,[8_a]_t\,,\\(s,u)&=[10]_s-\tfrac{5}{8}[8]_s\\&=[10]_u-\tfrac{5}{8}[8]_u\,.\end{aligned}\tag{27}$$

(ix) Solution for the process 9, $(8,1)+(10,4)\to(8,3)+(10,4)$.

$$W_s=\tfrac{3}{2},\quad\text{the same as Eq. (27).}\tag{28}$$

In the above we have suppressed the overall L dependence of the s,t and s,u terms. Crossing-invariant expressions for the processes 2 and 3 are given by the following equations.

(ii) Solution for the process 2, $(8,3)+(8,2) \to (8,3)+(10,4)$.

$W_t=1$:

$$\begin{aligned} -3(s,t) &= [8_{2/3}]_s - \tfrac{4}{5}[8_{5/3}]_s + [8_{-1/3}]_s + \tfrac{24}{25}\sqrt{30}\,[10]_s \\ &= \tfrac{6}{5}\sqrt{15}\,[8_s]_t + \tfrac{18}{5}\sqrt{3}\,[8_a]_t\,, \\ -3(s,u) &= [8_{2/3}]_s + \tfrac{2}{5}[8_{5/3}]_s - \tfrac{1}{2}[8_{-1/3}]_s + \tfrac{18}{25}\sqrt{30}\,[10]_s \\ &= -[8_{2/3}]_u - \tfrac{2}{5}[9_{5/3}]_u + \tfrac{1}{2}[8_{-1/3}]_u - \tfrac{18}{25}\sqrt{30}\,[10]_u\,. \end{aligned} \tag{29}$$

(iii) Solution for the process 3, $(8,3)+(10,4) \to (8,3)+(10,4)$.

$W_t=0$:

$$\begin{aligned} \sqrt{3}\,(s,t) &= [8_{2/3}]_s + [8_{5/3}]_s + \tfrac{5}{2}[8_{-1/3}]_s + \tfrac{18}{5}[10]_s \\ &= \tfrac{18}{5}\sqrt{5}\,[1]_t + \tfrac{9}{4}\sqrt{2}\,[8_s]_t + \tfrac{27}{20}\sqrt{10}\,[8_a]_t\,, \\ \sqrt{3}\,(s,u) &= [8_{2/3}]_s - \tfrac{1}{2}[8_{5/3}]_s - \tfrac{5}{4}[8_{-1/3}]_s + \tfrac{6}{5}[10]_s \\ &= [8_{2/3}]_u - \tfrac{1}{2}[8_{5/3}]_u - \tfrac{5}{4}[8_{-1/3}]_u + \tfrac{6}{5}[10]_u\,, \end{aligned} \tag{30}$$

$W_t=1$:

$$\begin{aligned} \tfrac{3}{5}\sqrt{10}\,(s,t) &= [8_{2/3}]_s + [8_{5/3}]_s + [8_{-1/3}]_s + \tfrac{12}{5}[10]_s \\ &= \tfrac{12}{5}\sqrt{5}\,[1]_t + \tfrac{3}{2}\sqrt{2}\,[8_s]_t + \tfrac{9}{10}\sqrt{10}\,[8_a]_t\,, \\ (s,u) &= 0\,. \end{aligned} \tag{31}$$

Note that the solutions for the processes 4, 5, 8, and 9 are crossing-invariant by themselves and those for the processes 6 and 7 cross into each other under s,u interchange.

We remark on the following points in the above expressions:

(i) The meson and baryon exchange-degenerate trajectories appear as $U(6)_W$ multiplets. The relative weights of the $\underline{56}$ and $\underline{70}$ representations in s,t and s,u terms differ by a factor -2, and hence they enable us to recover the harmonic-oscillator spectrum by Eq. (2).

(ii) The F/D ratios of the octet baryons at the PB and V_0B (V_0 denotes a longitudinally polarized vector meson) vertices are predicted to be $\frac{2}{3}$ and ∞ for quark-doublets belonging to $\underline{56}$, and they are $\frac{5}{3}$ and $\frac{1}{3}$ for quark doublets belonging to $\underline{70}$. The F/D ratio at the PB vertices is predicted to be $-\frac{1}{3}$ for quark quartets belonging to $\underline{70}$. These values agree with $SU(6)_W$.

