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Composite models of leptons

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We give ten criteria for composite models of leptons, and present two models which satisfy a number of these criteria. Both models use three triplets of leptonic analogs of quarks which we call "leptoquarks"; the first model uses fractionally charged leptoquarks and the second model uses integrally charged ones. In the second model, leptoquarks and quarks could be identical and there is a possibility of unifying the description of leptonic and hadronic phenomena. Both models assign leptons to nonsinglet representations of SU(3)″. A plausible SU(3)″ mass formula allows the known leptons to be less massive than baryons if leptoquarks and quarks are identical.

I. INTRODUCTION

The quark model[1] provides a picture of hadrons in which the algebras relevant to hadron symmetries can be abstracted from the commutation relations of bilinear expressions in quark fields, and the hadron states are composites of fundamental quark q and antiquark $\bar{q}$ constituents: baryons $\sim(qqq)$ and mesons $\sim(q\bar{q})$. For several reasons, especially the fact that the ground-state supermultiplet of the baryons is the symmetric $\underline{56}$ of SU(6),[2] it was suggested that quarks obey parastatistics of order three.[3] If the Green components[4] of the parafermion quark fields are considered independent and are Klein-transformed,[5] the parafermion quark model is exhibited[6] as a special case of the three-triplet model with three identical fermion quark triplets, which was later named the "color" model.[7] Another version of the three-triplet model allows the fundamental objects to have integral charge.[8] The three-triplet model has the static classification group SU(18).[9] Nambu and Han have discussed the relation between different three-triplet models and the paraquark model.[10]

The plethora of hadron states all falling into the $\underline{1}$, $\underline{8}$, or $\underline{10}$ of SU(3) provides strong motivation and support for the quark model of hadrons, both because of the economy of the quark-field expressions for currents compared to those using known hadron fields, and because of the success of the constituent quark model for hadron states. Pending the discovery of more leptons, this motivation is absent for a composite model of leptons. Nonetheless, there are motivations to consider such a model: If quarks are fractionally charged, as in the color model, the notion that all fundamental particles should have fractional charge provides some motivation for composite models of leptons[11,12]; such models also give a scheme in which none of the presently known particles, except the photon, are elementary, and *both* hadrons and leptons are composed of more basic objects. Such models allow the similarity between hadron and lepton algebras to be associated with similar (or perhaps identical) sets of presently unobserved fundamental fields. The SU(3)″ degree of freedom, in particular the nonsinglet representations, which are usually superfluous, can be used to describe properties of leptons, such as the difference between the (e, ν_e) and (μ, ν_μ) pairs. The suggested interpretation of the surprisingly large and constant cross section for electron-positron annihilation to hadrons as being due to extended structure of the electron and the suggestion that leptons should have a two-component scattering amplitude analogous to that of hadrons[13,14] also motivates consideration of such models. To the extent that gauge models introduce new leptons, they signal increasing usefulness of composite models of leptons. We suggest requirements for such models and give two examples of models satisfying some of these requirements.

A composite model of leptons should satisfy the following requirements: It should predict (1) the chiral algebra of lepton charges, (2) muon-electron universality, (3) separate conservation of muon and electron number, (4) that odd-half-integral (integral) spin particles have odd (even) values of lepton or baryon number,[15] (5) lepton magnetic moments, (6) G_V and G_A for weak lepton currents, (7) massless, two-component neutrinos, (8) mass relations for leptons, and the low mass of the known leptons, (9) lepton-hadron universality, and (10) absence of strong lepton interactions at low energy.

The chiral algebra of lepton charges[16] can be expressed in terms of currents bilinear in known lepton fields, using

$$\Lambda_L^+ = \int d^3x(e^\dagger \mu + \nu_e^\dagger \nu_\mu)_L, \quad \Lambda_L^- = (\Lambda_L^+)^\dagger, \tag{1a}$$

$$\Lambda_L^3 = \tfrac{1}{2}\int d^3x(e^\dagger e - \mu^\dagger \mu + \nu_e^\dagger \nu_e - \nu_\mu^\dagger \nu_\mu)_L, \tag{1b}$$

and

$$\Lambda_L^0 = \int d^3x(e^\dagger e + \mu^\dagger \mu + \nu_e^\dagger \nu_e + \nu_\mu^\dagger \nu_\mu)_L \tag{1c}$$

as generators of $U(2)_L$, and

$$\Lambda_R^+ = \int d^3x(e^\dagger \mu)_R, \quad \Lambda_R^- = (\Lambda_R^+)^\dagger, \tag{2a}$$

