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Baryon structure in the bag theory*

A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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A relativistic quark model of the hadron based on the bag theory is presented. The bag equations for massless, Dirac fields with quark quantum numbers are solved in three space dimensions for the special case of a static spherical boundary. These solutions are quantized in an approximation which neglects zero-point fluctuations. The spectrum of low-lying baryon resonances is constructed. The only parameter of the model is fixed by the average mass of the $N(938)$ and $\Delta(1236)$. The gyromagnetic ratio, axial-vector charge, and charge radius of the proton are computed and found to be 2.6, 1.09, and 1.0 fm, respectively. Refinements of the quantization procedure are discussed.

I. INTRODUCTION

We have proposed[1] a model for the hadron in which a strongly interacting particle consists of fields confined to a finite region of space, which we call a "bag." The confinement is accomplished in a Lorentz-invariant way by assuming that the bag possesses a constant, positive energy per unit volume, B. In this paper we shall explore the consequences of this model for the low-lying states of the hadron.

To begin, we review several pertinent features of our previous work. The effect of the energy density B is to add a term to the usual stress-energy tensor:

$$T^{\mu\nu} = T^{\mu\nu}_{\text{fields}} - g^{\mu\nu} B$$

inside the bag. Outside the bag $T_{\mu\nu}$ vanishes. Requiring energy-momentum conservation leads to boundary conditions on the fields at the surface of the bag. Here we specify the confined fields to be massless,[2] spin-$\frac{1}{2}$ fields carrying colored-quark[3] quantum numbers and interacting with massless, colored vector gluons. An exact consequence[1] of the bag boundary conditions for such an interaction is that only color singlet states (which have zero triality) can exist. The coupling constant need not be large to achieve this, and in this paper we shall neglect the gluons entirely (except that we shall limit our considerations to quark states which are color singlets).

Even when the quark fields are free inside the bag, the field equations plus boundary conditions are not exactly soluble in three space dimensions. Instead we solve them in what seems to be a reasonable zeroth approximation which is analogous to the "Bohr theory" for the hydrogen atom: The classical equations of motion admit a class of solutions in which the surface of the bag (in its rest frame) is a sphere of fixed radius. The boundary conditions require each quark to occupy a mode with total angular momentum $\frac{1}{2}$. We regard these modes in a fixed, spherical cavity as analogous to the circular orbits with fixed radius in the old quantum theory. The radius is then quantized by the condition that the quark-number operator take on integer values. For these states, the energy depends on which modes are occupied but not on the way the angular momenta or isospins of the individual quarks are added to obtain the total angular momentum and isospin of the hadron. Thus, for example, the lowest-lying[4] $N(\frac{1}{2}+)$ and $\Delta(\frac{3}{2}+)$ are degenerate. This aspect of the SU(6) quark model[5] is therefore a consequence of the dynamical approximation of fixed radius. However, in other aspects (for example, the mass spectrum and the value of g_A) this approximation to our model does not coincide with SU(6) or even broken SU(6).

In the remainder of this paper, we discuss in detail the consequences of this simple picture for the baryons. Mesons will be treated elsewhere. Since B is the only free parameter [to be fixed by the average mass of the $N(938)$ and $\Delta(1236)$] we are able to make predictions for dimensional quantities such as the proton magnetic moment and charge radius and the zeroth-order mass splittings in the baryon spectrum. In the conclusion we comment further about the fixed-radius approximation and how first-order corrections to it may be made.

II. CALCULATIONS

The equations of motion and boundary conditions for a massless, spin-$\frac{1}{2}$ field confined to a bag are[1]

$$\not{\partial}\psi_\alpha(x) = 0 \tag{1}$$

inside the bag and

$$i\not{n}\psi_\alpha(x)=\psi_\alpha(x), \tag{2a}$$

$$\sum_\alpha n\cdot\partial\bar{\psi}_\alpha(x)\psi_\alpha(x)=2B \tag{2b}$$

on the bag's surface. n_μ is the covariant, interior 4-normal to the bag's surface. α is an internal-symmetry index which we choose to designate isospin and color. We seek solutions for which the boundary is a static sphere of radius R_0, in which case $n_\mu=(0,-\hat{r})$ and Eqs. (2) become

$$-i\hat{r}\cdot\vec{\gamma}\psi_\alpha(x)=\psi_\alpha(x), \tag{3a}$$

$$-\sum_\alpha\frac{\partial}{\partial r}\bar{\psi}_\alpha(x)\psi_\alpha(x)=2B \tag{3b}$$

at $r=R_0$.