(iii) The octet F/D value of the mesons at the $\bar{B}B$ vertices are predicted to be ∞ for $W=0$ and $\frac{2}{3}$ for $W=1$. These values agree with $SU(6)_W$.

(iv) The ratio of the strengths of the baryons $\underline{8}$ and $\underline{10}$ belonging to $\underline{56}$ is predicted to be $1:\frac{48}{25}$ and $1:1$ for their couplings into PB and PD, respectively. Also, the ratios of the strengths of $\underline{1}$, $\underline{8}(W=\frac{1}{2})$, $\underline{10}$, and $\underline{8}(W=\frac{3}{2})$ belonging to $\underline{70}$ are predicted to be $1:\frac{2}{27}:\frac{1}{18}:\frac{1}{108}$, $0:1:\frac{8}{5}:\frac{5}{4}$, and $1:\frac{1}{6}:\frac{1}{2}:0$ for their couplings into PB, PD, and V_0B, respectively. These values also agree with $SU(6)_W$.

(v) The residue ratios of the meson trajectories $\underline{8}(W=0)$, $\underline{1}(W=1)$, and $\underline{8}(W=1)$ belonging to $\underline{35}$ are predicted to be $1:-\frac{8}{3}\sqrt{2}:\frac{1}{3}\sqrt{6}$ and $0:1:\frac{1}{8}\sqrt{3}$ for their couplings into PP and PV_0. Also, the ratios are predicted to be $1:-\frac{4}{3}\sqrt{3}:\frac{5}{3}$ and $1:-\frac{1}{3}\sqrt{10}:\frac{1}{3}\sqrt{5}$ for their couplings into $\bar{B}B$ and $\bar{D}D$. These values again agree with $SU(6)_W$.

(vi) The factorization property for the Regge residues,

$$[\beta(AB\to CD)]^2 = \beta(AB\to AB)\beta(CD\to CD)\,,$$

is always satisfied in our solutions. [Note that there is overall-sign ambiguity in our solutions for the SU(3)-inelastic processes, 2, 5, 6, 7, and 9. We have arbitrarily fixed their signs by taking the coefficients of the trajectories of $\underline{56}$ to be positive.]

Hence we have succeeded in constructing a consistent set of crossing invariant and $SU(6)_W$-symmetric solutions for all the processes involved in $\underline{35}+\underline{56}\to\underline{35}+\underline{56}$. [Solutions for the processes with the external mesons belonging to (1, 3) of $\underline{35}$ can automatically be made crossing-invariant and $SU(6)_W$-symmetric, since there do not occur any exotic states in these reactions.]

It is to be noted here, however, that while our constraint solutions predict the $SU(6)_W$-invariant vertices for each individual partial-wave amplitude in meson-baryon scattering, they do not share the predictions of the exact $SU(6)_W$ on the relations between the amplitudes involving different final-state relative orbital angular momenta for each resonance decay. These distinctions arise whenever a resonance decay can proceed via more than one partial waves into its final state. The reason is that the duality constraints apply only separately to each partial-wave amplitude, hence the relations between the amplitudes involving different final-state relative orbital angular momenta are left arbitrary and do not necessarily obey the constraints of the exact $SU(6)_W$.

In these respects our approach shares the same point of view with the l-broken $SU(6)_W$,[22-24] which uncouples the relations between different orbital angular momenta while preserving all other relations as in the exact $SU(6)_W$ limit. The distinction between the exact and broken versions of $SU(6)_W$ will be made clear when we discuss the signs of the resonant amplitudes in processes such as $\pi N \to \pi\Delta$ in Sec. V.

IV. PRESCRIPTION FOR THE CONSTRUCTION OF THE CONSTRAINT SOLUTIONS

Next let us consider a simple prescription for the construction of the above constraint solutions (17)–(31). Making use of this rule we can derive these solutions even without recourse to the SU(3) crossing matrices. The existence of such a prescription is not at all surprising, because our solutions possess strong consistency with $SU(6)_W$ and hence they can be derived directly from the constraint solutions at the SU(6) level.

