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Koba-Nielsen-Olesen scaling hypothesis and the total multiplicity distribution for produced hadrons in high-energy pp collisions*

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The observed high-energy scaling behavior of the charged-particle multiplicity distribution in pp collisions is shown to be consistent with the hypothesis that the total multiplicity distribution for all the produced hadrons satisfies the KNO scaling prediction at present energies. This hypothesis, plus simple, experimentally valid assumptions concerning leading-particle effects and neutral- to charged-particle correlations, is found to yield the measured multiplicity characteristics of particle production in high-energy pp collisions.

The theoretically motivated suggestion by Koba, Nielsen, and Olesen (KNO) that secondary-particle multiplicity distributions arising in high-energy inelastic hadron-hadron collisions should asymptotically follow scaling laws of the form[1]

$$P(n)=\frac{\sigma_n}{\sigma_{\text{inel}}}\xrightarrow[s\to\infty]{}\frac{1}{\langle n\rangle}\,\psi\left(\frac{n}{\langle n\rangle}\right), \tag{1}$$

with functions ψ which are energy-independent, has been shown to be in unexpectedly good agreement with charged-particle multiplicity data in the NAL energy range (50–300 GeV).[2] Moreover, a simple empirical modification of Eq. (1) has been found to be sufficient to extend this type of scaling to the description of low-energy data as well. Explicitly, scaling laws of the form

$$P(n)=\frac{1}{\langle n\rangle-\alpha}\,\psi\left(\frac{n-\alpha}{\langle n\rangle-\alpha}\right), \tag{2}$$

with energy-independent parameters α, have been successful in describing charged-particle multiplicity data from pp and π^+p collisions over a range of incident momenta spanning approximately two orders of magnitude.[3] (The optimum values of α are found to depend on the type of collision being fitted, but are of order unity for each of the above reactions; in particular $\alpha\approx0.9$ for pp collisions.)

One consequence of Eq. (2) is that the successive moments of the multiplicity distributions should satisfy equations of the form[4]

$$\langle(n-\alpha)^q\rangle=C_q\langle n-\alpha\rangle^q \quad\text{for } q=2,3,4,\ldots, \tag{3}$$

with the C_q being energy-independent constants. Consequently, Eq. (2) may be regarded as a generalization and refinement of an earlier observation by Wróblewski that the dispersion of the charged-particle multiplicity distribution could be written as a linear function of the mean of this distribution[5]:

$$D_{\text{ch}}=(\langle n_{\text{ch}}^2\rangle-\langle n_{\text{ch}}\rangle^2)^{1/2}=A(\langle n_{\text{ch}}\rangle-B)\,. \tag{4}$$

For pp collisions, Wróblewski found the parameter values $A=0.58$ and $B=1$.

Since Eq. (2) asymptotically approaches Eq. (1), the over-all conclusion which emerges is that the KNO scaling hypothesis is impressively well satisfied by all available charged-particle production data. Moreover, studies of neutral-particle production have also shown these data to be consistent with the KNO hypothesis, although, at present, the statistical accuracy of the available data on neutral-particle production is much less than that of the corresponding charged-particle data.[6]

Nevertheless, a very troublesome question arises as soon as one attempts to interpret the significance of this level of agreement between prediction and theory. This difficulty arises because the central assumption of the KNO derivation, namely, exact Feynman scaling at asymptotic energies, is very badly violated at the energies for which Eq. (2) is found to apply.[7] Consequently, there is no obvious explanation why such a simple modification of exact KNO scaling should suffice to fit low-energy data.

