(1957); M. T. Vaughan, R. Aaron, and R. D. Amado, Phys. Rev. 124, 1258 (1961); A. Salam, Nuovo Cimento 25, 224 (1962); S. Weinberg, Phys. Rev. 130, 776 (1963); D. Luriė and A. Macfarlane, Phys. Rev. 136, B816 (1964).

[16]S. Weinberg, Phys. Rev. 137, B672 (1965).

[17]G. B. West, Phys. Rev. Lett. 27, 762 (1971).

[18]Although viewing from the surface the discovery that gauge particles Reggeize seems to add weight to the contention that gauge fields are composite, we must caution any direct comparison of our work with the result obtained in Ref. 8: (1) We work only in the context of asymptotically free gauge theories; the theories discussed in Ref. 8 are not asymptotically free. (2) In asymptotically free theories not all gauge particles can be massive (at least in the framework of perturbation theory); hence the Born amplitudes of which the factorizability was examined in Ref. 8 are not well defined.

[19]P. E. Kaus and F. Zachariasen, Phys. Rev. 138, B1304 (1965).

[20]See, for example, R. Jackiw and K. Johnson, Phys. Rev. D 8, 2386 (1973); J. Cornwall and R. Norton, *ibid.* 8, 3338 (1973).

[21]Following the line of arguments presented in the last paragraph, we restrict our considerations to those theories without any scalar fields.

[22]This conclusion differs from those of Ref. 10.

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Asymptotic behavior of two-photon exchange in massive quantum electrodynamics*

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The Bethe-Salpeter equation is used in conjunction with the Froissart bound (assumed true off the mass shell) to put restrictions on two-photon exchange. Implications are discussed.

I. INTRODUCTION

We wish to study here the Regge asymptotic behavior of the sum of all two-photon reducible Feynman diagrams in massive quantum electrodynamics. This is motivated by the "tower" graph calculations of Frolov, Gribov, and Lipatov[1] and also Cheng and Wu.[2] This paper is an extension of previous work on π-π scatteting.[3] It is found in Refs. 1 and 2 that the sum of all "tower" graphs violates the Froissart bound by a power for all nonzero values of the coupling constant α. We ask here if it is likely that the sum of all crossed-channel, two-photon reducible graphs violates the Froissart bound in this manner. We feel that this question is important because of the following:

1. Massive quantum electrodynamics is a likely candidate for a quark-gluon field theory.

2. The fact that the tower graphs violate the Froissart bound by a power is essential to obtain Froissart-bound saturation in the Cheng-Wu eikonal model.[2]

3. If it seems probable or desirable that the full two-photon exchange amplitude does not violate the Froissart bound by a power, then a search for two-photon reducible graphs which cancel the leading behavior of the tower graphs is in order.

We begin with the Bethe-Salpeter equations for elastic γ-γ, e-γ, and e-e scattering. We then rewrite these equations in a particular way and make an assumption about the off-mass-shell behavior of the full amplitudes. We also make an assumption about certain moments of the Bethe-Salpeter (BS) equation. We find that these assumptions rule out the possibility that the sum of all two-photon reducible graphs violates the Froissart bound by a power. We first study γ-γ scattering which is the simplest case and then work our way up to γ-e and e-e scattering.

II. γ-γ SCATTERING

The invariant amplitudes for γ-γ scattering we write as

$$T(q,P,P',\lambda_1,\lambda_2,\lambda_3,\lambda_4)=T^{\mu\nu;\alpha\beta}(q,P,P')\epsilon_\mu(P+\tfrac{1}{2}q,\lambda_1)\epsilon_\nu(P-\tfrac{1}{2}q,\lambda_2)\epsilon_\alpha(P'-\tfrac{1}{2}q,\lambda_3)\epsilon_\beta(P'+\tfrac{1}{2}q,\lambda_4)\,, \tag{2.1}$$

where $\lambda_1, \ldots, \lambda_4$ are the polarization states of the external photons (see Fig. 1). We use the metric

$$A\cdot B = A_0 B_0 - \vec{A}\cdot\vec{B}. \tag{2.2}$$

Our normalization is such that for spin-nonflip scattering the optical theorem reads

$$\mathrm{Im}\, T(0,P,P') = 2|\vec{K}|\sqrt{s}\,\sigma_{\mathrm{tot}}, \tag{2.3}$$

where $|\vec{K}|$ is the c.m. momentum of one photon. We choose our polarization vectors to be real on the mass shell and to satisfy

$$\sum_{i=1}^{3} \epsilon_\mu(K,\lambda_i)\epsilon_\nu(K,\lambda_i) = -g_{\mu\nu} + \frac{K_\mu K_\nu}{K^2}. \tag{2.4}$$

It follows from gauge invariance, at least to any finite order in perturbation theory, that[4]

$$\begin{aligned}(P+\tfrac{1}{2}q)_\mu T^{\mu\nu;\alpha\beta}(q,P,P') &= (P-\tfrac{1}{2}q)_\nu T^{\mu\nu;\alpha\beta}(q,P,P')\\ &= (P'-\tfrac{1}{2}q)_\alpha T^{\mu\nu;\alpha\beta}(q,P,P')\\ &= (P'+\tfrac{1}{2}q)_\beta T^{\mu\nu;\alpha\beta}(q,P,P')\\ &= 0, \end{aligned} \tag{2.5}$$

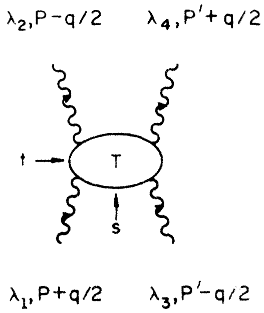

FIG. 1. Definition of the arguments of $T(q,P,P';\lambda_1,\lambda_2,\lambda_3,\lambda_4)$.

for all q, P, and P'. We assume that this result is correct for the true solution to the field equations. Let us now introduce the full two-photon irreducible kernel through the Bethe-Salpeter equation[5] for this process:

$$T^{\mu\nu;\alpha\beta}(q,P,P') = I^{\mu\nu;\alpha\beta}(q,P,P') - \frac{i}{2}\int \frac{d^4K'}{(2\pi)^4} I^{\mu\nu;\lambda\delta}(q,P,K')T^{\lambda'\delta';\alpha\beta}(q,-K',P')D_{\lambda\lambda'}(K'-\tfrac{1}{2}q)D_{\delta\delta'}(K'+\tfrac{1}{2}q). \tag{2.6}$$

Because the full amplitudes T are transverse as in (2.5), we are free to choose the full photon propagator $D_{\lambda\lambda'}$ transverse:

$$D_{\lambda\lambda'}(K) \to D^T_{\lambda\lambda'}(K) = \left(-g_{\mu\nu} + \frac{K_\mu K_\nu}{K^2}\right) D^T(K^2). \tag{2.7}$$

