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Elementary and composite particles in asymptotically free gauge theories

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We demonstrate that in asymptotically free gauge theories the wave-function renormalization constants for spin-1/2 fields do not vanish. (The scalar fields, if incorporated, also have this property.) However, there exists a subclass of such theories where the Z's for the gauge fields themselves tend to zero in the limit of infinite cutoff. These features are shown to be gauge-independent. This suggests the potentiality of constructing asymptotically free strong-interaction theories in which the only elementary fields are "quarks" and all other hadrons are bound states.

I. INTRODUCTION

Gauge-invariant quantum field theories based on non-Abelian groups are being actively investigated. A number of rather unique properties have been discovered about such theories. These features, when interpreted optimistically, indicate that non-Abelian gauge theories may provide the framework within which theories of strong interactions (or event unified theories of all elementary-particle forces) may be constructed.

Gauge theories are renormalizable.[1] It has been shown, by way of the renormalization-group equation,[2] that the origin of the coupling-constant space is an ultraviolet-stable fixed point[3] only in non-Abelian gauge theories.[4] This asymptotically free nature of the theory provides us with a field-theoretical explanation of Bjorken scaling—rather the explanation of *how* Bjorken scaling is approached in the deep Euclidean limit.[5] This same property indicates that the effective couplings can be large in the infrared limit—it just may provide the desired quark-confinement mechanism.[6]

There are also a number of works suggesting intriguing connections of gauge theories to dual models and relativistic string models of hadrons.[7] This confluence of field-theoretical and S-matrix approaches to strong-interaction physics is also indicated by the works of Grisaru, Schnitzer, and Tsao.[8] These authors have demonstrated that vector mesons and spin-$\frac{1}{2}$ fermions in such field theories satisfy the usual criteria of Reggeization: factorization of Born amplitudes and Mandelstam countings. However, as possible candidates for strong-interaction theories, the class of gauge theories investigated in their works may have some drawbacks: These gauge theories are not asymptotically free and while spin-$\frac{1}{2}$ particles (quarks?) lie on Regge trajectory, scalar fields do not. (It would seem an unattractive picture of having spin-$\frac{1}{2}$ fermions composite, but not all other particles.) In this paper we shall use another criterion for the compositeness of fields appearing in a Lagrangian field theory, i.e., the vanishing of the wave-function renormalization constants. Our results suggest the possibility of constructing strong-interaction field theories which are asymptotically free and in which the only elementary fields are quarks and all other hadrons are composite.

In Sec. II we shall demonstrate, through a straightforward exercise of solving the renormalization-group equation, that in asymptotically free gauge theories the renormalization constants for spin-0 and spin-$\frac{1}{2}$ fields do not vanish in the limit of infinite cutoff Λ. In particular we show that there exists a subclass of such theories where the Z's of the gauge fields vanish.[9] This property is gauge-independent since we can show that the effective gauge parameter α is always driven to some fixed value (for example, $\alpha=0$, the Landau gauge) in much the same manner as the effective couplings are driven to zero in the deep Euclidean region.[9,10]

The final section is devoted to a discussion of the implications of such gauge-independent features. Some of the arguments for the connection of vanishing renormalization constant and compositeness of particles are reviewed. Possible implications for incorporating scalars as composite Goldstone-Higgs particles are also briefly discussed.

II. EVALUATION OF RENORMALIZATION CONSTANTS IN ASYMPTOTICALLY FREE GAUGE THEORIES

To illustrate our point we shall restrict our considerations to gauge theories with two dimensionless coupling constants: the gauge coupling g and the quartic scalar self-coupling λ. [For example, we may consider the group O(N) or SU(N) with scalars in a single vector representation.] Generalization to cases involving more couplings is straightforward.

