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Canonical transforms between "current" and "constituent" quarks

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Certain transforms recently and not so recently proposed to convert current algebras into Hamiltonian symmetries are studied in the quark model with particular attention to their Fock-space properties. Expectation values and probability amplitudes are calculated which unveil the structure of a "current" quark in terms of a "constituent" quark. It is questioned whether these distributions admit a parton interpretation.

I. INTRODUCTION

Canonical transformations of the Foldy-Wouthuysen (FW) type have recently been revived[1-4] in the context of $SU(6)_W$,[5] the free-quark model, and a conjectured[6] relation between "current" quarks and "constituent" quarks. Gell-Mann[6] has suggested that the simple constituent-quark picture (in which a hadron is composed of two or three quarks) can be reconciled with the parton picture [in which an infinite number of parts (current quarks) are present] if a pair-creating transformation between current and constituent quarks can be constructed. Possible candidates for such a mechanism are those of Melosh[1] and Gomberoff, Horwitz, and Ne'eman,[7] who find transformations on the integrated local current densities which take these equal-time charge algebras into Hamiltonian symmetries. That a simple FW transform performs this task for $SU(6)_W$ was already pointed out by Gürsey[8] in 1965; the newer transforms exploit the fact that the requirement that the transformed charge be conserved does not uniquely specify the transform. Hence other requirements may be imposed, such as z-boost invariance, leading to modified FW transforms tailored to the kinematical configuration of the infinite-momentum limit instead of the nonrelativistic limit as is the case with the FW transform.

A property of the conventional equal-time formulation is that those $SU(6)_{W,\text{currents}}$ charges (the local integrated densities) which are not conserved create pairs and do not have single-particle eigenstates, whereas the $SU(6)_{W,\text{strong}}$ charges, although nonlocal, do not create pairs but do have single-particle eigenstates. Thus the transformations have the property of shifting a kind of nonlocality, in particle number as well as space, from states to operators, depending on the representation. Although not stated in second-quantized and Fock-space language, this state of affairs is implicit already in the classic paper of FW,[9] who point out that their transformation takes the naive and local position operator $\vec{x}$ into the nonlocal "mean-position" operator $\vec{X}$ of Newton and Wigner[10] (which does have localized eigenstates).

In this paper, we inquire into the Fock-space realization of these formal transformations, check whether and when they can be unitarily implemented, and in the process construct eigenstates of $SU(6)_{W,\text{current}}$ charges and the overlap between current and constituent quark states in the free-quark model. This overlap is zero unless the theory involves a cutoff in momentum. Suitable remarks are made concerning nonseparable infinite tensor-product spaces and the lack of unitary implementability. Certain averages and moments, however, are well defined. These are calculated and discussed in terms of a parton interpretation. Finally, nonvanishing overlap distributions are obtained for a cutoff theory which sheds light on the nature of the transformations and may be suggestive of what can occur in an interacting theory if the interactions supply the effective cutoff.

It should be stressed that we study those transformations which lead to exact symmetries of the quark model in the absence of interactions and in the equal-time formalism. Corresponding transforms in the lightlike formalism[2] *are* unitary, do not create pairs, and produce at most a spin rotation.

The paper is organized as follows: In Sec. II we list some FW-type transformations and discuss their uniqueness and consequences; in Sec. III, in a simplified formalism, we show how these transformations are implemented in the Hilbert space; in Sec. IV we obtain the eigenstates of the $SU(6)_{W,\text{currents}}$ charges and calculate distributions of current quarks in a strong quark. Section V contains the conclusions.

II. CANONICAL TRANSFORMS IN THE FREE-QUARK MODEL

The Melosh[1] transformation V_M is one of a large class of canonical formally unitary transforms V_i

$=e^{iY_i}$ ($Y_i=Y_i^\dagger$), which takes the free spin-$\frac{1}{2}$ theory in the Dirac representation

$$H=\int d^3x\,q^\dagger(x)(-i\vec{\alpha}\cdot\vec{\partial}+\beta m)q(x), \tag{2.1}$$

with

$$\{q^\dagger(\vec{x},t),q(\vec{x}',t)\}=\delta^3(\vec{x}-\vec{x}'), \tag{2.2}$$

into various forms

$$H_i=V_i^\dagger H V_i. \tag{2.3}$$

The existence of the transformation is determined by the same rules which govern unitary equivalence of matrices. Therefore the task is relatively simple if H_i involves only one of the matrices β, γ^5, α^k, and $\gamma^k\gamma^5$, with $k=1,2,3$.

The classic example is that of Foldy and Wouthuysen,[9] who found a transformation leading to a diagonal H_i,

$$H_{\rm FW}=\int d^3x\,q^\dagger(x)\beta(m^2-\vec{\partial}^2)^{1/2}q(x), \tag{2.4}$$

which is generated by

$$Y_{\rm FW}=\tfrac{1}{2}\int d^3x\,q^\dagger(x)\,\frac{\vec{\gamma}\cdot\vec{\partial}}{|\vec{\partial}|}\arctan\frac{|\vec{\partial}|}{m}\,q(x), \tag{2.5}$$

where

$$|\vec{\partial}|=(-\vec{\partial}^2)^{1/2}.$$

In this representation the "kinetic energy" $\vec{\alpha}\cdot\vec{p}$ term is effectively rotated into a momentum-dependent "mass term" proportional to β. The zero-momentum component is not affected and the representation yields the Pauli equations directly in the nonrelativistic limit. Alternatively, as observed by Cini and Touschek[11] and Bose, Gamba, and Sudarshan,[12] one can rotate the mass term $m\beta$ into a mass-dependent kinetic energy term and obtain

$$H_U=\int d^3x\,q^\dagger(x)(-i\vec{\alpha}\cdot\vec{\partial})\,\frac{(m^2-\vec{\partial}^2)^{1/2}}{|\vec{\partial}|}\,q(x) \tag{2.6}$$

using a transformation generated by

$$Y_U=-\tfrac{1}{2}\int d^3x\,q^\dagger(x)\frac{\vec{\gamma}\cdot\vec{\partial}}{|\vec{\partial}|}\arctan\frac{m}{|\vec{\partial}|}\,q(x). \tag{2.7}$$

Infinite-momentum terms are not affected and the representation is appropriate to the ultrarelativistic limit $p/m\to\infty$.

Other transformations can be constructed relevant to a nonrelativistic or ultrarelativistic limit in a particular direction. The Melosh transformation is generated by

$$Y_M=\tfrac{1}{2}\int d^3x\,q^\dagger(x)\,\frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{|\vec{\partial}_\perp|}\arctan\frac{|\vec{\partial}_\perp|}{m}\,q(x), \tag{2.8}$$

where

$$|\vec{\partial}_\perp|=(-\partial_x{}^2-\partial_y{}^2)^{1/2},$$

leading to

$$H_M=\int d^3x\,q^\dagger(x)[-i\alpha^3\partial_3+\beta(m^2-\vec{\partial}_\perp{}^2)^{1/2}]q(x), \tag{2.9}$$

which is diagonal in the FW sense for transverse motion and does not affect particles moving in the z direction. Yet another transformation, leading to

$$H_{UZ}=-i\int d^3x\,q^\dagger(x)\,\frac{\alpha^3\partial_3}{|\partial_3|}(m^2-\vec{\partial}^2)^{1/2}q(x) \tag{2.10}$$

appropriate to the kinematical configuration $E\sim p_z\gg M$, is generated by

$$V_{UZ}=e^{iY_{\rm FW}}e^{-iY_{UZ}}, \tag{2.11}$$

where

$$Y_{UZ}=\frac{\pi}{4}\int d^3x\,q^\dagger(x)\frac{\gamma^3\partial_3}{|\partial_3|}\,q(x). \tag{2.12}$$

A common feature of all such transformations V_i is that they are arbitrary up to a unitary transform $\tilde{V}$, which commutes with the Hamiltonian. An example is the transformation

$$V_G=e^{iY_M}e^{iY_G}, \tag{2.13}$$

where

$$Y_G=\tfrac{1}{2}\int d^3x\,q^\dagger(x)\frac{\gamma^3\partial_3}{|\partial_3|}\arctan\frac{|\vec{\partial}_\perp|}{(m^2-\vec{\partial}_\perp{}^2)^{1/2}}\,q(x), \tag{2.14}$$

constructed by Gomberoff, Horwitz, and Ne'eman,[7] which leads to $H_{\rm FW}$ even though $V_G\neq V_{\rm FW}$.

