

that resonance production could swamp the η 's and K 's with pions. While particle-production ratios thus lose some of their appeal for measuring the average quark-charge leakage \mathcal{Q} , we have suggested more general properties of the Pomeron that may be related to such leakage.

One helpful experiment would be to measure the production of resonances directly (for example, via their leptonic decays.) While one could not

check whether these resonances were produced "directly" or as decay products of some other resonances, one would at least expect the production of (say) ϕ and ρ to be much more comparable to one another than that of (say) η' and π if the ideas presented here are valid.

One of us (J. R.) is indebted to Dr. M. Einhorn for an enlightening conversation.

*Work supported in part by the U. S. Atomic Energy Commission under Contracts Nos. AT(11-1)-68 and AT(11-1)-1764.

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Factorization and shielding of hard Regge surfaces*

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(Received 13 May 1974)

Hard branch-point trajectories require shielding cuts in order to be compatible with elastic unitarity. For coupled channels like $\pi\bar{\pi}$ and $N\bar{N}$, shielding of the two-pion threshold is required for all amplitudes if the hard branch point factorizes. Explicit examples are given, for the shielding of the lowest as well as the higher thresholds.

I. INTRODUCTION

In previous papers,¹ one of us has shown how a Regge trajectory $j = \alpha(t)$ can be made compatible with two-particle t -channel unitarity in case it is not a simple pole surface with the appropriate branch point at the threshold $t = t_0$. Shielding cuts were introduced, which make the limit $j = \alpha(t)$ and the continuation of the partial-wave amplitude $F(t, j)$ around the branch point $t = t_0$ noninterchangeable. These shielding cuts may well be important phenomenologically at medium high ener-

gies. In Ref. 1 the discussion was restricted to elastic amplitudes like $\pi\bar{\pi} \rightarrow \pi\bar{\pi}$, but it is of interest to extend the shielding mechanism to sets of coupled amplitudes.

It is the purpose of this paper to discuss the shielding problem in cases with two or more coupled channels. In order to restrict our considerations to the essential points, we assume that the same singular surface $j = \alpha(t)$ is present in all of the coupled amplitudes, and that it has a t -independent character. We are mainly interested in the lowest threshold (e.g., $t_0 = 4m_\pi^2$) of a set of

coupled amplitudes like, for example, $\pi\bar{\pi} \rightarrow \pi\bar{\pi}$, $\pi\bar{\pi} \rightarrow N\bar{N}$, and $N\bar{N} \rightarrow N\bar{N}$. We find that it is always necessary to have a shielding cut for this threshold in the amplitude $\pi\bar{\pi} \rightarrow N\bar{N}$, but the requirements for $N\bar{N} \rightarrow N\bar{N}$ depend upon the factorization property of the coupled amplitudes in the limit $j \rightarrow \alpha(t)$. We show that for complete factorization an appropriate shielding cut must be present in all amplitudes, and we give an explicit model which satisfies the coupled unitarity equations. We also consider briefly the shielding of higher two-particle thresholds.

From our considerations it is apparent that the factorization of a hard singular surface $j = \alpha(t)$ in the complex angular momentum plane should be discussed in connection with the threshold properties of the function $\alpha(t)$ and with the shielding requirements.

II. SINGLE CHANNEL

It is quite possible that the Pomeron is not an ordinary Regge-pole trajectory $\alpha(t)$ which interpolates physical particle states and/or resonances for positive values of t . Ordinary Regge-pole surfaces generally become complex above the lowest threshold in the t channel at $t = t_0$. This property of a trajectory $\alpha(t)$ is related to the fact that single-particle "states" with $t \geq t_0$ are unstable. They are resonances described by complex poles in the secondary Riemann sheets of the partial-wave amplitude $F(t, j)$.

The existence of appropriate branch points in the pole trajectory $\alpha(t)$ makes it possible for the amplitude $F(t, j)$ to be compatible with the unitarity condition in the t channel. In particular, the two-particle unitarity relation

$$F^{11-1}(t, j) - F^{-1}(t, j) = 2i\rho(t), \quad (1)$$

with

$$\rho(t) = \left(\frac{t - t_0}{t} \right)^{1/2}, \quad (2)$$

$$\begin{aligned} (t - t_0)^j F^{-1}(t, j) = & \{ [j - \alpha_1(t)][j - \alpha_2(t)] \}^\beta \\ & \times \left(\phi(t, j) - \frac{1}{\pi} \{ [j - \alpha_1(t)][j - \alpha_2(t)] \}^\nu \right. \\ & \left. \times \int_{-\infty}^{\alpha_c(t)} d\lambda \rho(t, \lambda) \frac{\chi(t, j; \lambda)}{[\lambda - \alpha_+(t) - i\epsilon] \{ [j - \lambda - \alpha_-(t)][j - \lambda + \alpha_-(t)] \}^{\beta+\nu}} \right), \end{aligned} \quad (4)$$