$$\Lambda_R^3 = \tfrac{1}{2}\int d^3x(e^\dagger e - \mu^\dagger \mu)_R, \tag{2b}$$

and

$$\Lambda_R^0 = \int d^3x(e^\dagger e + \mu^\dagger \mu)_R \tag{2c}$$

as generators of $U(2)_R$, where

$$\begin{aligned}(e^\dagger e)_L &= e^\dagger \tfrac{1}{2}(1+\gamma_5)e,\\ (e^\dagger e)_R &= e^\dagger \tfrac{1}{2}(1-\gamma_5)e,\end{aligned} \tag{3}$$

etc. These generators commute with the weak, electromagnetic, and lepton number currents

$$j_\mu^{(+)} = (\bar\nu_e \gamma_\mu e + \bar\nu_\mu \gamma_\mu \mu)_L, \quad j_\mu^{(-)} = j_\mu^{(+)\dagger}, \tag{4}$$

$$j_\mu^{\text{em}} = -(\bar e \gamma_\mu e + \bar\mu \gamma_\mu \mu), \tag{5}$$

and

$$j_\mu = \bar e \gamma_\mu e + \bar\mu \gamma_\mu \mu + \bar\nu_e \gamma_\mu \nu_e + \bar\nu_\mu \gamma_\mu \nu_\mu . \tag{6}$$

This commutativity implies requirements (2) and (3) above. The current-current theory of weak interactions of leptons and the quantum electrodynamics of leptons are $U(2)_L \times U(2)_R$-invariant up to mass terms (this caveat can presumably be removed by generating masses by spontaneous symmetry breaking). The currents $j_\mu^{(+)}$, j_μ^{em}, and j_μ provide a $U(2)_L \times U(1)_R$ algebra with generators

$$I_L^+ = \int d^3x(\nu_e^\dagger e + \nu_\mu^\dagger \mu)_L, \quad I_L^- = (I_L^+)^\dagger, \tag{7a}$$

$$I_L^3 = \tfrac{1}{2}\int d^3x(\nu_e^\dagger \nu_e - e^\dagger e + \nu_\mu^\dagger \nu_\mu - \mu^\dagger \mu)_L, \tag{7b}$$

$$I_L^0 = \Lambda_L^0, \tag{7c}$$

$$I_R^0 = \Lambda_R^0. \tag{7d}$$

These lepton algebras can be realized in a composite model of leptons in terms of leptonic analogs of quark fields, just as hadron algebras can be realized in terms of quark fields, and the first three requirements above can be satisfied in a natural way, as we will illustrate in detail for specific models below. Since quarks were introduced specifically to deal with hadrons, the expression "leptonic analog quark" seems clumsy. We suggest that the word "quark" be reserved for a real or fictitious fundamental field or particle for hadron physics, and suggest the word "leptoquark" as the leptonic analog of quark.

Requirement (4) is automatically satisfied in any composite model in which leptons and baryons are composed of spin-$\frac{1}{2}$ constituents by proper nonzero choice of lepton number for leptoquarks and baryon number for quarks.

We discuss the remaining requirements, which are model-dependent, below.

II. FRACTIONALLY CHARGED LEPTOQUARK MODEL

Consider a three-triplet model of fractionally charged leptoquarks. We label the leptoquarks in the static limit by $A = 1, 2, 3$, or $\mathcal{P}, \mathcal{N}, \lambda$, the usual SU(3) index, $a = 1, 2$, or $\uparrow, \downarrow$, the $SU(2)_S$ spin index, and $i = 1, 2, 3$, the SU(3)″ or color index. We consider a static leptoquark as belonging to the fundamental representation of the group SU(18) which reduces to SU(6)×SU(3)″, with SU(6) reducing to $SU(3)\times SU(2)_S$. The static leptoquarks correspond to the constituent point of view in our model, and transform as (3, 2, 3) under $SU(3)\times SU(2)_S \times SU(3)''$.