The transformations typically generate conserved (or otherwise desirable) quantities from nonconserved ones. If $[H,F]\neq 0$ but a transformation V_i can be found such that $[H_i,F]=[V_i^\dagger HV_i,F]=0$, then $[H,V_iFV_i^\dagger]=0$ and $W_i=V_iFV_i^\dagger$ is conserved. This property has been used both by Melosh and by Gomberoff, Horwitz, and Ne'eman to find a conserved $\mathrm{SU}(6)_W$ generated by $W=V_MFV_M^\dagger$ and $\mathrm{U}(6)\times\mathrm{U}(6)$ generated by $W=V_GFV_G^\dagger$ and $\tilde{W}=V_G\tilde{F}V_G^\dagger$, respectively, starting with the "current generators"[13]

$$F_a=\int d^3x\,q^\dagger(x)\,\frac{\lambda_a}{2}\,q(x), \tag{2.15a}$$

$$F_a^{1,2}=\tfrac{1}{2}\int d^3x\,q^\dagger(x)\,\beta\sigma^{1,2}\,\frac{\lambda_a}{2}\,q(x), \tag{2.15b}$$

$$F^3_a = \tfrac{1}{2}\int d^3x\, q^\dagger(x)\sigma^3 \frac{\lambda_a}{2} q(x)\,, \tag{2.15c}$$

and

$$\tilde{F}_a = \int d^3x\, q^\dagger(x)\,\beta\, \frac{\lambda_a}{2} q(x)\,, \tag{2.16a}$$

$$\tilde{F}^{1,2}_a = \tfrac{1}{2}\int d^3x\, q^\dagger(x)\sigma^{1,2} \frac{\lambda_a}{2} q(x)\,, \tag{2.16b}$$

$$\tilde{F}^3_a = \tfrac{1}{2}\int d^3x\, q^\dagger(x)\,\beta\sigma^3 \frac{\lambda_a}{2} q(x)\,. \tag{2.16c}$$

The first set commutes with

$$\int d^3x\, q^\dagger(x)(\alpha^3, \beta) f(\vec{\partial}) q(x)$$

and all of them commute with

$$\int d^3x\, q^\dagger(x)\,\beta g(\vec{\partial}) q(x)\,,$$

where $f(\vec{\partial})$ and $g(\vec{\partial})$ are arbitrary functions of spatial derivatives, that is,

$$[F^{1,2,3}_a, H_{(M,\,\mathrm{FW})}] = 0 \tag{2.17}$$

and

$$[\tilde{F}^{1,2,3}_a, H_{\mathrm{FW}}] = 0\,. \tag{2.18}$$

Again, the above transformations, leading to conserved quantities, are unique only up to a unitary transform $\tilde{V}$ which commutes with H and a unitary transform $\hat{V}$ which commutes with the F that is to be conserved. If one finds one transformation V_i such that $[V_i^\dagger H V_i, F] = 0$, all other transformations leading to a vanishing commutator are given by

$$V_i' = \tilde{V} V_i \hat{V}\,. \tag{2.19}$$

Integrated bilinear densities of the type

$$D = \int d^3x : q^\dagger(x)\,\Gamma f(\vec{\partial}) q(x):\,,$$

where Γ is a Dirac matrix, have the following forms in momentum space:

$$\begin{aligned} D = \sum_{r,s}\int d^3k\, \frac{M^2}{E^2} f(\vec{k})\big[& a^{\dagger(r)}_{\vec{k}} a^{(s)}_{\vec{k}} u^{\dagger(r)}(\vec{k})\Gamma u^{(s)}(\vec{k}) - b^{\dagger(s)}_{\vec{k}} b^{(r)}_{\vec{k}} v^{\dagger(r)}(\vec{k})\Gamma v^{(s)}(\vec{k}) \\ &+ a^{\dagger(r)}_{-\vec{k}} b^{\dagger(s)}_{\vec{k}} u^{\dagger(r)}(-\vec{k})\Gamma v^{(s)}(\vec{k}) e^{2iEx_0} + b^{(r)}_{-\vec{k}} a^{(s)}_{\vec{k}} v^{\dagger(r)}(-\vec{k})\Gamma u^{(s)}(\vec{k}) e^{-2iEx_0}\big]\,, \end{aligned} \tag{2.20}$$

with the convention

$$\{a^{(r)}_{\vec{p}}, a^{\dagger(s)}_{\vec{p}'}\} = \{b^{(r)}_{\vec{p}}, b^{\dagger(s)}_{\vec{p}'}\} = \frac{E}{M}\delta_{r,s}\,\delta^3(\vec{p} - \vec{p}\,') \tag{2.21}$$

for the nonvanishing anticommutators. The densities D are conserved (nonconserved) according to whether they do not (do) create pairs. If the operators create pairs, they may further be classified according to whether these pairs, when arising in connected z graphs, contribute (bad operators) or do not contribute (good operators) to current-algebra sum rules saturated at infinite momentum ($p_z \to \infty$). Good (bad) operators commute (do not commute) with α^3. If Γ is one of $\gamma^{1,2,5}$, $\gamma^{1,2}\gamma^5$, σ^3, or α^3, the operators are good; F^α_a are good, $\tilde{F}^\alpha_a$ are bad. Accordingly, V_M was chosen in a way which preserves the goodness of the $\mathrm{SU}(6)_{W,\,\mathrm{currents}}$ generators, while V_G, itself bad, takes the entire $\mathrm{U}(6)\times\mathrm{U}(6)$ algebra of currents [generated by $(1\pm\beta)\vec{\sigma}\lambda_a$] into a symmetry generated by good operators.

A slightly more complicated operator of interest is the position operator

$$\begin{aligned} \hat{x}^i &= \int d^3x\, q^\dagger(x)\, x^i q(x) \\ &= \sum_{r,s}\int d^3k\, \frac{M}{E}\Big\{ i a^{\dagger(r)}_{\vec{k}} \frac{d}{dk^i} a^{(s)}_{\vec{k}} \delta_{r,s} + i\left(\frac{d}{dk^i} b^{\dagger(s)}_{\vec{k}}\right) b^{(r)}_{\vec{k}} \delta_{r,s} \\ &\qquad + a^{\dagger(r)}_{\vec{k}} a^{(s)}_{\vec{k}} \left[x_0 \frac{k^i}{E}\delta_{r,s} - i\frac{k^i}{2E^2}\delta_{r,s} - \frac{1}{2E(E+M)} u^{\dagger(r)}(0)(\vec{\sigma}\times\vec{k})^i u^{(s)}(0)\right] \\ &\qquad + b^{\dagger(s)}_{\vec{k}} b^{(r)}_{\vec{k}} \left[-x_0 \frac{k^i}{E}\delta_{r,s} - i\frac{k^i}{2E^2}\delta_{r,s} - \frac{1}{2E(E+M)} v^{\dagger(r)}(0)(\vec{\sigma}\times\vec{k})^i v^{(s)}(0)\right] \\ &\qquad + i b^{(r)}_{-\vec{k}} a^{(s)}_{\vec{k}} \left[-\frac{1}{2E} v^{\dagger(r)}(0)\gamma^i u^{(s)}(0) + \frac{k^i}{2E^2(E+M)} v^{\dagger(r)}(0)\vec{\gamma}\cdot\vec{k}\, u^{(s)}(0)\right] e^{-2iEx_0} \\ &\qquad - i a^{\dagger(r)}_{-\vec{k}} b^{\dagger(s)}_{\vec{k}} \left[\frac{1}{2E} u^{\dagger(r)}(0)\gamma^i v^{(s)}(0) - \frac{k^i}{2E^2(E+M)} u^{\dagger(r)}(0)\vec{\gamma}\cdot\vec{k}\, v^{(s)}(0)\right] e^{2iEx_0}\Big\}\,, \end{aligned} \tag{2.22}$$

which is a local function of the fields, creates pairs in momentum space, and does not possess single-particle eigenstates. Its Foldy transform is the Newton-Wigner[10] position operator which is a nonlocal function of the fields but possesses truly localized single-particle states,

$$|X\rangle=\int d^3p\,\frac{M}{E}\,e^{ipX}\,a_{\vec{p}}^{\dagger(r)}|0\rangle\,. \tag{2.23}$$

The Foldy-Wouthuysen-type transformations, when implemented in Fock space or larger spaces, provide a technical tool for constructing eigenstates of nonconserved, pair-producing operators. If the eigenstates $|q_s\rangle$ (the quantum numbers are suppressed in this notation) of a conserved charge W_i, $W_i=V_i FV_i^\dagger$, are denoted as constituent or strong quarks, which are single-particle Fock-space states in the Dirac representation, then the eigenstates of F are defined by $|q_c\rangle=V_i^\dagger|q_s\rangle$. Their overlap, the distribution of current quarks in a strong quark, if that colorful language may be used, is given by

$$\langle q_c|q_s\rangle=\langle q_s|V|q_s\rangle\,, \tag{2.24}$$

leading to what we shall call exclusive distributions. A similar object is the overlap of an eigenstate of $\hat{x}$ with an eigenstate of $\hat{X}$

$$\langle x|X\rangle=\langle x|V_{\rm FW}|x\rangle \tag{2.25}$$

which gives a measure of the kind of nonlocality implicit in the $V_{\rm FW}$. Additionally, we shall be interested in certain "inclusive" probabilities, that is, expectation values of current quark operators in constituent quark states, such as

$$\langle q_s|N_c|q_s\rangle\,, \tag{2.26}$$

the average number of current quarks in a constituent quark.

The next two sections are devoted to a discussion of the nature of these probability amplitudes and distributions.

