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Functional approach to strong-coupling theory in static models. I. Charged-scalar model*

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The strong-coupling theory in static models is formulated in terms of functional integrations. The method is demonstrated for the charged-scalar model. The expression of elastic and inelastic meson-nucleon scattering amplitudes is obtained to leading order in the strong-coupling expansion ($1/g$ expansion), while the isobar energy levels are obtained up to the next to the leading order.

I. INTRODUCTION

There are two distinct approaches to the strong-coupling theory in static models: the canonical field-theoretical method and the S-matrix method.

In the field-theory approach, which has a long history[1] since the first work of Wentzel,[2] one applies successive canonical transformations in the Hamiltonian formalism. On the other hand, in the S-matrix approach,[3] one starts with a set of Chew-Low equations for scattering amplitudes and solves them in the strong-coupling limit, presupposing various properties of the strong-coupling results, known from the field-theoretical method, such as the absence of production amplitudes in the strong-coupling limit. It is remarkable, however, that the final results are expressed in terms of operators in the isobar space which obey relatively simple algebraic equations.

We investigate the strong-coupling theory using the method of functional integration, with the intent of clarifying the perturbative nature of the strong-coupling theory ($1/g^2$ expansion). In this method, a canonical transformation is described by a corresponding change of variables of the phase-space functional integration, and a subsidiary condition on the state vector in the conventional formalism is described by a restriction of the Feynman integration path, which can be realized easily by inserting an appropriate δ functional in the integrand.[5] Thus, this method would be suitable for the description of strong-coupling theory.

In this paper we show the essence of the method using the charged-scalar model as an example and leave the general case to the following paper.

II. FUNCTIONAL-INTEGRAL REPRESENTATION OF GENERATING FUNCTIONAL OF GREEN'S FUNCTION IN STATIC MODEL

We define the generating functional by

$$Z(\eta) = \langle n_f | S_\eta | n_i \rangle \,, \tag{1}$$

where $|n_i\rangle$ and $|n_f\rangle$ are initial and final nucleon states, respectively, and S_η is the S operator with the meson external-source function $\eta_\alpha(\vec{x}, t)$ ($\alpha = 1, 2$). The reaction amplitudes are obtained by taking functional derivatives of $Z(\eta)$ with respect to η_α in the appropriate manner and by using the Lehmann-Symanzik-Zimmermann (LSZ) reduction formalism. For example, the elastic scattering amplitude is given by

$$T_{\alpha\beta}(\vec{k}', \vec{k}; n_f, n_i) = \int e^{-ik'y+ikx} d^4x\, d^4y (\Box_x - \mu^2)(\Box_y - \mu^2) \left. \frac{\delta^2 Z(\eta)}{\delta\eta_\alpha(x)\delta\eta_\beta(y)} \right|_{\eta_\alpha = 0} . \tag{2}$$

$Z(\eta)$ can be expressed in terms of the Feynman path integral as

$$Z(\eta) = \int D(\text{path}) \Psi_f^* \exp\left[i \int dt\, L(t) \right] \Psi_i , \tag{3}$$

where

$$L(t) = L_\varphi + L_{\text{int}} + \int \eta_\alpha(\vec{x}, t)\, \varphi_\alpha(\vec{x}, t)\, d\vec{x} , \tag{4}$$

$$L_\varphi(t) = \tfrac{1}{2} \int (\dot{\varphi}_\alpha \dot{\varphi}_\alpha - \vec{\nabla}\varphi_\alpha \cdot \vec{\nabla}\varphi_\alpha - \mu^2 \varphi_\alpha \varphi_\alpha)\, d\vec{x} , \tag{5}$$

$$L_{\text{int}} = g\, \tau_\alpha(t)\, q_\alpha(t) , \tag{6}$$

and

$$q_\alpha(t) = \int \rho(\vec{x})\, \varphi_\alpha(\vec{x}, t)\, d\vec{x} . \tag{7}$$

Ψ_i and Ψ_f are the initial and final nucleon wave functions, respectively.

III. FREEZING OF THE NUCLEON ISOSPIN IN THE STRONG-COUPLING LIMIT

In Eq. (3) the Feynman integration path should be taken both in the field variable $\varphi_\alpha(\vec{x}, t)$ and the nucleon isospin variable. The latter path can be specified in terms of two eigenvalues of L_{int} for all t. However, only one path will contribute in the strong-coupling limit, the path due to the positive eigenvalue of L_{int} for all t. On this path, the bare nucleon isospin is always parallel to the (isotopic) direction of the meson cloud. Since any switch from one eigenvalue to another involves a phase change of order g, one can show as in Appendix A that the contributions of other paths to reaction amplitudes are at most of order g^{-3}, while the contributions to energy levels are at most of order g^{-6}.

Thus, we replace L_{int} in (4) by

$$g q(t) = g \left[\left(\int \rho \varphi_\alpha d\, \vec{x} \right)^2 \right]^{1/2} , \tag{8}$$

and integrate only on the paths in $\varphi_\alpha(\vec{x}, t)$ space:

$$Z(\eta) = \int \prod_{\vec{x}, t, \alpha} [d\varphi_\alpha(\vec{x}, t)] \Psi_f^* \left[\exp\left(i \int L(t)\, dt \right) \right] \Psi_i , \tag{9}$$

where $\prod_{\vec{x}, t, \alpha}$ is the product symbol,

$$L(t) = \tfrac{1}{2} \int [\dot{\varphi}_\alpha{}^2(\vec{x}, t) - V(\varphi)]\, d\vec{x} + \int \eta_\alpha(\vec{x}, t)\, \varphi_\alpha(\vec{x}, t)\, d\vec{x} , \tag{10}$$

and $V(\varphi)$ is the effective potential energy of the meson cloud (including the coupling to the nucleon):

$$V(\varphi) = \tfrac{1}{2} \int d\vec{x} [(\vec{\nabla}\varphi_\alpha)^2 + \mu^2 \varphi_\alpha{}^2] - g q(t) . \tag{11}$$