The photon propagator appearing above is renormalized to behave as

$$\frac{1}{K^2 - m_\gamma^2}\left(-g_{\mu\nu} + \frac{K_\mu K_\nu}{K^2}\right)$$

near the physical photon mass m_γ. Making the replacement (2.7) in (2.6) yields

$$\begin{aligned} T^{\mu\nu;\alpha\beta}(q,P,P') = I^{\mu\nu;\alpha\beta}(q,P,P') - \frac{i}{2}\int \frac{d^4K'}{(2\pi)^4} I^{\mu\nu;\lambda\delta}(q,P,K')T^{\lambda'\delta';\alpha\beta}(q,-K',P')\\ \times\left(-g_{\lambda\lambda'} + \frac{(K'-\frac{1}{2}q)_\lambda (K'-\frac{1}{2}q)_{\lambda'}}{(K'-\frac{1}{2}q)^2}\right)\left(-g_{\delta\delta'} + \frac{(K'+\frac{1}{2}q)_\delta (K'+\frac{1}{2}q)_{\delta'}}{(K'+\frac{1}{2}q)^2}\right)\\ \times D^T((K'-\tfrac{1}{2}q)^2)\,D^T((K'+\tfrac{1}{2}q)^2). \end{aligned} \tag{2.8}$$

It follows from (2.8) and (2.5) that the irreducible kernel is also transverse:

$$\begin{aligned}(P+\tfrac{1}{2}q)_\mu I^{\mu\nu;\alpha\beta}(q,P,P') &= (P-\tfrac{1}{2}q)_\nu I^{\mu\nu;\alpha\beta}(q,P,P')\\ &= (P'-\tfrac{1}{2}q)_\alpha I^{\mu\nu;\alpha\beta}(q,P,P')\\ &= (P'+\tfrac{1}{2}q)_\beta I^{\mu\nu;\alpha\beta}(q,P,P')\\ &= 0. \end{aligned} \tag{2.9}$$

The values of the tensor $T^{\mu\nu;\alpha\beta}(q,P,P')$ are uniquely determined for all values of the external momenta in perturbation theory. No ambiguity appears when one goes off the mass shell. The irreducible kernel $I^{\mu\nu;\alpha\beta}$ is also uniquely determined.

The normalization which we have chosen for the polarization vectors ϵ^μ in (2.4) need only be true for on-shell photons. We choose this normalization even for virtual photons. This we do for convenience since it enables us to write the Bethe-Salpeter equation in terms of invariant amplitudes only. The longitudinal polarization vector can be chosen to be

$$\epsilon(q,\lambda_L) = \frac{1}{\sqrt{q^2}}(|\vec{q}|, 0, 0, q_0),$$

with the z axis in the direction of $\vec{q}$. Note that for spacelike q there is an ambiguity in the sign of $\epsilon(q,\lambda_L)$ due to the $\sqrt{q^2}$ factor. The transverse-polarization states do not have this ambiguity. Let us therefore choose a definite sign for $\sqrt{q^2}$ in $\epsilon(q,\lambda_L)$ in the following. The particular sign we choose is irrelevant so long as we are consistent.

Equation (2.8) can be rewritten, using (2.1) and (2.4), as

$$T(q,P,P';\lambda_1,\lambda_2,\lambda_3,\lambda_4)=I(q,P,P';\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\frac{i}{2}\int\frac{d^4K'}{(2\pi)^4}\sum_{\lambda,\lambda'}I(q,P,K';\lambda_1,\lambda_2,\lambda,\lambda')\times T(q,-K',P';\lambda,\lambda',\lambda_3,\lambda_4)\times D^T((K'-\tfrac{1}{2}q)^2)D^T((K'+\tfrac{1}{2}q)^2)\,. \tag{2.10}$$

We now take the absorptive part of (2.10) in the variable $(P+P')^2$. We use the Cutkosky rules.[6] This is illustrated in Fig. 2. A dashed line through a graph represents the absorptive part of that graph with support only over positive values of the mass^2 cut by the dashed line. With this definition one must sum two identical contributions for different routing of the Cutkosky cut. Equation (2.10) then becomes

$$\mathrm{Abs}\,T(q,P,P';\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\mathrm{Abs}\,I(q,P,P';\lambda_1,\lambda_2,\lambda_3,\lambda_4)+\sum_{\lambda,\lambda'}\int\frac{d^4K'}{(2\pi)^4}\mathrm{Abs}\,I(q,P,K';\lambda_1,\lambda_2,\lambda,\lambda')\mathrm{Abs}\,T(q,-K',P';\lambda,\lambda',\lambda_3,\lambda_4)\times D^T((K'+\tfrac{1}{2}q)^2)D^T((K'-\tfrac{1}{2}q)^2)\,\theta((P+K')^2)\,\theta((P'-K')^2)\,. \tag{2.11}$$

The normalization of the above absorptive parts is defined by the forward, elastic, spin-nonflip, on-shell relation .

$$\mathrm{Abs}\,T=2\,\mathrm{Im}\,T\,. \tag{2.12}$$

It is convenient to work with (2.11) rather than (2.10) since the absorbed equation is guaranteed to converge under simple iteration (assuming that no zero mass bound states appear in the final solution). In arriving at (2.11) we have assumed that the external photons are stable, and we restrict their masses to values below the threshold for inelastic photon decay.

It is well known that (2.11) diagonalizes in the proper "angular momentum" variable.[7] This formalism is elegant and powerful, but for our purposes it is sufficient and simpler to work with (2.11) in momentum space. Let us condense our notation by introducing the operation $\times_t$ defined by

$$a\times_t b=\sum_{\delta,\delta'}\int\frac{d^4K'}{(2\pi)^4}\,a(q,P,K';\lambda_1,\lambda_2,\delta,\delta')b(q,-K',P';\delta,\delta',\lambda_3,\lambda_4)\times D^T((K'+\tfrac{1}{2}q)^2)D^T((K'-\tfrac{1}{2}q)^2)\,\theta((K'+P)^2)\,\theta((P'-K')^2)\,, \tag{2.13}$$

with

$$a\times_t a=a^2 \tag{2.14}$$

and

$$a^{n-1}\times_t a=a\times_t a^{n-1}=a^n\,. \tag{2.15}$$

Equation (2.10) becomes

$$\mathrm{Abs}\,T=\mathrm{Abs}\,I+\mathrm{Abs}\,I\times_t\mathrm{Abs}\,T\,, \tag{2.16}$$

which is solved by iteration yielding

$$\mathrm{Abs}\,T=\sum_{n=1}^{\infty}(\mathrm{Abs}\,I)^n\,. \tag{2.17}$$

Let us now introduce the following functions which are crucial to the development:

$$\mathrm{Abs}\,T(\xi)=\sum_{n=1}^{\infty}\xi^n(\mathrm{Abs}\,I)^n\,. \tag{2.18}$$

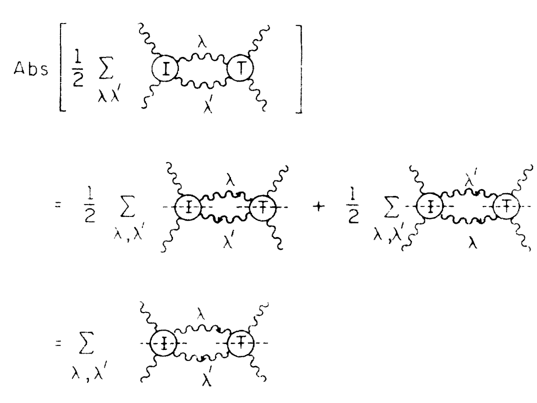

FIG. 2. The Cutkosky rules.