III. STRUCTURE OF THE TRANSFORMATIONS IN A SIMPLIFIED FORMALISM

The Hilbert-space implementation of the Foldy-Wouthuysen operator and its variants is best discussed in a formalism stripped of inessentials such as spinor SU(3) indices. Let us consider a (box normalized) discrete but infinite set of (spinless) fermion annihilation and creation operators obeying the anticommutation relations

$$\{a_k, a_{k'}^\dagger\}=\delta_{k,k'}\,, \tag{3.1a}$$

$$\{b_{-k}, b_{-k'}^\dagger\}=\delta_{k,k'}\,, \tag{3.1b}$$

all other anticommutators vanishing. Then in terms of pair creation and annihilation operators,

$$A_k=b_{-k}a_k\,, \tag{3.2a}$$

$$A_k^\dagger=a_k^\dagger b_{-k}^\dagger\,, \tag{3.2b}$$

with commutator

$$[A_k, A_{k'}^\dagger]=\delta_{k,k'}(1-a_k^\dagger a_k-b_{-k}^\dagger b_{-k})\,, \tag{3.3}$$

the operators of interest have, apart from counting terms as $a_k^\dagger a_k$, etc., the form

$$V=\exp\Big[i\sum_k(\gamma_k A_k^\dagger+\gamma_k^* A_k)\Big]=\prod_k V_k\,, \tag{3.4}$$

where γ_k is an arbitrary function of k. The operator

$$V_k=\exp[i(\gamma_k A_k^\dagger+\gamma_k^* A_k)] \tag{3.5}$$

acts on the simple kth-momentum-mode space built on the vacuum $|0_k\rangle$, a tensor product of the vacuum of a_k and the vacuum of b_{-k}, with base states $|0_k\rangle$, $|1_k\rangle=a_k^\dagger|0_k\rangle$, $|\bar{1}_k\rangle=b_{-k}^\dagger|0_k\rangle$, and $|2_k\rangle=a_k^\dagger b_{-k}^\dagger|0_k\rangle$ and arbitrary normalized state $|f_k\rangle$. V acts on the nonseparable infinite-term product space of elements

$$|f\rangle=\prod_k|f_k\rangle \tag{3.6}$$

with action

$$V|f\rangle=\prod_k V_k|f_k\rangle \tag{3.7}$$

and inner product

$$\langle g|f\rangle=\prod_k\langle g_k|f_k\rangle\,, \tag{3.8}$$

which usually vanishes. To a fixed vector $\prod_i|f_i\rangle$ corresponds an equivalence class composed of all others which differ from it by at most a finite number of factors. If the Fock vacuum $\prod_k|0_k\rangle$ belongs to this class, the class spans the Fock space. If a state of definite but infinite occupation number belongs to this class, the class spans a space which does not contain the vacuum but still carries a so-called discrete representation of the algebra. If no state of definite occupation number belongs to the class, the class spans a space which carries a "continuous" representation of the algebra; each member of the space is orthogonal to any state of definite occupation number.[14]

From the commutation relation, Eqs. (3.1) and (3.3), and by a method to be introduced in the next section, it is easy to show that the V transformed vacuum

$$V|0\rangle=\prod_k\left(\cos|\gamma_k||0_k\rangle+\frac{i\gamma_k}{|\gamma_k|}\sin|\gamma_k||2_k\rangle\right) \tag{3.9}$$

is a normed state whose overlap with the vacuum is

$$\langle 0|V|0\rangle = \prod_k \cos|\gamma_k| , \tag{3.10}$$

which vanishes unless all but a finite number of the γ_k are different from zero. Since this is not the case for the Foldy-Wouthuysen transforms, which affect an infinite number of modes, $|0\rangle$ and $V|0\rangle$ lie in different spaces corresponding to distinct equivalence classes, and V is not unitarily implementable; V takes Fock-space states into a new space which carries a continuous representation of (3.1), each of whose vectors are orthogonal to any state of definite occupation number, finite or not.

Thus all "exclusive" probability amplitudes, that is, overlaps of a V-transformed Fock-space state with any state of definite occupation number, are zero, and this zero can be evaded in a free theory only if the number of momentum modes is made finite, corresponding to a cutoff in momentum space. Certain "inclusive" probabilities, however, are well defined, such as the probability $P(n_p)$ of finding n pairs of momentum p and anything else, X, which is given by[15]

$$P(n_p) = \sum_X |\langle n_p, X|V|0\rangle|^2 , \tag{3.11}$$

with

$$P(0_p) = \cos^2|\gamma_p| , \tag{3.12}$$

$$P(1_p) = \sin^2|\gamma_p| , \tag{3.13}$$

as well as the number-operator expectation values in the transformed states, e.g.,

$$\langle 0|V^\dagger N_k V|0\rangle = \sin^2|\gamma_k| , \tag{3.14}$$

$$\langle 0|V^\dagger N V|0\rangle = \sum_k \langle 0|V^\dagger N_k V|0\rangle = \sum_k \sin^2|\gamma_k| = \langle N\rangle , \tag{3.15}$$

$$\langle 0|V^\dagger N^2 V|0\rangle = \langle N\rangle^2 + \langle N\rangle - \sum_k \sin^4|\gamma_k| = \langle N^2\rangle , \tag{3.16}$$

and

$$\langle N^2\rangle - \langle N\rangle^2 = \frac{1}{4}\sum_k \sin^2|2\gamma_k| , \tag{3.17}$$

indicating that even N is a bounded operator in the space built on $V|0\rangle$ if $\sum_k \sin^2|\gamma_k|$ converges.

Thus, when in the next section we calculate exclusive probabilities of "finding a current quark in a constituent quark" it should come as no surprise that these probabilities vanish unless the high momenta are cut off, whereas inclusive probabilities associated with these distributions, such as averages and moments, are well defined.

IV. DISTRIBUTIONS AND OVERLAPS

In this section we implement the Foldy-Wouthuysen and Melosh transforms on Fock or larger spaces. To avoid cumbersome SU(3) indices, which can be easily incorporated, we limit the discussion to the W-spin subalgebra of the $SU(6)_W$ charges, writing $F^{1,2,3}$ for $F_0^{1,2,3}$ and $W_i^{1,2,3}$ for $V_i F_0^{1,2,3} V_i^\dagger$, where i stands for FW or M.

As mentioned in Sec. II, the W_i commute with the Dirac Hamiltonian and create no pairs; they also commute with the momentum operator $\vec{\mathrm{P}}$ and with the quark number operator

$$N_q = \sum_r \int d^3p \, \frac{M}{E} \, a_{\vec{p}}^{\dagger(r)} a_{\vec{p}}^{(r)}$$

and its charge conjugate, the antiquark number operator $N_{\bar{q}}$. In addition, the W^3_{FW} and W^3_M commute with the z component of the angular momentum J_z. If $a_{s,i}^{\dagger(r)}(p)$ denotes a creation operator for a single strong quark, which is a simultaneous eigenstate of the classifying charges $(\vec{\mathrm{W}}_i)^2$, W_i^3, H, $\vec{\mathrm{P}}$, N_q, and $N_{\bar{q}}$, and belongs to the W_i^3 eigenvalue $r = \pm\frac{1}{2}$, i standing for FW or M, then the corresponding state is given by

$$|q_s, \vec{\mathrm{p}}, r, i\rangle = \mathfrak{N}_i \, a_{s,i}^{\dagger(r)}(\vec{\mathrm{p}})|0\rangle \tag{4.1}$$

provided

$$[W_i^3, a_{s,i}^{\dagger(\pm)}(\vec{\mathrm{p}})] = \pm\tfrac{1}{2} a_{s,i}^{\dagger(\pm)}(\vec{\mathrm{p}}) , \tag{4.2}$$

where $\mathfrak{N}_i$ is a normalization factor. In general, r would include the SU(3) indices. In view of the above discussion the most general form of $a_{s,i}^{\dagger(\pm)}(\vec{\mathrm{p}})$ is a linear combination of the usual fermion operators $a_{\vec{p}}^{\dagger(r)}$,

$$a_{s,i}^{\dagger(\pm)}(\vec{\mathrm{p}}) = \mu_i^\pm(\vec{\mathrm{p}}) \, a_{\vec{p}}^{\dagger(+)} + \nu_i^\pm(\vec{\mathrm{p}}) \, a_{\vec{p}}^{\dagger(-)} , \tag{4.3}$$

and similarly

$$b_{s,i}^{\dagger(\pm)}(\vec{\mathrm{p}}) = \lambda_i^\pm(\vec{\mathrm{p}}) \, b_{\vec{p}}^{\dagger(+)} + \kappa_i^\pm(\vec{\mathrm{p}}) \, b_{\vec{p}}^{\pm(-)} , \tag{4.4}$$

where μ, ν, λ, and κ are complex functions of the momentum. Since $a_{s,i}^{\dagger(+)}$ and $a_{s,i}^{\dagger(-)}$ belong to distinct W_i^3 eigenvalues, we have

$$\mu_i^+(\mu_i^-)^* + \nu_i^+(\nu_i^-)^* = 0 \tag{4.5}$$

and

$$\lambda_i^+(\lambda_i^-)^* + \kappa_i^+(\kappa_i^-)^* = 0 . \tag{4.6}$$

Furthermore, if

$$|\mu_i^\pm|^2 + |\nu_i^\pm|^2 = |\lambda_i^\pm|^2 + |\kappa_i^\pm|^2 = 1 , \tag{4.7}$$