where $\alpha_1(t)$ and $\alpha_2(t)$ are two crossing trajectories like $\alpha_{1,2} = \alpha_\pm \pm (at)^{1/2}$, and

$$\begin{aligned} \alpha_\pm &= \frac{1}{2}(\alpha_1 \pm \alpha_2), \\ \alpha_c(t) &= \alpha_+(t) + c(t - t_0), \end{aligned}$$

with c being a positive constant. The trajectories

is satisfied if $\alpha(t)$ has a branch point corresponding to^{2,3}

$$\alpha(t) = \alpha(t_0) + \text{const} \times (t_0 - t)^{\alpha(t_0) + 1/2} + \dots \quad (3)$$

at $t = t_0$. The power $\alpha(t_0)$ is due to the fact that $F(t, j) \propto (t - t_0)^j$ for $t \rightarrow t_0$.

If the Pomeron is not directly associated with resonances in the t channel, it is possible that it does not implement t -channel unitarity in the same way as an ordinary Regge pole. In fact, if it is not a simple pole trajectory but, for example, a hard branch-point surface, then compliance with the unitarity condition becomes a more complicated problem. Usually, it is then no more possible to satisfy unitarity and analyticity requirements by simply introducing a branch point into the function $\alpha(t)$, as was the case for a pole trajectory. Superficially, whenever $\alpha^{11}(t) \neq \alpha(t)$ for $t \geq t_0$, there appears to be no violation of the continuity theorem of functions of two or more complex variables³ if we continue the amplitude $F(t, j)$ into the second sheet around $t = t_0$. However, a closer examination of the unitarity conditions (1) and the analyticity requirements of $F(t, j)$ shows that there are difficulties and that a branch point of $\alpha(t)$ is not enough. We will discuss these problems elsewhere.⁴ Here we concentrate on the shielding-cut method, which may well be more natural for the Pomeron.

In this paper we restrict ourselves to trajectories with $\alpha^{11}(t) = \alpha(t)$, although we may, of course, have a mixed situation, where there are hard branch-point trajectories with branch points at $t = t_0$ and in addition shielding cuts in order to prevent a contradiction with unitarity. Care must be taken in these examples to maintain the correct analytic properties of the amplitude, which should not inherit the branch point of the trajectories at $t = t_0$.

In previous papers,¹ we have considered several examples of shielding cuts.⁵ A rather general ansatz for the amplitude $F(t, j)$ satisfying the unitarity condition (1) is given by

$\alpha_{1,2}(t)$ have no branch point at $t = t_0$: $\alpha_{1,2}^{11} = \alpha_{1,2}$. Although we have written Eq. (4) with a pair of crossing trajectories, it is applicable to all other types of singular surfaces for which $F(t, j \rightarrow \alpha(t)) \rightarrow \infty$. The functions ρ and χ are regular as required and satisfy the conditions

$$\begin{aligned}\rho(t, \alpha_+(t)) &= \rho(t), \\ \chi(t, j; \alpha_+(t)) &= (t - t_0)^j.\end{aligned}\quad (5)$$

Explicit examples have been given in Ref. 1, where also the mathematical features of representations like Eq. (4) are discussed.

III. COUPLED CHANNELS

Let us consider the amplitudes for $\pi\pi$, πN , and NN elastic scattering in the s channel. We can ignore spin and charge variables, and we are interested in the high-energy limits of these amplitudes as described by the leading singularities in the complex angular momentum plane of the crossed channel. There are three coupled amplitudes which are relevant,

$$\begin{aligned}F(\pi\bar{\pi} \rightarrow \pi\bar{\pi}) &= F(t, j), \\ G(\pi\bar{\pi} \rightarrow N\bar{N}) &= G(t, j), \\ N(N\bar{N} \rightarrow N\bar{N}) &= N(t, j).\end{aligned}\quad (6)$$