We also define

$$\mathrm{Abs}\,T_2(\xi)=\sum_{n=2}^{\infty}\xi^n(\mathrm{Abs}\,I)^n\,. \tag{2.19}$$

The following relation may be verified by substitution:

$$\left(\xi\frac{d}{d\xi}-1\right)\mathrm{Abs}\,T(\xi)=\left(\xi\frac{d}{d\xi}-1\right)\mathrm{Abs}\,T_2(\xi)=\mathrm{Abs}\,T(\xi)\times_t\mathrm{Abs}\,T(\xi) \tag{2.20}$$

From this follows the following Taylor's expansion for $\mathrm{Abs}\,T(\xi)$:

$$\mathrm{Abs}\,T(\xi)=\mathrm{Abs}\,T(\xi=1)+\sum_{n=1}^{\infty}(\xi-1)^n\{[\mathrm{Abs}\,T(\xi=1)]^n+[\mathrm{Abs}\,T(\xi=1)]^{n+1}\}\,. \tag{2.21}$$

Written out in full Eq. (2.20) is

$$\left(\xi\frac{d}{d\xi}-1\right)\mathrm{Abs}\,T_2(q,P,P',\lambda_1,\lambda_2,\lambda_3,\lambda_4;\xi)=\sum_{\lambda,\lambda'}\int\frac{d^4K'}{(2\pi)^4}\mathrm{Abs}\,T(q,P,K',\lambda_1,\lambda_2,\lambda,\lambda';\xi)\mathrm{Abs}\,T(q,-K',P',\lambda,\lambda',\lambda_3,\lambda_4;\xi)$$
$$\times D^T((K'+\tfrac{1}{2}q)^2)\,D^T((K'-\tfrac{1}{2}q)^2)\,\theta((P+K')^2)\,\theta((P'-K')^2)\,. \tag{2.22}$$

We must now evaluate integrals of the form $[\mathrm{Abs}\,T(\xi=1)]^n$. In order to do this we note the following. $\mathrm{Abs}\,T(\xi=1)$ on the mass shell corresponds to a physical scattering amplitude. Since we have no zero-mass particles in our picture (photons are massive) the Froissart bound[8] must be satisfied by these amplitudes for all photon polarizations. It is our conjecture, and we feel that it is physically reasonable, that the Froissart bound is also satisfied off shell by these amplitudes with the masses held fixed and $s\to\infty$. We also conjecture that the virtual photon masses in (2.22), $(K'+\frac{1}{2}q)^2$ and $(K'-\frac{1}{2}q)^2$, can be taken as soft.

With these assumptions let us consider the integral illustrated in Fig. 3, with

$$s_0=(P+K)^2,\quad s'=(P'-K')^2,\quad s=(P+P')^2\,. \tag{2.23}$$

We choose our coordinate system so that

$$(P+\tfrac{1}{2}q)=(P_0+\tfrac{1}{2}q_0,0,0,P_z+\tfrac{1}{2}q_z),\quad \vec{P}_\perp=-\tfrac{1}{2}\vec{q}_\perp\,, \tag{2.24}$$

$$(P'-\tfrac{1}{2}q)=(P_0'-\tfrac{1}{2}q_0,0,0,P_z'-\tfrac{1}{2}q_z),\quad \vec{P}_\perp'=\tfrac{1}{2}\vec{q}_\perp\,, \tag{2.25}$$

with $P_z+\frac{1}{2}q_z=-(P_z'-\frac{1}{2}q_z)>0$. We define light-cone variables by

$$P_+=P_0+P_z\,,\quad P_-=P_0-P_z\,. \tag{2.26}$$

Ignoring spin indices, the integral of Fig. 3 is

$$a\times_t b=\frac{1}{2}\int\frac{dK_+dK_-d^2K_\perp}{(2\pi)^4}a(q,P,K)b(q,-K,P')D^T((K-\tfrac{1}{2}q)^2)\,D^T((K+\tfrac{1}{2}q)^2)\,\theta((P+K)^2)\,\theta((P'-K)^2)\,. \tag{2.27}$$

Let us define

$$u_1=(P+\tfrac{1}{2}q)^2,\quad u_2=(P-\tfrac{1}{2}q)^2\,, \tag{2.28}$$

$$u_3=(P'-\tfrac{1}{2}q)^2,\quad u_4=(P'+\tfrac{1}{2}q)^2\,, \tag{2.29}$$

$$u_1'=(K+\tfrac{1}{2}q)^2,\quad u_2'=(K-\tfrac{1}{2}q)^2\,. \tag{2.30}$$

One finds

$$s_0=\tfrac{1}{2}(u_1+u_2-\tfrac{1}{2}q_+q_-+\vec{q}_\perp^{\,2})+K_+K_-+P_+K_-+P_-K_+-(\vec{K}_\perp-\tfrac{1}{2}\vec{q}_\perp)^2\,, \tag{2.31}$$

$$s'=\tfrac{1}{2}(u_3+u_4-\tfrac{1}{2}q_+q_-+\vec{q}_\perp^{\,2})+K_+K_--P_+'K_--P_-'K_+-(\tfrac{1}{2}\vec{q}_\perp-\vec{K}_\perp)^2\,, \tag{2.32}$$

$$s=P_+P_-'+P_-P_+'+\tfrac{1}{2}(u_1+u_2+u_3+u_4)-\tfrac{1}{2}q_+q_-+\vec{q}_\perp^{\,2}\,, \tag{2.33}$$

$$u_1=P_+P_-+\tfrac{1}{4}q_+q_-+\tfrac{1}{2}P_+q_-+\tfrac{1}{2}P_-q_+\,, \tag{2.34}$$

$$u_2=P_+P_-+\tfrac{1}{4}q_+q_--P_+\tfrac{1}{2}q_--P_-\tfrac{1}{2}q_+-\vec{q}_\perp^{\,2}\,, \tag{2.35}$$

$$u_3=P_+'P_-'+\tfrac{1}{4}q_+q_-+P_+'\tfrac{1}{2}q_--P_-'\tfrac{1}{2}q_+\,, \tag{2.36}$$

$$u_4=P_+'P_-'+\tfrac{1}{4}q_+q_-+P_+'\tfrac{1}{2}q_-+P_-'\tfrac{1}{2}q_+-\vec{q}_\perp^{\,2}\,, \tag{2.37}$$

$$u_1'=(K_++\tfrac{1}{2}q_+)(K_-+\tfrac{1}{2}q_-)-(\vec{K}_\perp+\tfrac{1}{2}\vec{q}_\perp)^2\,, \tag{2.38}$$

$$u_2'=(K_+-\tfrac{1}{2}q_+)(K_--\tfrac{1}{2}q_-)-(\vec{K}_\perp-\tfrac{1}{2}\vec{q}_\perp)^2\,. \tag{2.39}$$