the strong operators obey the usual equal-time anticommutation relations. Equation (4.2) and a similar relation for the strong antiquark operators combined with Eq. (4.7) yield

$$a^{\dagger(r)}_{s,\mathrm{FW}}(\vec{p}) = a^{\dagger(r)}_{\vec{p}}, \tag{4.8}$$

$$b^{\dagger(r)}_{s,\mathrm{FW}}(\vec{p}) = b^{\dagger(r)}_{\vec{p}}, \tag{4.9}$$

and

$$a^{\dagger(+)}_{s,M}(\vec{p}) = \left[\frac{(E+\omega)(M+\omega)}{2\omega(E+M)}\right]^{1/2} \times \left[a^{\dagger(+)}_{\vec{p}} - \frac{p^3 p_+}{(E+\omega)(M+\omega)}\, a^{\dagger(-)}_{\vec{p}}\right], \tag{4.10}$$

$$b^{\dagger(+)}_{s,M}(\vec{p}) = \left[\frac{(E+\omega)(M+\omega)}{2\omega(E+M)}\right]^{1/2} \times \left[b^{\dagger(+)}_{\vec{p}} + \frac{p^3 p_+}{(E+\omega)(M+\omega)}\, b^{\dagger(-)}_{\vec{p}}\right], \tag{4.11}$$

where $\omega = (M^2 + p_\perp{}^2)^{1/2}$ and $p_\pm = p^1 \pm ip^2$. Thus, the eigenstates of the FW-transformed $W^3_{\mathrm{FW}} = V_{\mathrm{FW}} F^3 V^\dagger_{\mathrm{FW}}$ are simply the naive spin states created by the usual fermion operators, whereas the eigenstates of W^3_M differ from these by a momentum-dependent spin rotation which does not affect particles moving in the z direction or in the transverse plane. As Melosh has pointed out, this is just a Wigner rotation arising when a state of a given transverse momentum is boosted in the z direction; the Wigner rotation is necessary if the W_M-spin classification is to be z-boost invariant.

We also introduce creation and annihilation operators for current quarks and antiquarks related to the strong operators by

$$a^{\dagger(r)}_{c,i}(\vec{p}) = V^\dagger_i\, a^{\dagger(r)}_{s,i}(\vec{p})\, V_i\,, \tag{4.12}$$

with an analogous relation for $b^{\dagger(r)}_{c,i}(\vec{p})$. The corresponding single-current quark states created by these operators,

$$|q_c, \vec{p}, r, i\rangle = \mathfrak{N}_i\, a^{\dagger(r)}_{c,i}(\vec{p})|0, i\rangle_c = V^\dagger_i |q_s, \vec{p}, r, i\rangle\,, \tag{4.13}$$

are time-dependent eigenstates of $(\vec{F})^2$, F^3, $V^\dagger_i H V_i$, $\vec{P}$, and of the current quark number operators

$$V^\dagger_i N_q V_i = \sum_r \int d^3p\, \frac{M}{E}\, a^{\dagger(r)}_{c,i}(\vec{p})\, a^{(r)}_{c,i}(\vec{p})$$

and

$$V^\dagger_i N_{\bar{q}} V_i\,.$$

The current vacuum

$$|0, i\rangle_c = V^\dagger_i |0\rangle \tag{4.14}$$

is annihilated by $a^{(r)}_{c,i}(\vec{p})$, $b^{(r)}_{c,i}(\vec{p})$, $F^{1,2,3}$, and $V^\dagger_i H V_i$, and carries no momentum. Since the V_i act as homogeneous operator transforms the $a^{\dagger(r)}_{c,i}$ are linear combinations of ordinary creation/annihilation operators, with all the complexity and possible diseases of the transforms contained in the transformed vacuum $|0, i\rangle_c$ as exposed in Sec. III.

Before obtaining these linear combinations we must introduce some notation. If we define

$$A(g) \equiv \int d^3p\, \frac{M}{E}\, g(\vec{p}) A_{\vec{p}} \tag{4.15}$$

with $A_{\vec{p}}$ a Fock-space operator on a $\vec{p}$-mode subspace and $g(\vec{p})$ a complex function of momentum, then the generators Y_i have the form

$$Y_i = -iC^\dagger(\alpha_i) - iD^\dagger(\beta_i) + iC(\alpha_i) + iD(\beta_i) + F(\gamma_i)\,, \tag{4.16}$$

where

$$C^\dagger_{\vec{p}} = (a^{\dagger(-)}_{\vec{p}} b^{\dagger(+)}_{-\vec{p}} - a^{\dagger(+)}_{\vec{p}} b^{\dagger(-)}_{-\vec{p}})\, e^{2iEx_0}\,,$$

$$D^\dagger_{\vec{p}} = \frac{1}{|p_\perp|}(p_+ a^{\dagger(-)}_{\vec{p}} b^{\dagger(-)}_{-\vec{p}} + p_- a^{\dagger(+)}_{\vec{p}} b^{\dagger(+)}_{-\vec{p}})\, e^{2iEx_0}\,,$$

$$F_{\vec{p}} = F^\dagger_{\vec{p}} = \frac{i}{|p_\perp|}\left[p_+(a^{\dagger(-)}_{\vec{p}} a^{(+)}_{\vec{p}} - b^{\dagger(-)}_{-\vec{p}} b^{(+)}_{-\vec{p}}) - p_-(a^{\dagger(+)}_{\vec{p}} a^{(-)}_{\vec{p}} - b^{\dagger(+)}_{-\vec{p}} b^{(-)}_{-\vec{p}})\right],$$

$C_{\vec{p}}$ and $D_{\vec{p}}$ are Hermitian conjugates of $C^\dagger_{\vec{p}}$ and $D^\dagger_{\vec{p}}$, respectively, and the functions α, β, and γ are

$$\alpha_{\mathrm{FW}}(\vec{p}) = \frac{1}{2}\, \frac{p^3}{|p|} \arctan \frac{|p|}{M}\,, \tag{4.17}$$

$$\beta_{\mathrm{FW}}(\vec{p}) = -\frac{1}{2}\, \frac{|p_\perp|}{|p|} \arctan \frac{|p|}{M}\,, \tag{4.18}$$

$$\gamma_{\mathrm{FW}}(\vec{p}) = 0 \tag{4.19}$$

for the FW transform, and

$$\alpha_M(\vec{p}) = \frac{1}{2}\, \frac{p^3|p_\perp|}{E(E+M)} \arctan \frac{|p_\perp|}{M}\,, \tag{4.20}$$

$$\beta_M(\vec{p}) = -\frac{1}{2}\, \frac{ME+\omega^2}{E(E+M)} \arctan \frac{|p_\perp|}{M}\,, \tag{4.21}$$

$$\gamma_M(\vec{p}) = \frac{1}{2}\, \frac{p^3}{E} \arctan \frac{|p_\perp|}{M} \tag{4.22}$$

for the Melosh transform. It is then straightforward to show that

$$V_i a^{\dagger(r)}_{\vec{p}} V_i^\dagger = e^{iY_i} a^{\dagger(r)}_{\vec{p}} e^{-iY_i}$$

$$= \cos\Gamma_i \left[a^{\dagger(+)}_{\vec{p}} - \frac{\gamma_i p_+}{\Gamma_i |p_\perp|} \tan\Gamma_i \, a^{\dagger(-)}_{\vec{p}} + e^{-2iEx_0} \left(\frac{\alpha_i}{\Gamma_i} b^{(-)}_{-\vec{p}} - \frac{\beta_i p_+}{\Gamma_i |p_\perp|} b^{(+)}_{-\vec{p}} \right) \tan\Gamma_i \right] \delta_{r,+}$$

$$+ \cos\Gamma_i \left[a^{\dagger(-)}_{\vec{p}} + \frac{\gamma_i p_-}{\Gamma_i |p_\perp|} \tan\Gamma_i \, a^{\dagger(+)}_{\vec{p}} - e^{-2iEx_0} \left(\frac{\alpha_i}{\Gamma_i} b^{(+)}_{-\vec{p}} + \frac{\beta_i p_-}{\Gamma_i |p_\perp|} b^{(-)}_{-\vec{p}} \right) \tan\Gamma_i \right] \delta_{r,-} , \tag{4.23}$$

where $\Gamma_i = (\alpha_i^2 + \beta_i^2 + \gamma_i^2)^{1/2}$, and hence

$$a^{\dagger(+)}_{c,\mathrm{FW}}(\vec{p}) = \left(\frac{E+M}{2E} \right)^{1/2} \left[a^{\dagger(+)}_{\vec{p}} - \left(\frac{p_+}{E+M} b^{(+)}_{-\vec{p}} + \frac{p^3}{E+M} b^{(-)}_{-\vec{p}} \right) e^{-2iEx_0} \right] \tag{4.24}$$

and

$$a^{\dagger(+)}_{c,M}(\vec{p}) = \frac{1}{2E} \left(\frac{E+\omega}{E+M} \right)^{1/2} \left[(2E+M-\omega) a^{\dagger(+)}_{\vec{p}} + \frac{p^3 p_+}{\omega+E} a^{\dagger(-)}_{\vec{p}} - \left(p_+ b^{(+)}_{-\vec{p}} + p^3 \frac{\omega - M}{\omega + M} b^{(-)}_{-\vec{p}} \right) e^{-2iEx_0} \right] . \tag{4.25}$$

Note that at this point we have three generally different kinds of creation and annihilation operators: the ordinary fermion operators as $a^{\dagger(r)}_{\vec{p}}$, strong operators as $a^{\dagger(r)}_{s,i}(\vec{p})$, and current quark operators as $a^{\dagger(r)}_{c,i}(\vec{p})$.

