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Isolating the 3-quark component of the proton's wave function

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The 3-quark component of the proton's wave function is extracted from deep-inelastic proton and neutron scattering data. We employ theoretical results based upon a theory of the hadronic wave function which is consistent with the interchange theory of fixed-angle and high-transverse-momentum processes.

INTRODUCTION

In this paper we describe a theoretical extraction of the quark distribution functions of the proton, using the deep-inelastic scattering data for neutrons and protons.[1] The present analysis differs from those previously given[2] in that the Pomeron and Regge contributions are assumed to have specific, theoretically motivated, threshold damping. This enables us to subtract these contributions in a well-defined manner, isolating those portions of the distribution functions most closely related to the simplest three-quark component of the proton's wave function. Among other results we find that for this component the $\mathcal{N}$-quark distribution function is a single power of $(1-x)$ (x is the fraction of the proton's momentum carried by the quark) times that for the $\mathcal{P}$ quark, for all x.

I. THEORETICAL CONSIDERATIONS

We begin by considering the probability $u_i(x)$ for finding a quark i of a given type, carrying a fraction x of the proton's total linear momentum (in a frame in which the proton's linear momentum is large). In general each such probability function may be thought of as having three contributions:

(i) It has a contribution $\hat{u}_i(x)$ arising from the simplest possible quark state consistent with the nucleon's (or meson's) quantum numbers: for instance, $\mathcal{P}\mathcal{P}\mathcal{N}$ for the proton, $\mathcal{P}\overline{\mathcal{N}}$ for a π^+, etc. The wave function (which when integrated over transverse momentum gives the probability distribution function) for this simplest constituent state might, for instance, obey a relatively simple[3] Bethe-Salpeter type of integral equation. Such a component will not exhibit either Regge behavior or Pomeron behavior $[u_i(x) \underset{x\to 0}{\sim} 1/x^\alpha$, with $\alpha=\frac{1}{2}$ or 1, respectively$]$. It should exhibit a maximum when the quarks present have approximately equal shares of the hadron's momentum ($x\sim\frac{1}{3}$ for a nucleon) and should, of course, be absent for quarks not required to be a part of the *simplest* quark state of the given hadron (e.g., $\overline{\mathcal{P}}\overline{\mathcal{N}}\lambda\overline{\lambda}$ for a proton).

(ii) It also has a contribution which exhibits non-Pomeron Regge behavior. In general, both this contribution and the Pomeron contribution (iii) will be present provided the quark-proton scattering amplitude exhibits these respective types of high-energy behavior (this connection is discussed in Ref. 3, for instance), as expected of a strong-interaction amplitude. In general, however, the non-Pomeron Regge contribution, which we denote by $r_i(x)$, need not be present for every quark type i. In fact, extension of conventional duality arguments to the quark-proton scattering amplitude suggests that for the proton $r_i(x) \neq 0$ only for $i = \mathcal{P}$ and $\mathfrak{N}$. According to the duality argument non-Pomeron Regge behavior should only appear in a quark-proton amplitude when the outgoing quark is able to arise from one of the initial proton quarks, in the sense of the usual dual diagrams (Fig. 1).[4] As we shall see, it is likely that this should only be regarded as a good approximation.

(iii) The probability function also has an SU(2), and perhaps SU(3), symmetric contribution $s_i(x)$, arising from the Pomeron behavior of the quark-proton scattering amplitude. Both this and the Regge component presumably arise from proton constituent states containing more than the simplest three quarks. Thus we shall speak of a "sea" of quark-antiquark pairs.

Consistent with the above separation we write

$$\begin{aligned} u_{\mathcal{P}}^p(x) &= \hat{u}_{\mathcal{P}}^p(x) + s^p(x) + r_{\mathcal{P}}^p(x)\,, \\ u_{\mathfrak{N}}^p(x) &= \hat{u}_{\mathfrak{N}}^p(x) + s^p(x) + r_{\mathfrak{N}}^p(x)\,, \\ u_{\bar{\mathcal{P}}}^p(x) &= u_{\bar{\mathfrak{N}}}^p(x) = s^p(x)\,, \\ u_{\lambda}^p(x) &= u_{\bar{\lambda}}^p(x) = s'^p(x)\;. \end{aligned} \tag{1}$$

For the Pomeron and Regge contributions we take the forms

$$\begin{aligned} s(x) &= \frac{a(1-x)^n}{x}\,, \quad s'(x) = \frac{a'(1-x)^n}{x}\,, \\ r_{\mathcal{P}}(x) &= \frac{a_{\mathcal{P}}(1-x)^{n_{\mathcal{P}}}}{\sqrt{x}}\,, \quad r_{\mathfrak{N}}(x) = \frac{a_{\mathfrak{N}}(1-x)^{n_{\mathfrak{N}}}}{\sqrt{x}}\,. \end{aligned} \tag{2}$$

In what follows we shall motivate specific values for n, $n_{\mathcal{P}}$, and $n_{\mathfrak{N}}$.

In the picture of the Pomeron in which it arises from a quark-antiquark "sea" component to the hadron wave function, the minimal such state for the proton corresponds to a $qqq\bar{q}q$ state.[5] The quark-antiquark pair can always be thought of as a meson or combination of mesons. The extraction of a sea-component quark could thus be thought of as occurring in two steps. The proton first emits, by a bremsstrahlung process, a meson with probability dictated by a full strong-interaction amplitude with Regge behavior. The meson in turn emits the sea quark according to the simple Bethe-Salpeter $q\bar{q}$ component of its wave function. This is illustrated in Fig. 2. In general one should include other intermediate hadrons H with the result

$$u_i^A(x) = \sum_H \int_x^1 \frac{dz}{z} G_{H/A}(z)\hat{u}_i^H(x/z)\,. \tag{3}$$