This replacement is equivalent to the following condition on the Schrödinger state vector $|\Psi\rangle$ of the system in the conventional operator formalism:

$$\tau_\alpha q_\alpha |\Psi(t)\rangle = q(t) |\Psi(t)\rangle . \tag{12}$$

IV. EXPANSION OF POTENTIAL ENERGY AROUND MINIMUM

Following Ref. 1, we first evaluate the value of the field strength φ_α^0 for which the potential energy of the system is minimum. φ_α^0 is determined by

$$\left. \frac{\delta V(\varphi)}{\delta \varphi_\alpha(\vec{x}, t)} \right|_{\varphi_\alpha = \varphi_\alpha^0} = 0 \tag{13}$$

and expressed as

$$\varphi_\alpha^0(\vec{x}, t) = g u A_\alpha , \tag{14}$$

where u is the solution of

$$-(\nabla^2 - \mu^2) u = \rho , \tag{15}$$

and A_α ($\alpha = 1, 2$) are the components of a unit vector. The corresponding q_α^0 is therefore given by

$$q_\alpha^0(t) = g a A_\alpha(t) , \tag{16}$$

where

$$a = \int \rho u\, d\vec{x} . \tag{17}$$

The minimum of V has a symmetry, since A_α in (14) is arbitrary. We change the integration variables φ_α into the variables of the symmetry and the variables orthogonal to them. For this purpose, in what follows, we use a method similar to that of Popov and Faddeev.[5]

We insert the identity

$$\int \prod_{t,\alpha} [dQ_\alpha(t)] \prod_{t,\alpha} \delta\left(Q_\alpha(t) - \int v\varphi_\alpha(\vec{x}, t)\, d\vec{x}\right) = 1 \tag{18}$$

[the function $v(\vec{x})$ being arbitrary] into Eq. (9) and make the following change of variables:

$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha - \varphi^0_\alpha , \tag{19}$$

where φ^0_α is given by Eq. (14), with A_α given by

$$A_\alpha = \frac{Q_\alpha}{Q} . \tag{20}$$

We then expand the interaction term around $q = q^0$. Since q^0 is of order g, it is an expansion in powers of $1/g$:

$$\begin{aligned} L_{\text{int}}(t) &= gq(t) \\ &= g\left(q^0 + \frac{\partial q}{\partial q_\alpha}\bigg|_{q_\alpha = q^0_\alpha} q'_\alpha + \frac{1}{2}\frac{\partial^2 q}{\partial q_\alpha \partial q_\beta}\bigg|_{q_\alpha = q^0_\alpha} q'_\alpha q'_\beta + \frac{1}{6}\frac{\partial^3 q}{\partial q_\alpha \partial q_\beta \partial q_\gamma}\bigg|_{q_\alpha = q^0_\alpha} q'_\alpha q'_\beta q'_\gamma \right) \\ &= g^2 a + gA_\alpha(t) q'_\alpha(t) + \frac{1}{2a}\Lambda_{\alpha\beta} q'_\alpha(t) q'_\beta(t) - \frac{1}{6a^2 g}\Xi_{\alpha\beta\gamma} q'_\alpha q'_\beta q'_\gamma + O\left(\frac{1}{g^2}\right) , \end{aligned} \tag{21}$$

where

$$\frac{\partial q}{\partial q_\alpha}\bigg|_{q_\alpha = q^0_\alpha} = A_\alpha(t) , \tag{22}$$

$$q_0 \frac{\partial^2 q}{\partial q_\alpha \partial q_\beta}\bigg|_{q_\alpha = q^0_\alpha} = \Lambda_{\alpha\beta} = \delta_{\alpha\beta} - A_\alpha A_\beta , \tag{23}$$

$$\begin{aligned} -q_0^2 \frac{\partial^3 q}{\partial q_\alpha \partial q_\beta \partial q_\gamma}\bigg|_{q = q^0_\alpha} &= \Xi_{\alpha\beta\gamma} \\ &= \Lambda_{\alpha\beta} A_\gamma + \Lambda_{\beta\gamma} A_\alpha + \Lambda_{\gamma\alpha} A_\beta . \end{aligned} \tag{24}$$

Thus we obtain

$$\begin{aligned} L(t) = {}& \tfrac{1}{2} g a^2 + L_{\varphi'} + \int d\vec{x}\,\eta_\alpha \varphi'_\alpha + \tfrac{1}{2} g^2 b \mathring{A}_\alpha \mathring{A}_\alpha + g\int d\vec{x}\,\eta_\alpha A_\alpha u + g\int \mathring{A}_\alpha u \mathring{\varphi}'_\alpha\, d\vec{x} + \frac{1}{2a}\Lambda_{\alpha\beta}(A) \int \rho\varphi'_\alpha\, d\vec{x} \int \rho\varphi'_\beta\, d\vec{y} \\ & - \frac{1}{6a^2 g}\Xi_{\alpha\beta\gamma} \int \rho\varphi'_\alpha\, d\vec{x} \int \rho\varphi'_\beta\, d\vec{y} \int \rho\varphi'_\gamma\, d\vec{z} , \end{aligned} \tag{25}$$

where

$$b = \int u^2 d\vec{x} . \tag{26}$$

Since the first term is a constant, we will omit it in the following discussion.

V. SUBSIDIARY CONDITIONS

Introducing a vector $\vec{B}$ such that $B_\alpha A_\alpha = 0$, we write the δ functional of Eq. (18) as

$$\prod_{t,\alpha} dQ_\alpha(t)\, \delta\left(Q_\alpha(t) - \int v\varphi_\alpha(\vec{x}, t)\, d\vec{x}\right) = \prod_t Q(t)\, dQ(t)\, \delta\left(Q(t) - A_\alpha \int v\varphi_\alpha(\vec{x}, t)\, d\vec{x}\right) d\theta(t)\, \delta\left(B_\alpha(t) \int v\varphi_\alpha(\vec{x}, t) d\vec{x}\right) . \tag{27}$$

Since $L(t)$ is independent of Q, we can integrate over Q. Inserting $\varphi_\alpha = \varphi^0_\alpha + \varphi'_\alpha$ into (27) we obtain the following effective integration measure:

$$\prod_t d\theta(t) \left[g\int vu\, d\vec{x} + A_\alpha(t) \int v\varphi'_\alpha(\vec{x}, t)\, d\vec{x} \right] \delta\left(B_\alpha(t) \int v\varphi'_\alpha(\vec{x}, t)\, d\vec{x}\right) . \tag{28}$$

Notice that A_α is a function of θ; accordingly B_α is also.