In this paper we wish to investigate whether the observed multiplicity data for the high-energy production of hadrons are consistent with an hypothesis which avoids this problem—namely, that the over-all multiplicity distribution for all the produced hadrons (neutral as well as charged) satisfies the KNO hypothesis at present energies. Naturally, this type of scaling behavior would require an explanation quite different from that presented by KNO, but the empirical success of Eq. (2) in itself strongly suggests the need for a different theoretical basis for the KNO hypothesis than the particular arguments which led these authors originally to suggest it. Our procedure will be to investigate whether the above hypothesis, plus reasonable assumptions concerning leading-particle effects and for the production of neutrals, is sufficient to reproduce the observed behavior of the charged-particle multiplicity distribution, and also the experimental neutral- to charged-particle correlation data.[8]

We begin with the assumption that, over the energy range spanned by present experiments, a general inelastic proton-proton collision may be adequately described by the equation

$$p+p \to 2 \text{ baryons} + n \text{ mesons}. \qquad (5)$$

We will regard the final-state baryons as leading particles, while the mesons will be assumed to be produced particles. We will not distinguish between mesons which are fragmentation products, and those which are created approximately at rest in the collision center-of-mass system. Consequently, we will be assuming that the combined effect of all the dynamical processes which lead to the high-energy production of hadrons is to produce a smooth over-all hadron multiplicity distribution to which we can attempt to apply the KNO scaling hypothesis. Finally, while in this note we will confine our explicit discussion to pp collisions only, for which the available experimental data are the most extensive, an analogous analysis could clearly also be applied to the other measured types of hadron-hadron collisions.

We will calculate the net probability for any particular configuration of final-state particles from the product of three individual probabilities which will be discussed separately.

(1) $P(n)$—the probability that a total of n mesons (both charged and neutral) will be produced in a given inelastic pp collision. Our goal will be to investigate whether the assumption that the distribution $P(n)$ obeys the KNO scaling hypothesis [Eq. (1)] at present energies is compatible with the observation that only in an empirically modified form [Eq. (2)] does such a scaling law describe charged-particle data at these same energies. More precisely, we will examine whether the assumption

$$\langle n^2\rangle = C_2\langle n\rangle^2, \qquad (6)$$

with an energy-independent constant C_2, leads naturally to Eq. (4), when coupled with the observed degree of correlation between neutral- and charged-particle production at these energies. We choose to confine our analysis to just the lowest moments of the multiplicity distribution because these are experimentally the most accurately determined, and, as a result, Eq. (4) represents an economical summary of the observed regularities in the energy variation of the charged-particle multiplicity distribution arising in pp collisions.

The particular parameterization which we will here employ for $P(n)$ is the following[9]:

$$P(n) \propto \int_{n-1}^{n} z^{\alpha} e^{-\beta z^2} dz \qquad (7)$$

for $n=1, 2, 3, \ldots$. [We note that $P(0)=0$, since we are restricting our analysis to inelastic collisions only.] The over-all normalization, plus the numerical values of the parameters α and β, is fixed at each energy by Eq. (6) plus the two constraints

$$\sum_n P(n) = 1, \qquad (8a)$$

$$\sum_n nP(n) = \langle n\rangle. \qquad (8b)$$

To avoid introducing a largely superfluous assumption concerning the precise energy dependence of the average total multiplicity $\langle n\rangle$, throughout this paper $\langle n\rangle$ will be employed as a measure of the collision energy. The constant C_2 in Eq. (6) was treated as an adjustable parameter; the curves to be presented were calculated using the fitted value $C_2=1.3$.

The known information concerning the correlation between neutral- and charged-meson production in pp collisions was incorporated via the following distributions:

(2) $P(q)$—the probability that the net charge of all the mesons produced in a given pp collision will equal q. Through over-all charge conservation, we may estimate $P(q)$ from measurements of the average number of leading protons in such collisions. This quantity, which is not at present experimentally very well determined, appears to decrease slowly with energy from a value of approximately 1.7 at 12 GeV/c to about 1.2 at the highest NAL energies (~400 GeV).[10] If we define P_{cex} to be the probability that in a given pp collision one of the incident protons will become a neutral baryon, and if we assume that each of the incident protons fragments independently, then $P(q)$ is binomial in form and is given by

$$P(q) = \frac{2}{q!(2-q)!} P_{\text{cex}}{}^{q}(1-P_{\text{cex}})^{2-q} \qquad (9)$$