Some of the renormalization constants we shall compute are defined to be

$$A_\mu = Z_V^{-1/2} A_{0\mu}\ ,\quad \psi = Z_f^{-1/2}\psi_0\ ,\quad \phi = Z_S^{-1/2}\phi_0\ , \tag{1}$$

and

$$g = Z_V^{3/2} Z_g^{-1} g_0\ ,\quad \lambda = Z_S^2 A^{-1}\lambda_0\ . \tag{2}$$

A, ψ, and ϕ denote the gauge, spinor, and scalar fields, respectively and the subscript 0 indicates unrenormalized quantities. The bare propagator for the gauge boson takes on the form of

$$\Delta^{ab}_{\mu\nu}(k) = \left(\frac{-g_{\mu\nu}+k_\mu k_\nu/k^2}{k^2} - \alpha_0\frac{k_\mu k_\nu}{k^4}\right)\delta^{ab}\ . \tag{3}$$

Since the higher-order correction terms do not modify the longitudinal part, the renormalized gauge parameter is related to α_0 by

$$\alpha = Z_V^{-1}\alpha_0\ . \tag{4}$$

The renormalization constants are functions of the cutoff Λ,[11] renormalization point M, couplings, and the gauge parameter

$$Z_i = Z_i(\Lambda/M, g, \lambda, \alpha),\quad i = V, f, S\ .$$

They satisfy the renormalization-group equations, which may be derived most simply by differentiating (with respect to M) both sides of the trivial identity $Z_i^{-1}Z_i=1$ and by applying the chain rule,

$$\left(\frac{\partial}{\partial t} - \beta_g\frac{\partial}{\partial g} - \beta_\lambda\frac{\partial}{\partial\lambda} - \beta_\alpha\frac{\partial}{\partial\alpha} + \gamma_i\right)Z_i = 0\ , \tag{5}$$

where

$$t = \ln(\Lambda/M)\ ,\qquad \beta_{g_l} = M\frac{\partial}{\partial M}g_l\bigg|_{\Lambda, g_{0l}}\ ,\quad \{g_l\}\equiv g, \lambda, \alpha\qquad \gamma_i = +M\frac{\partial}{\partial M}\ln Z_i\bigg|_{\Lambda, g_{0l}}\ . \tag{6}$$

From Eq. (4) it immediately follows that

$$\beta_\alpha = -\alpha\gamma_V\ . \tag{7}$$

The renormalization equation in (5) may be solved in the standard manner[12] by first defining the effective coupling constants and effective gauge parameter: $\{\bar{g}_l\} = \bar{g}, \bar{\lambda}, \bar{\alpha}$. They are functions of the physical couplings, gauge parameters, and t. And they satisfy the differential equations

$$\frac{d\bar{g}_l}{dt} = \beta_{g_l}(\{\bar{g}_m\})\ , \tag{8}$$

with the normalization conditions that $\{\bar{g}(t=0)\} = \{g_l\}$. It then follows that Eq. (5) has the solution

$$Z_i(t, g_l) = Z_i(0, \bar{g}_l(t))\exp\left[-\int_0^t dt'\,\gamma_i(\bar{g}_l(t'))\right]\ . \tag{9}$$

In an asymptotically free theory all effective coupling constants are driven to zero in the limit of infinite t (i.e., $\bar{g}, \bar{\lambda}\to 0$). Being interested in this class of theories only, we shall expand β's and γ's to the lowest order in the coupling constants:

$$\beta_g = -bg^3\ , \tag{10}$$

$$\beta_\lambda = A\lambda^2 + B\lambda g^2 + Cg^4\ , \tag{11}$$

$$\gamma_i = -(\Gamma_i + \tilde{\Gamma}_i\alpha)g^2\ , \tag{12}$$

and

$$\beta_\alpha = -(\Gamma_V + \tilde{\Gamma}_V\alpha)\alpha g^2\ . \tag{13}$$

The condition for asymptotic freedom is $b>0$, $B-b<0$, $(B-b)^2-4AC>0$, with A, C being always positive.[13,14a] The constants b, A, B, C, and Γ_i depend on the group and particle representation. In general