We label relativistic Dirac leptoquark fields in a similar way, except that the Dirac index, which we omit, replaces the $SU(2)_S$ index. If we assume that the SU(3) leptonic vector and axial charges are diagonal in SU(3)″, then the chiral SU(3)×SU(3) charge algebra requires the vector charges to be SU(3)″ singlets:

$$L_\alpha = \tfrac{1}{2}\int \mathcal{L}_\alpha^0 \, d^3x, \quad \alpha = 0, 1, \ldots, 8, \quad \lambda_0 = (\tfrac{2}{3})^{1/2}1, \tag{8a}$$

where

$$\mathcal{L}_\alpha^\mu = \bar l_{Ai} \gamma^\mu (\lambda_\alpha)^A_B \, l^{Bi}, \tag{8b}$$

and the axial charges to have the form

$$L_\alpha^5 = \tfrac{1}{2}\int \mathcal{L}_\alpha^{05} \, d^3x, \quad \alpha = 0, 1, \ldots, 8 \tag{9a}$$

where

$$\mathcal{L}_\alpha^{\mu 5} = \bar l_{Ai} \gamma^\mu \gamma_5 \eta^i_j (\lambda_\alpha)^A{}_B \, l^{Bj}, \tag{9b}$$

and η is diagonal with elements ±1. We assume that, in first approximation, these charges generate groups identical to the constituent groups mentioned above.

The S-wave bound states of three Fermi leptoquarks belong to the antisymmetric representation $\underline{816}$ of SU(18). Under SU(18) → SU(6)×SU(3)″,

$$\underline{816} = (\underline{56}, \underline{1}) + (\underline{70}, \underline{8}) + (\underline{20}, \underline{10}). \tag{10}$$

The magnetic moments of the spin-$\frac{1}{2}$ bound states in the $(\underline{56}, \underline{1})$ and the $(\underline{20}, \underline{10})$ were studied in connection with SU(6) symmetry[17,18]; bound states in these representations which are eigenstates of isospin and hypercharge cannot be identified with the known leptons because in both cases the neutral particles have nonvanishing magnetic moments. For this reason, we assign the known leptons to the ($\underline{70}$, 8). In particular, we place the $(\mu^+, \bar{\nu}_\mu)$ and $(e^+, \bar{\nu}_e)$ doublets in $I=\frac{1}{2}$, $Y=1$, SU(3) octets in two different SU(6) $\underline{70}$'s associated with the $Y''=0$, $I''=1$, $Q''=\pm 1$ (i.e., $\overline{\Sigma''^{\pm}}$) states in the SU(3)$''$ octet with spin $\frac{1}{2}$. For concreteness, we assume $(e^+, \bar{\nu}_e)$ are in the $\underline{70}$ associated with Σ''^-, and $(\mu^+, \bar{\nu}_\mu)$ are in the $\underline{70}$ associated with Σ''^+.

We now show that these assignments, together with the freedom allowed for the SU(3)$''$ structure of the axial charges, satisfy our first four criteria. The known muon-type lepton states are

$$|\mu^+\uparrow\rangle = \frac{1}{\sqrt{3}}\,|(p_3^\downarrow n_1^\uparrow - p_1^\downarrow n_3^\uparrow - p_3^\uparrow n_1^\downarrow)p_1^\uparrow\rangle, \qquad (11a)$$

$$|\mu^+\downarrow\rangle = \frac{1}{\sqrt{3}}\,|(p_3^\downarrow n_1^\uparrow + p_1^\uparrow n_3^\downarrow - p_3^\uparrow n_1^\downarrow)p_1^\downarrow\rangle, \qquad (11b)$$

$$|\bar{\nu}_\mu\downarrow\rangle = \frac{1}{\sqrt{3}}\,|(p_3^\downarrow n_1^\uparrow + p_1^\uparrow n_3^\downarrow - p_1^\downarrow n_3^\uparrow)n_1^\downarrow\rangle, \qquad (11c)$$

and the corresponding electron-type lepton states differ from the above by having SU(3$''$) 1 indices replaced by 2's. The magnetic moments of the leptons are computed in the same way as in the quark model for hadrons:

$$\mu_l = \left\langle l\uparrow \left| \frac{e\hbar}{2M_L c}\,(q\sigma_z)\right| l\uparrow\right\rangle, \qquad (12a)$$

where

$$(q\sigma_z) = l^\dagger_{Aai}\, q^A{}_B (\sigma_z)^a{}_b l^{Bbi}, \qquad (12b)$$

with q diagonal with elements $(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$, and σ_z the usual Pauli matrix. The result is

$$\mu_{l^+} = \tfrac{1}{3}\frac{e\hbar}{2M_L c} = \frac{e\hbar}{2m_l c} = \mu_{\rm Dirac}\,,\ e>0$$
$$\mu_{\bar{\nu}_l} = 0, \qquad (12c)$$

provided we assume that the effective mass of a leptoquark in a lepton composed of three leptoquarks is $\frac{1}{3}$ the mass of the lepton.[19] Using these values of μ, and the SU(3) formula

$$\mu = \alpha Q + \beta[U(U+1) - \tfrac{1}{4}Q^2 - \tfrac{1}{6}C_2^{(3)}]$$

valid in each octet, we find $\alpha = \mu_{\rm Dirac}$ and $\beta = 0$, so that all particles in these octets have pointlike magnetic moments.