for $q=0, 1, 2$. The probability P_{cex} may be directly calculated using the equation

$$P_{\text{cex}} = 1 - \tfrac{1}{2}\langle P_l\rangle, \qquad (10)$$

where $\langle P_l\rangle$ is the average number of leading protons at a given collision energy. Since the measured data are consistent with a linear decrease of $\langle P_l\rangle$ versus $\log(s)$ over the range of energies of interest to this analysis, we will assume a relationship of the form

$$P_{\text{cex}} = a\langle n\rangle + b. \qquad (11)$$

(This implies that the parameter $\langle n\rangle$ may also be regarded as a linear function of log(s) to within the accuracy of the above assumption concerning the energy variation of $\langle P_l\rangle$.) Deviations from the simple linear relationship assumed in Eq. (11) must necessarily be present at both small and large $\langle n\rangle$ values by virtue of the bounded nature of the probability P_{cex}. (As $\langle n\rangle\to 0$, the value of P_{cex} must smoothly approach 0, while as $\langle n\rangle\to\infty$, it is not unreasonable to anticipate that P_{cex} may approach $\frac{1}{2}$.) While it would be straightforward to modify Eq. (11) in a phenomenological manner to include these effects, they are not of significance in the energy range of interest to us here; therefore, in the interest of over-all simplicity, such refinements have not been introduced. The values of the constants a and b in Eq. (11) are fixed by the measured $\langle P_l\rangle$ values; the values employed in our analysis were $a=0.025$, and $b=0.1$.

(3) $P(n_0|n,q)$—the probability that n_0 neutral mesons will be produced in a pp collision in which a total of n mesons are emitted and have a net charge q. Given values for n_0, n, and q, we may directly calculate the number of positive and negative mesons which were also produced via the relationships

$$n_+=\tfrac{1}{2}(n-n_0+q)\,, \tag{12a}$$

$$n_-=\tfrac{1}{2}(n-n_0-q)\,. \tag{12b}$$

(Only n_0, n, and q combinations which yield nonnegative, integral values of n_+ and n_- are permissible.)

The existence of hadron resonances, plus the available experimental data concerning π^0/π^- correlations, both clearly indicate the presence of positive correlations between the charged and neutral mesons produced in any hadron-hadron collision.[8] Moreover, the very large number of low-mass, strongly decaying, resonant states which are known to exist strongly suggests that the net effect of all the possible production processes which may physically occur in a high-energy pp collision is to result in an over-all neutral- to charged-meson ratio which can be adequately reproduced by a very simple model employing just the following two assumptions:

(i) Each produced meson in a given event has an equal probability for being positive, negative, or neutral.

(ii) The resulting probability for any particular configuration of charged mesons is, for fixed n and q, simply proportional to the number of distinguishable charge permutations, viz.,

$$N(n_+,n_0,n_-)=\frac{(n_++n_0+n_-)!}{n_+!\,n_0!\,n_-!}\,, \tag{13}$$

where n_+ and n_- are related to n_0, n, and q by Eqs. (12a) and (12b).[11]

A parameter-free expression for $P(n_0|n,q)$ subsequently results:

$$P(n_0|n,q)=\frac{N(n_+,n_0,n_-)}{\sum_{n_0}N(n_+,n_0,n_-)}\,, \tag{14}$$

where $N(n_+,n_0,n_-)$ is given by Eq. (13), and the sum includes only those n_0 values for which Eqs. (12a) and (12b) apply.

The combined probability for any particular configuration of final-state mesons is given by the product of the preceding three separate probabilities:

$$P(n_+,n_0,n_-)=P(n_0|\,n,q)P(n)P(q)\,. \tag{15}$$

The consequences of this expression for $P(n_+,n_0,n_-)$ have been investigated using Monte Carlo methods. The figures to be presented were obtained using samples of 10 000 generated events at each of eight uniformly spaced values of $\langle n\rangle$ ranging from 2 through 16.