$G_{H/A}(z)$ is the probability for a hadron A to emit a secondary hadron H with fraction z of A's initial linear momentum P (in a frame in which $P \to \infty$). The above picture corresponds to the two-component (hadron reducible–hadron irreducible) picture of a hadronic wave function introduced by Blankenbecler, Brodsky, Gunion, and Savit.[3] $G_{H/A}$ has Pomeron and, in general, Regge behavior for $z \to 0$:

$$G_{H/A}(z) \underset{z\to 0}{\sim} \frac{1}{z^{\alpha}}\,, \tag{4}$$

which gives rise to Pomeron and Regge behavior for $u_i^A(x)$. $G_{H/\text{proton}}$ appears in the interchange-theory discussion[6] of production of high-transverse-momentum pions at the CERN ISR and at NAL.[7] It was found that

$$G_{M/\text{proton}} \propto \frac{(1-z)^5}{z} \quad (M = \text{meson}) \tag{5}$$

gives an excellent fit to the data. This form corresponds to Pomeron $1/z$ behavior with a specific threshold damping. The above threshold dependence can be easily motivated using the quark picture discussed earlier.

Consider the minimum proton state from which a pion or other meson may emerge, $qqq\bar{q}q$. In order to discuss meson emission from such a state we must introduce interactions between the quarks. We choose for spin-$\frac{1}{2}$ quarks to assume that vector gluon forces or similar interactions with dimensionless coupling constant on the quark level are relevant. This assumption together with the finiteness of wave functions at the origin (or, more naively, a weak binding limit for the quarks within a hadron) results in simple scaling laws, for asymptotic hadronic form factors and fixed-angle scattering and for high-transverse-momentum particle production, in good agreement with experi-

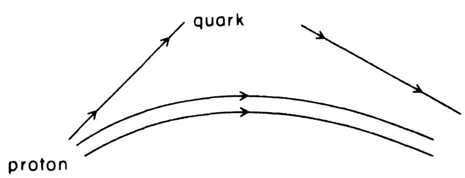

FIG. 1. Naive duality graph for the quark-proton amplitude.

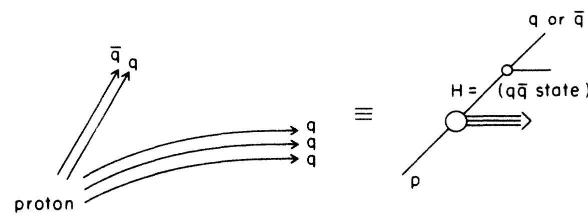

FIG. 2. The $qqq\bar{q}q$ state of the proton considered as a meson, M, plus three quarks.

ment.[8]

The production of a $q\bar{q}$ state with all the momentum, P, of the initial proton state may (for example) proceed with minimal suppression as in Fig. 3. Beginning with a weakly bound $qqqq\bar{q}$ state in which each quark carries $\frac{1}{5}$ (as stated earlier, one really only requires that the quark wave functions introduce no anomalous dimensions, i.e., are finite at the origin) of the proton's momentum P, various vector gluons are exposed in order to transfer this linear momentum entirely to the $q\bar{q}$ meson state (the q and $\bar{q}$ now carry $\frac{1}{2}P$ each in the weak-binding approximation). It is apparent, however, that this process requires propagation of quarks with 0 linear momentum, i.e., these remaining quarks are far off the mass shell. This actually results in the emission being suppressed by a well-determined number of multiplicative zeros.

This number is most easily determined by performing the calculation of a diagram such as that of Fig. 3 using time-ordered perturbation theory in the $P\to\infty$ frame. Any energy denominator

$$\left(\frac{1}{E_i-E_1},\ \frac{1}{E_i-E_2},\ \frac{1}{E_i-E_3}\ \text{in Fig. 3}\right)$$

containing a quark with $0\times P$ is 0 since the quark energy, proportional to $m^2/0\times P$, is infinite. Since nonoverlapping gluon denominators are always compensated by trace factors in this theory with no anomalous dimensions on the quark level, we ignore them. Thus the emission *amplitude* of Fig. 3 (in which we must include the initial- to final-state energy difference—we imagine operating on the particle in question with the number operator $a^\dagger a$) has a triple zero resulting in an emission *probability*

$$G(z) \sim \frac{(1-z)^{2\times 3}}{1-z}\ . \tag{6}$$

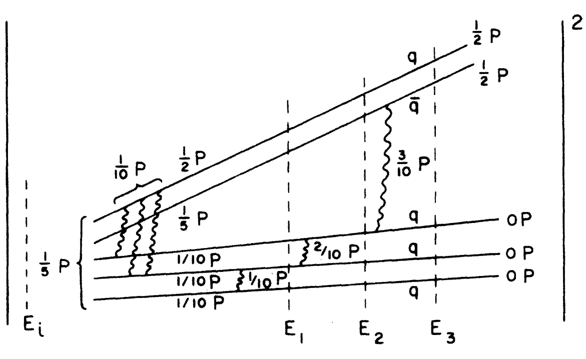

FIG. 3. Vector-gluon exposures required in order to transfer the full proton momentum, P, to the $q\bar{q}$ pair of a $qqqq\bar{q}$ proton state. Vertical dashed lines represent uncompensated intermediate states with infinite energy in time-ordered perturbation theory.

[The $1/(1-z)$ results from an uncompensated creation-annihilation energy factor in this approach.]

As a simpler example consider that of Fig. 4 (a complete calculation of which appears in Appendix A), in which a meson emits a quark carrying all the initial momentum. According to the above technique this configuration should be suppressed as

$$\hat{u}^M \propto (1-x)\,. \tag{7}$$

This corresponds to the Drell-Yan relation[9] if the mesons have monopole form factors as argued from interchange theory and the scaling laws mentioned earlier.

Combining Eqs. (6), (7), and (3) we obtain

$$s'^{p},\ s^{p}(x) \underset{x\to 1}{\simeq} (1-z)^{5+1+1=7}, \tag{8}$$

the extra single power arising from the limits of the convolution integral of Eq. (3).