From the point of view of this model, Dirac's prediction of the magnetic moment of the electron in a theory in which the electron is a point particle is correct only because of the strong binding and the specific form of the composite structure of the electron. We speculate that in the strong-binding limit, the electrodynamics of the composite leptons, including radiative corrections, will be pointlike.

A careful calculation of the effects of compositeness upon the properties of the electron involves dynamical problems which we do not want to deal with in this article. For scalar binding of Dirac particles in a potential, the effective mass of the bound particle enters the formula for the magnetic moment of the bound state in the way we assumed above.[20] Following a suggestion of Schnitzer that gauge theories of leptons Reggeize with a trajectory of slope G_F,[21] we take $G_F{}^{1/2} \sim 6\times 10^{-17}$ cm as a characteristic length associated with composite lepton structure. Since quantum electrodynamics has been checked experimentally only for distances greater than the order of 10^{-15} cm, we expect composite structure on the scale of $G_F{}^{1/2}$ not to cause immediate conflict with our present empirical knowledge of quantum electrodynamics.[22]

Since the leptons are $I=\frac{1}{2}$ doublets, $G_V = 1$. We calculate $G_A(\mu)$ for $(\mu^+, \bar{\nu}_\mu)$ as in the quark model:

$$G_A(\mu) = \langle \mu^+\uparrow|(I_+\sigma_+)|\bar{\nu}_\mu\downarrow\rangle, \qquad (13a)$$

where

$$(I_+\sigma_+) \equiv l^\dagger_{Aai}(I_+)_B{}^A(\sigma_+)_b{}^a \eta_j{}^i\, l^{Bbj}, \qquad (13b)$$

with (I_+) and (σ_+) the usual isospin- and spin-raising operators, and η as defined above. The result is

$$G_A(\mu) = \tfrac{1}{3}(2\eta_1{}^1 - \eta_3{}^3). \qquad (13c)$$

A similar calculation for $(e^+, \bar{\nu}_e)$ gives

$$G_A(e) = \tfrac{1}{3}(2\eta_2{}^2 - \eta_3{}^3). \qquad (13d)$$

The requirement that both G_A's are unity fixes $\eta_1{}^1 = \eta_2{}^2 = -\eta_3{}^3 = 1$. With this choice of axial-vector currents, (e, ν_e) and (μ, ν_μ) are each the sum of a left-handed doublet and a right-handed singlet under the $U(2)_L \times U(1)_R$ whose generators, equivalent to the I's of Sec. I, are given by

$$I_L^+ = L_{1+i2} + L^5_{1+i2},\quad I_2^- = (I_L^+)^\dagger, \qquad (14a)$$

$$I_L^3 = L_3 + L_3^5,\quad I_L^0 = 6^{1/2}(L_0 + L_0^5), \qquad (14b)$$

$$I_R^0 = 6^{1/2}(L_0 - L_0^5). \qquad (14c)$$

Since $G_V = G_A = 1$, our model allows two-component neutrinos, but does not predict them.

Our model has muon-electron universality, neglecting masses as usual, since the vector and axial-vector currents are invariant under the $U(2)_L \times U(2)_R$ whose generators, equivalent to the Λ's of Sec. I, are given by

$$\Lambda^+_{L,R} = L''_{1+i2} \pm L''^5_{1+i2},\quad \Lambda^-_{L,R} = (\Lambda^+_{L,R})^\dagger, \qquad (15a)$$

$$\Lambda^3_{L,R} = L''_3 \pm L''^5_3,\quad \Lambda^0_{L,R} = 6^{1/2}(L''_0 \pm L''^5_0), \qquad (15b)$$

where

$$L''_\alpha = \tfrac{1}{2}\int \mathcal{L}''^{0}_\alpha d^3x, \quad \alpha = 0, 1, \ldots, 8 \tag{16a}$$

$$\mathcal{L}''^{\mu}_\alpha = \bar{l}_{Ai}\gamma^\mu(\lambda_\alpha)^i{}_j\, l^{Aj}, \tag{16b}$$

$$L''^{5}_\alpha = \tfrac{1}{2}\int \mathcal{L}''^{\mu 5}_\alpha d^3x, \quad \alpha = 0, 1, \ldots, 8 \tag{17a}$$

and

$$\mathcal{L}''^{\mu 5}_\alpha = \bar{l}_{Ai}\gamma^\mu\gamma_5(\lambda_\alpha)^i{}_j\, l^{Aj}. \tag{17b}$$