In Fig. 1 we present the resulting D_{ch} versus $\langle n_{\text{ch}}\rangle$ plot. The straight line represents the results of a least squares fit to the eight data points, and has a χ^2 of 3.6. Clearly, the Wróblewski parameterization is well reproduced. (The value of 0.9 for the x intercept is in agreement with the aforementioned recent fit to the pp data.[3]) We have therefore succeeded in our goal of obtaining Eq. (4) for n_{ch} from assuming Eq. (6) for n.

Our calculation also yields results which are consistent with the other known multiplicity characteristics of high-energy pp collisions. This agreement will be discussed in an essentially qualitative manner, however, since experimentally these other features of the data are much less well determined than D_{ch} or $\langle n_{\text{ch}}\rangle$, and consequently place far weaker quantitative constraints on any

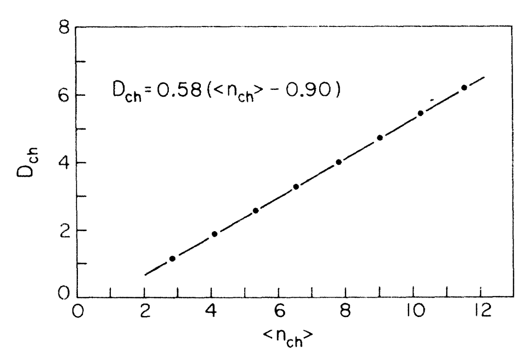

FIG. 1. The dispersion (D_{ch}) of the generated charged-particle multiplicity distribution plotted versus the mean ($\langle n_{\text{ch}}\rangle$) of this distribution. The points were calculated using the model described in the text; the statistical error bars are in each case less than the size of the plotted point. The straight line is a least-squares fit to these generated points.

empirical model.

In Fig. 2 we present the various mean multiplicities for secondary particles plotted versus the parameter $\langle n\rangle$. The following observations can be made:

(1) $\langle n_{\text{ch}}\rangle$ is an effectively linear function of $\langle n\rangle$ over the range presented. Therefore, since $\langle n_{\text{ch}}\rangle$ is measured to be an approximately linear function of log(s), our assumption that $\langle n\rangle$ was also an approximately linear function of log(s) is internally consistent and experimentally justified.

(2) $\langle p_l\rangle$ is 1.6 for $\langle n_{\text{ch}}\rangle=4$ and 1.2 for $\langle n_{\text{ch}}\rangle=9$; consequently, Eq. (11), with the indicated values of a and b, successfully reproduces the measured data.

(3) $\langle n_+\rangle$ is always greater than $\langle n_-\rangle$, in agreement with what is experimentally observed. The relative magnitudes of these averages are also consistent with the measured values except that for large $\langle n\rangle$, the curves in Fig. 2 continue to separate, whereas the measured values appear to approach a constant separation. This discrepancy, however, is caused entirely by our neglect of the high-energy flattening of the $\langle p_l\rangle$ curve which sets in at approximately $\langle n_{\text{ch}}\rangle=9$. When this effect is included, the $\langle n_+\rangle$ and $\langle n_-\rangle$ curves generated from Eq. (15) also rise in parallel.

(4) $\langle n_0\rangle$ is approximately equal to $\langle n_+\rangle$ for $\langle n_{\text{ch}}\rangle$ values from 4 to 9. This is also in agreement with the experimental data. Above $\langle n_{\text{ch}}\rangle=9$, the combined influences of increasing η production, plus the effect of the flattening out of $\langle p_l\rangle$, should combine to preserve this approximate equality between $\langle n_0\rangle$ and $\langle n_+\rangle$ (In contrast to the behavior of the curves in Fig. 2).

The remaining feature of the multiplicity data to be compared is the $\langle n_0\rangle$ to n_- correlation results. In Fig. 3 we plot this correlation for our generated data. The trend of these data is in complete agreement with that observed experimentally. It should be recalled that no low-multiplicity clustering was built into Eq. (14) even though the presence of such clusters in physical pp data is well established. The effect of introducing such clusters on Fig. 3 was investigated using alternate expressions for Eq. (14). The following assumptions were examined:

(i) All produced particles are ρ mesons which subsequently decay into π mesons (an example of 2-particle clustering).