$$b=\frac{1}{16\pi^2}\left[\tfrac{11}{3}S_1(v)-\tfrac{4}{3}S_3(f)-\tfrac{1}{6}S_3(s)\right], \tag{14}$$

$$\Gamma_V=-\frac{1}{8\pi^2}\left[\tfrac{13}{3}S_1(v)-\tfrac{8}{3}S_3(f)-\tfrac{1}{3}S_3(s)\right], \tag{15a}$$

$$\tilde\Gamma_V=\frac{1}{8\pi^2}S_1(v)>0, \tag{15b}$$

$$\Gamma_S=\frac{-3}{8\pi^2}S_2(s)<0, \tag{16a}$$

$$\tilde\Gamma_S=\frac{-1}{16\pi^2}S_2(s)<0, \tag{16b}$$

$$\Gamma_f=0, \tag{17a}$$

$$\tilde\Gamma_f=\frac{1}{8\pi^2}S_2(f)>0, \tag{17b}$$

where

$$\begin{aligned} &S_1(v)\,\delta^{ab}=C^{acd}C^{bcd},\\ &S_2(s)\,\delta_{ij}=(\theta^a\theta^a)_{ij},\\ &S_2(f)\,\delta_{ij}=(t^at^a)_{ij},\\ &S_3(s)\,\delta^{ab}=\mathrm{tr}(\theta^a\theta^b),\\ &S_3(f)\,\delta^{ab}=\mathrm{tr}(t^at^b), \end{aligned} \tag{18}$$

C^{abc} being the structure constant of the group; θ^a, t^a being the representation matrices of the scalar and spin-$\frac{1}{2}$ particles, respectively. Specifically, for O(N), $S_1=\frac{1}{2}(N-2)$, and $S_2=\frac{1}{4}(N-1)$, $S_3=\frac{1}{2}$ for the vector representation; for SU(N), $S_1=N$, and $S_2=(N^2-1)/2N$, $S_3=\frac{1}{2}$ for the (real) vector representations.

When the condition for asymptotic freedom is satisfied

$$\lim_{t\to\infty}\bar g\sim(2bt)^{-1/2}\to 0,$$

and

$$\lim_{t\to\infty}\bar\lambda\sim\left\{\frac{(-B+b)-[(B-b)^2-4AC]^{1/2}}{4Ab}\right\}t^{-1}\to 0\,.$$

The ultraviolet behavior of the gauge parameter $\bar\alpha$ in asymptotically free theories is discussed in the Appendix. Depending on the value of Γ_V two distinctive cases obtain. For $\Gamma_V>0$, the only fixed point is the stable one at $\alpha_\infty=0$ (the Landau gauge); for $\Gamma_V>0$, $\bar\alpha$ will be driven to some finite positive value $\alpha_\infty=-\Gamma_V/\tilde\Gamma_V$, while $\alpha=0$ is an unstable fixed point.

The discussion of wave-function renormalization constants will be divided into these two cases: (i) $\Gamma_V>0$ and (ii) $\Gamma_V<0$.

(i) When $\Gamma_V>0$ ($\alpha_\infty=0$), it immediately follows [see Eq. (9)] that the wave-function renormalization constants for gauge fields vanish in the limit of infinite cutoff, while the Z's for scalar fields do not:

$$\begin{aligned} &Z_V\sim t^{-\Gamma_V/2b}\to 0\,,\\ &Z_S\sim t^{-\Gamma_S/2b}\to\infty\,. \end{aligned} \tag{19}$$

The behavior of the wave-function renormalization constant for a spin-$\frac{1}{2}$ field, Z_f, will need a separate discussion. Since the one-loop contribution is proportional to the gauge parameter, i.e., $\Gamma_f=0$ [see Eq. (17)], then $\gamma_f(g_i(t))$ is driven to zero in the asymptotic limit faster than $\bar g^2(t)$. It then follows that the Z_f's do not vanish in the $t\to\infty$ limit. Explicitly, we have (see Appendix) $\bar\alpha\sim t^{-\Gamma_V/2b}\to 0$; then Eq. (19) takes on the form

$$\begin{aligned} Z_f&\sim\exp\left[-\tilde\Gamma_f\int^t dt'\,\bar\alpha(t')\bar g^2(t')\right]\\ &\quad+\text{terms constant in the large-}t\text{ limit}\\ &=e^{\text{const}\,t^{-\Gamma_V/2b}}+\text{constant}\\ &\to\text{constant}. \end{aligned} \tag{20}$$