It is often helpful to write V_i as a product of exponential operators with all operators which annihilate the vacuum appearing to the right of those which do not annihilate the vacuum. This ordering procedure is described in the Appendix. The result is

$$\begin{aligned} V_i &= \exp[C^\dagger(\alpha_i) + D^\dagger(\beta_i) - C(\alpha_i) - D(\beta_i) + iF(\gamma_i)] \\ &= \exp[C^\dagger(f_i^{(1)}) + D^\dagger(f_i^{(2)})] \exp[C(g_i^{(1)} + D(g_i^{(2)})] \\ &\quad \times \exp[iF(h_i)] \exp[K(m_i)] , \end{aligned}$$

where

$$K(fg) \equiv [C(f), C^\dagger(g)] , \tag{4.26a}$$

$$K_{\vec{p}} = \frac{2E}{M} \delta(0) - a^{\dagger(+)}_{\vec{p}} a^{(+)}_{\vec{p}} - a^{\dagger(-)}_{\vec{p}} a^{(-)}_{\vec{p}} - b^{\dagger(+)}_{-\vec{p}} b^{(+)}_{-\vec{p}} - b^{\dagger(-)}_{-\vec{p}} b^{(-)}_{-\vec{p}} , \tag{4.26b}$$

$$f_i^{(1)} = \left(1 + \frac{\gamma_i^2}{\Gamma_i^2} \tan^2\Gamma_i \right)^{-1} \left(\frac{\alpha_i}{\Gamma_i} - \frac{\beta_i \gamma_i}{\Gamma_i^2} \tan\Gamma_i \right) \tan\Gamma_i , \tag{4.26c}$$

$$f_i^{(2)} = \left(1 + \frac{\gamma_i^2}{\Gamma_i^2} \tan^2\Gamma_i \right)^{-1} \left(\frac{\beta_i}{\Gamma_i} + \frac{\alpha_i \gamma_i}{\Gamma_i^2} \tan\Gamma_i \right) \tan\Gamma_i , \tag{4.26d}$$

$$g_i^{(1)} = -(1 + \tan^2\Gamma_i)^{-1} \left(\frac{\alpha_i}{\Gamma_i} - \frac{\beta_i \gamma_i}{\Gamma_i^2} \tan\Gamma_i \right) \tan\Gamma_i , \tag{4.26e}$$

$$g_i^{(2)} = -(1 + \tan^2\Gamma_i)^{-1} \left(\frac{\beta_i}{\Gamma_i} + \frac{\alpha_i \gamma_i}{\Gamma_i^2} \tan\Gamma_i \right) \tan\Gamma_i , \tag{4.26f}$$

$$h_i = \arctan \left(\frac{\gamma_i}{\Gamma_i} \tan\Gamma_i \right) , \tag{4.26g}$$

and

$$m_i = -\tfrac{1}{2} \ln[1 + (f_i^{(1)})^2 + (f_i^{(2)})^2] . \tag{4.26h}$$

Then, finally we can write

$$V_i|0\rangle = \exp[C^\dagger(f^{(1)}) + D^\dagger(f^{(2)})] |0\rangle \times \exp \left\{ -\delta^3(0) \int d^3p \ln[1 + (f_i^{(1)})^2 + (f_i^{(2)})^2] \right\} , \tag{4.27}$$

a form which strikingly illustrates that $V_i|0\rangle$ is orthogonal to any state of *definite particle number*. [Crudely speaking, any such state $|\Psi\rangle$ cannot develop an infinity from

$$\langle \Psi | \exp[C^\dagger(f_i^{(1)}) + D^\dagger(f_i^{(2)})] |0\rangle$$

to cancel the zero coming from the strongly divergent negative exponent.] Nonetheless, as discussed earlier, $V_i|0\rangle$ is normed, but it does not reside in Fock space. Consequently, the probability of finding n strong quarks and m strong antiquarks in a current quark,

$$|\langle n q_s, m \bar{q}_s | q_c \rangle|^2 ,$$

vanishes even if n and/or m are infinite. This state of affairs persists even if we quantize in a finite volume $(2L)^3$ leading to discrete momenta $\vec{k} = \vec{n} \pi L^{-1}$, where $\vec{n} = (n_x, n_y, n_z)$ are integers. With

$$\{a^{\dagger(r)}_{\vec{k}}, a^{(s)}_{\vec{k}'}\}=\{b^{\dagger(r)}_{\vec{k}}, b^{(s)}_{\vec{k}'}\}$$
$$=\delta_{r,s}\,\delta_{\vec{k},\vec{k}'}, \tag{4.28}$$

a shorthand for

$$\{a^{\dagger(r)}(n_x,n_y,n_z), a^{(s)}(m_x,m_y,m_z)\}$$
$$=\{b^{\dagger(r)}(n_x,n_y,n_z), b^{(s)}(m_x,m_y,m_z)\}$$
$$=\delta_{r,s}\,\delta_{n_x,m_x}\delta_{n_y,m_y}\delta_{n_z,m_z}, \tag{4.29}$$

the generators Y_i are given by

$$Y_i=\sum_{\vec{k}} Y_{i,\vec{k}}$$
$$=-i\sum_{\vec{k}}[\alpha_i(\vec{k})C^\dagger_{\vec{k}}+\beta_i(\vec{k})D^\dagger_{\vec{k}}-\alpha_i(\vec{k})C_{\vec{k}}$$
$$-\beta_i(\vec{k})D_{\vec{k}}+i\gamma_i(\vec{k})F_{\vec{k}}], \tag{4.16$'$}$$

with

$$V_i=\prod_{\vec{k}} V_{i,\vec{k}}=\prod_{\vec{k}} e^{iY_{i,\vec{k}}} \tag{4.30}$$

a bona fide infinite tensor product, each factor $V_{i,\vec{k}}$ of which can be put in "ordered" form

$$V_{i,\vec{k}}=\exp[f^{(1)}_i(\vec{k})C^\dagger_{\vec{k}}+f^{(2)}_i(\vec{k})D^\dagger_{\vec{k}}]\exp[g^{(1)}_i(\vec{k})C_{\vec{k}}+g^{(2)}_i(\vec{k})D_{\vec{k}}]\exp[ih_i(\vec{k})F_{\vec{k}}]\exp[m_i(\vec{k})K_{\vec{k}}], \tag{4.26$'$}$$

where

$$K_{\vec{k}}=2-a^{\dagger(+)}_{\vec{k}}a^{(+)}_{\vec{k}}-a^{\dagger(-)}_{\vec{k}}a^{(-)}_{\vec{k}}-b^{\dagger(+)}_{-\vec{k}}b^{(+)}_{-\vec{k}}-b^{\dagger(-)}_{-\vec{k}}b^{(-)}_{-\vec{k}}$$

and the functions f, g, h, and m are given by Eqs. (4.26). Again,

$$V_i|0\rangle=\prod_{\vec{k}}\{1+[f^{(1)}_i(\vec{k})]^2+[f^{(2)}_i(\vec{k})]^2\}^{-1}$$
$$\times\exp[f^{(1)}_i(\vec{k})C^\dagger_{\vec{k}}+f^{(2)}_i(\vec{k})D^\dagger_{\vec{k}}]|0\rangle \tag{4.27$'$}$$

is orthogonal to all states of definite particle number unless a momentum cutoff $|\vec{k}|\le\Lambda$ is introduced, which makes the number of degrees of freedom finite.

However, there are distributions which do not vanish even if $\Lambda\to\infty$, such as the average number of current quarks in a strong state. Even though these can be calculated directly by evaluating the appropriate matrix elements, it is more instructive to find these from the expansion of a strong-quark state in terms of current-quark states

$$|q_s,\vec{p},+,i\rangle=a^{\dagger(+)}_{s,i}(\vec{p})|0\rangle$$
$$=\{1+[f^{(1)}_i(\vec{p})]^2+[f^{(2)}_i(\vec{p})]^2\}^{1/2}\left[1+\frac{\gamma_i{}^2(\vec{p})}{\Gamma_i{}^2(\vec{p})}\tan^2\Gamma_i\right]^{-1/2}\left[a^{\dagger(+)}_{c,i}(\vec{p})-\frac{\gamma_i p_+}{\Gamma_i|p|}\tan\Gamma_i\,a^{\dagger(-)}_{c,i}(\vec{p})\right]V_i|0,i\rangle_c, \tag{4.31}$$