The general solution to Eqs. (1) and (3) is a superposition (with coefficients a_α) of solutions to the free Dirac equation:

$$\psi_\alpha(x,t)=\sum_{n\kappa jm}N(\omega_{n\kappa j})a_\alpha(n\kappa jm)\psi_{n\kappa jm}(x,t). \tag{4}$$

j and m label the mode's angular momentum and its z component. κ is the Dirac quantum number,[6] $\kappa=\pm(j+\frac{1}{2})$, which differentiates the two states of opposite parity for each value of j. The index n labels frequencies which are to be determined by the linear boundary condition, Eq. (3a). The quadratic boundary condition (3b) restricts the modes which may be excited. Among other things, Eq. (3b) allows only $j=\frac{1}{2}$ solutions to Dirac's equation. For $j=\frac{1}{2}$, either $\kappa=-1$,

$$\psi_{n\,-1\,1/2\,m}(x,t)=\frac{1}{\sqrt{4\pi}}\begin{pmatrix} ij_0(\omega_{n,-1}r/R_0)U_m \\ -j_1(\omega_{n,-1}r/R_0)\sigma\cdot\hat{r}U_m\end{pmatrix}\times e^{-i\omega_{n,-1}t/R_0}, \tag{5a}$$

or $\kappa=1$,

$$\psi_{n\,1\,1/2\,m}(x,t)=\frac{1}{\sqrt{4\pi}}\begin{pmatrix} ij_1(\omega_{n,1}r/R_0)\sigma\cdot\hat{r}U_m \\ j_0(\omega_{n,1}r/R_0)U_m\end{pmatrix}\times e^{-i\omega_{n,1}t/R_0}. \tag{5b}$$

U_m is a two-component Pauli spinor and $j_l(z)$ are conventional spherical Bessel functions. We have dropped the index j on $\omega_{n\kappa}$ since only $j=\frac{1}{2}$ is of interest at present. $N(\omega_{n\kappa})$ is a normalization constant chosen for future convenience:

$$N(\omega_{n\kappa})\equiv\left(\frac{\omega_{n\kappa}{}^3}{2R_0{}^3(\omega_{n\kappa}+\kappa)\sin^2\omega_{n\kappa}}\right)^{1/2}. \tag{6}$$

The linear boundary condition (3a) generates an eigenvalue condition for the mode frequencies $\omega_{n\kappa}$,

$$j_0(\omega_{n\kappa})=-\kappa j_1(\omega_{n\kappa}),$$

or

$$\tan\omega_{n\kappa}=\frac{\omega_{n\kappa}}{\omega_{n\kappa}+\kappa}. \tag{7}$$

[By convention we choose positive (negative) n sequentially to label the positive (negative) roots of Eq. (7).] The first few solutions to Eq. (7) are

$$\begin{aligned}&\kappa=-1:\ \ \omega_{1-1}=2.04;\ \ \omega_{2-1}=5.40\\ &\kappa=+1:\ \ \omega_{11}=3.81;\ \ \omega_{21}=7.00.\end{aligned} \tag{8}$$

The quadratic boundary condition requires $\sum_\alpha(\partial/\partial r)\bar{\psi}_\alpha(x)\psi_\alpha(x)$ to be time- and direction-independent for $r=R_0$. Angular independence requires $j=\frac{1}{2}$. To obtain time independence, set

$$\sum_\alpha a^*_\alpha(n\,\kappa\,j=\tfrac{1}{2}\,m)a_\alpha(n'\,\kappa'\,j=\tfrac{1}{2}\,m')=0, \tag{9}$$

unless $n=n'$, $\kappa=\kappa'$ or $n=-n'$, $\kappa=-\kappa'$, in which cases there is no restriction since the time-dependent terms cancel. Equation (9) is a severe restriction on the modes which are to be occupied. We shall implement Eq. (9) by requiring that for each internal degree of freedom α only one normal mode, $a_\alpha(n\,\kappa\,j=\frac{1}{2}m)$, be excited.[7] This will automatically be the case for three-quark baryons if they are required to be color singlets.