We note, if we had $\bar\alpha\to 0$ faster than $\bar g^2$ then γ_f should be properly calculated to include higher orders in coupling constants. Also in this paper we have excluded Yukawa couplings (by some discrete symmetry, for example). However, we would like to emphasize that the result in Eq. (20) holds even under these situations, since its validity is dependent solely on the condition that $\gamma_f(t)$ vanish faster than t^{-1}. (In the following discussion, we shall denote such a situation by "$t^0+\cdots$ const.") This clearly holds when higher-order couplings are included. The Yukawa coupling contribution also satisfies this condition since it is known that in order for the gauge theory to remain asymptotically free, the Yukawa coupling must be driven to zero at a rate *greater* than that of the gauge coupling.[14a]

Clearly there are a wide class of gauge theories which can satisfy the above conditions. For example, in an SU(3) theory with 16 sets of fermions and one set of complex scalars in triplet representations,[14a] we have $b=1/48\pi^2$, $\Gamma_V=-2/3\pi^2$, $\Gamma_S=-1/2\pi^2$, $A=7/8\pi^2$, $B=-1/\pi^2$, $C=13/48\pi^2$.

(ii) When $\Gamma_V<0$, the ultraviolet value of the gauge parameter is finite, $\alpha_\infty=-\Gamma_V/\tilde\Gamma_V$ (see Appendix). The solutions in Eq. (9) take on the following asymptotic values[14b]:

$$Z_V\sim t^0+\cdots\text{ constant}, \tag{21}$$

$$Z_S\sim t^{-(\Gamma_S-\tilde\Gamma_S\Gamma_V/\tilde\Gamma_V)/2b}\to\infty, \tag{22}$$

and

$$Z_f\sim t^{\Gamma_f\Gamma_V/2b\tilde\Gamma_V}\to\infty. \tag{23}$$

Since $\alpha=0$ is also a fixed point (albeit an unstable one) we must consider this situation also:

$$Z_V \sim t^{-\Gamma_V/2b} \to \infty \,, \tag{21a}$$

$$Z_S \sim t^{-\Gamma_S/2b} \to \infty \,, \tag{22a}$$

and

$$Z_f \sim t^0 + \cdots \text{ constant} . \tag{23a}$$

To summarize, we have shown that, independent of the choices of gauges, the renormalization constants in asymptotically free gauge theories take on the values as

$$Z_f \neq 0 \,,$$

$$Z_S \neq 0 \,,$$

and

$$Z_V = 0, \text{ for } \Gamma_V > 0$$

$$Z_V \neq 0, \text{ for } \Gamma_V > 0 \,.$$

III. DISCUSSION

Since the wave-function renormalization constant Z is usually identified as the probability of the particle in its bare state, it has been suggested by a number of authors that Z equal to zero be taken as a general criterion for composite systems.[15] The validity of this connection has been verified in a number of toy field-theory models. Phenomenological analysis also suggests that the Z's for the deuteron[16] and proton[17] are consistent with the value zero. This is compatible with an expectation that they are composite systems.

Furthermore, the vanishing of renormalization constants has been shown to be related to the compositeness criteria as formulated in the S-matrix language,[18] i.e., the particles appear on a Regge trajectory and the absence of a Kronecker δ in the J plane. In this context, Kaus and Zachariasen[19] have shown that an added requirement will be the vanishing of the ratio of vertex renormalization constant to wave-function renormalization. (Crudely speaking, this requires that the interaction terms vanish faster than the kinetic energy term.) We note that in asymptotically free gauge theories, these additional criteria are automatically satisfied. One may check this either by explicit computation or by noting that[2] $\bar{g}(\infty) = g_0 \to 0$. Then Eq. (2) clearly requires that

$$Z_g / Z_V \to 0$$

in order to keep the physical coupling constants on the left-hand side of the equation finite.