We should note at this point that the same result for $s\ (x)$ obtains for H=baryon in Eq. (3). It is not difficult to see that for many baryons

$$G_{\text{baryon/proton}} \propto (1-z)^3\ . \tag{9}$$

Combining this with

$$\hat{u}_i^{\text{baryon}} \propto (1-x)^3 \tag{10}$$

for at least some of the quarks within a baryon, we again obtain contributions to the quark sea with the threshold damping of Eq. (8). In Appendix B we give complete rules for hadronic bremsstrahlung.

The result of Eq. (8) agrees with an alternative argument which applies the Drell-Yan relation to the form factor of a parton which is part of a five-quark proton state.[10] The absence of anomalous dimensions on the quark level implies that

$$F(t) \sim \frac{1}{t^4} \tag{11}$$

for such a parton. According to the Drell-Yan relation, this quark's distribution function should then behave as in Eq. (8). Direct vector-gluon diagram (Fig. 5) calculation verifies this result. Four infinite ΔE's appear. The net result is that in Eq. (2) we take $n=7$. No other choice can be motivated in as simple and self-consistent a theoretical approach. One should note that for a pion the mini-

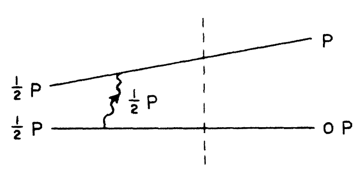

FIG. 4. A meson emitting a quark with the entire initial momentum P of the meson. See also Fig. 3 caption.

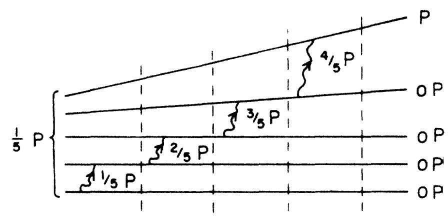

FIG. 5. An alternative diagram for calculating the probability of a proton emitting a quark carrying all the proton's initial momentum when arising from a $qqq\bar{q}q$ state.

mal sea component is $q\bar{q}q\bar{q}$ which yields a threshold suppression for the sea component of $(1-x)^5$.

The Regge distribution components $r_{\mathcal{P}}$ and $r_{\mathcal{N}}$ of Eq. (2) are discussed in the same way. One expects that the simplest Bethe-Salpeter-type component to the proton's wave function does not display Regge behavior and hence such behavior must arise, via a bremsstrahlung type of process, from the Regge behavior of hadronic amplitudes. The question arises, however, as to why Regge behavior should appear only in the $\mathcal{P}$- and $\mathcal{N}$-quark distribution functions. It might seem that since all hadrons H produced in the bremsstrahlung process are summed over, Regge behavior should appear in the other distribution functions as well. Indeed, this is the case in general; however, there is an important factor which serves to enhance the $\mathcal{P}$- and $\mathcal{N}$-quark Regge components above those of other quarks. Recall that according to the duality diagram rules proposed by Lipkin and others,[11] the magnitude of the Regge component in the amplitude for $A\bar{H}$ scattering (the probability for H *emission* is proportional to the imaginary part of the $A\bar{H}$ amplitude) is proportional to the number of single $q\bar{q}$ annihilations possible between quarks of A and antiquarks of $\bar{H}$. That is, Regge behavior arises from dual diagrams of the form of Fig. 6.

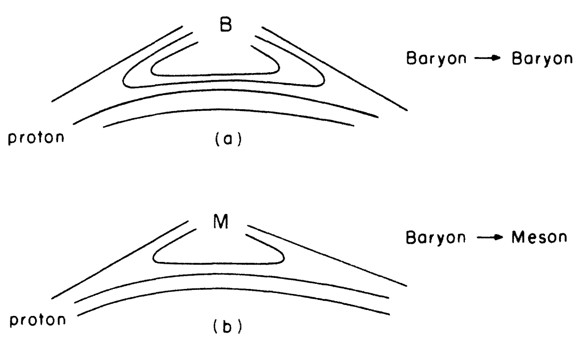

FIG. 6. (a) Baryon (B)-proton duality diagrams; Regge behavior is present for proton emission of B only when such a planar diagram can be drawn. (b) As in (a), only for mesons (M).

For instance, $p\bar{p}$ scattering (i.e., emission of a proton by a proton) is characterized by 5 possible annihilations, $p\bar{n}$ by 4, $p\bar{\Delta}^{++}$ by 6, pp by 0, etc. Thus for Regge components, emitted baryons having the maximum number of quarks in common with the proton are greatly enhanced. Meson emission is clearly suppressed relative to baryon emission since fewer annihilations are possible with a $q\bar{q}$ state and because there are fewer mesons [35 versus 56 for the standard SU(6) multiplet] than baryons. More precise statements are possible and will appear elsewhere.[12] For the present purposes we regard Eq. (1) as an adequate approximation.

We must now discuss the expected threshold damping. We assume that bremsstrahlung (characterized by Regge behavior) of a baryon or meson proceeds *dynamically* according to the dual diagrams of Fig. 6, so that threshold behaviors can be obtained by exposing the minimum number of gluons required to accomplish the momentum transfer. For instance, for baryon (B) bremsstrahlung we have Fig. 7(a), which by the now-familiar counting rules requires

$$G^{\text{Regge}}_{B/\text{proton}}(z) \underset{z\to 1}{\sim} (1-z)^3 \,. \tag{12}$$