(ii) All produced particles are A mesons which subsequently decay by the chain $A\to\rho\pi\to\pi\pi\pi$ (an example of 3-particle clustering).

In either case the same general trends were observed that are displayed in Fig. 3. The only significant difference is that the fewer the number of particles making up the clusters, the steeper is the rise of the correlation function. (The points displayed in Fig. 3 can be regarded as resulting from the assumption of a single n-body cluster per event.) Consequently, the effect of adding the effects of low-multiplicity cluster-

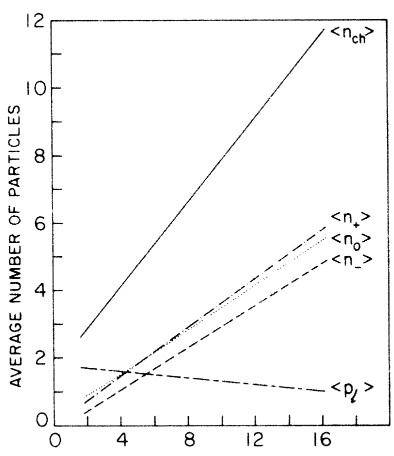

FIG. 2. The dependence of the average number of final-state particles of a particular type upon $\langle n\rangle$, the average total number of produced mesons; $\langle n_{\text{ch}}\rangle$ is the average number of charged secondaries (including leading protons); $\langle p_l\rangle$ is the average number of leading protons; $\langle n_+\rangle$, $\langle n_0\rangle$, and $\langle n_-\rangle$ refer, respectively, to the average number of positive, neutral, and negative mesons produced.

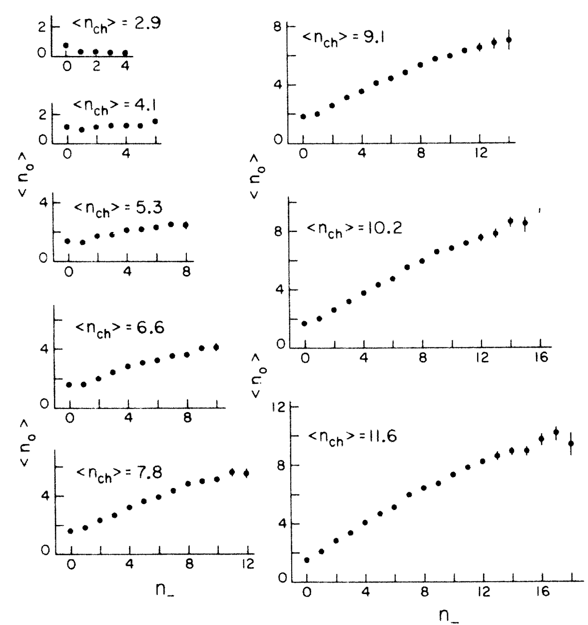

FIG. 3. The correlation between $\langle n_0\rangle$, the average number of neutral mesons produced, and n_-, the number of negative mesons, for the model described in the text. Where error bars are not shown, they are less than the size of the plotted point.

ing to our calculation would be to increase the slope of the points displayed in Fig. 3. (The slope of the measured data does appear steeper than the rates of rise in Fig. 3, although the experimental error bars are too large to establish this point solidly.)[12]

In summary, therefore, we conclude that all available data concerning secondary hadron multiplicities in high-energy pp collisions are consistent with the hypothesis that the over-all multiplicity distribution for all the produced hadrons satisfies the KNO scaling hypothesis at finite energies.[31] Besides being a useful observation from the point of view of those attempting to construct simple empirical models of high-energy production processes, this conclusion is also important because it would appear to require the presence of long-range correlations between the secondary hadrons produced in high-energy collisions. [We note that Eq. (6) implies that $f_2 = (C_2 - 1)\langle n\rangle^2 - \langle n\rangle$, and hence asymptotically this correlation moment will increase as $\langle n\rangle^2$, rather than as $\langle n\rangle$.]