With a and b as functions of invariants (we omit writing their dependence on the external masses $u_1\cdots u_4$) (2.27) becomes

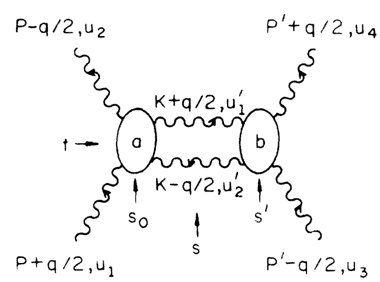

FIG. 3. Kinematics of the integral $a\times_t b$.

$$a\times_t b=\frac{1}{2}\int\frac{dK_+dK_-d^2K_\perp}{(2\pi)^4}a(q^2,s_0,u_1',u_2')b(q^2,s',u_1',u_2')\times D^T(u_1')D^T(u_2')\theta(s_0-m_0^2)\theta(s'-m_0^2)\,. \tag{2.40}$$

For convenience we have introduced the actual lower limit m_0^2 of the s_0 and s' integrations, below which a and b have no support. Written out in full (2.40) is

$$\begin{aligned}a\times_t b=\frac{1}{2}\int\frac{dK_+dK_-d^2K_\perp}{(2\pi)^4}\,&a\big(q^2,\tfrac{1}{2}(u_1+u_2-\tfrac{1}{2}q_+q_-+\vec{q}_\perp^{\,2})+K_+K_-+P_+K_-+P_-K_+-(K_\perp-\tfrac{1}{2}\vec{q}_\perp)^2,\\&(K_++\tfrac{1}{2}q_+)(K_-+\tfrac{1}{2}q_-)-(\vec{K}_\perp+\tfrac{1}{2}\vec{q}_\perp)^2,(K_+-\tfrac{1}{2}q_+)(K_--\tfrac{1}{2}q_-)-(\vec{K}_\perp-\tfrac{1}{2}\vec{q}_\perp)^2\big)\\&\times b\big(q^2,\tfrac{1}{2}(u_3+u_4-\tfrac{1}{2}q_+q_-+\vec{q}_\perp^{\,2})+K_+K_--P_+'K_--P_-'K_+-(\vec{K}_\perp-\tfrac{1}{2}\vec{q}_\perp)^2,\\&(K_++\tfrac{1}{2}q_+)(K_-+\tfrac{1}{2}q_-)-(\vec{K}_\perp+\tfrac{1}{2}\vec{q}_\perp)^2,(K_+-\tfrac{1}{2}q_+)(K_--\tfrac{1}{2}q_-)-(\vec{K}_\perp-\tfrac{1}{2}\vec{q}_\perp)^2\big)\\&\times D^T\big((K_++\tfrac{1}{2}q_+)(K_-+\tfrac{1}{2}q_-)-(\vec{K}_\perp+\tfrac{1}{2}\vec{q}_\perp)^2\big)D^T\big((K_+-\tfrac{1}{2}q_+)(K_--\tfrac{1}{2}q_-)-(\vec{K}_\perp-\tfrac{1}{2}\vec{q}_\perp)^2\big)\\&\times\theta\big(\tfrac{1}{2}(u_1+u_2-\tfrac{1}{2}q_+q_-+\vec{q}_\perp^{\,2})+K_+K_-+P_+K_-+P_-K_+-(\vec{K}_\perp-\tfrac{1}{2}q_\perp)^2-m_0^2\big)\\&\times\theta\big(\tfrac{1}{2}(u_3+u_4-\tfrac{1}{2}q_+q_-+\vec{q}_\perp^{\,2})+K_+K_--P_+'K_--P_-'K_+-(\vec{K}_\perp-\tfrac{1}{2}\vec{q}_\perp)^2-m_0^2\big)\,.\end{aligned} \tag{2.41}$$

We are interested in s very large and therefore q can be taken as purely transverse. Throughout most of the phase space the following inequalities are satisfied:

$$s\gg s_0,\quad s'\gg\text{all other mass variables}, \tag{2.42}$$

provided we agree to treat the virtual photons as soft. Taking these inequalities to be true greatly simplifies (2.41). We find (with $l=-K_+K_-$)

$$u_1'\approx-l-(\vec{K}_\perp+\tfrac{1}{2}\vec{q}_\perp)^2,\quad u_2'\approx-l-(\vec{K}_\perp-\tfrac{1}{2}\vec{q}_\perp)^2, \tag{2.43}$$

$$s_0\approx P_+K_-,\quad s'\approx-P_-'K_+,\quad s\approx P_+P_-' \tag{2.44}$$

$$s_0s'\approx sl\,, \tag{2.45}$$

$$\begin{aligned}a\times_t b=\frac{1}{2}\frac{1}{(2\pi)^4}\int_{m_0^2}^{s}\frac{ds_0}{s_0}\int_0^\infty dl\int d^2K_\perp\,&a\big(q^2,s_0,-l-(\vec{K}_\perp+\tfrac{1}{2}\vec{q}_\perp)^2,-l-(\vec{K}_\perp-\tfrac{1}{2}\vec{q}_\perp)^2\big)\\&\times b\big(q^2,sl/s_0,-l-(\vec{K}_\perp+\tfrac{1}{2}\vec{q}_\perp)^2,-l-(\vec{K}_\perp-\tfrac{1}{2}\vec{q}_\perp)^2\big)\\&\times D^T\big(-l-(\vec{K}_\perp+\tfrac{1}{2}\vec{q}_\perp)^2\big)D^T\big(-l-(\vec{K}_\perp-\tfrac{1}{2}\vec{q}_\perp)^2\big)\,.\end{aligned} \tag{2.46}$$

Note that with these kinematic approximations the virtual photon momenta are restricted to spacelike values. The expression of (2.46) can be iterated to calculate the leading behavior of $(\mathrm{Abs}\,T)^n$. Putting in the Froissart bound $\mathrm{Abs}\,T(s)=\beta s(\ln s)^2$, with β depending on the external masses, q^2, and the external spin states, we find

$$(\mathrm{Abs}\,T)^n=C_n\,s\,(\ln s)^{2+3(n-1)}+\text{nonleading terms}, \tag{2.47}$$

where $C_1=\beta$. Putting this result into (2.20) and (2.21) yields

$$\Big(\xi\frac{d}{d\xi}-1\Big)\mathrm{Abs}\,T(\xi)\Big|_{\xi=1}=\Big(\xi\frac{d}{d\xi}-1\Big)\mathrm{Abs}\,T_2(\xi)\Big|_{\xi=1}\leq C_2 s(\ln s)^5\,, \tag{2.48}$$

$$\frac{d^n}{d\xi^n}\mathrm{Abs}\,T(\xi)\Big|_{\xi=1}=\frac{d^n}{d\xi^n}\mathrm{Abs}\,T_2(\xi)\Big|_{\xi=1}\leq n!\,[C_n s(\ln s)^{2+3(n-1)}+C_{n+1}s(\ln s)^{2+3n}]\,,\quad n\geq 2\,. \tag{2.49}$$