Recalling that the quark Bethe-Salpeter component for baryons behaves as in Eq. (10), we obtain[13]

$$r_i(x) \sim (1-x)^7 \quad \text{(baryon source)}. \tag{13}$$

From Fig. 7(b) we see that for meson (M) emission this same argument leads to

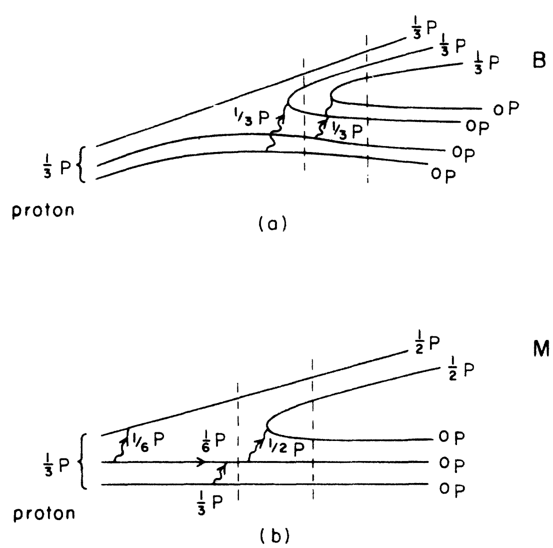

FIG. 7. (a) Explicit dynamical mechanism for transferring the entire proton momentum P to a baryon, B, while preserving duality diagram topology. (b) As in (a), but for a meson M.

$$G^{\text{Regge}}_{M/\text{proton}} \sim (1-z)^3 \quad (14)$$

or

$$r_i(x) \sim (1-x)^5 \quad \text{(meson emission).} \quad (15)$$

Despite this preference for the meson terms as $x \to 1$, the $x \to 0$ region is dominated by the baryon emission, as stated earlier, so that we shall take

$$n_{\pi} = n_{\mathcal{P}} = 7 \quad (16)$$

in Eq. (2). (The results given later are not sensitive to the choice of 7 as opposed to 5.)

II. ANALYSIS OF THE DATA

We now turn to an analysis of existing deep-inelastic scattering data for neutrons and protons in order to ascertain what support there is for the picture proposed here. For "data" we use those given in Fig. 8(a) and 8(b). The points given represent an average of existing experimental data at

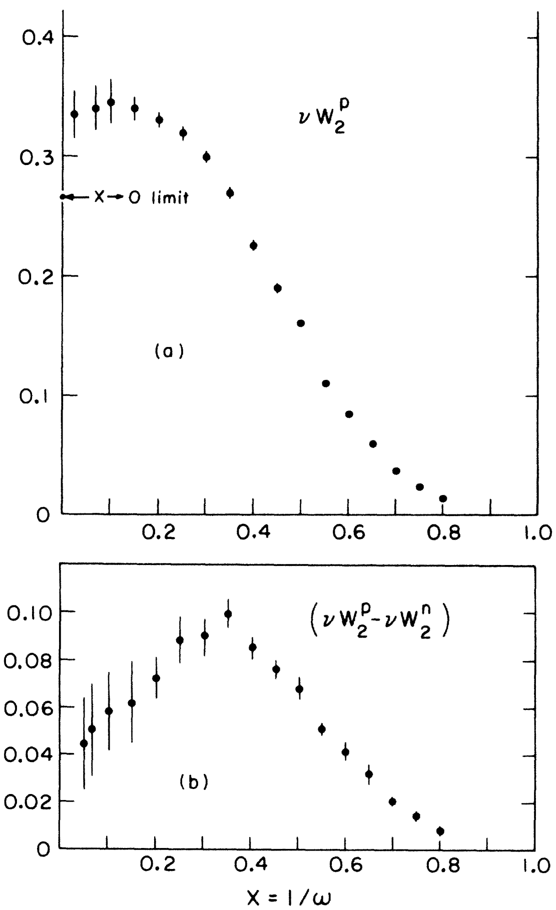

FIG. 8. (a) νW_2^p $(x = 1/\omega)$ for the proton. (b) $\nu W_2^p - \nu W_2^n$, the difference between the deep-inelastic structure functions for proton and neutron. These "data" represent an average of existing experimental data for the two quantities.

and near any particular $x = 1/\omega$ value, with an error estimated from the dispersion and statistical errors of those same points.[14] In the notation of Eq. (1) we have

$$\nu W_2^p(x) \equiv F_2^p(x) = x\left[\tfrac{4}{9}(\hat{u}_{\mathcal{P}} + r_{\mathcal{P}}) + \tfrac{1}{9}(\hat{u}_{\pi} + r_{\pi}) + \tfrac{10}{9}s + \tfrac{2}{9}s'\right] \quad (17)$$

(we drop the proton superscripts),

$$F_2^{pn} \equiv F_2^p(x) - F_2^n(x) = \tfrac{1}{3}x(\hat{u}_{\mathcal{P}} + r_{\mathcal{P}} - \hat{u}_{\pi} - r_{\pi}). \quad (18)$$

Since we cannot, without additional input, determine a and a' [Eq. (2)] separately, we define the combination

$$\tfrac{10}{9}s + \tfrac{2}{9}s' \equiv \tfrac{12}{9}\frac{A(1-x)^7}{x}. \quad (19)$$

For any given value of A define

$$F_2^{p(\text{NP})} \equiv F_2^p(x) - \tfrac{12}{9}\frac{A(1-x)^7}{x} \quad (20)$$

(NP stands for non-Pomeron). Then

$$\hat{u}_{\mathcal{P}} + r_{\mathcal{P}} = \tfrac{3}{5}(3F_2^{p(\text{NP})} + F_2^{pn}),$$
$$\hat{u}_{\pi} + r_{\pi} = \tfrac{3}{5}(3F_2^{p(\text{NP})} - 4F_2^{pn}). \quad (21)$$

We determine the values of a_{π} and $a_{\mathcal{P}}$ (for any A) by assuming the Regge forms to hold by $\omega = 20$, i.e., $x\hat{u}_p(x)|_{x=0.05} \approx 0$. The appropriate value of A is then determined by demanding that

$$\int(\hat{u}_{\mathcal{P}} + r_{\mathcal{P}})dx \approx 2,$$
$$\int(\hat{u}_{\pi} + r_{\pi})dx \approx 1, \quad (22)$$

as required if the quark distributions are to result in the correct quantum numbers for the proton. The assumed Regge forms are, of course, employed to calculate the integral contributions with $x < 0.05$. As emphasized by Bloom,[1] both sum rules cannot be fully saturated, though with new σ_n/σ_p data, incorporated in Fig. 8(a), one can come quite close. We find, using our Regge extrapolation,

$$\tfrac{1}{3}\int(\hat{u}_{\mathcal{P}} - \hat{u}_{\pi} + r_{\mathcal{P}} - r_{\pi})dx \approx 0.293 \pm (?), \quad (23)$$

where $? > 0.04$. This value is A-independent. Taking $A = 0.2$ we obtain

$$\int_0^1 (\hat{u}_{\pi} + r_{\pi}) \approx 0.997, \quad (24)$$

with $a_{\mathcal{P}}$ and a_{π} determined to be

$$a_{\mathcal{P}} = 1.88, \quad a_{\pi} = 1.023. \quad (25)$$

From Eq. (21) and the now completely determined forms of $r_{\mathcal{P}}$ and r_{π} we may extract the simplest Bethe-Salpeter components of the proton-quark