Our prescription is as follows: Start from the constraint solution (1). Choose a particular reaction involved in $\underline{35}+\underline{56}\to\underline{35}+\underline{56}$. Replace the $SU(6)_W$ multiplets in Eq. (1) by the sum of their SU(3) $\times$SU(2) contents and multiply each of them by the product of its respective SU(3) scalar factors appropriate for the process. Then, we obtain our constraint solution for the reaction. Crossing-invariant expressions are acquired if we choose a definite W spin in the t channel.

Let us next illustrate this prescription in the case of the pseudoscalar-meson–baryon scattering. We start with the constraint solution at the SU(6) level:

$$\begin{aligned}(s,t)&=f(L)(15[56]_s+16[70]_s),\\ &=f(L)(14\sqrt{10}[1]_t+8\sqrt{6}\,[35_D]_t\\ &\quad-12\sqrt{3}[35_F]_t),\\ (s,u)&=g(L)(15[56]_s-8[70]_s),\\ &=g(L)(15[56]_u-8[70]_u).\end{aligned}\tag{1'}$$

Next we replace $\underline{56}$ and $\underline{70}$ or $\underline{1}$, $\underline{35}_D$, and $\underline{35}_F$ by their SU(3)$\times$SU(2) contents multiplied by the product of SU(3) scalar factors.[20] We illustrate this for the two cases $W_s=\frac{1}{2}$ and $W_t=0$.

(i) $W_s=\frac{1}{2}$:

$[56]_s$ and $[70]_s$ are replaced as follows.

$$\begin{aligned}[56]_s\to{}&\left(\begin{matrix}\underline{35}&\underline{56}\\(8,3)&(8,2)\end{matrix}\middle|\begin{matrix}\underline{56}\\(8,2)\end{matrix}\right)_s^2[8_{ss}]_s+\left(\begin{matrix}\underline{35}&\underline{56}\\(8,3)&(8,2)\end{matrix}\middle|\begin{matrix}\underline{56}\\(8,2)\end{matrix}\right)_s\left(\begin{matrix}\underline{35}&\underline{56}\\(8,3)&(8,2)\end{matrix}\middle|\begin{matrix}\underline{56}\\(8,2)\end{matrix}\right)_a([8_{sa}]_s+[8_{as}]_s)\\ &+\left(\begin{matrix}\underline{35}&\underline{56}\\(8,3)&(8,2)\end{matrix}\middle|\begin{matrix}\underline{56}\\(8,2)\end{matrix}\right)_a^2[8_{aa}]_s=\tfrac{5}{54}[8_{2/3}]_s,\end{aligned}$$

$$\begin{aligned}[70]_s\to{}&\left(\begin{matrix}\underline{35}&\underline{56}\\(8,3)&(8,2)\end{matrix}\middle|\begin{matrix}\underline{70}\\(1,2)\end{matrix}\right)^2[1]_s+\left(\begin{matrix}\underline{35}&\underline{56}\\(8,3)&(8,2)\end{matrix}\middle|\begin{matrix}\underline{70}\\(8,2)\end{matrix}\right)_s^2[8_{ss}]_s\\ &+\left(\begin{matrix}\underline{35}&\underline{56}\\(8,3)&(8,2)\end{matrix}\middle|\begin{matrix}\underline{70}\\(8,2)\end{matrix}\right)_s\left(\begin{matrix}\underline{35}&\underline{56}\\(8,3)&(8,2)\end{matrix}\middle|\begin{matrix}\underline{70}\\(8,2)\end{matrix}\right)_a([8_{as}]_s+[8_{sa}]_s)\\ &+\left(\begin{matrix}\underline{35}&\underline{56}\\(8,3)&(8,2)\end{matrix}\middle|\begin{matrix}\underline{70}\\(8,2)\end{matrix}\right)_a^2[8_{aa}]_s+\left(\begin{matrix}\underline{35}&\underline{56}\\(8,3)&(8,2)\end{matrix}\middle|\begin{matrix}\underline{70}\\(10,2)\end{matrix}\right)^2[10]_s=\tfrac{3}{4}[1]_s+\tfrac{1}{18}[8_{5/3}]_s+\tfrac{1}{24}[10]_s.\end{aligned}$$