Let us now define

$$K_1=\lim_{s\to\infty}\Big(\xi\frac{d}{d\xi}-1\Big)\mathrm{Abs}\,T_2(s,\xi)\Big|_{\xi=1}\Big/\mathrm{Abs}\,T_2(s)\,, \tag{2.50}$$

$$K_n = \lim_{s\to\infty} \frac{d^n}{d\xi^n} \mathrm{Abs}T_2(s,\xi)\Big|_{\xi=1} \Big/ \mathrm{Abs}T_2(s), \quad n \geq 2\,. \tag{2.51}$$

Our final assumption is that at least one of the numbers K_n above is not equal to zero. The consequences of all the K_n vanishing are discussed in Sec. V of this paper. Let $K_i \neq 0$. Combining this with (2.48) and (2.49) we find

$$K_i \leq \lim_{s\to\infty} i\,![C_{i+1}s(\ln s)^{2+3i}]/\mathrm{Abs}T_2(s)\,. \tag{2.52}$$

Since K_i is not zero by assumption, we have the bound

$$\mathrm{Abs}T_2(s) \leq Ks(\ln s)^{2+3i}\,, \tag{2.53}$$

for some K and s large.

We wish to compare this result to the leading-log approximation for the tower graphs.[1,2] The tower graphs for light scattering on light are pictured in Fig. 4. The absorptive part of these graphs has an asymptotic behavior of the form (for no spin flip)

$$\mathrm{Abs}T_{2\,\mathrm{tower}}(s,t) = \beta(t)\frac{s^{1+11\alpha^2\pi/32}}{(\ln s)^2}\,. \tag{2.54}$$

If this is the true asymptotic behavior of $\mathrm{Abs}T_2$, even qualitatively, then one of our assumptions is wrong, since (2.54) is in contradiction with (2.53). We admit this possibility. On the other hand, one can easily imagine sets of Feynman graphs which grow faster than the tower graphs by log factors, and which are two-photon reducible. These graphs (we expect that there are many of them) are of higher order in α, the coupling constant, and it is hoped that their net contribution is small if α is small. Our assumptions, culminating in (2.53), demand that these extra contributions exactly cancel off the leading behavior of the tower graphs and yield a two-photon reducible amplitude which satisfies the Froissart bound up to logarithms.

The situation is summarized as follows: Froissart bound for off shell photons + soft virtual photons in BS equation + one nonvanishing moment

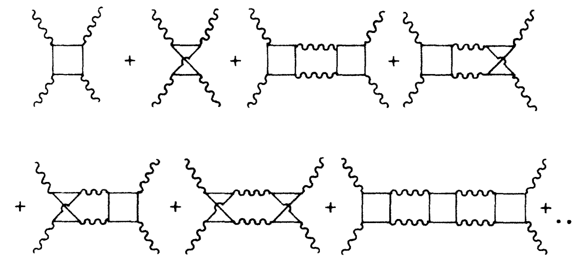

FIG. 4. The "tower" graphs.

(K_i) of BS equation → Tower graphs are canceled by higher-order terms in perturbation expansion which are two-photon reducible in the t channel. The remainder satisfies the Froissart bound up to logarithms.

In the next section we discuss the γ-e case.

III. γ-e SCATTERING

In this section we will discuss photons scattering with fermions. In the entire discussion the external fermions will always be kept on the mass shell. We write the scattering amplitudes as

$$T_{e\gamma}(q,P,P',S_1,S_2,\lambda_1,\lambda_2) = T^{\mu\nu}_{e\gamma}(q,P,P';S_1,S_2) \times \epsilon_\mu(P'-\tfrac{1}{2}q,\lambda_1) \times \epsilon_\nu(P'+\tfrac{1}{2}q,\lambda_2), \tag{3.1}$$

where S_1 and S_2 denote the initial and final spin states of the fermion. The external photons of (3.1) must end on either a closed fermion loop or a fermion line which zig-zags through the graph. This causes $T^{\mu\nu}_{e\gamma}$ to be transverse in any finite order in perturbation theory. We assume that the true solution has this property also. Thus we have

$$(P'-\tfrac{1}{2}q)_\mu T^{\mu\nu}_{e\gamma}(q,P,P') = (P'+\tfrac{1}{2}q)_\nu T^{\mu\nu}_{e\gamma}(q,P,P') = 0. \tag{3.2}$$

We next introduce the Bethe-Salpeter equation

$$T^{\mu\nu}_{e\gamma}(q,P,P',S_1,S_2) = I^{\mu\nu}_{e\gamma}(q,P,P',S_1,S_2) - \frac{i}{2}\int\frac{d^4K'}{(2\pi)^4}\, T^{\lambda\delta}_{e\gamma}(q,P,K';S_1,S_2) I^{\lambda'\delta';\mu\nu}(q,-K',P') \times D_{\lambda\lambda'}(K'-\tfrac{1}{2}q)D_{\delta\delta'}(K'+\tfrac{1}{2}q)\,. \tag{3.3}$$

$I^{\lambda'\delta';\mu\nu}$ above is the same kernel as in (2.6). $I^{\mu\nu}_{e\gamma}(q,P,P';S_1,S_2)$ is the sum of all Feynman graphs which contribute to γ-e scattering, and which do not have a two-photon cut in the crossed channel. Note that single-photon exchange does not contribute because of Furry's theorem. It follows from (3.3), (3.2), and (2.9) that $I^{\mu\nu}$ is transverse. It also follows that we may choose the renormalized propagators $D_{\lambda\lambda'}(K)$ to be transverse. Equation (3.3) thus becomes

$$T^{\mu\nu}_{e\gamma}(q,P,P';S_1,S_2)=I^{\mu\nu}(q,P,P';S_1,S_2)-\frac{i}{2}\int\frac{d^4K'}{(2\pi)^4}T^{\lambda\delta}_{e\gamma}(q,P,K',S_1,S_2)I^{\lambda'\delta';\mu\nu}(q,-K',P')$$
$$\times\left(-g_{\lambda\lambda'}+\frac{(K'-\frac{1}{2}q)_\lambda(K'-\frac{1}{2}q)_{\lambda'}}{(K'-\frac{1}{2}q)^2}\right)\left(-g_{\delta\delta'}+\frac{(K'+\frac{1}{2}q)_\delta(K'+\frac{1}{2}q)_{\delta'}}{(K'+\frac{1}{2}q)^2}\right)$$
$$\times D^T((K'-\tfrac{1}{2}q)^2)D^T((K'+\tfrac{1}{2}q)^2)\,. \tag{3.4}$$

Expressing (3.4) in terms of invariant amplitudes we find

$$T_{e\gamma}(q,P,P';S_1,S_2,\lambda_1,\lambda_2)=I_{e\gamma}(q,P,P';S_1,S_2,\lambda_1,\lambda_2)$$
$$-\frac{1}{2}\sum_{\lambda,\lambda'}\int\frac{d^4K'}{(2\pi)^4}T_{e\gamma}(q,P,K';S_1,S_2,\lambda,\lambda')$$
$$\times I(q,-K',P';\lambda,\lambda',\lambda_1,\lambda_2)D^T((K'+\tfrac{1}{2}q)^2)D^T((K'-\tfrac{1}{2}q)^2)\,. \tag{3.5}$$