If this connection of vanishing renormalization constant to the dynamical nature of the particle is indeed a correct one, then the result of this calculation could have a number of implications in our attempts of constructing asymptotically free gauge theories for strong interactions.

So far no one has succeeded in incorporating scalars in such theories in a way one can demonstrate that all gauge fields acquire mass through usual Higgs mechanism (at the tree diagram level) and still retain asymptotic freedom.[13,14a] Attempts have been made to attack the problem through nonperturbative approach, and it is hoped that the fermion and antifermion will form bound-state Goldstone particles which will in turn be eliminated by the massless gauge field to form a massive spin-1 particle.[20] Our result may suggest that asymptotic freedom and dynamical symmetry breaking as envisaged by some authors may not be realizable in such theories. If this is indeed the case, it would appear that the only option for viable theories for strong interactions would be the type as proposed in Ref. 6, i.e., gauge theories with fermions based on exact non-Abelian local symmetries.

Our calculation also suggests that there exist a subclass of asymptotically free theories[21] which are consistent with the following picture of strong-interaction dynamics: Strong interactions are indeed described by a non-Abelian gauge theory with fermions in which the only bona fide elementary fields are fermions (quarks), in which all other particles, including the gauge particles, are dynamical bound states. Furthermore a sort of bootstrap world will result if the fundamental fermion fields are never present in the asymptotical states as free particles: All observed particles are dynamical ones.

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APPENDIX

The renormalization-group equation for effective gauge parameter $\bar{\alpha}(t)$ is as follows:

$$\frac{d\bar{\alpha}}{dt} = -\gamma_V(\bar{g}_i)\,\bar{\alpha} \,. \tag{A1}$$

When the right-hand side is expanded in terms of effective couplings, we have [see Eq. (12)]

$$\frac{d\bar{\alpha}}{dt} = -(\Gamma_V \bar{\alpha} + \bar{\Gamma}_V \bar{\alpha}^2)\,\bar{g}^2 + \text{higher-order terms in effective couplings} . \tag{A2}$$

As we are only interested in the asymptotically free theories, the higher-order terms may be ignored. Then Eq. (A2) may be solved by a simple change of variable from t to u with $u=(2b)^{-1}\times\ln(g^{-2}+2bt)$:

$$\frac{d\bar{\alpha}(u)}{du}=-\Gamma_V\,\bar{\alpha}(u)-\tilde{\Gamma}_V\,\bar{\alpha}^2(u)\ , \tag{A3}$$

which has the general solution of

$$\bar{\alpha}^{-1}=Ce^{\Gamma_V u}-\tilde{\Gamma}_V/\Gamma_V\ . \tag{A4}$$

The constant C may be fixed by the boundary condition $\bar{\alpha}(t=0)=\alpha$ to be

$$C=g^{\Gamma_V/b}\left(\frac{1}{\alpha}+\frac{\tilde{\Gamma}_V}{\Gamma_V}\right)\ . \tag{A5}$$

The limit of $\bar{\alpha}$ for large t (i.e., large u) depends on the sign of Γ_V.

(i) $\Gamma_V>0$. The $e^{\Gamma_V u}$ term in (A4) tends to infinity, resulting in

$$\bar{\alpha}\sim\text{const}\times t^{-\Gamma_V/2b}\to 0\ . \tag{A6}$$

(ii) $\Gamma_V<0$. The $e^{\Gamma_V u}$ factor tends to zero, resulting in

$$\bar{\alpha}\to-\Gamma_V/\tilde{\Gamma}_V\ . \tag{A7}$$

In the following we shall give a somewhat more detailed discussion of the nature of these fixed points.