In other words, the $(\mu^+, \bar{\nu}_\mu)$ and $(e^+, \bar{\nu}_e)$ differ only in having the SU(3)″ structure 113 or 223, and all currents have the $SU(2)''_I$ singlet structure $\bar{1}1 + \bar{2}2$. Provided, as we assume, all observables are diagonal in SU(3)″, muon and electron number will be separately conserved. The electromagnetic and weak interaction Lagrangians for leptons can be written as

$$\mathcal{L}_I^{\text{em}} = e\mathcal{L}^{\mu}_{3+(1/\sqrt{3})8}A_\mu,$$
$$\mathcal{L}_I^{\text{weak}} = g(\mathcal{L}^{\mu}_{1+i2} + \mathcal{L}^{\mu 5}_{1+i2})W_\mu + \text{H.c.}, \tag{18}$$

where A_μ and W_μ are the electromagnetic potential and W-meson field.

We have now completed our discussion showing that this model satisfies the first six of the requirements we listed in Sec. I. Although it is possible to write down a two-body mass operator which lowers the known leptons relative to all other states in the $(\underline{70}, \underline{8})$,

$$M = a[\tfrac{1}{3}C_2^{(3)''} - I''(I''+1) + \tfrac{1}{4}Y''^2]$$
$$+ b[1 - Q''^2] + c[J(J+1) - \tfrac{3}{4}]$$
$$+ d[e - (e + \tfrac{1}{2})Y + I(I+1) - \tfrac{1}{4}Y^2], \tag{19}$$

with a, b, c, and d large and positive, and e positive, we do not consider this as satisfying our eighth requirement.

III. INTEGRALLY CHARGED LEPTOQUARK MODEL

Consider a three-triplet model of integrally charged Fermi leptoquarks. We label the leptoquarks as in Sec. II, except that now the static leptoquarks transform as (3, 2, 3*) under $SU(3) \times SU(2)_S \times SU(3)''$. We label the relativistic Dirac fields l^i_A, where A is the SU(3) index and i is the SU(3)″ index, and suppress the Dirac indices. The SU(3) currents now are[23]

$$\mathcal{L}^{(\mu 5)}_\alpha = \bar{l}^A{}_i\Gamma^{(\mu 5)}(\lambda_\alpha)_A{}^B l_B{}^i, \tag{20a}$$

where $\Gamma^{(\mu 5)} = \gamma^\mu$ or $\gamma^\mu\gamma_5$ for vector or axial-vector currents. Both vector and axial charges, which are space integrals of $\mathcal{L}^{(05)}_\alpha$, are SU(3)″ singlets, and we assume these charges generate the corresponding static constituent groups. The SU(3)″ currents are

$$\mathcal{L}''^{(\mu 5)}_\alpha = -\bar{l}^A{}_i\Gamma^{(\mu 5)}(\lambda^*_\alpha)^i{}_j l_A{}^j,$$
$$\Gamma^{(\mu 5)} = \gamma^\mu \text{ or } \gamma^\mu\gamma_5 \tag{20b}$$

where the asterisk stands for complex conjugate. The electromagnetic current is

$$j^{\mu}_{\text{em}} = \tfrac{1}{2}(\mathcal{L}^{\mu}_{3+(1/\sqrt{3})8} + \mathcal{L}''^{\mu}_{3+(1/\sqrt{3})8}), \tag{21}$$

and the electric charge is $Q = \int j^0_{\text{em}} d^3x$. Note that the SU(3)″ currents contribute only to j^{μ}_{em}. With this choice of Q, the electric charges of the leptoquarks are

$$Q\begin{pmatrix} \mathcal{P}_1 & \mathcal{P}_2 & \mathcal{P}_3 \\ \mathfrak{N}_1 & \mathfrak{N}_2 & \mathfrak{N}_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}. \tag{22}$$

The weak charge-raising and -lowering currents are

$$j^{\mu}_{(\pm)} = \mathcal{L}^{\mu}_{1\pm i2} + \mathcal{L}^{\mu 5}_{1\pm i2}. \tag{23}$$