We assume that the Pomeron trajectory $j = \alpha(t)$ is a singular surface which is present in every one of these continued partial-wave amplitudes, and we are interested in the shielding of the lowest two-particle threshold at $t = t_0 = 4m_\pi^2$. Let us denote by the superscript II the continuations of F , G , and N through the two-pion cut in the interval $t_0 \leq t < t_i$, where $t_i = (4m_\pi)^2$ is the next higher threshold. Written in an analytic form, the exact unitarity relations in this interval are then given by

$$\begin{aligned}F - F^{\text{II}} &= 2i\rho F F^{\text{II}}, \\ G - G^{\text{II}} &= 2i\rho G F^{\text{II}}, \\ N - N^{\text{II}} &= 2i\rho G G^{\text{II}},\end{aligned}\quad (7)$$

with ρ as defined in Eq. (2). Solving for the amplitudes in sheet II, we obtain³

$$\begin{aligned}F^{\text{II}} &= \frac{F}{1 + 2i\rho F}, \\ G^{\text{II}} &= \frac{G}{1 + 2i\rho F}, \\ N^{\text{II}} &= N - \frac{2i\rho G^2}{1 + 2i\rho F},\end{aligned}\quad (8)$$

where we note that $FG^{\text{II}} = GF^{\text{II}}$ as required in (7).

As we have reviewed in Sec. II, a hard singular surface $j = \alpha(t)$, with $\alpha^{\text{II}}(t) = \alpha(t)$, of the amplitude $F(t, j)$ necessarily requires shielding in order to be compatible with the first relation in Eqs. (7). For simplicity, we make the natural assumption in this section that the character of the surface $j = \alpha(t)$ is the same for the three amplitudes. The question then is to what extent the coupled unitarity equations (7) require shielding cuts to be present

also in G and N .

According to the assumptions made, the limit

$$\lim_{j \rightarrow \alpha(t)} \frac{G(t, j)}{F(t, j)} = g(t, \alpha(t)) \quad (9)$$

is finite. But then it follows from Eq. (7) that

$$G^{\text{II}^{-1}}(t, \alpha(t)) = 2i\rho g^{-1}(t, \alpha(t)), \quad (10)$$

while $G(t, j \rightarrow \alpha(t)) \rightarrow \infty$. Hence, as in $F(t, j)$, we would have a sudden change in the singular character of the surface $\alpha(t)$ as we continue G into the second sheet. Since this behavior is not compatible with the continuity theorem, a shielding cut is certainly also required for $G(t, j)$. Actually, this result is independent of the character of the singularity in G . Since $G/G^{\text{II}} = 1 + 2i\rho F$, the functions G and G^{II} behave differently for $j \rightarrow \alpha(t)$ if $F \rightarrow \infty$ in this limit.

Next we consider $N(t, j)$, the partial-wave amplitude for $N\bar{N} \rightarrow N\bar{N}$. By assumption, the limit

$$\lim_{j \rightarrow \alpha(t)} \frac{N(t, j)}{F(t, j)} = n(t, \alpha(t)) \quad (11)$$

is again finite. With Eqs. (9) and (11), we find then from the unitarity relation

$$\frac{N}{G} - \frac{N^{\text{II}}}{G} = 2i\rho G^{\text{II}}$$

that

$$\frac{n(t, \alpha(t))}{g(t, \alpha(t))} - \lim_{j \rightarrow \alpha} \frac{N^{\text{II}}}{G} = g(t, \alpha(t)). \quad (12)$$

In general, the ratio N^{II}/G may approach a finite limit for $j \rightarrow \alpha(t)$. Then $N^{\text{II}}(t, j)$ could have the same singularity as $N(t, j)$, with

$$\lim_{j \rightarrow \alpha} \frac{N^{\text{II}}}{N} = 1 - \frac{g^2}{n}, \quad (13)$$

and there would be no difficulty with the continuity theorem. But we see that the finiteness of the ratio N^{II}/N is directly related to the question of the factorization of the singular term proportional to $(j - \alpha)^{-\beta}$ in the limit $j \rightarrow \alpha(t)$. Exact factorization at the branch point or pole requires

$$g^2(t, \alpha(t)) = n(t, \alpha(t)), \quad (14)$$

and implies $N^{\text{II}}/N \rightarrow 0$ for $j \rightarrow \alpha$. In this case, we have a sudden change in the character of the singular surface $j = \alpha(t)$ as we continue $N(t, j)$, and hence a specific shielding cut is then called for. Of course, since we know already that $G(t, j)$ must have a shielding cut, it is expected from the unitarity relation (7) for N that the discontinuity of N along the two-pion cut has an additional branch point $\alpha_c(t)$, with $\alpha_c(t_0) = \alpha(t_0)$, regardless of factorization.