From Eq. (A1) it is clear that $\alpha=0$ is always a fixed point. In Eq. (A6) we have shown that for $\Gamma_V>0$, it is a stable one; but for $\Gamma_V<0$, the zero is an ultraviolet-unstable fixed point.

On the other hand, while Eq. (A7) demonstrated that when $\Gamma_V<0$ $\alpha=-\Gamma_V/\tilde{\Gamma}_V$ is a stable fixed point, *it will be incorrect to conclude that this is an unstable fixed point when* $\Gamma_V>0$ [as Eqs. (A3) and (A4) would lead us to believe].[22] This comes about because (unlike the $\alpha=0$ situation) the higher-order terms in Eq. (A2) do not vanish simultaneously when $\alpha=-\Gamma_V/\tilde{\Gamma}_V$. Consequently at the point of $\alpha=-\Gamma_V/\tilde{\Gamma}_V$ the subdominant terms have the effect of moving α away from this point; but once α is dislocated, the g^2 term is once again present and is naturally the dominant driving force; when $\Gamma_V>0$, it drives $\bar{\alpha}$ to zero; when $\Gamma_V<0$ it tends to restore $\bar{\alpha}$ back to $-\Gamma_V/\tilde{\Gamma}_V$.

To summarize the asymptotic behaviors of the effective gauge parameter:

(i) $\Gamma_V>0$; the *only* fixed point is the stable one at $\alpha=0$.

(ii) $\Gamma_V<0$; there are two fixed points: $\alpha=-\Gamma_V/\tilde{\Gamma}_V$ is the stable one; $\alpha=0$ is the unstable one.

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Asymptotic behavior of two-photon exchange in massive quantum electrodynamics*

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The Bethe-Salpeter equation is used in conjunction with the Froissart bound (assumed true off the mass shell) to put restrictions on two-photon exchange. Implications are discussed.

I. INTRODUCTION

We wish to study here the Regge asymptotic behavior of the sum of all two-photon reducible Feynman diagrams in massive quantum electrodynamics. This is motivated by the "tower" graph calculations of Frolov, Gribov, and Lipatov[1] and also Cheng and Wu.[2] This paper is an extension of previous work on π-π scatteting.[3] It is found in Refs. 1 and 2 that the sum of all "tower" graphs violates the Froissart bound by a power for all nonzero values of the coupling constant α. We ask here if it is likely that the sum of all crossed-channel, two-photon reducible graphs violates the Froissart bound in this manner. We feel that this question is important because of the following:

1. Massive quantum electrodynamics is a likely candidate for a quark-gluon field theory.

2. The fact that the tower graphs violate the Froissart bound by a power is essential to obtain Froissart-bound saturation in the Cheng-Wu eikonal model.[2]

3. If it seems probable or desirable that the full two-photon exchange amplitude does not violate the Froissart bound by a power, then a search for two-photon reducible graphs which cancel the leading behavior of the tower graphs is in order.

We begin with the Bethe-Salpeter equations for elastic γ-γ, e-γ, and e-e scattering. We then rewrite these equations in a particular way and make an assumption about the off-mass-shell behavior of the full amplitudes. We also make an assumption about certain moments of the Bethe-Salpeter (BS) equation. We find that these assumptions rule out the possibility that the sum of all two-photon reducible graphs violates the Froissart bound by a power. We first study γ-γ scattering which is the simplest case and then work our way up to γ-e and e-e scattering.

II. γ-γ SCATTERING

The invariant amplitudes for γ-γ scattering we write as

$$T(q,P,P',\lambda_1,\lambda_2,\lambda_3,\lambda_4)=T^{\mu\nu;\alpha\beta}(q,P,P')\epsilon_\mu(P+\tfrac{1}{2}q,\lambda_1)\epsilon_\nu(P-\tfrac{1}{2}q,\lambda_2)\epsilon_\alpha(P'-\tfrac{1}{2}q,\lambda_3)\epsilon_\beta(P'+\tfrac{1}{2}q,\lambda_4)\,, \tag{2.1}$$