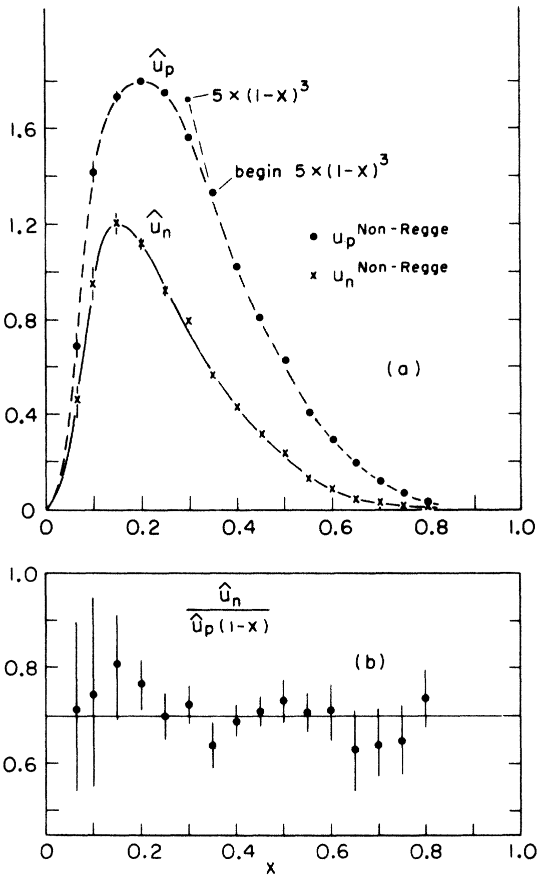

FIG. 9. (a) The simple 3-quark component of the proton's $\mathcal{P}$- and $\mathfrak{N}$-quark distribution functions ($\hat{u}_{\mathcal{P}}$ and $\hat{u}_{\mathfrak{N}}$). Both Pomeron and Regge contributions have been eliminated. (b) The ratio $[\hat{u}_{\mathfrak{N}}(x)/\hat{u}_{\mathcal{P}}(x)]/(1-x)$ as a function of x. The average value is ≈ 0.7.

distribution functions, $\hat{u}_{\mathcal{P}}$ and $\hat{u}_{\mathfrak{N}}$. These are plotted in Fig. 9(a), and have a number of very reasonable properties.

One should note, for instance, that $\hat{u}_{\mathcal{P}}$ is exactly proportional to $(1-x)^3$ for $x \gtrsim 0.35$, as expected from the Drell-Yan relation as well as from the simple models with canonical dimensions on the quark level referred to earlier. In addition, $\hat{u}_{\mathcal{P}}$ appears to vanish as $x \to 0$ as a single power of x (though assuming pure Regge behavior for $x < 0.05$ in $\hat{u}_{\mathcal{P}} + r_{\mathcal{P}}$ leads to some uncertainty in this statement). This is expected for the simple three-quark component of the proton's wave function since only one linear zero is required when two quarks carry the entire momentum of the proton. This is apparent from Fig. 10, where only one nongluon

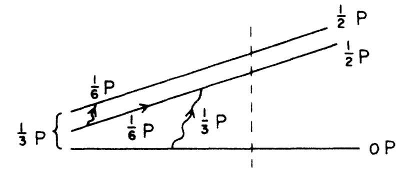

FIG. 10. Explicit mechanism for producing a single quark (from a qqq proton state) with none of the initial proton's momentum.

time-ordered perturbation-theory denominator is infinite.

Of greatest interest, however, is the behavior of $\hat{u}_{\mathfrak{N}}(x)/\hat{u}_{\mathcal{P}}(x)$. It is well known that as $x \to 1$, this ratio must vanish if $F_2^n(\omega=1)/F_2^p(\omega=1)=\frac{1}{4}$. From Fig. 9(b) we see that within errors

$$\hat{u}_{\mathfrak{N}}(x)/\hat{u}_{\mathcal{P}}(x) = c(1-x) \tag{26}$$

for all x. Any other simple power is ruled out. In addition any substantial change in the threshold behavior of s and s' destroys this result for $x < 0.5$. It also does not hold for $x < 0.2$ if one uses $(\hat{u}_{\mathfrak{N}} + r_{\mathfrak{N}})/(\hat{u}_{\mathcal{P}} + r_{\mathcal{P}})$. The Regge contributions must be removed. This result, Eq. (26), adds weight to the identification of $\hat{u}_{\mathfrak{N}}$ and $\hat{u}_{\mathcal{P}}$ with the purely 3-quark component of the proton's wave function. In a model in which the $\mathfrak{N}$ quark is more strongly bound to one of the $\mathcal{P}$ quarks than the remaining $\mathcal{P}$ quark is to either,[15] Eq. (26) seems the simplest possible result for the relative distribution function.

We should also point out that

$$\langle x\rangle = \int x\hat{u}_{\mathcal{P}}(x)dx = 0.3 \ , \tag{27}$$

as one might have expected for a weakly bound 3-quark system, and that

$$\int \hat{u}_{\mathcal{P}}(x)dx \approx 0.67, \quad \int \hat{u}_{\mathfrak{N}}(x)dx \approx 0.34 \ , \tag{28}$$

i.e., there are twice as many $\mathcal{P}$ quarks in the 3-quark portion of the proton wave function as there are $\mathfrak{N}$ quarks.