As an example of a set of amplitudes F , G , and N which satisfy the unitarity equations (7), we may make the ansatz

$$\begin{aligned} G(t, j) &= g(t, j)F(t, j), \\ N(t, j) &= n(t, j)F(t, j), \end{aligned} \quad (15)$$

where $F(t, j)$ is given by an expression like Eq. (4) with a hard singular surface $j = \alpha(t)$ and a corresponding shielding cut. The functions g and n are assumed to have no singularities in the neighborhood of the points $j = \alpha(t)$. Since the ratio G/F has no unitarity branch point at $t = t_0$, the same must be true for $g(t, j)$; hence $g'' = g$. For the function $n(t, j)$, the unitarity relation (7) implies

$$n''(t, j) - n(t, j) = 2i\rho(t)F(t, j)[n(t, j) - g^2(t, j)], \quad (16)$$

and we see that choosing $n = n''$ requires complete factorization: $n(t, j) = g^2(t, j)$ in our example. Note that the functions n and g must contain the kinematic threshold factors appropriate for G and N .

The model described above is, of course, not meant for the complete amplitudes, but it may be used as an ansatz for F , G , and N in the neighborhood of $j = \alpha(t)$, and hence for the description of the high-energy limits of meson-meson, meson-nucleon, and nucleon-nucleon scattering amplitudes. Phenomenological calculations using these expressions will be reported elsewhere by one of us (S.P.).⁶

IV. HIGHER THRESHOLDS

In the previous section we have considered only the lowest branch point at $t = t_0$, using the exact unitarity condition in the interval $t_0 \leq t < t_i$, where t_i is the next higher threshold. The shielding mechanism we have described can, however, be generalized to higher thresholds. As an example, we consider here briefly the $\pi\bar{\pi}$ and $K\bar{K}$ thresholds. We ignore, *a priori*, all other branch points in an interval $t_0 \leq t < t_i$, where $t_i > t_K$, $t_\pi = 4m_\pi^2$, $t_K = 4m_K^2$, at least in that part of the amplitude which

$$\begin{aligned} F^{-1}(t, j) &= [j - \alpha(t)]^\beta \left\{ \varphi(t, j) - \frac{1}{\pi} [j - \alpha(t)]^\nu \int_{-\infty}^{\alpha_{c\pi}} d\lambda \rho_\pi(t, \lambda) \frac{\chi_\pi(t, j; \lambda)}{[\lambda - \alpha(t) - i\epsilon](j - \lambda)^{\beta-\nu}} \right. \\ &\quad \left. - \frac{k}{\pi} [j - \alpha(t)]^\mu \int_{-\infty}^{\alpha_{cK}} d\lambda \rho_K(t, \lambda) \frac{\chi_K(t, j; \lambda)}{[\lambda - \alpha(t) - i\epsilon](j - \lambda)^{\beta+\mu}} \right\}. \end{aligned} \quad (22)$$

Here ν and μ are positive constants. The functions $k(t, j)$ and $n(t, j)$ contain the appropriate kinematical threshold factors, but they are regular for $j = \alpha(t)$. We have two shielding cuts with branch-point surfaces $\alpha_{c\pi}(t)$ and $\alpha_{cK}(t)$. They must satisfy the conditions

$$\alpha_{c\pi}(t_\pi) = \alpha(t_\pi), \quad \alpha_{cK}(t_K) = \alpha(t_K), \quad (23)$$

is singular for $j = \alpha(t)$, where $\alpha(t)$ is a hard branch point or pole.

We write the coupled amplitudes in the matrix form

$$\underline{T} = \begin{pmatrix} F & G_{\pi K} & G_{\pi N} \\ G_{\pi K} & K & G_{KN} \\ G_{\pi N} & G_{KN} & N \end{pmatrix}, \quad (17)$$

where $F(t, j)$, $K(t, j)$, and $N(t, j)$ are the amplitudes for $\pi\bar{\pi} \rightarrow \pi\bar{\pi}$, $K\bar{K} \rightarrow K\bar{K}$, and $NN \rightarrow NN$, respectively, and $G_{\pi K}$, $G_{\pi N}$, G_{KN} correspond to $\pi\bar{\pi} \rightarrow K\bar{K}$, $\pi\bar{\pi} \rightarrow NN$, $KK \rightarrow NN$. In view of our assumptions, we write the restricted unitarity condition in the interval $t_K \leq t \leq t_i$ in the form⁷