Once Eq. (9) is satisfied, the time-independent terms in Eq. (3b) may be collected,

$$\sum_{\alpha n\kappa m}\omega_{n\kappa}a^*_\alpha(n\,\kappa\,\tfrac{1}{2}\,m)a_\alpha(n\,\kappa\,\tfrac{1}{2}\,m)=4\pi BR_0{}^4, \tag{10}$$

where it is understood that only one $a_\alpha(n\,\kappa\,\frac{1}{2}m)$ is different from zero for each choice of α. The relation between the energy and size of a bag in its rest frame is known from a virial theorem,[1]

$$E=4B\langle V\rangle=\frac{16\pi}{3}BR_0{}^3. \tag{11}$$

Therefore Eqs. (10) and (11) express the Hamiltonian of the static bag in terms of the normal-mode coefficients $a_\alpha(n\,k\,\frac{1}{2}m)$. Equations (4)–(11) complete the solution to the classical, static, spherical Dirac bag. We have so far ignored a familiar problem with Dirac systems, namely that the left-hand side of Eq. (10) is not positive-definite since $\{\omega_{n\kappa}\}$ are not bounded below. This problem is best treated at the quantum level, which we now describe.

We wish now to quantize this limited class of solutions to a classical problem. Guided by analogy to Bohr's quantization of the circular orbits of the hydrogen atom, we shall quantize the action integrals of our solutions. As will be evident from Eq. (14) below, this amounts to requiring an integer number of quarks of each type inside the bag. Equivalently we could treat the static spherical bag as a "field in a box (of radius R_0)" and quantize the dynamical variables via Poisson brackets. The quantum theory developed this way requires $[R_0, H]=0$ [see Eq. (11)]. We preserve from the Poisson-bracket approach an operator formalism which allows us to keep track of quantum superposition of states.

Specifically, we define

$$\begin{aligned} a_\alpha(n\,\kappa\, j=\tfrac{1}{2}m) &\equiv b_\alpha(n\,\kappa\, m)\,, \quad n>0 \\ a_\alpha(n\,\kappa\, j=\tfrac{1}{2}m) &\equiv d_\alpha^\dagger(-n,-\kappa,m)\,, \quad n<0 \end{aligned} \tag{12}$$

with

$$\{b_\alpha(n\kappa m), b_\alpha^\dagger(n\kappa m)\}=\{d_\alpha(n\kappa m), d_\alpha^\dagger(n\kappa m)\}=1 \tag{13}$$

and all other anticommutators zero. We define a state $|0\rangle$ such that $b_\alpha(n\kappa m)|0\rangle = d_\alpha(n\kappa m)|0\rangle = 0$. With these definitions the action integrals become

$$\begin{aligned} N_\alpha &\equiv \int_{\text{bag}} d^3x\, \psi_\alpha^\dagger(x)\psi_\alpha(x) \\ &= \sum_{n\kappa m} [b_\alpha^\dagger(n\kappa m)b_\alpha(n\kappa m) - d_\alpha^\dagger(n\kappa m)d_\alpha(n\kappa m)]\,. \end{aligned} \tag{14}$$