Using the Cutkosky rules to take the absorptive part of (3.5) we find

$$\text{Abs}T_{e\gamma}(q,P,P';S_1,S_2,\lambda_1,\lambda_2)=\text{Abs}I_{e\gamma}(q,P,P';S_1,S_2,\lambda_1,\lambda_2)+\sum_{\lambda,\lambda'}\int\frac{d^4K'}{(2\pi)^4}\text{Abs}T_{e\gamma}(q,P,K';S_1,S_2,\lambda,\lambda')$$
$$\times\text{Abs}I(q,-K',P';\lambda,\lambda',\lambda_1,\lambda_2)$$
$$\times D^T((K'+\tfrac{1}{2}q)^2)D^T((K'-\tfrac{1}{2}q)^2)\,. \tag{3.6}$$

As in Sec. II we write (3.6) in the abbreviated form

$$\text{Abs}T_{e\gamma}=\text{Abs}I_{e\gamma}+\text{Abs}T_{e\gamma}\times_t\text{Abs}I\,. \tag{3.7}$$

We next consider the generalized equation

$$\text{Abs}T_{e\gamma}(\xi)=\xi\text{Abs}I_{e\gamma}+\xi\text{Abs}T_{e\gamma}(\xi)\times_t\text{Abs}I\,. \tag{3.8}$$

This equation is guaranteed to be solved by iteration:

$$\text{Abs}T_{e\gamma}(\xi)=\xi\text{Abs}I_{e\gamma}+\sum_{n=1}^{\infty}\xi^{n+1}\text{Abs}I_{e\gamma}\times_t(\text{Abs}I)^n\,. \tag{3.9}$$

We now introduce the two-photon reducible amplitude

$$\text{Abs}T_{2e\gamma}(\xi)=\text{Abs}T_{e\gamma}(\xi)-\xi\text{Abs}I_{e\gamma}\,. \tag{3.10}$$

The following relations may be checked by substitution:

$$\left(\xi\frac{d}{d\xi}-1\right)\text{Abs}T_{e\gamma}(\xi)=\left(\xi\frac{d}{d\xi}-1\right)\text{Abs}T_{2e\gamma}(\xi)=\text{Abs}T_{e\gamma}(\xi)\times_t\text{Abs}T(\xi)\,, \tag{3.11}$$

$$\text{Abs}T_{e\gamma}(\xi)=\text{Abs}T_{e\gamma}(\xi=1)+(\xi-1)[\text{Abs}T_{e\gamma}(\xi=1)+\text{Abs}T_{e\gamma}(\xi=1)\times_t\text{Abs}T(\xi=1)]$$
$$+\sum_{n=1}^{\infty}(\xi-1)^{n+1}\{\text{Abs}T_{e\gamma}(\xi=1)\times_t[\text{Abs}T(\xi=1)]^n+\text{Abs}T_{e\gamma}(\xi=1)\times_t[\text{Abs}T(\xi=1)]^{n+1}\}\,. \tag{3.12}$$

Written out in full, Eq. (3.11) is

$$\left(\xi\frac{d}{d\xi}-1\right)\text{Abs}T_{2e\gamma}(q,P,P';S_1,S_2,\lambda_1,\lambda_2;\xi)=\sum_{\lambda,\lambda'}\int\frac{d^4K'}{(2\pi)^4}\text{Abs}T_{e\gamma}(q,P,K';S_1,S_2,\lambda,\lambda';\xi)$$
$$\times\text{Abs}T(q,-K',P';\lambda,\lambda',\lambda_1,\lambda_2;\xi)$$
$$\times D^T((K'+\tfrac{1}{2}q)^2)D^T((K'-\tfrac{1}{2}q)^2)$$
$$\times\theta((P+K')^2)\,\theta((P'-K')^2)\,. \tag{3.13}$$

It follows from (3.11) and (3.12) that

$$\left(\xi\frac{d}{d\xi}-1\right)\text{Abs}T_{2e\gamma}(\xi)\Big|_{\xi=1}=\text{Abs}T_{e\gamma}\times_t\text{Abs}T, \tag{3.14}$$

$$\frac{d^n}{d\xi^n}\text{Abs}T_{2e\gamma}(\xi)\Big|_{\xi=1}=n![\text{Abs}T_{e\gamma}\times_t(\text{Abs}T)^{n-1}+\text{Abs}T_{e\gamma}\times_t(\text{Abs}T)^n]\,, \tag{3.15}$$

for $n \geq 2$. Let us introduce the following moments:

$$K_1 = \lim_{s\to\infty} \left(\xi \frac{d}{d\xi} - 1\right) \mathrm{Abs}\, T_{2e\gamma}(\xi)\Big|_{\xi=1} \Big/ \mathrm{Abs}\, T_{2e\gamma}\,, \tag{3.16}$$

$$K_n = \lim_{s\to\infty} \frac{d^n}{d\xi^n} \mathrm{Abs}\, T_{2e\gamma}(\xi)\Big|_{\xi=1} \Big/ \mathrm{Abs}\, T_{2e\gamma}\,, \quad n \geq 2. \tag{3.17}$$

We assume, as in Sec. II, that at least one of the K_n above is not zero. We also assume that the four photon amplitudes Abs$T$ and the photon electron amplitudes Abs$T_{e\gamma}$ satisfy the Froissart bound off shell. We take the virtual photons in (3.14) and (3.15) as soft. This leads, as before, to the asymptotic results (the kinematics is the same as in II)

$$\left(\xi \frac{d}{d\xi} - 1\right) \mathrm{Abs}\, T_{2e\gamma}(\xi)\Big|_{\xi=1} \leq C_{1e\gamma}\, s\,(\ln s)^5\,, \tag{3.18}$$

$$\frac{d^n}{d\xi^n} \mathrm{Abs}\, T_{2e\gamma}(\xi)\Big|_{\xi=1} \leq n!\,[\,C_{ne\gamma}\, s\,(\ln s)^{2+3n}\,]\,, \quad n \geq 2\,. \tag{3.19}$$

Choosing i such that $K_i \neq 0$, we have from (3.16), (3.17), (3.18), and (3.19)

$$\mathrm{Abs}\, T_{2e\gamma}(s,t) \leq K_{e\gamma}\, s\,(\ln s)^{2+3i}\,, \quad s\to\infty\,. \tag{3.20}$$

The tower-graph contribution to Abs$T_{2e\gamma}$ does not satisfy (3.20) as in the photon-photon case[1,2]

$$\mathrm{Abs}\, T_{2e-\gamma\,\mathrm{tower}} = \beta_{e-\gamma}(t)\, s^{1+11\pi\alpha^2/32}/(\ln s)^2\,. \tag{3.21}$$

We next study the e-e case.