where, by Eq. (4.27′),

$$V_{\rm FW}|0,{\rm FW}\rangle_c=|0\rangle$$
$$=\prod_{\vec{k}}\{1+[f^{(1)}_{\rm FW}(\vec{k})]^2+[f^{(2)}_{\rm FW}(\vec{k})]^2\}^{-1}$$
$$\times\Big(1+f^{(1)}_{\rm FW}(\vec{k})[a^{\dagger(-)}_{c,{\rm FW}}(\vec{k})b^{\dagger(+)}_{c,{\rm FW}}(-\vec{k})-a^{\dagger(+)}_{c,{\rm FW}}(\vec{k})b^{\dagger(-)}_{c,{\rm FW}}(-\vec{k})]e^{2iEx_0}$$
$$+\frac{1}{|k_\perp|}f^{(2)}_{\rm FW}(\vec{k})[k_+a^{\dagger(-)}_{c,{\rm FW}}(\vec{k})b^{\dagger(-)}_{c,{\rm FW}}(-\vec{k})+k_-a^{\dagger(+)}_{c,{\rm FW}}(\vec{k})b^{\dagger(+)}_{c,{\rm FW}}(-\vec{k})]e^{2iEx_0}$$
$$-\{[f^{(1)}_{\rm FW}(\vec{k})]^2+[f^{(2)}_{\rm FW}(\vec{k})]^2\}a^{\dagger(-)}_{c,{\rm FW}}(\vec{k})b^{\dagger(+)}_{c,{\rm FW}}(-\vec{k})a^{\dagger(+)}_{c,{\rm FW}}(\vec{k})b^{\dagger(-)}_{c,{\rm FW}}(-\vec{k})e^{4iEx_0}\Big)|0,{\rm FW}\rangle_c, \tag{4.32}$$

and

$$V_M|0,M\rangle_c = |0\rangle$$

$$= \prod_{\vec{k}} \{1+[f_M^{(1)}(\vec{k})]^2+[f_M^{(2)}(\vec{k})]^2\}^{-1}\left[1+\frac{\gamma_M{}^2(\vec{k})}{\Gamma_M{}^2(\vec{k})}\tan^2\Gamma_M(\vec{k})\right]^{-1}$$

$$\times\Big\{1+\frac{\omega\gamma_M(\vec{k})}{E\,\Gamma_M(\vec{k})}\tan^2\Gamma_M(\vec{k})[a_{c,M}^{\dagger(-)}(\vec{k})b_{c,M}^{\dagger(+)}(-\vec{k})-a_{c,M}^{\dagger(+)}(\vec{k})b_{c,M}^{\dagger(-)}(-\vec{k})]\,e^{2iEx_0}$$

$$-\frac{\omega}{E}\frac{1}{|k_\perp|}\tan^2\Gamma_M(\vec{k})[k_+a_{c,M}^{\dagger(-)}(\vec{k})b_{c,M}^{\dagger(-)}(-\vec{k})+k_-a_{c,M}^{\dagger(+)}(\vec{k})b_{c,M}^{\dagger(+)}(-\vec{k})]\,e^{2iEx_0}$$

$$-\frac{\omega^2}{E^2}\tan^2\Gamma_M(\vec{k})a_{c,M}^{\dagger(-)}(\vec{k})b_{c,M}^{\dagger(+)}(-\vec{k})a_{c,M}^{\dagger(+)}(\vec{k})b_{c,M}^{\dagger(-)}(-\vec{k})\,e^{4iEx_0}\Big\}\;|0,M\rangle_c\,. \tag{4.33}$$

The picture of a strong quark of momentum $\vec{p}$ as composed of current quarks, which emerges from Eq. (4.31), is that of a "leading particle" carrying the momentum $\vec{p}$ and a cloud of pairs, with momenta $\vec{k}$, $-\vec{k}$, clustering *about the origin* in momentum space (not about $\vec{p}$), with zero overlap on states containing a definite number of current quarks. This picture can be viewed from any frame, including the infinite-momentum frame, and is not conspicuously amenable to a parton interpretation, if only because the "parts" do not follow the "whole." The latter effect is due to the fact that the transformation is bilinear in the fields *and* commutes with the momentum; it could be remedied by changing one of these two properties. It is interesting to note that the noncovariant cutoff procedure not only cures the zero overlap problem but also forces the choice of a natural frame in which to view the strong quark ($\vec{p}=0$) where the current quark pairs cluster about the leading particle.

The probability $P(n_c,\vec{k},0_s,i)$ of finding in the vacuum, $|0\rangle$, n_c current quarks in the $\vec{k}$th mode and anything else in other modes can be easily found from Eqs. (4.32) and (4.33),

$$P(0_c,\vec{k},0_s,i)=\{1+[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}^{-2}\,, \tag{4.34}$$

$$P(1_c,\vec{k},0_s,i)=2\{[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}P(0_c,\vec{k},0_s,i)\,, \tag{4.35}$$

and

$$P(2_c,\vec{k},0_s,i)=\{[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}^2P(0_c,\vec{k},0_s,i)\,. \tag{4.36}$$

(Due to the exclusion principle, n_c cannot be larger than 2.) This enables us to determine the average number of current quarks of momentum $\vec{k}$ in the vacuum:

$$\langle N(c,\vec{k},0_s,i)\rangle=\Big\langle 0\Big|\sum_r a_{c,i}^{\dagger(r)}(\vec{k})a_{c,i}^{(r)}(\vec{k})\Big|0\Big\rangle$$

$$=\sum_{n_c} n_c P(n_c,\vec{k},0_s,i)$$

$$=2\{[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}$$

$$\times\{1+[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}^{-1}\,. \tag{4.37}$$

By a similar procedure, we can write down the probabilities $P(n_c,\vec{k},1_s,\vec{p},i)$ of finding in a strong quark $|q_s,\vec{p},+,i\rangle$ n_c current quarks in the $\vec{k}$th mode and anything else in other modes:

$$P(0_c,\vec{k},1_s,\vec{p},i)=(1-\delta_{\vec{p},\vec{k}})P(0_c,\vec{k},0_s,i)\,, \tag{4.38}$$

$$P(1_c,\vec{k},1_s,\vec{p},i)=\{1+[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}\delta_{\vec{p},\vec{k}}P(0_c,\vec{k},0_s,i)+(1-\delta_{\vec{p},\vec{k}})P(1_c,\vec{k},0_s,i)\,, \tag{4.39}$$

and

$$P(2_c,\vec{k},1_s,\vec{p},i)=\tfrac{1}{2}\{1+[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}\delta_{\vec{p},\vec{k}}P(1_c,\vec{k},0_s,i)+(1-\delta_{\vec{p},\vec{k}})P(2_c,\vec{k},0_s,i)\,, \tag{4.40}$$

yielding the average number of current quarks of momentum $\vec{k}$ in a strong quark

$$N(c,\vec{k},1_s,\vec{p},i)=\sum_{n_c}n_cP(n_c,\vec{k},1_s,\vec{p},i)$$

$$=\{1+[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}^{-1}\{\delta_{\vec{p},\vec{k}}+2[f_i^{(1)}(\vec{k})]^2+2[f_i^{(2)}(\vec{k})]^2\}\,. \tag{4.41}$$

The fluctuations D of the number distributions, $D=\langle N^2\rangle-\langle N\rangle^2$, are given by

$$D(c,\vec{k},0_s,i)=P(1_c,\vec{k},0_s,i) \tag{4.42}$$

and

$$D(c,\vec{k},1_s,\vec{p},i)=(1-\tfrac{1}{2}\delta_{\vec{p},\vec{k}})P(1_c,\vec{k},0_s,i). \tag{4.43}$$

The values of the quantities (4.34)–(4.37) and (4.41)–(4.43) for FW, Melosh, and UZ transformations are given in Table I. As expected from the form of the transformation, the average number of current quarks in a strong quark is infinite. While the distribution of the current quarks for the FW case is spherically symmetric in momentum space, the transverse direction is preferred for the current pairs of Melosh, with none of them moving in the z direction—another aspect making the parton interpretation difficult. Also, the high-momentum region is more populated by the pairs than the low-momentum one, with the maximum population reached at infinite momentum (infinite transverse momentum for Melosh and UZ).

Distributions related to matrix elements of Fock-space operators between a current and a strong state are meaningful only if Λ is finite. From Eq. (4.31), for example, we can read off the probability of finding a current quark of momentum $\vec{k}$ (and nothing else) in $|q_s,\vec{p},+,i\rangle$,

$$\sum_r |\langle q_c,\vec{k},r,i\,|q_s,\vec{p},+,i\rangle|^2=\{1+[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}\,\delta_{\vec{p},\vec{k}}\,|\langle 0|V_i|0\rangle|^2, \tag{4.44}$$

the probability of finding a current quark of momentum $\vec{k}$ and a current quark-antiquark pair of a given momentum $\vec{k}^{(1)}$, $-\vec{k}^{(1)}$,

$$\sum_{r,s,t} |{}_c\langle 0,i\,a_{c,i}^{(r)}(\vec{k})a_{c,i}^{(s)}(\vec{k}^{(1)})b_{c,i}^{(t)}(-\vec{k}^{(1)})|q_s,\vec{p},+,i\rangle|^2$$

$$=\{1+[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}\,\delta_{\vec{p},\vec{k}}\,2\{[f_i^{(1)}(\vec{k}^{(1)})]^2+[f_i^{(2)}(\vec{k}^{(1)})]^2\}(1-\tfrac{1}{2}\delta_{\vec{k},\vec{k}^{(1)}})|\langle 0|V_i|0\rangle|^2, \tag{4.45}$$

or the probability that $|q_s,\vec{p},+,i\rangle$ will contain a current quark of momentum $\vec{k}$ accompanied by two pairs of momentum $\vec{k}^{(1)}$,

$$\sum_{r,s,t,m,n} |{}_c\langle 0,i\,|a_{c,i}^{(r)}(\vec{k})a_{c,i}^{(s)}(\vec{k}^{(1)})b_{c,i}^{(t)}(-\vec{k}^{(1)})a_{c,i}^{(m)}(\vec{k}^{(1)})b_{c,i}^{(n)}(-\vec{k}^{(1)})|q_s,\vec{p},+,i\rangle|^2$$

$$=\{1+[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}\,\delta_{\vec{p},\vec{k}}\{[f_i^{(1)}(\vec{k}^{(1)})]^2+[f_i^{(2)}(\vec{k}^{(1)})]^2\}^2(1-\delta_{\vec{k},\vec{k}^{(1)}})|\langle 0|V_i|0\rangle|^2. \tag{4.46}$$

TABLE I. Pair distributions as a function of momentum.