Finally, if we assume an SU(3)-symmetric Pomeron ($s = s'$), then the total momentum carried by λ (or $\bar{\lambda}, \overline{\mathcal{P}}\overline{\mathfrak{N}}$) quarks in the proton is

$$\int_0^1 x\frac{A(1-x)^7}{x}dx = 0.025 \ , \tag{29}$$

relative to the total momentum carried by all quarks:

$$\int_0^1 x(u_{\mathcal{P}} + u_{\mathfrak{N}} + u_{\overline{\mathcal{P}}} + u_{\overline{\mathfrak{N}}} + u_\lambda + u_{\bar{\lambda}})dx = \begin{cases}\text{3-quark piece} \\ 0.33\end{cases} + \begin{matrix}\text{Regge}\\ 0.11\end{matrix} + \begin{matrix}\text{Pomeron}\\ 0.15\end{matrix} = 0.59 \ . \tag{30}$$

Note that $\int (u_{\mathcal{P}}+u_{\bar{\mathcal{P}}}+u_{\mathfrak{N}}+u_{\bar{\mathfrak{N}}})dx \approx 0.54$, consistent with the experimental value determined from neutrino data[16] of 0.49 ± 0.07, and that the four-momentum of antipartons relative to that of all partons is ≈ 0.125 compared to the experimental value from $\sigma^{\bar{\nu}}/\sigma^{\nu}$ (Ref. 16) of 0.1 ± 0.03.

The above results seem sufficiently simple, so that the proposed picture can, at least, be said to be consistent.

CONCLUSION

We review the basic conclusions of this paper. Referring to Eqs. (1) and (2) we have found that in simple theories with no anomalous dimensions on the quark level, the Pomeron contribution to the quark distribution functions is expected to behave as

$$s(x)[s'(x)] \sim (1-x)^7 \quad \text{as } x \to 1.$$

The same power suppression applies to the non-Pomeron Regge component as well. Assuming specific forms with this behavior for the Pomeron and Regge-behaved portion of the quark distributions, we extracted the remaining, simplest 3-quark components of the proton's wave function, $\hat{u}_{\mathcal{P}}$ and $\hat{u}_{\mathfrak{N}}$. The $\hat{u}_{\mathcal{P}}$ and $\hat{u}_{\mathfrak{N}}$ so extracted have all the properties that simple, Bethe-Salpeter-type, wave-function contributions should have. In particular,

$$\hat{u}_{\mathfrak{N}}(x)/\hat{u}_{\mathcal{P}}(x) \propto (1-x)$$

for all x, the simplest possible result consistent with $F_2^n/F_2^p \to \frac{1}{4}$ for $\omega \to 1$.

This last result lends credence to the belief that there is a tightly bound $[\mathcal{P}\mathfrak{N}]$ pair within the proton to which the remaining $\mathcal{P}$ quark is less strongly bound. To the extent that this is the case, important implications for high-transverse-momentum phenomenology follow: (i) In exclusive pp scattering, there will be a strong component such that

$$\frac{d\sigma}{dt} \underset{\text{fixed angle}}{\overset{s\to\infty}{\sim}} \frac{1}{s^{12}}$$

(as originally proposed[6]), instead of the naive dimensional result

$$\frac{d\sigma}{dt} \sim \frac{1}{s^{10}},$$

which assumes the $[\mathcal{P}\mathfrak{N}]$ pair capable of being broken apart. (ii) The contribution to the proton form factor (by the Drell-Yan relation) from either quark of the $\mathcal{P}$-$\mathfrak{N}$ pair should behave as $1/t^{5/2}$ instead of the usual dipole result. (iii) The ut graph in the interchange version of π^--p fixed-angle phenomenology [which essentially measures $(F_q(t))^2$, where q is required for this topology to be an $\mathfrak{N}$ quark] will be suppressed relative to the st topology, with resultant (though minor) modification of the predicted angular distribution. (iv) The K^-p fixed-angle cross section (which requires extraction of the $\mathfrak{N}$ quark in the proton) will fall more rapidly by a single power of s than the K^+p fixed-angle cross section; i.e.,

$$\frac{d\sigma^{K^-p}}{dt} \sim \frac{1}{s^n},$$

with $n=9$ at fixed angle, instead of the canonical result which has $n=8$. A similar result holds for such reactions as $K_Lp \to K_sp$ (K_L and K_s contain only $\mathfrak{N}$, $\bar{\mathfrak{N}}$, λ, and $\bar{\lambda}$), where the power measured experimentally ($n=8.5\pm1.4$) is inconclusive.

It will be interesting to see if any of these effects can be observed. Of course, it may be that for x very near 1, $\hat{u}_{\mathfrak{N}}$ behaves like $(1-x)^3$ just as does $\hat{u}_{\mathcal{P}}$, but with a much reduced coefficient. If this is the case, i.e., if the $[\mathcal{P}\mathfrak{N}]$ pair behaves canonically when probed at sufficiently short distances, all of the standard interchange results will become valid.

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APPENDIX A

In time-ordered perturbation theory the absolute square of the diagram of Fig. 4 may be calculated as follows:

(i) We presume that the initial quark-antiquark pair couples with naive γ_5 coupling to the pion.

(ii) We take, for simplicity, the quark mass to be $\frac{1}{2}$ the pion mass m_H, as for the (easily relaxed) weak-binding limit.

(iii) We associate momenta $p=(p+m_H^2/2p, 0, p)$ ($p\to\infty$) with the pion and $k=(xp+(m_H^2+k_\perp^2)/8xp, \frac{1}{2}k_\perp, xp)$ ($x\to1$) with the upper outgoing quark line ($k^2=\frac{1}{4}m_H^2$).