Then Eq. (1) turns into

$$\begin{aligned}(s,t)&=f(L)(\tfrac{75}{54}[8_{2/3}]_s+12[1]_s+\tfrac{8}{9}[8_{5/3}]_s+\tfrac{2}{3}[10]_s),\\ (s,u)&=g(L)(\tfrac{75}{54}[8_{2/3}]_s-6[1]_s-\tfrac{4}{9}[8_{5/3}]_s-\tfrac{1}{3}[10]_s).\end{aligned}$$

These expressions are proportional to Eq. (17).

(ii) $W_t=0$:

$[1]_t$, $[35_D]_t$, and $[35_F]_t$ are replaced as follows.

$$[1]_t\to\left(\begin{matrix}\underline{56}^*&\underline{56}\\(8,2)&(8,2)\end{matrix}\middle|\begin{matrix}\underline{1}\\(1,1)\end{matrix}\right)\left(\begin{matrix}\underline{35}&\underline{35}\\(8,3)&(8,3)\end{matrix}\middle|\begin{matrix}\underline{1}\\(1,1)\end{matrix}\right)[1]_t=-\tfrac{4}{35}\sqrt{15}\,[1]_t,$$

$$[35_D]_t \to \begin{pmatrix} \underline{56}^* & \underline{56} & \big| & \underline{35}_D \\ (8,2) & (8,2) & \big| & (8,1) \end{pmatrix}_a \begin{pmatrix} \underline{35} & \underline{35} & \big| & \underline{35}_D \\ (8,3) & (8,3) & \big| & (8,1) \end{pmatrix}_s [8_{as}]_t = -\tfrac{1}{8}\sqrt{5}\,[8_{as}]_t\,,$$

$$[35_F]_t \to \begin{pmatrix} \underline{56}^* & \underline{56} & \big| & \underline{35}_F \\ (8,2) & (8,2) & \big| & (8,1) \end{pmatrix}_a \begin{pmatrix} \underline{35} & \underline{35} & \big| & \underline{35}_F \\ (8,3) & (8,3) & \big| & (8,1) \end{pmatrix}_a [8_{aa}]_t = \tfrac{1}{4}\sqrt{2}\,[8_{aa}]_t\,.$$

Equation (1) changes into

$$(s,t) = f(L)(-8\sqrt{6}\,[1]_t - \sqrt{30}\,[8_{as}]_t - 3\sqrt{6}\,[8_{aa}]_t)\,,$$

and its corresponding (s,u) term is given by

$$(s,u) = -\tfrac{25}{54}\sqrt{6}\; g(L)(-\tfrac{108}{25}[1]_s + [8_{2/3}]_s - \tfrac{8}{25}[8_{5/3}]_s - \tfrac{2}{25}[8_{-1/3}]_s + \tfrac{18}{5}[10]_s)\,.$$

Then the above equation is indeed proportional to the crossing-invariant expression (8).

Readers can easily check that all of our constraint solutions (17)–(31) can be reconstructed in this way from a single constraint solution, Eq. (1).

When expressed mathematically, our prescription is equivalent to the following statement: "If we have

$$\sum_i [\mathrm{SU}(6)\text{ crossing matrix}]_{ji}\,\beta^i = 0 \qquad \text{for } j \neq \underline{56},\ \underline{70} \text{ or } \underline{1},\ \underline{35}\,,$$

then it follows that

$$\sum_i \sum_\gamma [\mathrm{SU}(3)\text{ crossing matrix}]_{\delta\gamma}\,\beta^i C^i_{\gamma,c} = 0 \qquad \text{for } \delta \neq \underline{1},\ \underline{8},\ \underline{10} \text{ or } \underline{1},\ \underline{8}.$$

Here the summation with respect to i runs over different nonexotic $\mathrm{SU}(6)_W$ representations and the sum γ runs over all the SU(3) contents of the $\mathrm{SU}(6)_W$ multiplet i. The coefficient $C^i_{\gamma,c}$ is the product of the SU(3) scalar factors

$$\begin{pmatrix} \underline{35} & \underline{56} & \big| & \underline{i} \\ (\alpha,a) & (\beta,b) & \big| & (\gamma,c) \end{pmatrix} \begin{pmatrix} \underline{35} & \underline{56} & \big| & \underline{i} \\ (\alpha',a') & (\beta',b') & \big| & (\gamma,c) \end{pmatrix}.$$

The above equations hold for both s,t and s,u crossing matrices and for any values of (α,a), (β,b), (α',a'), (β',b'), and c."