VI. CHOICE OF $v(\vec{x})$

Next we choose $v(\vec{x})$ so that $\mathring{A}_\alpha \int u \mathring{\varphi}'_\alpha\, d\vec{x}$ in (25) can be neglected in the strong-coupling limit, thus decoupling the kinetic-energy terms of φ'_α and A_α. The choice[6] is

$$v(\vec{x}) = u(\vec{x}) . \tag{29}$$

Then, because of the δ function in (28), i.e., $B_\alpha \int v\varphi'_\alpha\, d\vec{x} = 0$, we can write

$$\begin{aligned} g\mathring{A}_\alpha \frac{d}{dt}\left(\int u\varphi'_\alpha\, d\vec{x}\right) &= g\mathring{A}_\alpha \frac{d}{dt}\left[A_\alpha\left(A_\beta \int u\varphi'_\beta\, d\vec{x}\right)\right] \\ &= g\mathring{A}_\alpha \mathring{A}_\alpha \left(A_\beta \int u\varphi'_\beta\, d\vec{x}\right) , \end{aligned} \tag{30}$$

since $A_\alpha \dot{A}_\alpha = 0$. Thus, this term is negligible in the strong-coupling limit compared to $\frac{1}{2}g^2 b \dot{A}_\alpha \dot{A}_\alpha$. But, since we are interested in the next-order term in $1/g$, we keep (30). Combining with the fourth term in (25) we obtain

$$\tfrac{1}{2}g^2 b \dot{A}_\alpha \dot{A}_\alpha \left(1 + \frac{2}{gb} A_\alpha \int u \varphi'_\alpha \, d\vec{x}\right). \tag{31}$$

We neglect the term $g\eta_\alpha A_\alpha u$ of (25), since it cannot produce poles in the external lines of Green's functions, so that it does not contribute to the reaction amplitudes.[7]

Thus we obtain the following $Z(\eta)$ in the strong-coupling limit:

$$Z(\eta) = \int \cdots \int \prod_t \left\{ d\theta(t) \left[gb + A_\alpha \int u \varphi'_\alpha(\vec{x}, t)\, d\vec{x}\right] \prod_{\vec{x},\alpha} d\varphi'_\alpha(\vec{x}, t)\, \delta\left(B_\alpha \int u \varphi'_\alpha(\vec{x}, t)\, d\vec{x}\right) \right\}$$
$$\times \exp\left[i \int_{-\infty}^{\infty} L(t)\, dt\right] \Psi_f(\vec{A}(\infty)) \Psi_i(\vec{A}(-\infty)), \tag{32}$$

where

$$L(t) = L_A + L_{\varphi'} + L'_I + \int \eta_\alpha(\vec{x}, t)\, \varphi'_\alpha(\vec{x}, t)\, d\vec{x},$$

$$L_A = \tfrac{1}{2}g^2 b \dot{A}_\alpha \dot{A}_\alpha \left[1 + \frac{2}{gb} A_\alpha \int u \varphi'_\alpha(\vec{x}, t)\, d\vec{x}\right],$$

$$L_I = \frac{1}{2a} \Lambda_{\alpha\beta} \int d\vec{x}\, \rho(\vec{x})\, \varphi'_\alpha(\vec{x}, t) \int d\vec{y}\, \rho(\vec{y})\, \varphi'_\beta(\vec{y}, t),$$

$$L'_I = -\frac{1}{6a^2 g} \Xi_{\alpha\beta\gamma} \int d\vec{x}\, \rho(\vec{x})\, \varphi'_\alpha(\vec{x}, t) \int d\vec{y}\, \rho(\vec{y})\, \varphi'_\beta(\vec{y}, t)$$
$$\times \int d\vec{z}\, \rho(\vec{z})\, \varphi'_\gamma(\vec{z}, t). \tag{33}$$

This Lagrangian represents an interacting system of a nucleon, described by the dynamical variable A_α, with the meson field φ'_α. In the strong-coupling limit, the Lagrangian of the nucleon system, L_A, has the same form as a rod rotator in a plane with moment of inertia $g^2 b$, so that there exist many excited states (isobars). The initial and final nucleon states are assumed to be these, so Ψ_i and Ψ_f in (32) are functions of A_α alone.

VII. GENERATING FUNCTIONAL IN OPERATOR FORMALISM

In order to go over to the operator formalism it is necessary to make a transition to phase-space functional integration. The momenta conjugates to θ and φ'_α are introduced by inserting the following two identities into (32):

$$\prod_t gb(1 + \rho_A) = \int \exp\left[-\frac{i}{2g^2 b} \int \frac{p_\theta^2}{(1+\rho_A)^2}\, dt\right] \prod_t \sqrt{b}\, dp_\theta(t), \tag{34}$$

$$\int \prod_{\vec{x},t} \left[\sqrt{b} \prod_\alpha d\pi'_\alpha(\vec{x}, t)\right] \prod_t \delta\left(B_\alpha \int u(\vec{x})\, \pi'_\alpha(\vec{x}, t)\, d\vec{x}\right) \exp\left(-\frac{i}{2} \int \pi'_\alpha \pi'_\alpha\, d^4x\right) = 1. \tag{35}$$

Then we change variables as follows:

$$\begin{aligned} p_\theta &\to p_\theta - \dot{\theta} g^2 b (1+\rho_A)^2, \\ \pi'_\alpha &\to \pi'_\alpha - \varphi'_\alpha, \end{aligned} \tag{36}$$

where

$$\rho_A = \frac{1}{gb} \int A_\alpha u \varphi'_\alpha(\vec{x}, t)\, d\vec{x}. \tag{37}$$