(iv) The particle energies in this frame are

(a) pion energy $E_p = p + m_H^2/2p$,

(b) gluon energy

$$E_G = [\tfrac{1}{2}-(1-x)]p + \tfrac{1}{2}(m_G^2+\tfrac{1}{4}k_\perp^2)/[\tfrac{1}{2}-(1-x)]p,$$

(c) lower quark energy

$$E_{p-k} = (1-x)p + (m_H^2+k_\perp^2)/8(1-x)p.$$

The result, integrated over transverse momenta, is then

$$u(x) \equiv \int d^2k_\perp |\mathcal{A}_{\text{Fig. 4}}|^2 \alpha \int d^2k_\perp \frac{\text{Tr}[(\not k + \frac{1}{2}m_H)\gamma_\alpha(\frac{1}{2}\not p + \frac{1}{2}m_H)\gamma_5(\frac{1}{2}\not p - \frac{1}{2}m_H)\gamma_\alpha(\not p - \not k - \frac{1}{2}m_H)\gamma'_\alpha(\frac{1}{2}\not p - \frac{1}{2}m_H)\gamma_5(\frac{1}{2}\not p + \frac{1}{2}m_H)\gamma'_\alpha]}{(2E_G)^2[(E_p - E_G - E_{p-k})]^2 2E_k 2E_{p-k}[(E_p - E_k - E_{p-k})]^2}, \tag{A1}$$

which in the limit of $x \to 1$ reduces to

$$u(x) \propto \int d^2k_\perp \frac{4m_H^2 \,\text{Tr}[\not k \frac{1}{2}\not p(\not p - \not k)\frac{1}{2}\not p]}{p^2[-(m_H^2 + k_\perp^2)^2/8(1-x)p]^2 4(1-x)p^2[-(m_H^2+k_\perp^2)/8(1-x)p]^2}$$

$$\propto (1-x)\int d^2k_\perp \frac{m_H^2}{(m_H^2+k_\perp^2)^2} = (1-x)\ . \tag{A2}$$

The following general features of this calculation should be noted. The reduced trace factor explicitly cancels the denominators involving the gluon energy E_G. A creation-annihilation factor $2E_{p-k} \propto 2(1-x)p$ must be included for the lower quark. The final answer is dimensionless as it must be. These features are completely general, barring peculiar cancellations and spin correlations. In fact, such spin correlations could, however, be at the root of the difference between the $\mathcal{P}$- and $\mathfrak{N}$-quark wave functions.

APPENDIX B

This appendix presents general rules for the threshold behavior of hadronic bremsstrahlung probabilities. We define $G_{H'/H}(z)$ to be the probability that hadron H can emit a hadron, H', carrying a fraction z of H's initial linear momentum. Our results are for the behavior of $G(z)$ as $z \to 1$.

The primary ingredient is Eq. (3). Suppose that in order to emit H', H must be in a state of n quarks (plus antiquarks). The threshold suppression of an n-quark state distribution function is

$$u(x) \propto (1-x)^{2n-3}, \tag{B1}$$

corresponding to an n-quark state for m factors of $1/t^{n-1}$.

Quark emission from such a state may also be viewed, according to Eq. (3), as proceeding in two steps:

(i) emission of the secondary hadron H' according to $G_{H'/H}(z)$, which we suppose behaves as

$$G_{H'/H}(z) \propto (1-z)^m, \tag{B2}$$

as $z \to 1$,

(ii) followed by emission of the quark from the valence (i.e., lowest quark number) state of H'. Such emissions are characterized by

$$\hat{u}(x) \propto (1-x)^{2p_{H'}-3}\ , \tag{B3}$$

with $p_{H'} = 2$ for H' = meson, $p_{H'} = 3$ for H' = baryon. Consistency of the two approaches requires, according to Eq. (3), that

$$2n - 3 = m + 2p_{H'} - 3 + 1, \qquad m = 2(n - p_{H'}) - 1\ , \tag{B4}$$

the extra +1 arising from the limits of the integral in Eq. (3).

Clearly the minimal quark-state number n for $H \to H'$ depends upon how many constituents the emitted hadron has in common with the initial hadron. The following cases occur:

(a) H = baryon ($p_H = 3$), H' = meson ($p_{H'} = 2$). Two possible cases occur: (i) The meson's quark is common to the simple 3-quark valence part of H in which case only the antiquark must be supplied by the sea ($q\bar{q}$) component of H. A five-quark $qqqq\bar{q}$ H state is sufficient.

$$n = 5 \Rightarrow m = 5,$$

$$G(z) \propto (1-z)^5.$$

Example: $p(\mathcal{P}\mathcal{P}\mathfrak{N}) \to \pi^+(\mathcal{P}\bar{\mathfrak{N}})$. (ii) The meson H''s quark is not contained in the valence part of H. A seven-quark $qqqq\bar{q}q\bar{q}$ H state is needed.

$$n = 7 \Rightarrow m = 9,$$

$$G(z) \propto (1-z)^9.$$

Example: $p(\mathcal{P}\mathcal{P}\mathfrak{N}) \to K^-(\bar{\mathcal{P}}\lambda)$.

(b) H = baryon ($p_H = 3$), H' = baryon ($p_{H'} = 3$). Three nontrivial cases are possible depending upon whether H' has $n_c = 2$, 1, or 0 quarks in common with the valence part of H. The minimal H state required consists of

$$n = p_H + 2(p_{H'} - n_c)$$

quarks and antiquarks, so that

$$m = 4(3 - n_c) - 1.$$

Examples:

$$p(\mathcal{P}\mathcal{P}\mathfrak{N}) \to \Sigma^0(\mathcal{P}\mathfrak{N}\lambda),\quad n_c = 2,\quad m = 3$$

$$\left.\begin{array}{l} p(\mathcal{P}\mathcal{P}\mathfrak{N}) \to \Sigma^-(\mathfrak{N}\mathfrak{N}\lambda) \\ p(\mathcal{P}\mathcal{P}\mathfrak{N}) \to \Xi^0(\mathcal{P}\lambda\lambda) \end{array}\right\},\quad n_c = 1,\quad m = 7$$

or

$$\left.\begin{array}{l} p(\mathcal{P}\mathcal{P}\mathfrak{N}) \to \Omega^-(\lambda\lambda\lambda) \\ \text{baryon} \to \text{antibaryon} \end{array}\right\},\quad n_c = 0,\quad m = 11.$$