Our experience tells us the validity of this statement. Its mathematical proof may be in order; however, we have not yet attempted it.

V. AMPLITUDE SIGNS IN INELASTIC PROCESSES

As we have remarked in Sec. III, our constraint solutions do not predict the $\mathrm{SU}(6)_W$ invariance for the vertices when a resonance decay can proceed via more than one final-state relative orbital angular momentum. In this respect our predictions are similar to those of the extended version of $\mathrm{SU}(6)_W$ in which the amplitudes involving different angular momenta for each resonance decay are uncoupled from the other while all other relations are preserved as in the exact $\mathrm{SU}(6)_W$ limit. It is known that in the case of the pseudoscalar-meson emission vertices the predictions of this modified version of $\mathrm{SU}(6)_W$ are equivalent to those implied by the transformation between currents and constituents in the free-quark model.[25-27]

Let us next consider the signs of the resonant amplitudes in $PB \to P'D'$ in order to illustrate the above points. In this reaction the baryon resonances of spin J with $L = J \mp \frac{1}{2}$ can decay through two relative orbital angular momenta $l = J \pm \frac{1}{2}$ and $J \mp \frac{3}{2}$ into their final states. Each partial-wave amplitude in the extended version of $\mathrm{SU}(6)_W$ is written as

$$\begin{aligned} T_J^{J\pm 1/2,\,J\pm 1/2} &\sim \mp a^{J\pm 1/2}_{J\mp 1/2}\, a^{J\pm 1/2}_{J\mp 1/2}\ [\text{products of SU(3), SU(2) scalar factors}]\,, \\ T_J^{J\mp 3/2,\,J\pm 1/2} &\sim \pm(-1)^{S-1/2}\, a^{J\mp 3/2}_{J\mp 1/2}\, a^{J\pm 1/2}_{J\mp 1/2}\ [\text{products of SU(3), SU(2) scalar factors}]\,. \end{aligned} \qquad (32)$$

Here $T_J^{l',l}$ denotes the resonant amplitude of spin J with the initial and final orbital angular momenta l and l', respectively. The coefficients a^l_L are the reduced $\mathrm{SU}(6)_W$ matrix elements and are taken to be universal for the couplings within a given multiplet. The $L_z = 0$ constraint of the exact $\mathrm{SU}(6)_W$ implies a specific relation between a_L^l and $a_L^{l'}$. Factors put into parentheses in the above equations are the abbreviated form of the expression

$$\left(\begin{array}{cc|c} \underline{35} & \underline{56} & \underline{R} \\ (8,3) & (10,4) & (\alpha,a) \end{array}\right)\left(\begin{array}{cc|c} \underline{8} & \underline{10} & \underline{\alpha} \\ P' & D' & R \end{array}\right)\left(\sum_i \left(\begin{array}{cc|c} \underline{35} & \underline{56} & \underline{R} \\ (8,3) & (8,2) & (\alpha,a) \end{array}\right)_i \left(\begin{array}{cc|c} \underline{8} & \underline{8} & \underline{\alpha} \\ P & B & R \end{array}\right)_i\right),$$

where $\underline{R}$ and (α, a) label the $SU(6)_W$ and SU(3) $\times$SU(2) classifications of the resonance and the sum over i corresponds to d and f coupling when α is an octet.