From the definitions of (12) and (13) it is direct that N_α has integer eigenvalues. Furthermore, Eq. (10) may be rewritten (with the help of the identity $\omega_{n,1} = -\omega_{-n,-1}$) as

$$\begin{aligned} 4\pi B R_0^{\,4} = \sum_{\alpha n\kappa m} \omega_{n\kappa}[b_\alpha^\dagger(n\kappa m)b_\alpha(n\kappa m) \\ + d_\alpha^\dagger(n\kappa m)d_\alpha(n\kappa m)]\,, \end{aligned} \tag{15}$$

in which only positive $\omega_{n\kappa}$ occur. We have dropped from Eq. (15) a normal-ordering term corresponding to zero-point fluctuations in the field. By fixing R_0 in the first place, we have ignored fluctuations induced by the fields. Dropping this term is at the same level of approximation. A prescription for including these fluctuations is discussed in the conclusion. The field ψ of Eq. (4) may be rewritten in terms of the operators $b_\alpha(n\kappa m)$ and $d_\alpha(n\kappa m)$. The Hamiltonian of the quantum theory is obtained from Eqs. (11) and (15).

It is interesting to note that the second bag boundary condition [Eq. (3b)] can be replaced by an energy-variational principle. Equations (1) and (3a) define a massless Dirac field confined to an infinite, spherical "square well" potential of radius R_0. Solutions to this problem are given by Eqs. (5), (6), and (7) (for $j=\frac{1}{2}$). They may be quantized in the usual canonical way, leading to an energy

$$E_{\text{field}} = \sum_{\alpha n\kappa jm} N_\alpha(n\kappa jm)\omega_{n\kappa j}/R_0\,,$$

where $N_\alpha(n\kappa jm)$ are integers. This system is clearly unstable, since the energy decreases as R_0 increases: There is no confinement. The bag theory introduces a "pressure" B which stabilizes the system [but only for $j=\frac{1}{2}$ modes commensurate with Eq. (9) does the system remain spherical]. The total energy is then

$$E(R_0) = E_{\text{field}} + \frac{4\pi B}{3} R_0^{\,3}\,.$$

Equilibrium is obtained if $E(R_0)$ is a minimum. $\partial E/\partial R_0 = 0$ immediately yields Eqs. (10) and (11) and thereby the entirety of the "static" bag model we have developed.[8]

A quark model is specified by choosing the symmetries associated with the index α. Since we ignore strangeness, our quarks are six in number: three colors (1, 2, 3) each of isospin $\pm\frac{1}{2}$ ($\mathcal{P}, \mathfrak{N}$ species). As discussed above, three-quark baryons are to be color singlets [thereby satisfying Eq. (9)] and are therefore to be constructed of quarks in totally symmetric spin-isospin-spatial states. Several spatial states (i.e., quantum "modes") are available (in order of increasing energy):

$1S_{1/2}$ with $\omega_{1,-1} = 2.04$,

$1P_{1/2}$ with $\omega_{11} = 3.84$,

$2S_{1/2}$ with $\omega_{2,-1} = 5.4$,

etc .

It is a simple problem in group representations to find the totally symmetric ways to distribute $\mathcal{P}$ and $\mathfrak{N}$ quarks with spin $=\frac{1}{2}$ among these orbitals. The resulting spectrum is given in Fig. 1. The lowest states, $(1S_{1/2})^3$, are to be identified with the $N(938)$ and $\Delta(1236)$. The masses quoted in Fig. 1 are determined from Eqs. (11) and (15). The parameter B is chosen to fit the average[9] mass of the $N(938)$-$\Delta(1236)$ system:

$$\tfrac{4}{3}(4\pi B)^{1/4}(3\omega_{1,-1})^{3/4} = 1180 \text{ MeV}\,,$$

whence $B^{1/4} = 120$ MeV.

It is clear from Fig. 1 that the symmetry scheme of the bag model is not the SU(6) of the nonrelativistic quark model. For example, the low-lying negative-parity multiplet does not include a $J^\pi = \frac{5}{2}^-$ state since each quark is in a $j=\frac{1}{2}$ state. Baryons with three quark modes occupied and $J \geq \frac{5}{2}$ must be states in which the surface is not static. The lowest negative-parity multiplet in Fig. 1 corresponds rather well with the low-lying $J \leq \frac{3}{2}$ nonstrange baryon resonances. Higher-energy states are less well isolated and presumably mix with one another. Which states are strongly excited in π-N scattering depends on detailed