From the δ functional in (32) one sees that

$$B_\alpha \int u \dot{\varphi}'_\alpha\, d\vec{x} = A_\alpha \dot{\theta} \int u \varphi'_\alpha\, d\vec{x}. \tag{38}$$

In order to isolate the θ dependence from the meson wave functions, we write them in the body-fixed coordinate system:

$$\begin{aligned} \varphi''_1 &= A_\alpha \varphi'_\alpha, \\ \varphi''_2 &= B_\alpha \varphi'_\alpha, \\ \pi''_1 &= A_\alpha \pi'_\alpha, \\ \pi''_2 &= B_\alpha \pi'_\alpha. \end{aligned} \tag{39}$$

At this stage, in addition to $p_\theta \dot{\theta}$, there still remain undesired terms containing $\dot{\theta}$ in both the action integral and the subsidiary condition. These are eliminated by performing the change of variables

$$\begin{aligned} \pi''_2 &\to \pi''_2 + \dot{\theta} \frac{u(\vec{x})}{b} \int u(\vec{x}')\, \varphi''_1(\vec{x}', t)\, d\vec{x}', \\ p_\theta &\to p_\theta - T'', \end{aligned} \tag{40}$$

with

$$T'' = \int (\pi_2'' \varphi_1'' - \pi_1'' \varphi_2'')\, d\vec{x}\,. \tag{41}$$

The final result of all the manipulations described above is the following expression for the generating functional (we drop the double primes):

$$Z(\eta, \epsilon) = \int \cdots \int \prod_t \left\{ d\theta(t)\, dp_\theta(t) \left[\prod_{\vec{x},\alpha} d\varphi_\alpha(\vec{x}, t)\, d\pi_\alpha(\vec{x}, t) \right] \delta\left(\int u \varphi_2\, d\vec{x} \right) \delta\left(\int u \pi_2\, d\vec{x} \right) \right\}$$
$$\times \exp \left\{ i \int dt \left[p_\theta \dot{\theta} + \int d\vec{x}\, \pi_\alpha \dot{\varphi}_\alpha - H + \int (\pi_\alpha \epsilon_\alpha + \varphi_\alpha \bar{\eta}_\alpha)\, d\vec{x} \right] \right\}, \tag{42}$$

where

$$H = \frac{1}{2g^2 b} \frac{(p_\theta - T)^2}{(1+\rho)^2} + H_\varphi - \bar{L}_I - \bar{L}_I',$$
$$\rho = \frac{1}{gb} \int u \varphi_1(\vec{x}, t)\, d\vec{x},$$
$$H_\varphi = \int d\vec{x} \left(\tfrac{1}{2} \pi_\alpha \pi_\alpha + \tfrac{1}{2} \vec{\nabla} \varphi_\alpha \cdot \vec{\nabla} \varphi_\alpha + \tfrac{1}{2} \mu^2 \varphi_\alpha \varphi_\alpha \right), \tag{43}$$
$$\bar{L}_I = \frac{1}{2a} \left[\int d\vec{x}\, \rho(\vec{x})\, \varphi_2(\vec{x}, t) \right]^2,$$
$$\bar{L}_I' = -\frac{1}{2a^2 g} \int d\vec{x}\, \rho(\vec{x})\, \varphi_1(\vec{x}, t) \left[\int d\vec{y}\, \rho(\vec{y})\, \varphi_2(\vec{y}, t) \right]^2,$$

and

$$\bar{\eta}_1 = A_\alpha \eta_\alpha\,,$$
$$\bar{\eta}_2 = B_\alpha \eta_\alpha\,. \tag{44}$$

We have introduced the source functions ϵ_α for the momenta π_α in order to be able to perform a perturbative expansion.

We need also to rewrite (42) in a form more suitable for a perturbative treatment:

$$Z(\eta, \epsilon) = \int \cdots \int \prod_t [d\theta(t)\, dp_\theta(t)]\, \Psi_f^* \exp \left[i \int dt \left(p_\theta \dot{\theta} - \frac{p_\theta^2}{2g^2 b} \right) \right]$$
$$\times \exp \left[-i \int dt H_I' \left(p_\theta, -i \frac{\delta}{\delta \bar{\eta}_\alpha}, -i \frac{\delta}{\delta \epsilon_\alpha} \right) \right] \tilde{Z}(\tilde{\eta}, \epsilon)\, \Psi_i\,, \tag{45}$$

where

$$H_I'(p_\theta, \varphi_\alpha, \pi_\alpha) = \frac{1}{2g^2 b} \frac{T^2 - 2p_\theta T}{(1+\rho)^2} + \frac{3}{2g^2 b} p_\theta^2 \rho^2 - \bar{L}_I' + O\left(\frac{1}{g^5} \right), \tag{46}$$

$$\tilde{\eta}_2 = \bar{\eta}_2\,,$$
$$\tilde{\eta}_1 = \bar{\eta}_1 + \frac{p_\theta^2}{g^3 b^2} u\,, \tag{47}$$

and

$$\tilde{Z}(\tilde{\eta}, \epsilon) = \int \cdots \int \prod_{\vec{x}, t, \alpha} [d\varphi_\alpha(\vec{x}, t)\, d\pi_\alpha(\vec{x}, t)] \prod_t \delta\left(\int u \varphi_2\, d\vec{x} \right) \delta\left(\int u \pi_2\, d\vec{x} \right)$$
$$\times \exp \left\{ i \int dt \left[-H_\varphi + \bar{L}_I + \int (\pi_\alpha \epsilon_\alpha + \varphi_\alpha \tilde{\eta}_\alpha)\, d\vec{x} \right] \right\}. \tag{48}$$

If we use the connection between Feynman path integrals and the standard operator formalism of quantum mechanics,[8] we obtain

$$Z(\eta, \epsilon) = \left\langle n_f \,\middle|\, T \exp \left[-i \int dt\, H_I' \left(p_\theta, -i \frac{\delta}{\delta \bar{\eta}_\alpha}, -i \frac{\delta}{\delta \epsilon_\alpha} \right) \right] \tilde{Z}(\tilde{\eta}, \epsilon) \,\middle|\, n_i \right\rangle. \tag{49}$$