To allow the possibility that leptoquarks and quarks are identical, and because the lepton magnetic moments in the singlet would be incorrect, we reserve the SU(3)″ singlet for baryons, and assign the known leptons to S-wave three-leptoquark states as follows:

$$|e^+\uparrow\rangle = |\mathcal{P}_2^\uparrow\lambda_2^\uparrow\lambda_2^\downarrow\rangle, \tag{24a}$$

$$|\bar{\nu}_e\downarrow\rangle = |\mathfrak{N}_2^\downarrow\lambda_2^\uparrow\lambda_2^\downarrow\rangle, \tag{24b}$$

$$|\mu^+\uparrow\rangle = |\mathcal{P}_3^\uparrow\lambda_3^\uparrow\lambda_3^\downarrow\rangle, \tag{24c}$$

$$|\bar{\nu}_\mu\downarrow\rangle = |\mathfrak{N}_3^\downarrow\lambda_3^\uparrow\lambda_3^\downarrow\rangle. \tag{24d}$$

These states belong to $U''_3 = \pm\frac{3}{2}$ states in the SU(3)″ $\underline{10}$*, to $I = \frac{1}{2}$, $Y = -1$ states in SU(3) $\underline{8}$'s, and have spin $\frac{1}{2}$. The magnetic moments, calculated as before, are

$$\mu_{l^+} = \frac{e\hbar}{2M_L c}, \quad \mu_{l^0} = 0, \tag{25}$$

where M_L is the leptoquark mass. The λ leptoquarks in these states play the role of a neutral core and do not contribute to matrix elements of $j^{\mu}_{(\pm)}$, thus $G_V = G_A = 1$, and $(e^+, \bar{\nu}_e)$ are again each the sum of a left-handed doublet and a right-handed singlet under the $U(2)_L \times U(1)_R$ whose generators are

$$I_L^{\pm} = L_{1\pm i2} + L^5_{1\pm i2}, \quad I^3_L = L_3 + L^5_3, \tag{26a}$$

$$I^0_L = 6^{1/2}(L_0 + L^5_0), \quad I^0_R = 6^{1/2}(L_0 - L^5_0). \tag{26b}$$

Again neglecting masses, this model has μ-e universality, since the SU(3) currents are invariant under SU(3)″ transformations. The $U(2)_L \times U(2)_R$ group which transforms between e and μ states has generators

$$\Lambda^{+}_{L,R}=L''_{6+i7}\pm L''^{5}_{6+i7}, \quad \Lambda^{-}_{L,R}=(\Lambda^{+}_{L,R})^{\dagger}, \tag{27a}$$

$$\Lambda^{3}_{L,R}=-\tfrac{1}{2}(L''_3\pm L''^{5}_3)+\tfrac{1}{2}\sqrt{3}\,(L''_8\pm L''^{5}_8), \tag{27b}$$

$$\Lambda^{0}_{L,R}=6^{1/2}(L''_0\pm L''^{5}_0). \tag{27c}$$

Now the e and μ states differ only in having SU(3)″ structure 222 or 333. Since we take all observables diagonal in SU(3)″, electron and muon number are separately conserved. The electromagnetic and weak Lagrangians are

$$\mathcal{L}_I^{\text{em}}=\tfrac{1}{2}e(\mathcal{L}^{\mu}_{3+(1/\sqrt{3})8}+\mathcal{L}''^{\mu}_{3+(1/\sqrt{3})8})A_\mu \tag{28a}$$

and

$$\mathcal{L}_I^{\text{weak}}=g(\mathcal{L}^{\mu}_{1+i2}+\mathcal{L}^{\mu 5}_{1+i2})W_\mu+\text{H.c.} \tag{28b}$$

So far, we have shown that this model satisfies requirements (1), (2), (3), (4), and (6) above.

We give a two-body mass formula which lowers the known leptons relative to other states in the (20, 10*), again without considering this formula as satisfying requirement (8):

$$M=a[\tfrac{9}{4}-(U''_3)^2]+b(1+Y), \tag{29}$$

with a and b large and positive.

IV. PREDICTIONS FOR HEAVY LEPTON STATES

Both of these leptoquark models have many possible lepton states, while, to date, only four leptons and their antiparticles are known. Many properties of heavy leptons in such models are model-dependent; here we wish to comment on two features common to both models which may be of a more general character.