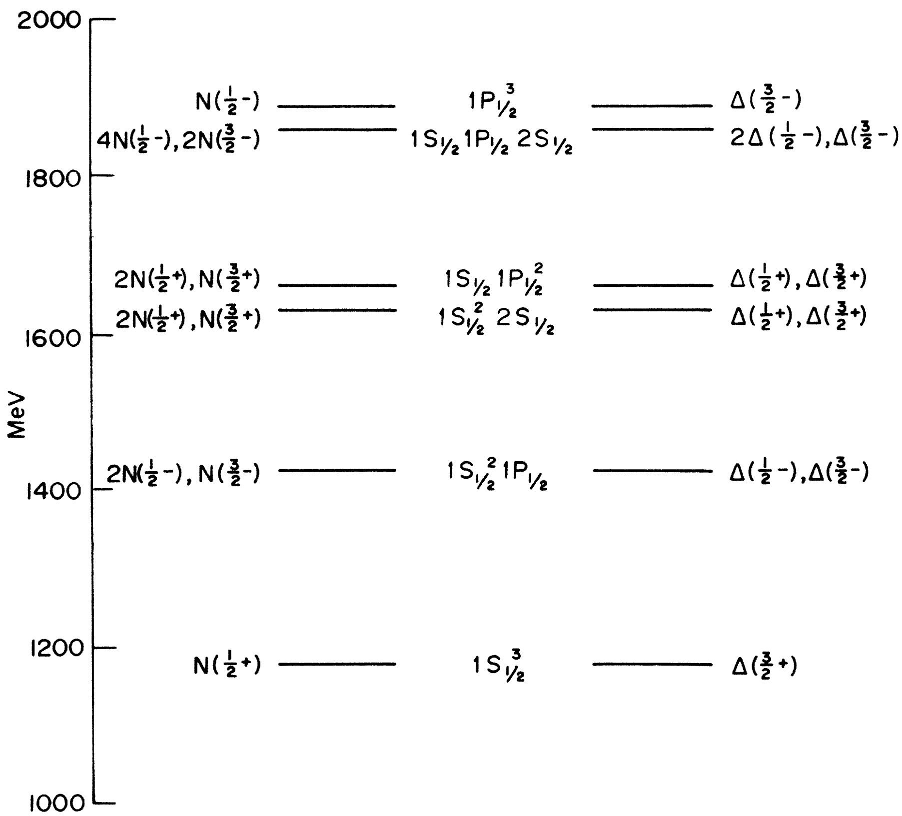

FIG. 1. Low-lying ($M < 2$ GeV) three-quark nonstrange baryon states with $J \le \frac{3}{2}$ in the bag quark model. Nucleons ($I=\frac{1}{2}$) are in the left column, Δ's ($I=\frac{3}{2}$) in the right.

dynamics beyond the scope of this paper. We expect energies to be shifted and degeneracies removed when quantum fluctuations are incorporated in the model.

The spin-isospin structure of the state vectors of our lowest states corresponds to that of the nonrelativistic quark model. For example, the $\Delta^{++}(1236)$ is given by

$$|\Delta^{++}, J_z=\tfrac{3}{2}\rangle = \mathcal{P}_1^\dagger(\uparrow)\mathcal{P}_2^\dagger(\uparrow)\mathcal{P}_3^\dagger(\uparrow)|0\rangle ,$$

where 1, 2, 3 denote color and all creation operators create quarks in the $1S_{1/2}$ mode whose "wave function" is Eq. (5a) with $n=1$. State vectors for other $(1S_{1/2})^3$ states may be found in (e.g.) Ref. 5.