Performing the integration over π_α in (48), we get

$$\tilde{Z}(\bar{\eta}, \epsilon) = \int \prod_{\vec{x}, t, \alpha} [d\varphi_\alpha(\vec{x}, t)] \prod_t \delta\left(\int u\varphi_2 \, d\vec{x} \right)$$
$$\times \exp\left\{ i\int dt \left[L_\varphi + \overline{L}_I + \int d\vec{x} [\varphi_\alpha(\bar{\eta}_\alpha - \dot{\epsilon}_\alpha) + \tfrac{1}{2}\epsilon_\alpha \epsilon_\alpha] - \frac{1}{2b}\left(\int u\epsilon_2 \, d\vec{x} \right)^2 \right] \right\} . \qquad (50)$$

Since the action in (50) is quadratic in φ_α, the $(\eta_\alpha - \dot{\epsilon}_\alpha)$ dependence can be factored out if one knows the solution of the equations of motion derived from this action with subsidiary conditions $(\eta'_\alpha = \eta_\alpha - \dot{\epsilon}_\alpha)$:

$$(\Box - \mu^2)\tilde{\varphi}_2(\vec{x}, t) + \frac{1}{a}\rho(\vec{x}) \int \rho(\vec{x}')\tilde{\varphi}_2(\vec{x}', t) \, d\vec{x}' + \lambda(t)u + \eta'_2(\vec{x}, t) = 0 , \qquad (51a)$$

$$(\Box - \mu^2)\tilde{\varphi}_1(\vec{x}, t) + \eta'_1(\vec{x}, t) = 0 , \qquad (51b)$$

and

$$\int u\tilde{\varphi}_2(\vec{x}, t) \, d\vec{x} = 0 , \qquad (52)$$

where $\lambda(t)$ is a Lagrange multiplier. Making the change of variables $\varphi_\alpha = \tilde{\varphi}_\alpha + \varphi'_\alpha$ in (50) we obtain

$$\tilde{Z}(\bar{\eta}, \epsilon) = \text{const} \times \exp\left\{ \frac{i}{2}\left[\int d^4x [(\bar{\eta}_\alpha - \dot{\epsilon}_\alpha)\tilde{\varphi}_\alpha + \epsilon_\alpha \epsilon_\alpha] - \frac{1}{b}\int dt \left(\int u\epsilon_2 \, d\vec{x} \right)^2 \right] \right\} . \qquad (53)$$

Using the Green's functions of the integro-differential equations (51) one can write

$$\tilde{\varphi}_1(\vec{x}, t) = \int \Delta(x, y)[\bar{\eta}_1(y) - \dot{\epsilon}_1(y)] \, d^4y ,$$
$$\tilde{\varphi}_2(\vec{x}, t) = \int G(x, y)[\bar{\eta}_2(y) - \dot{\epsilon}_2(y)] \, d^4y . \qquad (54)$$

The solution of (51) is discussed in Appendix B.

VIII. ENERGY LEVELS (ISOBARS)

The effective Hamiltonian which determines the isobar energy levels is obtained from the exponent of (45) by setting the external source functions η_α and ϵ_α equal to zero. The leading term is of course

$$H_0 = \frac{p_\theta^{\,2}}{2g^2 b} , \qquad (55)$$

which is of order $1/g^2$. The next corrections are obtained from

$$\exp\left[-i\int dt \, H'_I \right] \tilde{Z}(\bar{\eta}, \epsilon) .$$

The outline of the calculation is given in Appendix C. The result agrees with relation (49) of Nickle and Serber[6]:

$$H_{\text{eff}} = \frac{p_\theta^{\,2}}{2g^2 b}\left\{ 1 + \frac{1}{g^2 \pi b}\left[\int_\mu^\infty d\omega \, R(\omega) + \int_0^\infty d\omega \, S(\omega) \right] \right\} , \qquad (56)$$

$$R(\omega) = \text{Im}\frac{b}{H(\omega) - H(0)} + \frac{(d/d\omega^2)[H(\omega) - H(0)]}{H(\omega) - H(0)} , \qquad (57)$$

$$S(\omega) = \frac{-2\omega^2[d^2/(d\omega^2)^2][H(i\omega) - H(0)]}{H(i\omega) - H(0)} , \qquad (58)$$

and

$$H(\omega) = \frac{1}{(2\pi)^3}\int \frac{d\vec{k} \, |F(\vec{k})|^2}{\omega^2 - \omega_{\vec{k}}^{\,2} + i\epsilon} , \qquad (59)$$

$$F(\vec{k}) = \int d\vec{x} \, e^{-i\vec{k}\cdot\vec{x}} \rho(\vec{x}) . \qquad (60)$$

IX. ELASTIC SCATTERING AMPLITUDE

In the strong-coupling limit $\tilde{\eta}_\alpha = \bar{\eta}_\alpha$ and $H'_I = 0$, so that $Z(\eta, \epsilon = 0)$ is the exponential of a bilinear functional of $\bar{\eta}_\alpha$. Thus, it is obvious that meson multiproduction amplitudes vanish in the strong-coupling limit. In the next section we obtain the multiproduction amplitudes from the higher order terms in $1/g$ expansion of (49).

Using Eqs. (2) and (44), we obtain the elastic scattering amplitude

$$T_{\alpha\beta}(\vec{k}', \vec{k}; n_f, n_i) = \int e^{i(kx - k'y)} d^4x \, d^4y (\Box_x - \mu^2)(\Box_y - \mu^2) \times \langle n_f | G_{\alpha\beta}(x, y; A) | n_i \rangle , \qquad (61)$$

$$G_{\alpha\beta}(x, y; A) = A_\alpha A_\beta \Delta(x, y) + B_\alpha B_\beta G(x, y) . \qquad (62)$$

Using the expression of Green's function given in Appendix A, we obtain

$$T_{\alpha\beta}(\vec{k}', \vec{k}; n_f, n_i) = \delta(\omega - \omega')\frac{|F(\vec{k})|^2}{H(\omega) - H(0)}\langle n_f | \Lambda_{\alpha\beta} | n_i \rangle , \qquad (63)$$

with $\Lambda_{\alpha\beta}$ given by (23). Formula (63) agrees with the known result.[9]

X. PRODUCTION AMPLITUDES

The meson production processes are treated as perturbation corrections in the strong-coupling theory. In the computation of $Z(\eta)$ we kept the terms correctly up to the order of $1/g$. Therefore we can compute the one-meson production amplitude from $Z(\eta)$ given by (49). The leading term is due to the presence of L_I' in (49). For the two-meson production amplitude one has to keep the $1/g^2$ term in the expansion (21). Since this calculation is a straightforward extension of the one-meson production process we do not describe it here.

