Leakage of quark charge*

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A sequential model for the fragmentation of a quark into hadrons is constructed with the property that a net charge $\mathcal{Q}$ "leaks" down the multiperipheral chain. Specifically, for a process initially producing a quark, $\mathcal{Q} = \frac{1}{3}(1-r)/(2+r)$, where r is the ratio of directly produced $K^\pm$ to $\pi^\pm$ in the quark fragmentation plateau. This expression corrects an earlier one in which the consequences of such a model were misstated.

One interpretation of the large cross sections observed in deep-inelastic lepton scattering off nucleons is that they result from the existence of pointlike constituents of hadrons. The use of different beams $(e, \nu, \bar{\nu})$ and targets (p, n) then allows one, to some extent, to determine the charges and other properties of these constituents.

Some time ago, Feynman[1] suggested that the average charge of a constituent ("parton," quark, ...) may be measured directly by the average charge of the hadrons it emits after interaction with a lepton. This suggestion was shown to be model-dependent, and a particular counterexample was constructed.[2] In the model of Ref. 2, the struck quark was assumed to emit hadrons via a sequential process of quark pair creation with hadrons strictly ordered in rapidity. With sufficient distance in rapidity ($\Delta y \gtrsim 1-2$) from the struck quark, the hadrons were then assumed (as in Ref. 1) to form a plateau in rapidity space, called the "current fragment plateau." This plateau was assumed to be neutral with respect to isospin and hypercharge, so that (for example) as many π^+ as π^0 or π^-, and as many K^+ as K^- were produced. It was then found that unless the hadrons in the plateau were produced in SU(3)-invariant ratios, a net charge $\mathcal{Q}$ tended to "leak" through the plateau. This result can be visualized most simply as follows.

Consider the struck quark to be a quark and not an antiquark. (One can select for this situation in neutrino experiments, as shown in Refs. 1 and 2.) Suppose that some dynamical mechanism totally suppresses the pair creation of strange quarks. Then the struck quark q_1 will fragment into a meson and a nonstrange quark q_2. This last quark q_2 will fragment into another nonstrange meson and a nonstrange quark q_3 and so on down the rapidity chain (see Fig. 1). In a conventional quark model the average charge of nonstrange quarks is $(Q_u + Q_d)/2 = (\frac{2}{3} - \frac{1}{3})/2 = \frac{1}{6}$, and this charge *continues to propagate down the chain* even in the current fragmentation plateau. Hence the average charge $\langle Q \rangle$ of the parton fragments is $\frac{1}{6}$ less than the charge of the struck quark, on the average: $\langle Q \rangle = \frac{1}{2}$ for an initial u quark, and $\langle Q \rangle = -\frac{1}{2}$ for an initial d quark.

In the above example, and in what follows, we do not necessarily adopt a space-time picture in which the struck quark fragments *first*. (No such models have been found to give a plateau of fragments in rapidity space.[3,4])

In this note we construct a simple model for the cascade in which the charge leaking through the plateau is directly related to probabilities for observing hadrons in the plateau. This was our intent in our original paper,[2] but the result given there [in Eq. (9)] is based on incorrect reasoning. Here we derive the correct result and develop some of its consequences.

Let $W(M_i)$ be the probability that a given meson in the plateau is i (π, K, η, etc.). Let the probabilities of finding a given quark in the chain of Fig. 1(a) (in the plateau region) be given by $P(u) = P(d)$ and $P(s)$, where

$$2P(u) + P(s) = 1\,. \tag{1}$$

Independently of any production mechanisms, we expect

$$\mathcal{Q} = \sum_q Q_q P(q) + \sum_{\bar{q}} Q_{\bar{q}} P(\bar{q})\,. \tag{2}$$

Henceforth, we shall assume that if a quark is struck by the current, only quarks (i.e., not antiquarks) propagate toward lower rapidity, and vice versa if an antiquark is struck. In this case

$$\begin{aligned}\mathcal{Q} &= \tfrac{2}{3}P(u) - \tfrac{1}{3}P(d) - \tfrac{1}{3}P(s)\\ &= [1 - 3P(s)]/6\end{aligned} \tag{3}$$

in the conventional fractionally charged quark model. Note that Eq. (3) gives $\mathcal{Q} = 0$ when $P(s) = \frac{1}{3}$ [the SU(3) limit] and $\mathcal{Q} = \frac{1}{6}$ when $P(s) = 0$ (the example mentioned above).