IV. e-e SCATTERING

In this section we discuss fermions scattering on fermions. The external fermions are always kept on shell. We first introduce the "transposed" γ-e amplitudes

$$T_{\gamma e}(q,P,P';\lambda_1,\lambda_2,S_1,S_2) = T_{e\gamma}(-q,P',P;S_1,S_2,\lambda_1,\lambda_2)\,, \tag{4.1}$$

with the same definition for $I_{\gamma e}$. The Bethe-Salpeter equation for the invariant e-e scattering amplitude is

$$\begin{aligned}T_{ee}(q,P,P';S_1,S_2,S_3,S_4) &= I_{ee}(q,P,P';S_1,S_2,S_3,S_4) - \frac{i}{2}\sum_{\lambda,\lambda'}\int \frac{d^4K'}{(2\pi)^4}\, T_{e\gamma}(q,P,K';S_1,S_2,\lambda,\lambda') \\ &\qquad \times I_{\gamma e}(q,-K',P';\lambda,\lambda',S_3,S_4) \\ &\qquad \times D^T((K'-\tfrac{1}{2}q)^2)\, D^T((K'+\tfrac{1}{2}q)^2) \qquad (4.2)\\ &= I_{ee}(q,P,P';S_1,S_2,S_3,S_4) - \frac{i}{2}\sum_{\lambda,\lambda'}\int \frac{d^4K'}{(2\pi)^4}\, I_{e\gamma}(q,P,K';S_1,S_2,\lambda,\lambda') \\ &\qquad \times T_{\gamma e}(q,-K',P';\lambda,\lambda',S_3,S_4) \\ &\qquad \times D^T((K'-\tfrac{1}{2}q)^2)\, D^T((K'+\tfrac{1}{2}q)^2)\,. \qquad (4.3)\end{aligned}$$

Taking the absorptive part we find

$$\begin{aligned}\mathrm{Abs}\, T_{ee}(q,P,P';S_1,S_2,S_3,S_4) &= \mathrm{Abs}\, I_{ee}(q,P,P';S_1,S_2,S_3,S_4) \\ &\quad + \sum_{\lambda,\lambda'}\int \frac{d^4K'}{(2\pi)^4}\, \mathrm{Abs}\, T_{e\gamma}(q,P,K';S_1,S_2,\lambda,\lambda')\, \mathrm{Abs}\, I_{\gamma e}(q,-K',P';\lambda,\lambda',S_3,S_4) \\ &\quad \times D^T((K'+\tfrac{1}{2}q)^2)\, D^T((K'-\tfrac{1}{2}q)^2) \\ &\quad \times \theta((P+K')^2)\, \theta((P'-K')^2)\,, \end{aligned} \tag{4.4}$$

with a similar equation coming from (4.3). We again abbreviate these as

$$\begin{aligned}\mathrm{Abs}\, T_{ee} &= \mathrm{Abs}\, I_{ee} + \mathrm{Abs}\, T_{e\gamma} \times_t \mathrm{Abs}\, I_{\gamma e} \\ &= \mathrm{Abs}\, I_{ee} + \mathrm{Abs}\, I_{e\gamma} \times_t \mathrm{Abs}\, T_{\gamma e}\,.\end{aligned} \tag{4.5}$$

We next consider the generalized equation

$$\begin{aligned}\mathrm{Abs}\, T_{ee}(\xi) &= \xi\, \mathrm{Abs}\, I_{ee} + \xi\, \mathrm{Abs}\, T_{e\gamma}(\xi) \times_t \mathrm{Abs}\, I_{\gamma e} \\ &= \xi\, \mathrm{Abs}\, I_{ee} + \xi\, \mathrm{Abs}\, I_{e\gamma} \times_t \mathrm{Abs}\, T_{\gamma e}(\xi)\,,\end{aligned} \tag{4.6}$$

and the two-photon reducible part

$$\mathrm{Abs}\, T_{2ee}(\xi) = \mathrm{Abs}\, T_{ee}(\xi) - \xi\, \mathrm{Abs}\, I_{ee}\,. \tag{4.7}$$

We find the following relations:

$$\left(\xi \frac{d}{d\xi} - 1\right) \mathrm{Abs}\, T_{2ee}(\xi)\Big|_{\xi=1} = \mathrm{Abs}\, T_{e\gamma} \times_t \mathrm{Abs}\, T_{\gamma e}\,, \tag{4.8}$$

$$\begin{aligned}\frac{d^n}{d\xi^n} \mathrm{Abs}\, T_{2ee}(\xi)\Big|_{\xi=1} &= n!\,[\,\mathrm{Abs}\, T_{e\gamma} \times_t (\mathrm{Abs}\, T)^{n-2} \times_t \mathrm{Abs}_{\gamma e} \\ &\quad + \mathrm{Abs}\, T_{e\gamma} \times_t (\mathrm{Abs}\, T)^{n-1} \times_t \mathrm{Abs}\, T_{\gamma e}\,]\,, \quad n \geq 2.\end{aligned} \tag{4.9}$$

We next introduce the usual moments

$$K_1 = \lim_{s\to\infty} \left(\xi\frac{d}{d\xi}-1\right)\mathrm{Abs}\,T_{2ee}(\xi)\Big|_{\xi=1} \Big/ \mathrm{Abs}\,T_{2ee}\,, \tag{4.10}$$

$$K_n = \lim_{s\to\infty} \frac{d^n}{d\xi^n}\mathrm{Abs}\,T_{2ee}(\xi)\Big|_{\xi=1} \Big/ \mathrm{Abs}\,T_{2ee}\,,\quad n\geq 2, \tag{4.11}$$

and assume that at least one of these is nonzero. Putting the Froissart bound into (4.8) and (4.9) yields

$$\left(\xi\frac{d}{d\xi}-1\right)\mathrm{Abs}\,T_{2ee}(\xi)\Big|_{\xi=1} \leq C_{1ee}\, s(\ln s)^5\,, \tag{4.12}$$

$$\frac{d^n}{d\xi^n}\mathrm{Abs}\,T_{2ee}(\xi)\Big|_{\xi=1} \leq n!\,C_{nee}\, s(\ln s)^{2+3n}\,,\quad n\geq 2. \tag{4.13}$$

If K_i is nonzero then from (4.10), (4.11), (4.12), and (4.13) we must have

$$\mathrm{Abs}\,T_{2ee}(s) \leq K_{ee}\, s(\ln s)^{2+3i}\,, \tag{4.14}$$

for large s. The "tower" graphs again do not satisfy (4.14); for spin-nonflip scattering they give a contribution[1,2]

$$\mathrm{Abs}\,T_{2ee\,\mathrm{tower}} = \beta_{ee}(t)s^{1+11\alpha^2\pi/32}/(\ln s)^2\,. \tag{4.15}$$

The cases for $\bar{e}$-$\bar{e}$ and $\bar{e}$-γ scattering are identical to the e-e and e-γ ones. In the next sections we consider the consequences of these results.