On the other hand, the resonance couplings in our constraint solutions are predicted to be

$$T_J \sim \pm(u_\pm(L), v_\pm(L)) \quad [\text{products of SU(3), SU(2) scalar factors}]. \tag{33}$$

Here the $\pm$ sign in front of the equation signifies the overall-sign ambiguity of our constraint solutions for SU(3) inelastic processes. Note that with only the factorization condition

$$[T_J^{I',I}(AB \to CD)]^2 = T_J^I(AB \to AB)T_J^{I'}(CD \to CD)$$

we cannot rule out the functions like $(-1)^L u_\pm(L)$ in the above equation. These functions are, however, rejected because of their nonanalytic L dependence.

Then, by comparing the above expression with Eq. (32), one finds that the duality constraints on the amplitude signs in $PB \to P'D'$ imply a definite sign, irrespective of L, for each product of two reduced matrix elements a_L^l and $a_L^{l'}$. This is because the functions $u_\pm(L)$ and $v_\pm(L)$ are required to be positive-semidefinite for $L \geq 0$ and hence each product $a_L^l a_L^{l'}$ is also predicted to have a definite sign for $L \geq 0$ (positive-semidefiniteness of $u_\pm(L)$ and $v_\pm(L)$ is necessary in order to avoid the negative-norm states in elastic processes).

From the viewpoint of the algebraic structure of the pseudoscalar-meson emission vertices, our predictions imply that one of their two components, $L_z = 0$ or $L_z = 1$, is always dominating the other in the pseudoscalar-mesonic transitions of the baryons. Therefore, in these respects, we find a stronger predictive power of our approach than of the broken $SU(6)_W$ schemes.

Readers may easily convince themselves that the same situations as above also occur in all other reactions involved in $\underline{35} + \underline{56} \to \underline{35} + \underline{56}$. Each product of two reduced matrix elements is required to have a definite sign in order to fulfil the duality restrictions.

The experimental knowledge needed to test these predictions is as yet quite scant; however, there exist some preliminary analyses of the amplitude signs in $\pi N \to \pi\Delta$ (Refs. 28–30) deduced from the recent phase-shift data.[31,32] According to these, the present solutions of $\pi N \to \pi\Delta$ phase-shift analysis contradict our theoretical expectations, and are either in gross disagreement with the predictions of the broken $SU(6)_W$ (solution A) or in accord with them only with the dominance of $L_z = 1$ for $\underline{70}$ $L = 1$ and $L_z = 0$ for $\underline{56}$ $L = 2$ (solution B) (I thank J. L. Rosner for communicating with me on this point). Hence if these are the only solutions of $\pi N \to \pi\Delta$ phase-shift analysis, they will raise a serious difficulty for our theories. However, the experimental situations do not yet seem to be rigid enough at present to allow drawing definite conclusions on the success or failure of duality or the ideas associated with $SU(6)_W$.

VI. SUMMARY AND CONCLUSIONS

In this note we have shown that the exact duality constraints on the baryons, when taken with the baryon spectrum of the harmonic-oscillator quark model, allow $SU(6)_W$-symmetric and crossing-invariant solutions for a wide class of reactions. In our approach the classifications of the hadrons are the same as in the harmonic oscillator quark model, and the predictions of their interactions are equivalent to those of a specifically broken version of $SU(6)_W$.

This strong consistency of constraint solutions with the higher symmetry predictions has not been known in the broken-duality approach. In the foregoing sections, the mesons and baryons emerged to behave as if they were made up of quarks and antiquarks with spin $\frac{1}{2}$ when our constraint solutions were solved. These quarks should obey the para-Fermi statistics of rank three to fit in the harmonic-oscillator spectrum.

These phenomena seem to be the most intriguing aspect of our approach and are of great theoretical interest. A fascinating possibility is put forward by them of unifying the duality-bootstrap approach with the quark description of hadrons to form a consistent and complementary picture of strong interactions. Such a possibility has recently been advocated by Gell-Mann[33] in conjection with the distinctions of quarks. In such a view the nonexoticity assumption of duality corresponds to the approximation of the classification of the hadrons under U(6)$\times$U(6)$\times$O(3) generated by the charges of the constituent quarks and the rough symmetry under $U(6)_W \times O(2)_{L_z}$. We have seen in the previous sections that such was indeed the case in all our solutions for the reactions involved in $\underline{35}+\underline{56} \to \underline{35}+\underline{56}$. Situations on the meson trajectories in meson-meson scattering $\underline{35}+\underline{35} \to \underline{35}+\underline{35}$ are exactly similar, though we have not worked these out explicitly in the text.