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[1]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1972).

[2]P. Olesen, Phys. Lett. 41B, 602 (1972); P. Slattery, Phys. Rev. Lett. 29, 1624 (1972); Phys. Rev. D 7, 2073 (1973).

[3]A. J. Buras, J. Dias de Deus, and R. Møller, Phys. Lett. 47B, 251 (1973); R. Møller, Nucl. Phys. B74, 145 (1974). In addition, F. T. Dao, J. Lach, and J. Whitmore [Phys. Lett. 45B, 513 (1973)] have shown that charged-particle multiplicity distributions from relatively low-energy $\bar{p}p$ collisions are also consistent with the KNO scaling hypothesis.

[4]Strictly speaking, Eq. (3) is an exact consequence of Eq. (2) only if n is treated as a continuous, rather than as a discrete, variable. For a discussion of some of the mathematical subtleties of the KNO scaling hypothesis, see A. Chodos, M. Rubin, and R. L. Sugar, Phys. Rev. D 8, 1620 (1973).

[5]A. Wróblewski, Acta Phys. Pol. B4, 857 (1973). The proposal that $D_{\mathrm{ch}} = \frac{1}{2}\langle n_{\mathrm{ch}}\rangle$ was initially made by P. K. Malhotra [Nucl. Phys. 46, 559 (1963)], and was later investigated by O. Czyżewski and K. Rybicki [Nucl. Phys. B47, 633 (1972)].

[6]The experimentally observed correlation between neutral and charged pions was shown to be consistent with that expected on the basis of the KNO scaling hypothesis by F. T. Dao and J. Whitmore [Phys. Lett. 46B, 252 (1973)]. Similar results were obtained for K^0 and Λ^0 production by D. Cohen [Phys. Lett. 47B, 457 (1973)].

[7]The presence of significant and systematic low-energy deviations from exact Feynman scaling in the central region of secondary hadron production has been pointed out by T. Ferbel [Phys. Rev. D 8, 2321 (1973)].

[8]Throughout this paper we will employ the label "leading particle" to denote a final-state baryon (proton or neutron) which we associate with one of the colliding protons; we use this label for the purpose of contrasting these outgoing particles with "produced particles" (pions). The hypothesis that the parameter α is related to leading-particle effects has been independently investigated by A. Bassetto and J. Dias de Deus [Nuovo Cimento Lett. 9, 525 (1974)]. The presence of a positive correlation between $\langle n_{\pi^0}\rangle$ and $\langle n_{\pi^-}\rangle$ in high-energy hadron collisions was noted by F. T. Dao and J. Whitmore (Ref. 5). See also the reviews by J. Whitmore [Phys. Rep. 10C, 273 (1974)] and by H. Bøggild and T. Ferbel [Annu. Rev. Nucl. Sci. (to be published)].

[9]The mathematical form assumed for $P(n)$ is similar to that employed by H. Weisberg [Phys. Rev. D 8, 331 (1973)], and earlier by G. Bozóki, E. Gombosi, M. Posch, and L. Vanicsek [Nuovo Cimento 64A, 881 (1969)], to describe the corresponding charged-particle distribution function $P(n_{\mathrm{ch}})$. No fundamental significance is here being attributed to this particular functional form. It was chosen solely for its simplicity and it is not unlikely that the higher C_q values (for $q>2$) predicted by Eq. (7) will not precisely describe the experimental data.

[10]M. Antinucci, A. Bertin, P. Capiluppi, M. D'Agostino-Bruno, A. M. Rossi, G. Vannini, G. Giacomelli, and A. Bussiere, Nuovo Cimento Lett. 6, 121 (1973). The average number of final-state neutrons plus antineutrons produced in pp collisions at 303 GeV/c has been measured to be 0.8 ± 0.2 by E. Malamud *et al.*, Bull. Am. Phys. Soc. 19, 467 (1974). At CERN ISR energies (>500 GeV), the increasing production of baryon-antibaryon pairs raises the over-all average number of final-state protons above the average number of just *leading* protons, which is the quantity of interest in the present context.