V. GENERAL REMARKS

If in any of the preceding cases all the moments K_i vanish let us see what can be said. Recall that, in each case, this implies

$$K_1 = \lim_{s\to\infty} \left(\xi\frac{d}{d\xi}-1\right)\mathrm{Abs}\,T_2(s,\xi) \Big/ \mathrm{Abs}\,T_2(s) = 0, \tag{5.1}$$

$$K_n = \lim_{s\to\infty} \frac{d^n}{d\xi^n}\mathrm{Abs}\,T_2(s,\xi) \Big/ \mathrm{Abs}\,T_2(s) = 0,\quad n\geq 2. \tag{5.2}$$

Suppose we can interchange the order of limit and derivative above. Then we get a unique solution:

$$\lim_{s\to\infty} \mathrm{Abs}\,T_2(s,\xi)/\mathrm{Abs}\,T_2(s) = \xi\,, \tag{5.3}$$

in some region about $\xi=1$. Let us see how reasonable this is. Suppose $\mathrm{Abs}\,T_2(s,\xi)$, for large s, is of the form

$$\mathrm{Abs}\,T_2(s,\xi) = \beta(t,\xi)s^{\alpha_1(t,\xi)}(\ln s)^{\alpha_2(t,\xi)}[\ln(\ln s)]^{\alpha_3(t,\xi)} \times\cdots\{\ln[\ln(\cdots\ln s)]\}^{\alpha_n(t,\xi)}\,. \tag{5.4}$$

It is clear that for all the moments K_n to vanish we must demand all the α's independent of ξ. All the ξ dependence is in β. In this case we can certainly change the order of limit and differentiation in (5.1) and (5.2). The same is true if $\mathrm{Abs}\,T_2(s,\xi)$ is a finite sum of terms of this form. We feel that this exhausts all cases of physical interest. Therefore we feel safe in interchanging the limits, and we accept (5.3) as the only reasonable possibility. The reader may find our arguments of Ref. 3 on this matter entertaining. We obtained there the same result (5.3) from a somewhat different line of reasoning. There we expanded $\mathrm{Abs}\,T_2(s,\xi)$ in a Taylor series about $\xi=1$ by using the equivalent of Eq. (2.21). Assuming that the sum of the absolute values of this series was polynomially bounded in s led to (5.3).

Reexpressing $\mathrm{Abs}\,T_2(s,\xi)$ as a polynomial in ξ, we have from (5.3)

$$\lim_{s\to\infty} \sum_{n=2}^{\infty} (\xi)^n(\mathrm{Abs}\,I)^n \Big/ \mathrm{Abs}\,T_2(s) = \xi, \tag{5.5}$$

in some region about $\xi=1$. The reader can see the pathology of this situation. The left-hand side of (5.5) seems to contain no term linear in ξ, whereas the right-hand side is purely linear in ξ. There is no known example of this kind of behavior in existing studies of the Bethe-Salpeter equation. We therefore feel that the strongest assumption leading to the bounds here discussed for $\mathrm{Abs}\,T_2$ is that of the Froissart bound off shell.

We wish to point out that we have not made any positivity assumptions in this paper, nor have we used any positivity arguments.

VI. CONCLUSIONS

We have established reasonable conditions which limit the growth of two-photon exchange in massive QED. These conditions must be wrong if tower graphs are the main contribution to two-photon exchange in any process studied here. Assuming that our conditions are correct one is forced to ask whether perturbation theory breaks down in the asymptotic limit in massive QED. The situation is quite a bit more hopeful in φ^3 (and we expect also φ^4) field theory.[9] Here the leading-log contributions do not violate our conditions until the coupling is increased beyond a critical point. No problem is encountered for sufficiently small values of the coupling. In massive QED these conditions are contradicted by the leading-log tower graphs for arbitrarily small coupling. We find this situation distressing yet interesting.

The problem addressed here is not without physical interest. It is well known that total proton-proton cross sections are increasing with energy (at $s \approx 2500$ GeV^2). One model which attempts to explain (and in fact predicted) this behavior is the eikonal model of Cheng and Wu. This model is a perturbative calculation in massive QED, where the tower graphs are taken as a Born term in an eikonal expansion for the scattering amplitude. The result they find for total cross sections is $\sigma_{tot} \propto (\ln s)^2$, saturating the Froissart bound. This saturation depends critically on the fact that the tower graphs themselves (the Born term) violate the Froissart bound by a power. A model of this type will have difficulty coexisting with the conditions leading to the bounds here discussed. It may be that the CERN results coupled with the Cheng-Wu model offer experimental evidence that the virtual Compton amplitude violates the Froissart bound at energies of $s \approx 2500$ GeV^2 for spacelike photons.

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[1]G. V. Frolov, V. N. Gribov, and L. N. Lipatov, Phys. Lett. 31B, 34 (1970).
[2]H. Cheng and T. T. Wu, Phys. Rev. Lett. 24, 1456 (1970).
[3]M. Davidson, Phys. Rev. D 8, 4438 (1973).
[4]See, for example, James D. Bjorken and Sidney D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), pp. 198–200.
[5]H. A. Bethe and E. E. Salpeter, Phys. Rev. 84, 1232 (1951); M. Gell-Mann and F. Low, *ibid.* 84, 350 (1951).
[6]R. E. Cutkosky, J. Math. Phys. 1, 429 (1960).
[7]M. L. Goldberger, in *Subnuclear Phenomena*, Part A, edited by A. Zichichi (Academic, New York, 1970), p. 62.
[8]M. Froissart, Phys. Rev. 123, 1053 (1961); R. J. Eden, *High Energy Collisions of Elementary Particles* (Cambridge University Press, Cambridge, 1967).
[9]B. W. Lee and R. F. Sawyer, Phys. Rev. 127, 2266 (1962); D. Amati, S. Fubini, and A. Stangellini, Nuovo Cimento 26, 896 (1962).

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Possibility that massless Yang-Mills fields generate massive vector particles*

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It is examined whether the massless Yang-Mills field theories can describe massive vector particles which have a common mass. The nonzero mass is supposed to be accompanied by poles in vertex functions. It is demonstrated that such poles can be consistent with the Ward identities. Homogeneous linear integral equations describing approximately the residues of these poles are shown to have asymptotic nontrivial solutions.

I. INTRODUCTION

It was conjectured[1] some time ago that theories with a local gauge group may describe massive vector particles, where the mass is of purely dynamical origin. One way to achieve this is by means of the so-called "Higgs mechanism" leading to the classes of spontaneously broken gauge theories[2] which have received much interest in recent times. Because these models can be treated by perturbation theory, they are practical, at least for the description of weak interactions. Yet, because of the often large number of scalar fields that have to be introduced, there can be many free parameters resulting from the multitude of possible couplings of the scalar fields among themselves. It is therefore useful to investigate the possibility that vector particles may acquire a mass without the introduction of scalar mesons. This has been done,[3–5] essentially by exploiting the analogy with the Meissner effect in the theory of superconductivity, with the qualitative conclusion that this spontaneous mass generation is indeed possible.

The present investigation deals with the possibility that the pure Yang-Mills field, without other couplings, may generate massive vector particles, specifically, without breaking the global symmetry so that the particles have equal mass.

It is known[1,6] that the vector particles can only be massive if their proper self-energy function has a pole at $p^2=0$. In Sec. II a simple mechanism that may lead to the formation of such a pole is proposed, and it is reviewed how this pole