As examples of the application of our quark model we calculate the magnetic moment and charge radius of the proton and neutron and the axial-vector charge of the proton. The magnetic-moment operator is defined by

$$\vec{\mu} = \int_{\text{bag}} d^3x\, \tfrac{1}{2}\vec{r}\times\psi^\dagger\vec{\alpha}Q\psi \tag{16}$$

[Q is the matrix $\begin{pmatrix} 2/3 & 0 \\ 0 & -1/3\end{pmatrix}$ in the quark space]. Explicit calculation with the field of Eqs. (4)–(6) yields

$$\vec{\mu} = \frac{R_0}{12}\sum_{\alpha n\kappa m} f(\omega_{n\kappa})U_m^\dagger\vec{\sigma}U_m[b_\alpha^\dagger(n\kappa m)Q_\alpha b_\alpha(n\kappa m) + d_\alpha^\dagger(n\kappa m)Q_\alpha d_\alpha(n\kappa m)] , \tag{17}$$

where

$$f(\omega_{n\kappa}) = \frac{4\omega_{n\kappa}+3\kappa}{\omega_{n\kappa}(\omega_{n\kappa}+\kappa)} \tag{18}$$

and Q_α are the diagonal elements of the matrix Q. For the proton state constructed above,

$$\mu_p = \frac{R_0}{12}\,\frac{4\omega_{1,-1}-3}{\omega_{1,-1}(\omega_{1,-1}-1)} , \tag{19}$$

where R_0 is fixed by Eq. (15): $R_0=(3\omega_{1,-1}/4\pi B)^{1/4}$. From $B=120$ MeV and $\omega_{1,-1}=2.04$ we conclude $\mu_p = 1.4\ \text{GeV}^{-1}$, or a gyromagnetic ratio of

$$g_p \equiv 2m_p\mu_p = 2.6 . \tag{20}$$

The neutron's magnetic moment is calculated analogously and $g_n = -\frac{2}{3}g_p$ is obtained. It should be noted that the origin of the magnetic moment in our model is completely different from that of the nonrelativistic quark model. If it were not confined, the massless Dirac field would possess no magnetic moment [in Eq. (19) $\mu \to \infty$, as $R_0 \to \infty$]. Confinement sets a scale (via B) and a magnetic moment arises from the cross terms between the upper and lower components of the wave functions [Eq. (5)].

The charge-radius-squared operator is defined by

$$\langle r^2 \rangle \equiv \int_{\text{bag}} d^3x\, \psi^\dagger(x) Q \psi(x) |\vec{x}|^2, \tag{21}$$

and may be evaluated as was μ:

$$\langle r^2 \rangle = \frac{R_0^{\,2}}{6} \sum_\alpha g(\omega_{n\kappa}) [b_\alpha^\dagger(n\kappa m) Q_\alpha b_\alpha(n\kappa m) - d_\alpha^\dagger(n\kappa m) Q_\alpha d_\alpha(n\kappa m)], \tag{22}$$

where

$$g(\omega_{n\kappa}) = \frac{2\omega_{n\kappa}^{\,3} + 2\kappa\omega_{n\kappa}^{\,2} + 4\omega_{n\kappa} + 3\kappa}{\omega_{n\kappa}^{\,2}(\omega_{n\kappa} + \kappa)}. \tag{23}$$

For the proton we obtain

$$\langle r^2 \rangle_p^{1/2} = 1.04 \text{ fm}; \tag{24}$$

for the neutron

$$\langle r^2 \rangle_n = 0. \tag{25}$$

Finally we may calculate the axial-vector coupling constant of β decay defined by

$$g_A \equiv \left\langle PS_z = \tfrac{1}{2} \right| \int_{\text{bag}} d^3x\, \psi^\dagger(x) \tau_3 \sigma_z \psi(x) \left| PS_z = \tfrac{1}{2} \right\rangle, \tag{26}$$

where $\tau_3 = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$ in $(\mathcal{P}, \mathcal{N})$ space. Explicit evaluation yields

$$\int_{\text{bag}} d^3x\, \psi^\dagger(x) \tau_3 \sigma_z \psi(x) = \sum_{\alpha n\kappa m} h(\omega_{n\kappa}) U_m^\dagger \sigma_z U_m [b_\alpha^\dagger(n\kappa m)(\tau_3)_\alpha b_\alpha(n\kappa m) + d_\alpha^\dagger(n\kappa m)(\tau_3)_\alpha d_\alpha(n\kappa m)], \tag{27}$$

where

$$h(\omega_{n\kappa}) = 1 - \frac{2\omega_{n\kappa} + 3\kappa}{3(\omega_{n\kappa} + \kappa)} \tag{28}$$

and $(\tau_3)_\alpha$ are the diagonal matrix elements of τ_3. From Eq. (26) we obtain

$$g_A = \frac{5}{3}\left(1 - \frac{2\omega_{1,-1} - 3}{3(\omega_{1,-1} - 1)}\right) = 1.09. \tag{29}$$

In the nonrelativistic quark model $g_A = \frac{5}{3}$. Our result differs because the lower components in Eq. (5a) are important and have opposite spin orientation from the upper components.