Let p_1 be the four-momentum of the initial meson, and p_2 and p_3 be that of the final meson. The LSZ reduction formula for the production amplitude is then

$$T_{\alpha\beta\gamma}(p_1,p_2,p_3;n_i,n_f)=\int e^{i(p_1x_1-p_2x_2-p_3x_3)}d^4x_1\,d^4x_2\,d^4x_3(\Box_1-\mu^2)(\Box_2-\mu^2)(\Box_3-\mu^2)\left.\frac{\delta^3Z(\eta)}{\delta\eta_\alpha(x_1)\,\delta\eta_\beta(x_2)\,\delta\eta_\gamma(x_3)}\right|_{\eta=0}. \tag{64}$$

If we compute the functional derivative using (49), (43), and (44), we obtain several terms. However, only one term contributes to (64) because of the kinematics. We obtain

$$T_{\alpha\beta\gamma}=-\frac{1}{ga^2}\int e^{i(p_1x_1-p_2x_2-p_3x_3)}d^4x_1\,d^4x_2\,d^4x_3(\Box_1-\mu^2)(\Box_2-\mu^2)(\Box_3-\mu^2)$$
$$\times\int dt\langle n_f|\Xi_{\delta\epsilon\eta}K_{\delta\alpha}(x_1,t)K_{\epsilon\beta}(x_2,t)K_{\eta\gamma}(x_3,t)|n_i\rangle\,, \tag{65}$$

where

$$K_{\delta\alpha}(x_1,t)=\left.\int d\vec{y}\,\rho(\vec{y})\,G_{\delta\alpha}(x_1,y;A)\right|_{y_0=t}. \tag{66}$$

The rest of the calculation is straightforward:

$$T_{\alpha\beta\gamma}(p_1,p_2,p_3;n_i,n_f)$$
$$=\frac{2\pi\delta(\omega_1-\omega_2-\omega_3)}{g}F(\vec{p}_1)F(\vec{p}_2)F(\vec{p}_3)\left[\frac{\langle n_f|A_\alpha\Lambda_{\beta\gamma}|n_i\rangle}{[H(\omega_2)-H(0)][H(\omega_3)-H(0)]}+\frac{\langle n_f|A_\beta\Lambda_{\alpha\gamma}|n_i\rangle}{[H(\omega_1)-H(0)][H(\omega_3)-H(0)]}\right.$$
$$\left.+\frac{\langle n_f|A_\gamma\Lambda_{\alpha\beta}|n_i\rangle}{[H(\omega_1)-H(0)][H(\omega_2)-H(0)]}\right]. \tag{67}$$

XI. DISCUSSION

We formulated the strong-coupling theory in terms of the Feynman path integral for the charged-scalar static model. We then separated the meson field into two parts, the part of the meson cloud attached to the nucleon which is described in terms of A_α and the scattering part of φ_α'. In the strong-coupling limit we showed that the φ_α' integration can be performed and the higher order corrections in $1/g^2$ can be treated by the perturbation method.

The resulting expressions for reaction amplitudes and the isobar energy levels are then written in the form of operator expressions in which only A_α and the third component of isospin p_θ (or I_3) appear. From the definition of operators $p_\theta(I_3)$ and A_α it is obvious that they satisfy the following commutation relations:

$$[I_3,A_1]=iA_2;\quad [I_3,A_2]=iA_1;\quad [A_1,A_2]=0\,,$$

which are just the commutation relations of the strong-coupling group.[4] One can then use the results based on the group representation to evaluate physical quantities such as scattering amplitudes and isobar energy levels.

The extension of the present method to more complicated static models, such as the charge-symmetric pseudoscalar model can be done with a relatively small additional effort. We shall discuss the general static model in the following paper.

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APPENDIX A

We calculate here the leading-order contribution from other paths of nucleon isospin. This contribution is due to the paths on which

$$\tau_\alpha q_\alpha|\Psi(t)\rangle = q(t)|\Psi(t)\rangle \quad \text{for } t<t_1 \text{ and } t>t_2\,,$$
$$\tau_\alpha q_\alpha|\Psi(t)\rangle = -q(t)|\Psi(t)\rangle \quad \text{for } t_2>t>t_1\,. \tag{A1}$$

The contribution from these paths to the total generating functional is given by

$$Z'(\eta) = \left\langle n_f \left| \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{t_2} dt_1 \prod_{\vec{x},t,\alpha} d\varphi_\alpha(\vec{x},t) \exp\left[-2ig\int_{t_1}^{t_2} q(t)\,dt\right] \exp\left[i\int_{-\infty}^{\infty} L(t)\,dt\right] \right| n_i \right\rangle\,. \tag{A2}$$

We perform the integration over t_1 by performing a change of variable:

$$t_1 \to y(t_1,t_2) = \int_{t_1}^{t_2} q(t)\,dt\,. \tag{A3}$$

We can now perform successive partial integrations in y to get

$$Z'(\eta) = \left\langle n_f \left| \int_{-\infty}^{\infty} \left[-\frac{i}{2g}\frac{1}{q(t_2)} + O\left(\frac{1}{g^5}\right)\right] dt_2 \exp\left[i\int_{-\infty}^{\infty} L(t)\,dt\right] \right| n_i \right\rangle\,. \tag{A4}$$

We get then for the total generating functional

$$Z(\eta)_{\text{total}} = Z(\eta) + Z'(\eta) = \left\langle n_f \left| \exp\left\{ i\int_{-\infty}^{\infty}\left[L(t) - \frac{1}{2g}\frac{1}{q(t)}\right]dt \right\} \right| n_i \right\rangle\,, \tag{A5}$$

where we have neglected a *constant* term of order $1/g^4$.