[11]More rigorously, assumption (ii) is a reasonable consequence of assumption (i) only if just one type of meson exists. Consequently, a more accurate description of the experimental data would be expected from a model in which the above two assumptions were applied separately to pions and kaons. Furthermore, assumption (i) ignores those final-state pions which result from the electromagnetically decaying η meson. Since, instead of being in the vicinity of $\frac{1}{2}$, the neutral- to charged-pion ratio for the products of η decay is approximately $\frac{8}{3}$, any appreciable amount of η production at high energies will have significant effects. Nevertheless, the present state of our knowledge of neutral-particle production in high-energy pp collisions does not really justify introducing the above refinements. Consequently, in the interests of simplicity, the influence of kaons and η mesons was not explicitly included.

[12]It is perhaps worth emphasizing that the model em-

ployed in the text has been kept intentionally as simple as possible. For this reason, no explicit comparison with experimental data has been included. Reference 11 and the comments made in the text concerning the consequences of introducing low-multiplicity clustering into the model described were intended to support the contention that the model has more than enough flexibility to provide totally satisfactory fits to all of the presently available experimental data. Indeed, it is this very flexibility which makes such an experimental comparison fundamentally uninteresting at this time; the quality of the neutral-multiplicity data is simply not adequate at present to yield any significant new insights from such a parameter-fitting exercise.

[13]For a differently motivated, but technically somewhat similar, attempt to reconcile KNO scaling with available multiplicity data, see L.-N. Chang and N.-P. Chang, Phys. Rev. D 9, 660 (1974).

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Spin-rotation effects in meson-nucleon scattering*

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A discussion is given of the theoretical description of spin-rotation effects in the scattering of a spinless projectile by a spin-1/2 target. Description of these effects in terms of the polarization parameter P and the Wolfenstein rotation angle β is shown to be especially convenient for a variety of experimental and theoretical purposes. The information obtainable from rescattering experiments is shown to be maximized when these are cast in the form of null experiments for the deviation of β from its anticipated value.

Wolfenstein[1,2] advocated measurement of the independent components of the polarization of scattered polarized particles, so that from the rotation of the polarization vector, more information about the scattering process could be deduced than is obtainable from the parameter P alone. As a result of many technical advances with polarized targets and particle detectors, rescattering experiments are becoming increasingly attractive. A review of the theory of spin-rotation experiments is therefore timely, and we present this discussion to call attention once again to their usefulness for eliminating a continuum ambiguity in the scattering amplitude. In order that the most effective experimental configuration be chosen, we emphasize that theoretical studies in conjunction with such experiments are especially important.

In his original paper,[1] Wolfenstein defined a spin-rotation angle β and showed how this angle was related to observables as well as to the scattering amplitude. In most subsequent work, however, the discussion has been in terms of "R and A parameters" also introduced by Wolfenstein. One aim of our work is to point out, through a review of scattering from a polarized target, how useful the Wolfenstein angle β is, both for characterizing the structure of the scattering amplitude and for describing the results of experiments. As shown below, the angle β is, in fact, the relative phase of scattering eigenstates. Furthermore, kinematical effects associated, for example, with relativistic transformations can be accounted for directly as additions to β. By focusing on β, it is also easy to see how to best arrange experiments to obtain significant information about the scattering amplitudes. We show that if one uses previous theoretical estimates of the spin-rotation angle β and constructs null experiments, it is possible to minimize error propagation from the measured polarization component to the rotation angle.

In pion-nucleon scattering, determination of the scattering amplitudes from measured quantities has special features arising from isospin conservation. In the last part of this paper, we show how use of the rotation angle simplifies formal discussion of this scattering process, and permits a simple yet complete analysis of the ambiguities which may remain in amplitudes constructed from complete or partial data.

Polarization phenomena in the nonrelativistic scattering of a spin-$\frac{1}{2}$ beam by a spinless target were discussed in detail by Wolfenstein. His re-