We satisfied requirement (6), $G_A=G_V=1$, by assigning the known leptons to doublet representations of $\text{SU}(2)_L\times \text{U}(1)_R$ for which there is no leakage for a weak current between the observed lepton states. Thus, as a one-W-exchange process, the usual production reactions $\mu^- h\to l^*X$ and $\nu_\mu h\to l^*X$ are forbidden, where l^* is a heavy lepton state. Charged l^*'s can, of course, be produced by $e^+e^-\to\bar{l}^*l^*$ and by $pp\to\bar{l}^*l^*X$, both electromagnetic interactions, and the neutral l^*'s produced similarly by a virtual $Z^0\to\bar{l}^*l^*$. Thus single heavy-lepton production is prohibited in lowest order, and only (higher-threshold) pair production of charged heavy leptons is allowed in lowest order. These features may be relevant to the failure to detect heavy leptons up to now.

For both our models, the leptonic hypercharge is conserved by both weak and electromagnetic interactions, and thus the lightest heavy lepton for each hypercharge not equal to the hypercharge of the presently known leptons is stable.

V. POSSIBLE IDENTITY OF LEPTOQUARKS AND QUARKS IN THE INTEGRALLY CHARGED MODEL

Up to now we have assumed that leptoquarks are unrelated to quarks. Now we speculate about the possibility that leptoquarks and quarks are identical. If they are identical, then, although baryon (B) → lepton (l) + meson (M) vertices are prohibited, it seems to be impossible to isolate leptons from strong interactions, due to the fact that $\bar{l}l$, $\bar{B}B$, and M can all have the same quantum numbers. Let us suspend judgement about this serious difficulty[24] for the remainder of this paragraph and assume that leptoquarks and quarks are identical. This identity of leptoquarks and quarks fits well with the assignment of the baryons and mesons to the singlet and the leptons to the 10* of SU(3)″ in our second model. The octets of vector and axial-vector currents are singlets under SU(3)″, and thus satisfy the usual Gell-Mann algebra for strong interactions.[25] Thus all the usual quark-model and current-algebra results for strong interactions can be carried over for the second model.[26]

In addition to explaining the occurrence of the same algebras in both hadronic and leptonic physics, we want to point out a simplification of the structure of weak currents which can be made in this unified model of hadrons and leptons. The Cabibbo angle reflects the fact that the direction of breaking of SU(3) in hadron states differs from the direction relevant to weak interactions of hadrons. Up to now, the Cabibbo angle has been introduced explicitly into the hadronic weak currents, but does not appear in the hadronic states. We suggest introducing the Cabibbo angle into the states via the relation

$$\langle h\{\mathcal{P},\mathfrak{N},\lambda\}|j\{\mathcal{P},\mathfrak{N}_C,\lambda_C\}|h'\{\mathcal{P},\mathfrak{N},\lambda\}\rangle =\langle h\{\mathcal{P},\mathfrak{N}_{-C},\lambda_{-C}\}|j\{\mathcal{P},\mathfrak{N},\lambda\}|h'\{\mathcal{P},\mathfrak{N}_{-C},\lambda_{-C}\}\rangle, \tag{30}$$

where $\mathfrak{N}_{\pm C}=\mathfrak{N}\cos\theta_C\pm\lambda\sin\theta_C$ and $\lambda_{\pm C}=\mp\mathfrak{N}\sin\theta_C+\lambda\cos\theta_C$, and h and h' stand for hadron states. With this change, the weak currents for the hadrons and leptons are identical and lepton-hadron universality, requirement (9) above, is satisfied.

If leptoquarks and quarks are identified, it should be possible to relate the masses of leptons and baryons. The fact that the SU(3)″-singlet state can be made lowest has been discussed by several authors in the context of models of hadrons, where the known hadrons are assigned to the SU(3)″ singlet. Thus models such as the two we discussed earlier, in which the leptons are assigned to nonsinglet representations, seem to run the risk of predicting that leptons are more massive than hadrons.

TABLE I. Mass formula inequalities for nonsinglet SU(3)″ lepton assignments.