Expanding now around φ_α^0, we obtain the expansion of $L(t)$ described in the paper and in addition terms arising from the expansion of $1/q(t)$:

$$\frac{-1}{2g\,q(t)} \simeq -\frac{1}{2g^2a} + \frac{1}{2g^3a^2} A_\alpha \int \rho(\vec{x})\,\varphi_\alpha'(\vec{x},t)\,d\vec{x} - \frac{1}{2g^4a^3} A_\alpha A_\beta \int \rho(\vec{x})\,\varphi_\alpha'(\vec{x},t)\,d\vec{x} \int \rho(\vec{y})\,\varphi_\beta'(\vec{y},t)\,d\vec{y}$$
$$+ \frac{1}{4g^4a^3}(\delta_{\alpha\beta} - A_\alpha A_\beta)\int \rho\varphi_\alpha'(\vec{x},t)\,d\vec{x}\int\rho\varphi_\beta'(\vec{y},t)\,d\vec{y}\,. \tag{A6}$$

The first term contributes just with a constant shift of the energy levels. The other terms produce calculable corrections (by the perturbation method) to production amplitudes of order $1/g^3$ and higher, and to energy levels of order $1/g^6$ and higher.

APPENDIX B

Equation (51b) is trivially solved. $\Delta(x,y)$ in (54) is given by

$$\Delta(x,y) = -\frac{1}{(2\pi)^4}\int \frac{d\vec{k}\,d\omega\, e^{i\omega(x^0-y^0)}\, e^{-i\vec{k}\cdot(\vec{x}-\vec{y})}}{\omega^2 - \omega_{\vec{k}}^{\,2} + i\epsilon}\,. \tag{B1}$$

Taking the Fourier transform of (51a) we get

$$(\omega^2 - \omega_{\vec{k}}^{\,2})\,\varphi_2(\vec{k},\omega) + \frac{1}{a}F(\vec{k})\,\varphi(\omega) + \lambda(\omega)\frac{F(\vec{k})}{\omega_{\vec{k}}^{\,2}} + \eta_2'(\vec{k},\omega) = 0\,, \tag{B2}$$

where $\varphi_2(\vec{k},\omega)$ and $\eta_2'(\vec{k},\omega)$ are the Fourier transform of $\varphi_2(\vec{x},t)$ and $\eta_2'(\vec{x},t)$, respectively, while $\lambda(\omega)$ is the Fourier transform of $\lambda(t)$ and $\varphi(\omega)$ is defined by

$$\varphi(\omega) = \frac{1}{(2\pi)^3}\int \tilde{\varphi}_2(\vec{q},\omega)\,F(-\vec{q})\,d\vec{q}\,. \tag{B3}$$

Multiplying (B2) by $[1/(2\pi)^3]\,F(\vec{k})/(\omega^2 - \omega_{\vec{k}}^{\,2})$ and integrating over $\vec{k}$ we obtain a simple linear equation for $\varphi(\omega)$, the solution of which is given by

$$\varphi(\omega) = \frac{H(0)}{H(0) - H(\omega)}\left(\frac{\lambda(\omega)}{\omega^2}[H(\omega) - H(0)] + \eta(\omega)\right)\,, \tag{B4}$$

where

$$\eta(\omega) = \frac{1}{(2\pi)^3}\int \frac{\eta_2'(\vec{q},\omega)\,F(-\vec{q})}{\omega^2 - \omega_q^{\,2}}\,d\vec{q}\,, \tag{B5}$$

and $H(\omega)$ is given by (59).

We substitute (B4) into (B2) and obtain

$$\tilde{\varphi}_2(\vec{k},\omega) = -\frac{\lambda(\omega)\,F(\vec{k})}{\omega^2\,\omega_{\vec{k}}^{\,2}} - \frac{\eta_2'(\vec{k},\omega)}{\omega^2 - \omega_{\vec{k}}^{\,2}} + \frac{\eta(\omega)\,F(\vec{k})}{(\omega^2 - \omega_k^{\,2})[H(\omega) - H(0)]}\,. \tag{B6}$$

We then use the subsidiary condition (52) to obtain $\lambda(\omega)$:

$$\lambda(\omega) = \frac{1}{b}\frac{1}{(2\pi)^3}\int \frac{F(-\vec{k})\,\eta_2'(\vec{k},\omega)}{\omega_{\vec{k}}^{\,2}}\,d\vec{k}\,. \tag{B7}$$

If we define the Fourier transform of Green's function $G(x,y)$ by

$$G(\vec{k},\vec{k}',\omega)=\int e^{i\vec{k}\cdot\vec{x}-i\vec{k}'\cdot\vec{y}-i\omega t}G(x,y)\Big|_{x_0-y_0=t}\,dt\,d\vec{x}\,d\vec{y}\,, \tag{B8}$$

G is given by

$$G(\vec{k},\vec{k}',\omega)=\frac{-(2\pi)^3\delta^{(3)}(\vec{k}-\vec{k}')}{\omega^2-\omega_k^2}+F(\vec{k})F(\vec{k}')\left\{\frac{1}{b\omega^2\omega_k^2\omega_{k'}^2}+\frac{1}{[H(\omega)-H(0)](\omega^2-\omega_k^2)(\omega^2-\omega_{k'}^2)}\right\}. \tag{B9}$$

APPENDIX C

The starting point of the calculation are formulas (45), (46), (47), (53), and (54). One then expands

$$\exp\left(-i\int dt\,H_I'\right),$$

takes the functional derivatives, and sets $\eta_\alpha=\epsilon_\alpha=0$ at the end. The term $T^2/2g^2b(1+\rho)^2$ in (46) is found to contribute only an ignorable constant term to order $1/g^4$, while the term $p_\theta T(2\rho-3\rho^2)/g^2b$ is found not to contribute to this order. In the expansion of

$$\exp\left(i\int dt\,p_\theta T/g^2b\right)$$

the term $i\int dt p_\theta T/g^2b$ gives a zero contribution, while the term

$$-\int dt\,dt'\,p_\theta^2\,T(t)\,T(t')/2g^4b^2$$

produces a correction to energy levels of order $1/g^4$, the derivation of which is described below.