Quantity \ i	FW	M	UZ
$P(0_c,\vec{k},0_s,i)$	$\left(\frac{E+M}{2E}\right)^2$	$\left(\frac{\omega(\omega+M)}{2E^2}+\frac{k_z^2}{E^2}\right)^2$	$\left(\frac{E+\|k_z\|}{2E}\right)^2$
$P(1_c,\vec{k},0_s,i)$	$\frac{\|k\|^2}{2E^2}$	$\frac{\|k_\perp\|^2\omega}{E^2(\omega+M)}\left(\frac{\omega(\omega+M)}{2E^2}+\frac{k_z^2}{E^2}\right)$	$\frac{\omega^2}{2E^2}$
$P(2_c,\vec{k},0_s,i)$	$\left(\frac{\|k\|^2}{2E(E+M)}\right)^2$	$\left(\frac{\|k_\perp\|^2\omega}{2E^2(\omega+M)}\right)^2$	$\frac{\omega^2}{2E(E+\|k_z\|)}$
$\langle N(c,\vec{k},0_s,i)\rangle$	$1-\frac{M}{E}$	$\frac{\|k_\perp\|^2\omega}{E^2(\omega+M)}$	$1-\frac{\|k_z\|}{E}$
$D(c,\vec{k},0_s,i)$	$\frac{\|k\|^2}{2E^2}$	$\frac{\|k_\perp\|^2\omega}{E^2(\omega+M)}\left(\frac{\omega(\omega+M)}{2E^2}+\frac{k_z^2}{E^2}\right)$	$\frac{\omega^2}{2E^2}$
$\langle N(c,\vec{k},1_s,\vec{p},i)\rangle$	$\frac{E+M}{2E}\left(\delta_{\vec{k},\vec{p}}+2\frac{\|k\|^2}{(E+M)^2}\right)$	$\left(\frac{\omega(\omega+M)}{2E^2}+\frac{k_z^2}{E^2}\right)\left(\delta_{\vec{k},\vec{p}}+\frac{\|k_\perp\|^2\omega}{E^2(\omega+M)}\right)$	$\frac{E+\|k_z\|}{2E}\left(\delta_{\vec{k},\vec{p}}+\frac{2\omega^2}{(E+\|k_z\|)^2}\right)$
$D(c,\vec{k},1_s,\vec{p},i)$	$\frac{\|k\|^2}{2E^2}(1-\frac{1}{2}\delta_{\vec{k},\vec{p}})$	$\frac{\|k_\perp\|^2\omega}{E^2(\omega+M)}\left(\frac{\omega(\omega+M)}{2E^2}+\frac{k_z^2}{E^2}\right)(1-\frac{1}{2}\delta_{\vec{k},\vec{p}})$	$\frac{\omega^2}{2E^2}(1-\frac{1}{2}\delta_{\vec{k},\vec{p}})$

In general there is an additional multiplicative factor of

$$2\{[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}$$

for each current quark pair in the $\vec{k}$th mode and a factor

$$\{[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}^2$$

for each double pair in the $\vec{k}$th mode. The above quoted probability amplitudes strictly vanish unless a cutoff keeps the common factor

$$\langle 0|V_i|0\rangle=\prod_{\vec{k}}\{1+[f_i^{(1)}(\vec{k})]^2+[f_i^{(2)}(\vec{k})]^2\}^{-1}$$

from vanishing; they can be interpreted as relative probabilities, all of which vanish as $\Lambda\to\infty$, but with well-defined ratios.

V. SUMMARY AND DISCUSSION

We have discussed a class of operators V_i which lead to an exact SU(6) symmetry in the free-quark model. While not unitary on Fock space, they have a simple structure when discussed in terms of a larger nonseparable infinite tensor-product space, where they map, in an isometric but unitarily inequivalent way, one separable Hilbert space onto another separable Hilbert space, with image space orthogonal to object space. Nonetheless, Fock-space operators have well-defined matrix elements between members of the same equivalence class, or, expressed differently, V_i-transformed Fock-space operators remain well defined in Fock space. As a result, we found a finite nonvanishing mean number of current quarks of a given momentum in a strong state. We see no reason why such transformations should be proscribed even though they are technically nonunitary on a separable space; they appear ideally suited for producing pair distributions, which we have discussed in detail and whose possible relevance to parton-model pictures we have speculated on. It is, however, imperative to recognize that V_i-transformed Fock-space states are orthogonal to all Fock-space states (i.e., current quarks are orthogonal to strong quarks) unless a momentum cutoff limits the degrees of freedom to a finite number. Such a cutoff, which seems characteristic of most parton models,[16] may arise naturally in an interacting theory.

It is interesting that the distributions we have found show such a rich structure and complexity. It therefore seemed useful to us to seek a parton interpretation despite the fact that the V_i, which we studied in the equal-time formulation were found by requiring the transforms to induce symmetries in the free-quark model and need not have properties more general than the limited framework from which they sprang.

In our formulation (we define $|q_c\rangle=V^\dagger|q_s\rangle$, which is equivalent to diagonalizing F^3 and $V_i^\dagger H V_i$ simultaneously) the spin-averaged distributions (see Table I) depend solely on the form of $H_i=V_i^\dagger H V_i$ and are not sensitive to the arbitrariness inherent in V_i, which was discussed in Sec. II. As a result, the distributions produced by the transformation of Gomberoff, Horwitz, and Ne'eman are the same as those of FW. No pairs are created in momentum modes not affected by the transformation; pairs are produced "in proportion" to the difference between the forms of H and H_i within a given momentum mode. Thus, for example, there are no zero-momentum pairs but many infinite momentum pairs in $V_{\rm FW}|0\rangle$, no pairs moving in the z direction in $V_M|0\rangle$ but many moving in the transverse direction, and no $p_z\to\infty$ pairs in $V_{UZ}|0\rangle$. As a further consequence, none of the distributions have a transverse-momentum falloff. Even though a falloff could be produced by a transformation which leaves large transverse momenta unaffected, such a transformation would not result in a transformed Hamiltonian commuting with F_a^α and would therefore not lead to an exact symmetry.

The complete hadronic distribution would involve not only the distribution of current quarks in a constituent quark but also the distribution of constituent quarks in a hadron, that is, the naive quark model wave function. This kind of convolution has been carried out by Altarelli, Cabibbo, Maiani, and Petronzio[17] in a different context. It is clear, however, that this would not change our conclusions since the number of current quarks in a hadron would still be increasing with transverse momentum whatever the strong wave function or longitudinal momentum frame.

For these and other reasons cited in the text, the distributions we have calculated do not readily lend themselves to a parton interpretation. This is perhaps not surprising considering the lack of dynamics in the free-quark model. The effect of interactions, necessary for quark binding, on V_i and corresponding distributions can be answered only if a more complete theory is at hand. We entertained the possibility that the true distribution might be realistically approximated by the free quark model V_i, with interactions built only into the strong wave function; as discussed above this does not appear to be the case. Whether other means exist of imposing a dynamical input on these transforms in the context of the formalism we have developed, and whether the resulting distributions would prove phenomenologically useful remains an open and interesting question.