Lepton assignment	Nonzero M_i in Eq. (31)	Sufficient inequalities
Σ''	$M_1,\ M_3$	$M_1>0,\quad M_3>0,\quad -24M_1<7M_3<-21M_1$
Ω''	$M_1,\ M_3$	$M_3>12M_1>0$
Ξ''	$M_1,\ M_2$	$M_1>0,\quad M_2>0,\quad 12M_1>M_2>6M_1$
N''	$M_1,\ M_2,\ M_3$	$M_1>0,\quad M_2<0,\quad -3M_2<18M_1<-4M_2,\quad -3M_1<M_3<-12M_1-2M_2$
Δ''	$M_1,\ M_2,\ M_3$	$M_1>0,\quad -3M_1<M_3<-2M_1,\quad -12M_1+\frac{7}{2}M_3<M_2<6M_1+\frac{1}{2}M_3$

We have considered this question in general, using the mass formula

$$M=M_0+M_1C_2^{(3)''}+M_2Y''+M_3[I''(I''+1)-\tfrac{1}{4}Y''^2]\ , \tag{31}$$

which follows from octet-dominant one- and two-body mass operators using the SU(3)″ degree of freedom. We require that the leptons lie lowest in mass, followed by the baryons, with all other I'' multiplets in $\underline{8}''$ and $\underline{10}^{*}{}''$ lying higher. We label these multiplets by the names they have in the baryon octet, decuplet, and singlet: N'', Λ_8'', Σ'', Ξ'', Δ'', $\Sigma^{*}{}''$, $\Xi^{*}{}''$, Ω'', Λ_1''. Here we choose the fundamental nonet to transform as $(\underline{3},\underline{3}^*)$ under $SU(3)\times SU(3)''$. [Had we chosen the nonet to transform as $(\underline{3},\underline{3})$, the same conclusions would follow, but with $M_2\to -M_2$ and $N''\rightleftarrows\Xi''$.] There are five acceptable assignments of leptons: to N'', Σ'', Ξ'', Δ'', or Ω''. The other three assignments (to Λ_8'', $\Sigma^{*}{}''$, and $\Xi^{*}{}''$) are ruled out. For Σ'' and Ω'', only the M_1 and the M_3 coefficients need be nonzero; for Ξ'', only M_1 and M_2 need be nonzero; for N'' and Δ'', M_1, M_2, and M_3 must be nonzero. We list sufficient inequalities for these lepton assignments in Table I. We emphasize that leptons can be assigned to nonsinglet representations of SU(3)″ without having the known leptons more massive than the baryons.

VI. SUMMARY AND OUTLOOK

We have explored the usefulness of composite models of leptons to represent known features of lepton physics in an economical way. We found that composite models are useful to represent a number of the features of lepton physics which we listed as requirements in Sec. I. We gave two such models, each using three triplets of leptoquarks; the first with fractionally charged, and the second with integrally charged leptoquarks. In the second model leptoquarks could be identical to quarks, and this second model offers the possibility to unify the description of leptonic and hadronic phenomena.

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Functional approach to strong-coupling theory in static models. I. Charged-scalar model*

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The strong-coupling theory in static models is formulated in terms of functional integrations. The method is demonstrated for the charged-scalar model. The expression of elastic and inelastic meson-nucleon scattering amplitudes is obtained to leading order in the strong-coupling expansion ($1/g$ expansion), while the isobar energy levels are obtained up to the next to the leading order.

I. INTRODUCTION

There are two distinct approaches to the strong-coupling theory in static models: the canonical field-theoretical method and the S-matrix method.

In the field-theory approach, which has a long history[1] since the first work of Wentzel,[2] one applies successive canonical transformations in the Hamiltonian formalism. On the other hand, in the S-matrix approach,[3] one starts with a set of Chew-Low equations for scattering amplitudes and solves them in the strong-coupling limit, presupposing various properties of the strong-coupling results, known from the field-theoretical method, such as the absence of production amplitudes in the strong-coupling limit. It is remarkable, however, that the final results are expressed in terms of operators in the isobar space which obey relatively simple algebraic equations.

We investigate the strong-coupling theory using the method of functional integration, with the intent of clarifying the perturbative nature of the strong-coupling theory ($1/g^2$ expansion). In this method, a canonical transformation is described by a corresponding change of variables of the phase-space functional integration, and a subsidiary condition on the state vector in the conventional formalism is described by a restriction of the Feynman integration path, which can be realized easily by inserting an appropriate δ functional in the integrand.[5] Thus, this method would be suitable for the description of strong-coupling theory.

In this paper we show the essence of the method using the charged-scalar model as an example and leave the general case to the following paper.

II. FUNCTIONAL-INTEGRAL REPRESENTATION OF GENERATING FUNCTIONAL OF GREEN'S FUNCTION IN STATIC MODEL

We define the generating functional by

$$Z(\eta) = \langle n_f | S_\eta | n_i \rangle , \tag{1}$$

where $|n_i\rangle$ and $|n_f\rangle$ are initial and final nucleon