We have shown that the bag quark model incorporates many of the successful features of the nonrelativistic quark model, and where it is different improvements are made, as, for example, in the value of g_A. The bag model also makes an advance in that it allows one to calculate quantities (such as the gyromagnetic ratio and the mean square charge radius of the proton) which in conventional models are parameters fixed to agree with experiment.

These calculations (good to 10–20%) have been done assuming that the bag's surface is a sphere of fixed radius. This approximation bears some similarity to the Born-Oppenheimer approximation of molecular physics. There, one first calculates the electronic motion for fixed internuclear distance, and then uses this electronic energy as a potential to calculate the nuclear motion. The approximation is justified because the kinetic-energy term for the nuclei is small (being inversely proportional to the nuclear mass). In our case, we likewise calculate the quark energies for a given bag radius, using the equations of motion and the linear boundary conditions. However, for us the approximation is exact, because there is no kinetic-energy term for the bag's surface at all. Instead, there is an equation of constraint [the second boundary condition, Eq. (3b)], which fixes the radius at a particular value depending on the energy.

Thus, at the classical level, the solutions to the static spherical bag are exact. However, the assumption that the shape of the bag is sharply defined is incompatible with quantum mechanics. The boundary conditions imply that the variables describing the surface of the bag are functions of the constituent fields inside. At the quantum level these variables will then be operators which necessarily have fluctuations in energy eigenstates. Our prescription of quantizing the action integral to be integer multiples of $\hbar$ clearly ignores this subtlety. To rigorously evaluate the validity of our approximations, it is necessary to develop the quantum mechanics in the presence of the constraints implied by the boundary conditions. This is a formidable task. However, we expect that a study of the effects on our results of taking into account small oscillations of the bag's surface will yield some insight. For example, the nonspherical fluctuations would presumably lead to a splitting between the N and Δ states, which would be calculable. This program is presently under investigation.

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[1]A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974).

[2]SU(3) breaking may be incorporated by assigning the singlet quark a finite mass; however, in this paper we treat only nonstrange baryons and therefore ignore this possibility.

[3]H. Fritzsch and M. Gell-Mann, in *Proceedings of the International Conference on Duality and Symmetry in Hadron Physics*, edited by E. Gotsman (Weizmann Science Press, Jerusalem, 1971).

[4]N and Δ label states of isospin $\frac{1}{2}$ and $\frac{3}{2}$, respectively. The spin and parity are given in parentheses.

[5]See, for example, J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969).

[6]See, for example, J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[7]Actually two modes (n, κ and $-n$, $-\kappa$) could be excited consistent with Eq. (9). The negative-frequency mode corresponds to an antiquark. Such states are essential in the construction of mesons, but play no role in the baryon sector which we are studying here.

[8]After the completion of this paper we learned of the work of P. N. Bogoliubov [Ann. Inst. Henri Poincaré 8, 163 (1967)]. Bogoliubov considered massless quarks confined to an infinite, spherical "square well" potential of radius R_0. No confining "pressure" was introduced; instead the "square well" was motivated as an approximation to a "self-consistent" interquark potential. R_0 was fixed by equating the total energy of the quarks to the mass of the hadron. Bogoliubov's calculations are quite similar to ours; the primary differences stem from the absence in his model of a term $-g_{\mu\nu}B$ in the stress tensor, which is essential to achieve confinement in a Lorentz-covariant way. We wish to thank Dr. S.-H. H. Tye for bringing Bogoliubov's work to our attention.

[9]Weighted by the spin-isospin degeneracy of the states.