$$\underline{T} - \underline{T}^{\text{III}} = 2i \underline{T} \underline{\rho} \underline{T}^{\text{III}}, \quad (18)$$

where

$$\underline{\rho} = \begin{pmatrix} \rho_\pi & 0 & 0 \\ 0 & \rho_K & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (19)$$

with

$$\rho_{\pi, K} = \left(\frac{t - t_{\pi, K}}{t} \right)^{1/2}. \quad (20)$$

The general unitarity condition in this interval contains additional multipion thresholds, but we define by the superscript III the continuation into a secondary Riemann sheet with respect to the $\pi\bar{\pi}$ and $K\bar{K}$ branch points only.

A simple solution of Eq. (18) with a singular surface $j = \alpha(t)$ is obtained by assuming complete factorization. We write it in the form

$$\underline{T} = \begin{pmatrix} 1 & \sqrt{k} & \sqrt{n} \\ \sqrt{k} & k & \sqrt{nk} \\ \sqrt{n} & \sqrt{nk} & n \end{pmatrix} F. \quad (21)$$

In the neighborhood of the hard branch point $j = \alpha(t)$, the amplitude F is given by

and may be chosen to have the form

$$\begin{aligned} \alpha_{c\pi}(t) &= \alpha(t) + c_\pi(t - t_\pi), \\ \alpha_{cK}(t) &= \alpha(t) + c_K(t - t_K). \end{aligned} \quad (24)$$

The functions $\rho_\pi(t, \lambda)$ and $\rho_K(t, \lambda)$ reduce to $\rho_\pi(t)$ and $\rho_K(t)$ for $\lambda \rightarrow \alpha(t)$, respectively, and we require here

$$\chi_{\pi, K}(t, j; \lambda = \alpha(t)) = 1. \quad (25)$$

For example, ρ_K and χ_K may be of the form

$$\rho_K(t, \lambda) = \left[\frac{\alpha_{cK}(t) - \lambda}{\alpha_{cK}(t) + c_K t_K - \lambda} \right]^{1/2}, \quad (26)$$

$$\chi_K(t, j; \lambda) = \left[\frac{\alpha_{cK}(t) - \lambda}{c_K(t - t_K)} \right]^j,$$

with

$$k(t, j) \propto \left(\frac{t - t_K}{t - t_\pi} \right)^j. \quad (27)$$

In more general models, the functions χ , as well as k and n , may also contain multipion and other thresholds.

The ansatz (21) can be generalized to more complicated many-channel situations, but here we do not intend to go into further details. Since the shielding branch points have intercepts

$$\alpha_{c\pi, K}(0) = \alpha(0) - c_{\pi, K} t_{\pi, K}, \quad (28)$$

it is plausible that it is the two-pion threshold which may be most relevant for diffraction scattering.

V. REMARKS

We have seen that for singular surfaces of coupled amplitudes (like F , G , and N in Sec. II) which satisfy $\alpha^{II}(t) = \alpha(t)$, the unitarity condition does not imply factorization, even if $\alpha(t)$ is a pole trajectory. For ordinary Regge poles with $\alpha^{II}(t) \neq \alpha(t)$ and no shielding cuts, we know that factorization is implied by the equations (8).^{3,8} The surface $\alpha^{II}(t)$ is then a zero of $1 + 2i\rho(t)F(t, j)$, or correspondingly, $\alpha(t)$ is a zero of $1 - 2i\rho(t)F^{II}(t, j)$. Even if the surface $\alpha(t)$ does not have the appropriate branch point at the threshold $t = t_0$, as we have assumed in this paper, we still have $1 - 2i\rho F^{II} \rightarrow 0$ for $j \rightarrow \alpha(t)$. But this does not imply factorization unless $N^{II}/N \rightarrow 0$ for $j \rightarrow \alpha(t)$. On the other hand, if $N^{II}/N \rightarrow 0$, it becomes necessary to introduce an appropriate shielding cut also for N , and hence all the coupled amplitudes are required to have one. We then obtain a certain universality of the Pomeron together with its associated shielding surface.

The examples we have considered in this paper can be generalized in many ways. We have restricted our considerations to more simple situations in order to exhibit the principles rather than list all possibilities.

*Work supported in part by the National Science Foundation under Contract No. NSF GP 32904 X1.

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