Another interesting result in this article consists in the assurance of the s, u crossing invariance of the baryon spectrum. These crossing-invariance requirements have hitherto been a formidable difficulty standing in the way of a consistent theory of baryon exchange degeneracy. In this note we have overcome this difficulty by constructing constraint solutions in accord with $SU(6)_W$ symmetry and making appropriate combinations of the $W=\frac{1}{2}$ and $\frac{3}{2}$ solutions in the s channel to single out a definite W-spin exchange in the t channel. In these contexts the concept of quark's intrinsic spin plays an essential role, and but for it the formulation of crossing invariance would have hardly been possible. These points seem to choose $SU(6)_W$ as a relevant symmetry for duality arguments.

After all, we feel that we have made a step forward in our understanding of the duality constraints on meson-baryon scattering amplitudes, and the theory of baryon exchange degeneracy has begun to take a good shape. An attractive picture seems to be emerging in which the duality-bootstrap type of approach merges quite naturally with the constituent-quark characterization of the hadrons.

Note added in proof. A simple mathematical proof of the prescription of Sec. IV has been found and described in Ref. 34.

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APPENDIX: DERIVATION OF THE SOLUTION (5)

We assume the following exchange-degeneracy pattern:

$$(1+8_{F/D}+8_{F/D'}+10) \to (1+8_{F/D'}+10),$$

where the octets are discriminated by their signatures and F/D ratios. If we assign the above trajectories the strengths $c_1, \ldots, c_7$, respectively, duality constraint equations take the following form:

$$\tfrac{1}{8}(c_1+c_5) - \tfrac{8}{3}[D^2c_2 + D'^2(c_3+c_6)] + \tfrac{1}{4}(c_4+c_7) = 0,$$

$$\tfrac{1}{8}(c_1+c_5) + \tfrac{4}{3}[D^2c_2 + D'^2(c_3+c_6)] - 4[F^2c_2 + F'^2(c_3+c_6)] - \tfrac{1}{12}(c_4+c_7) = 0,$$

$$-\tfrac{1}{8}(c_1-c_5) + \tfrac{8}{3}[D^2c_2 + D'^2(c_3-c_6)] - 8[DFc_2 + D'F'(c_3-c_6)] + \tfrac{1}{4}(c_4-c_7) = 0,$$

$$\tfrac{1}{8}(c_1-c_5) + \tfrac{4}{3}[D^2c_2 + D'^2(c_3-c_6)] + 4[F^2c_2 + F'^2(c_3-c_6)] + \tfrac{1}{12}(c_4-c_7) = 0.$$

These equations allow the solution

$$F=\tfrac{2}{5},\quad D=\tfrac{3}{5},\quad F'=\tfrac{5}{8},\quad D'=\tfrac{3}{8},$$

$$c_1=\tfrac{54}{25},\quad c_2=1,\quad c_3=\tfrac{4}{25},\quad c_4=\tfrac{3}{25},\quad c_5=\tfrac{162}{25},\quad c_6=\tfrac{12}{25},\quad c_7=\tfrac{9}{25}.$$

The (s, t) and (s, u) terms are given by

$$(s,t) = c_2[8_{2/3}]_s + (c_1+c_5)[1]_s + (c_3+c_6)[8_{5/3}]_s + (c_4+c_7)[10]_s,$$

$$= [8_{2/3}]_s + \tfrac{216}{25}[1]_s + \tfrac{16}{25}[8_{5/3}]_s + \tfrac{12}{25}[10]_s.$$

$$(s,u) = c_2[8_{2/3}]_s + (c_1-c_5)[1]_s + (c_3-c_6)[8_{5/3}]_s + (c_4-c_7)[10]_s,$$

$$= [8_{2/3}]_s - \tfrac{108}{25}[1]_s - \tfrac{8}{25}[8_{5/3}]_s - \tfrac{6}{25}[10]_s.$$

The derivation of Eq. (6) is similar.

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