The first step of this calculation produces the expression

$$-i\int\Delta H_1dt=\frac{2}{g^4b^2}\int p_\theta^2\,d^4x\,d^4y\,\Delta(x-y)\,\frac{\partial^2}{\partial x^0\partial y^0}G(x,y)$$
$$+\frac{1}{2g^4b^2}\int p_\theta^2[\Delta(x-y)+G(x,y)]\,\delta^{(4)}(x-y)\,d^4x\,d^4y-\frac{1}{2g^4b^3}\int p_\theta^2\,dt\,u(\vec{x})u(\vec{y})\,\Delta(\vec{x}-\vec{y},0)\,d\vec{x}\,d\vec{y}\,. \tag{C1}$$

Using (B1) and (B9), we have, apart from terms that cancel among themselves, the following three terms [which we denote by (a), (b), and (c)]:

$$\text{(a)}\quad -i\int\Delta H_1^{(1)}dt=\frac{1}{2g^4b^2}\int dt\,p_\theta^2\,\frac{1}{(2\pi)^4}\int d\vec{k}\,d\omega|F(\vec{k})|^2\left[\frac{1}{H(\omega)-H(0)}\frac{1}{(\omega^2-\omega_{\vec{k}}^2+i\epsilon)^2}+\frac{1}{b\omega^2}\frac{1}{\omega_{\vec{k}}^4}\right]$$
$$=-i\int dt\,\frac{p_\theta^2}{2\pi g^4b^2}\int_\mu^\infty \operatorname{Im}\frac{(d/d\omega^2)[H(\omega)-H(0)]}{H(\omega)-H(0)}\,. \tag{C2}$$

(The last step follows by deforming the contour of integration in ω and picking up the discontinuity across the cut from μ to ∞.)

$$\text{(b)}\quad -i\int\Delta H_1^{(2)}dt=\frac{-2}{g^4b^2}\int dt\,p_\theta^2\,\frac{1}{(2\pi)^4}\int d\vec{k}\,d\omega\,\frac{|F(\vec{k})|^2}{[\omega^2-\omega_k^2+i\epsilon]^3}\,\frac{1}{H(\omega)-H(0)}$$
$$=i\int dt\,\frac{p_\theta^2}{\pi g^4b^2}\int_0^\infty d\omega\,\omega^2\,\frac{\{d^2/[d\omega^2]^2\}[H(i\omega)-H(0)]}{H(i\omega)-H(0)}\,, \tag{C3}$$

where the second step follows by deforming the countour of integration to lie along the imaginary axis and noting that the integrand is even in ω. The reason for this peculiar choice of contour becomes obvious if one analyzes the point source limit of (C3).

$$\text{(c)}\quad -i\int\Delta H_I^{(3)}dt$$
$$=\frac{3}{2g^4b^3}\int p_\theta^2\,dt\int d\vec{x}\,d\vec{y}\,u(\vec{x})u(\vec{y})\,\Delta(\vec{x}-\vec{y},0)\,. \tag{C4}$$

(C4) cancels the contribution of the term $3p_\theta^2\rho^2/2g^2b$ in (46). It is rather straightforward to determine the contribution of $-\overline{L}_I'$ in (46). It is found to be

$$-i\int \Delta H_2\,dt = -i\int \frac{p_\theta^2\,dt}{2\pi g^4 b}\int_\mu^\infty d\omega\,\mathrm{Im}\,\frac{1}{H(\omega)-H(0)}\,. \tag{C5}$$

Adding (C2), (C3), and (C5), exponentiating by means of the identity

$$1 - i\int dt\,\Delta H \approx \exp\left(-i\int dt\,\Delta H\right)\,, \tag{C6}$$

and combining with (55), we find (56). (56) agrees with the final result (49) of Ref. 6. The way to see that is to expand in powers of $1/g$ the integral form of their result (30), which produces (C5). If one evaluates (C2) and (C3) in the point source limit, which is a straightforward exercise, one finds that their sum is the same as the sum of (47) and (48) in Ref. 6.

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[7]$gA_\alpha u$ is the part of the pion field attached to the nucleon. It does not propagate to infinity, and therefore it does not contribute to the reaction amplitudes.

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Functional approach to strong-coupling theory in static models. II. General case*

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The functional approach to strong-coupling theory is extended to the general case. The necessary mathematical techniques are discussed. Properties of the matrix Λ_{DE} which appears in scattering amplitudes are analyzed.

I. INTRODUCTION

In the preceding paper[1] we described the functional-integral approach to the strong-coupling theory of the static model, taking the charged-scalar model as an example. In this paper we extend this method to the study of the strong-coupling limit in the general case, i.e., the static model of an arbitrary partial wave and internal symmetry. This can be achieved with relatively small effort by introducing appropriate mathematical techniques to deal with orbit invariants and generalized angles for the representation of the symmetry group of the theory.

We use the same definition of the generating functional $Z(\eta)$ of the isobar matrix elements of time-ordered products of pion fields, as in Ref. 1. The interaction Lagrangian in this case is written as

$$L_{\text{int}} = g\sum_{D=1}^{m} S_D q_D\,, \tag{1}$$

where S_D's are tensor operators of the space and internal group, and q_D is given by

$$q_D = \int [\rho(\vec{x})\varphi(\vec{x},t)]_D\,d\vec{x}\,. \tag{2}$$

The actual content of the index D is rich, and it may contain both the internal-symmetry-group index δ and the space-group index d. For example, the charge-symmetric pseudoscalar-pseudovector model is described as

$$\begin{aligned} &D=(\delta,d)\,,\quad S_{\delta d}=\tau_\delta\sigma_d\quad(\delta=1,2,3;\ d=1,2,3)\,,\\ &q_{\delta d}=\int[\nabla_d\rho(\vec{x})]\varphi_\delta(\vec{x},t)\,d\vec{x}\,. \end{aligned} \tag{3}$$