(c) $H=$ meson ($p_H=2$), $H'=$ meson ($p_{H'}=2$). The minimal H state required consists of

$$n=p_H+2(p_{H'}-n_c)$$

quarks and antiquarks (n_c is again the number of common constituents) so that

$$m=4(2-n_c)-1.$$

Examples:

$$\pi^+(\mathcal{P}\bar{\mathcal{N}})\to K^+(\mathcal{P}\bar{\lambda}),\ n_c=1,\ m=3$$

$$\left.\begin{array}{l}\pi^+(\mathcal{P}\bar{\mathcal{N}})\to\pi^-(\mathcal{N}\bar{\mathcal{P}})\\ K^+(\mathcal{P}\bar{\lambda})\to\pi^-(\mathcal{N}\bar{\mathcal{P}})\end{array}\right\},\ n_c=0,\ m=7\ .$$

(d) $H=$ meson ($p_H=2$), $H'=$ baryon ($p_{H'}=3$).

$$n=p_H+2(p_{H'}-n_c)\ ,$$

$$m=4(p_{H'}-n_c)+2(p_H-p_{H'})-1$$

$$=4(3-n_c)-3.$$

Examples:

$$\pi^+(\mathcal{P}\bar{\mathcal{N}})\to p(\mathcal{P}\mathcal{P}\mathcal{N}),\ n_c=1,\ m=5$$

$$\pi^+(\mathcal{P}\bar{\mathcal{N}})\to\Omega^-(\lambda\lambda\lambda),\ n_c=0,\ m=9.$$

Thus, to summarize, if hadron H has p_H quarks (plus antiquarks) in its valence state, while H' has $p_{H'}$, and n_c of these quarks are common to the H, H' valence states, the minimal quark state of H necessary to emit the H' valence state consists of

$$n=p_H+2(p_{H'}-n_c) \tag{B5}$$

quarks plus antiquarks. The behavior of $G(z)$ is then given by

$$\begin{aligned} m&=2(n-p_{H'})-1\\ &=4(p_{H'}-n_c)+2(p_H-p_{H'})-1. \end{aligned} \tag{B6}$$

There are a number of possible processes in which such bremsstrahlung behavior might be important (other than high-transverse-momentum processes); for instance, in the fragmentation region of normal inclusive scattering, e.g., $p+p\to H+X$. One proton might first emit H in a bremsstrahlung process, following which either H or the stuff remaining after p emits H would then scatter from the other proton. Evidence for such processes is being examined.[17]

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[1]For an up-to-date summary of experimental data see E. D. Bloom, in *Experiments on High Energy Particle Collisions—1973*, proceedings of the international conference on new results from experiments on high energy particle collisions, Vanderbilt University, 1973, edited by Robert S. Panvini (A.I.P., New York, 1973), p. 277.

[2]R. McElhaney and S. F. Tuan, Phys. Rev. D 8, 2267 (1973); V. Barger, University of Wisconsin report, 1973 (unpublished); P. V. Landshoff, J. C. Polkinghorne, Nucl. Phys. B19, 432 (1970); R. W. Fidler, Phys. Lett. 46B, 455 (1973).

[3]See, R. Blankenbecler, S. J. Brodsky, J. F. Gunion, and R. Savit, Phys. Rev. D 8, 4117 (1973), and work in preparation.

[4]This notation has a long history, beginning with J. Kuti and V. Weisskopf [Phys. Rev. D 4, 3418 (1971)] and H. Harari [Phys. Rev. Lett. 24, 286 (1970)]. For more recent discussions see M. Chaichian *et al.*, Nucl. Phys. B51, 221 (1973). See also P. V. Landshoff and J. C. Polkinghorne, *ibid.* B28, 225 (1971).

[5]It is possible that a higher particle-number state could be required. For instance, if one dynamically interprets (as in what follows) the usual "background" duality diagrams, the $(1-x)^7$ of Eq. (8) is altered to $(1-x)^9$.

[6]S. J. Brodsky, R. Blankenbecler, and J. F. Gunion, Phys. Lett. 42B, 461 (1973). See J. F. Gunion, Univ. of Pittsburgh, Report No. PITT-114, 1973 (unpublished) for further references.

[7]M. Banner *et al.*, Phys. Lett. 44B, 537 (1973); F. W. Büsser *et al.*, *ibid.* 46B, 471 (1973); B. Alper *et al.*, *ibid.* 44B, 521, 527 (1973); J. W. Cronin *et al.*, Phys. Rev. Lett. 31, 1426 (1973).

[8]S. J. Brodsky and G. Farrar, Phys. Rev. Lett. 31, 1153 (1973).

[9]S. D. Drell and T.-M. Yan, Phys. Rev. Lett. 24, 181 (1970).

[10]This argument from the Drell-Yan relation is due to S. J. Brodsky and G. Farrar (private communication).

[11]See for example Ref. 4 and also H. J. Lipkin, Report No. WIS-73/8 (unpublished).

[12]J. F. Gunion, in preparation.

[13]One should note that for $B=$ proton, $r_{\mathcal{N}}(x)/r_{\mathcal{P}}(x)\to 0$ as $x\to 1$, simply because $\hat{u}^p_{\mathcal{N}}/\hat{u}^p_{\mathcal{P}}\to 0$ as $x\to 1$ if $F_2^n/F_2^p\to\frac{1}{4}$ as $x\to 1$. However, other baryons, notably the neutron, yield just the opposite behavior.

[14]See Ref. 1 and references therein for the data available.

[15]This possibility has long been common knowledge, but should perhaps be referenced to M. Kugler, but a dynamical mechanism has not been proposed.

[16]D. H. Perkins, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 189.

[17]S. J. Brodsky and R. Blankenbecler (private communication).