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APPENDIX

As mentioned in Sec. IV, all the transformations we analyze can be written in the form

$$\exp[C^\dagger(\alpha)+D^\dagger(\beta)-C(\alpha)-D(\beta)+iF(\gamma)] \tag{A1}$$

with all symbols defined in Eqs. (4.15) and (4.16). If f_1,f_2,f_3,f_4,f_5,f_6 are arbitrary functions of momenta, the algebra of the set of operators $C^\dagger(f_1)$, $D^\dagger(f_2)$, $C(f_3)$, $D(f_4)$, $F(f_5)$ enlarged by $K(f_6)$, $K(fg)\equiv[C(f),C^\dagger(g)]$, closes and the commutation relations are as follows:

$$[C(f),D(g)]=[C^\dagger(f),D^\dagger(g)]=0\,, \tag{A2}$$

$$[C(f),C^\dagger(g)]=[D(f),D^\dagger(g)]=K(fg)\,, \tag{A3}$$

$$[K(f),C(g)]=2C(fg)\,, \tag{A4}$$

$$[K(f),C^\dagger(g)]=-2C^\dagger(fg)\,, \tag{A5}$$

$$[K(f),D(g)]=2D(fg)\,, \tag{A6}$$

$$[K(f),D^\dagger(g)]=-2D^\dagger(fg)\,, \tag{A7}$$

$$[D(f),C^\dagger(g)]=iF(fg)\,, \tag{A8}$$

$$[C(f),D^\dagger(g)]=-iF(fg)\,, \tag{A9}$$

$$[F(f),C^\dagger(g)]=-2iD^\dagger(fg)\,, \tag{A10}$$

$$[F(f),C(g)]=-2iD(fg)\,, \tag{A11}$$

$$[F(f),D^\dagger(g)]=2iC^\dagger(fg)\,, \tag{A12}$$

$$[F(f),D(g)]=2iC(fg)\,, \tag{A13}$$

$$[K(f),F(g)]=0\,. \tag{A14}$$

The operators C, D, and F annihilate the vacuum, $C^\dagger$ and $D^\dagger$ do not. K contains a c-number part and a part which annihilates the vacuum. We assert that Eq. (A1) can be written in an ordered form

$$\exp[C^\dagger(\alpha)+D^\dagger(\beta)-C(\alpha)-D(\beta)+iF(\gamma)]=\exp[C^\dagger(f_1)+D^\dagger(f_2)]\exp[C(g_1)+D(g_2)]\exp[iF(h)]\exp[K(m)]\,. \tag{A15}$$

This is a generalization of the well-known identity $e^{A+B}=e^Ae^Be^{-[A,B]/2}$, which holds only if both A and B commute with $[A,B]$. In order to relate the functions f, g, h, and m to the given functions α, β, and γ, we introduce a real parameter λ and define

$$\begin{aligned}G(\lambda)&=\exp[K(m)]\\&=\exp[-iF(h)]\exp[-C(g_1)-D(g_2)]\exp[-C^\dagger(f_1)-D^\dagger(f_2)]\exp\{\lambda[C^\dagger(\alpha)+D^\dagger(\beta)-C(\alpha)-D(\beta)+iF(\gamma)]\}\,;\end{aligned} \tag{A16}$$

f, g, h, and m are now functions of both the momentum and λ. Then

$$\begin{aligned}G'(\lambda)&=\frac{d}{d\lambda}G(\lambda)\\&=K(m')G(\lambda)\\&=\{-iF(h')-\exp[-iF(h)][C(g_1'+D(g_2')]\exp[iF(h)]\\&\quad-\exp[-iF(h)]\exp[-C(g_1)-D(g_2)][C^\dagger(f_1')+D^\dagger(f_2')]\exp[C(g_1)+D(g_2)]\exp[iF(h)]\\&\quad+\exp[-iF(h)]\exp[-C(g_1)-D(g_2)]\exp[-C^\dagger(g_1)-D^\dagger(g_2)][C^\dagger(\alpha)+D^\dagger(\beta)-C(\alpha)-D(\beta)+iF(\gamma)]\\&\qquad\times\exp[C^\dagger(f_1)+D^\dagger(f_2)]\exp[C(g_1)+D(g_2)]\exp[iF(h)]\}G(\lambda)\\&=(C\{[G_1-g_1'+g_1(f_1'g_1+f_2'g_2)-g_2(f_1'g_2-f_2'g_1)]\cos 2h\\&\qquad+[G_2-g_2'+g_2(f_1'g_1+f_2'g_2)+g_1(f_1'g_2-f_2'g_1)]\sin 2h\}\\&\quad+D\{[G_2-g_2'+g_2(f_1'g_1+f_2'g_2)+g_1(f_1'g_2-f_2'g_1)]\cos 2h\\&\qquad-[G_1-g_1'+g_1(f_1'g_1+f_2'g_2)-g_2(f_1'g_2-f_2'g_1)]\sin 2h\}\\&\quad+C^\dagger[(F_1-f_1')\cos 2h+(F_2-f_2')\sin 2h]+D^\dagger[(F_2-f_2')\cos 2h-(F_1-f_1')\sin 2h]\\&\quad+iF(H-h'+f_1'g_2-f_2'g_1)+K(M+f_1'g_1+f_2'g_2))\,G(\lambda)\,,\end{aligned} \tag{A17}$$

where the prime stands for the derivative with respect to the variable λ, and

$$F_1=\alpha+2f_2\gamma+f_1(f_1\alpha+f_2\beta)+f_2(f_1\beta-f_2\alpha)\,,$$

$$F_2=\beta+2f_1\gamma+f_2(f_1\alpha+f_2\beta)-f_1(f_1\beta-f_2\alpha)\,,$$

$$\begin{aligned}G_1=-\alpha-g_1[&(g_1\alpha+g_2\beta)+2(f_1\alpha+f_2\beta)]\\&-g_2[(2\gamma+g_1\beta-g_2\alpha)-2(f_1\beta-f_2\alpha)]\\&-(f_1\alpha+f_2\beta)[g_1(f_1g_1+f_2g_2)-g_2(f_1g_2-f_2g_1)]\\&-(2\gamma-f_1\beta+f_2\alpha)[g_1(f_1g_2-f_2g_1)+g_2(f_1g_1+f_2g_2)]\,,\end{aligned}$$

$$\begin{aligned}G_2=-\beta+g_1[&(2\gamma+g_1\beta-g_2\alpha)-2(f_1\beta-f_2\alpha)]\\&-g_2[(g_1\alpha+g_2\beta)+2(f_1\alpha+f_2\beta)]\\&-(f_1\alpha+f_2\beta)[g_2(f_1g_1+f_2g_2)+g_1(f_1g_2-f_2g_1)]\\&+(2\gamma-f_1\beta+f_2\alpha)[g_1(f_1g_1+f_2g_2)-g_2(f_1g_2-f_2g_1)]\,,\end{aligned}$$

$$\begin{aligned}H=\gamma&-(f_1\beta-f_2\alpha)+(g_1\beta-g_2\alpha)\\&-(f_1\alpha+f_2\beta)(f_1g_2-f_2g_1)\\&+(2\gamma-f_1\beta+f_2\alpha)(f_1g_1+f_2g_2)\,,\end{aligned}$$

$$\begin{aligned}M=&-(f_1\alpha+f_2\beta)-(g_1\alpha+g_2\beta)\\&-(f_1\alpha+f_2\beta)(f_1g_1+f_2g_2)\\&+(2\gamma-f_1\beta+f_2\alpha)(f_1g_2-f_2g_1)\,.\end{aligned}$$

Because of the linear independence of the operators C, D, $C^\dagger$, $D^\dagger$, F, and K, this leads to six linear differential equations of the first order,

$$G_1-g_1'+g_1(f_1'g_1+f_2'g_2)-g_2(f_1'g_2-f_2'g_1)=0\,, \tag{A18}$$

$$G_2-g_2'+g_2(f_1'g_1+f_2'g_2)+g_1(f_1'g_2-f_2'g_1)=0\,, \tag{A19}$$

$$F_1-f_1'=0\,, \tag{A20}$$

$$F_2-f_2'=0\,, \tag{A21}$$

$$H-h'+f_1'g_2-f_2'g_1=0 \tag{A22}$$

and

$$M-m'+f_1'g_1+f_2'g_2=0\,, \tag{A23}$$

which have a unique solution if the six boundary conditions at $\lambda=0$

$$f_1(0)=f_2(0)=g_1(0)=g_2(0)=h(0)=m(0)=0 \tag{A24}$$

are imposed. The solutions are

$$f_1=(\Gamma^2+\gamma^2\tan^2\lambda\Gamma)^{-1}(\alpha\Gamma-\beta\gamma\tan\lambda\Gamma)\tan\lambda\Gamma\,, \tag{A25}$$

$$f_2=(\Gamma^2+\gamma^2\tan^2\lambda\Gamma)^{-1}(\beta\Gamma+\alpha\gamma\tan\lambda\Gamma)\tan\lambda\Gamma\,, \tag{A26}$$

$$g_1=-\Gamma^{-2}(1+\tan^2\lambda\Gamma)^{-1}(\alpha\Gamma-\beta\gamma\tan\lambda\Gamma)\tan\lambda\Gamma\,, \tag{A27}$$

$$g_2=-\Gamma^{-2}(1+\tan^2\lambda\Gamma)^{-1}(\beta\Gamma+\alpha\gamma\tan\lambda\Gamma)\tan\lambda\Gamma\,, \tag{A28}$$

$$h=\arctan\left(\frac{\gamma}{\Gamma}\tan\lambda\Gamma\right), \tag{A29}$$

and

$$\begin{aligned}m&=\tfrac{1}{2}\ln[\Gamma^{-2}(1+\tan^2\lambda\Gamma)^{-1}(\Gamma^2+\gamma^2\tan^2\lambda\Gamma)]\\&=-\tfrac{1}{2}\ln(1+f_1^2+f_2^2)\,,\end{aligned} \tag{A30}$$

where $\Gamma=(\alpha^2+\beta^2+\gamma^2)$. Equation (A16) now holds for all values of λ, but we use it only at $\lambda=1$ to obtain Eq. (A15).

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