Suppose, now, that the probabilities of quark pair creation are *uncorrelated* with one another from one step to the next in the chain.[5,6] Then the ratio of K^+ to π^+ production is just given, in a model with pseudoscalar mesons, by

$$\frac{W(K^+)}{W(\pi^+)}=\frac{P(q_n=u)P(q_{n+1}=s)}{P(q_n=u)P(q_{n+1}=d)}$$
$$=\frac{P(s)}{P(d)}=\frac{2P(s)}{1-P(s)}\equiv r\,. \qquad (4)$$

Here the index n labels the step in the chain, and is assumed large enough that the hadron $q_n\bar{q}_{n+1}$ lies in the plateau.

Eliminating $P(s)$ from (3) and (4), we find

$$\mathcal{Q}=\frac{1}{3}\frac{1-r}{2+r}\,. \qquad (5)$$

When $r=1$ $\mathcal{Q}=0$ [the SU(3) limit], and when $r=0$ $\mathcal{Q}=\frac{1}{6}$ (the example of strange-quark suppression mentioned above). Given the tendency of kaon production to be highly suppressed relative to pion production, we might expect $\mathcal{Q}>0$.

A Mueller-Regge picture[7,8] yields similar results, as well as predictions for production of other particles. The total cross section for production of a given particle M in the plateau is represented in the present model by the absorptive part of the forward amplitude depicted in Fig. 2(a), summed over everything but M. The relevant subgraph is enclosed in the dashed rectangle in Fig. 2(a), and enlarged in Fig. 2(b).

Figure 2(b) has several features to which we wish to call attention:

(i) The Pomeron (P) is represented as a quark-antiquark pair. This is specific to our model, and is required for consistency with Figs. 1(a) and 2(a). The SU(3) structure of the Pomeron is therefore that of a 3×3 matrix:

$$\mathrm{P}=\begin{bmatrix} x & 0 & 0\\ 0 & x & 0\\ 0 & 0 & y\end{bmatrix}. \qquad (6)$$

We shall later relate x and y to quark probabilities. In the SU(3) limit, $x=y$. In general, however, we are assuming the Pomeron to have an *intrinsic octet component* as well.

(ii) The coupling in Fig. 2(b) is of the form

$$W(M_i)=C\,\mathrm{Tr}(\mathrm{P}M_i\mathrm{P}\tilde{M}_i)\,. \qquad (7)$$

This is not the most general sort of coupling leading to SU(3)-noninvariant probabilities for meson emission,[8] but it is in keeping with our assumption that intrinsic differences in quark pair creation somehow govern particle production ratios. As we shall see below, the magnitude of η production provides a test of this assumption. One can also

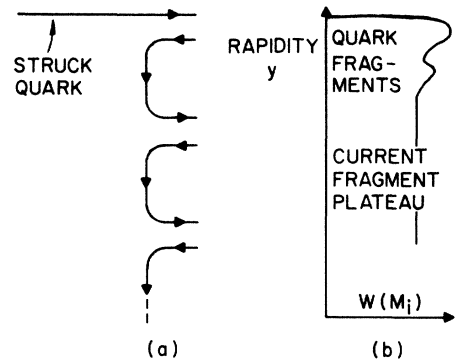

FIG. 1. Model for quark fragmentation. (a) Sequential chain leading to meson production. (b) Corresponding hadron distribution in rapidity space.

envision a Pomeron of the type (6) arising from density-matrix arguments like those of Feynman,[9,6] if differences in phase space are taken into account.

To interpret x and y in Eq. (6), we calculate, using Eq. (7),

$$\frac{W(K^+)}{W(\pi^+)}=\frac{\mathrm{Tr}(\mathrm{P}K^+\mathrm{P}K^-)}{\mathrm{Tr}(\mathrm{P}\pi^+\mathrm{P}\pi^-)}=\frac{xy}{x^2}=\frac{y}{x}\equiv r \qquad (8)$$

in a model in which the basic mesons are pseudoscalar. However, since our assumptions imply independent quark-pair-creation probabilities,

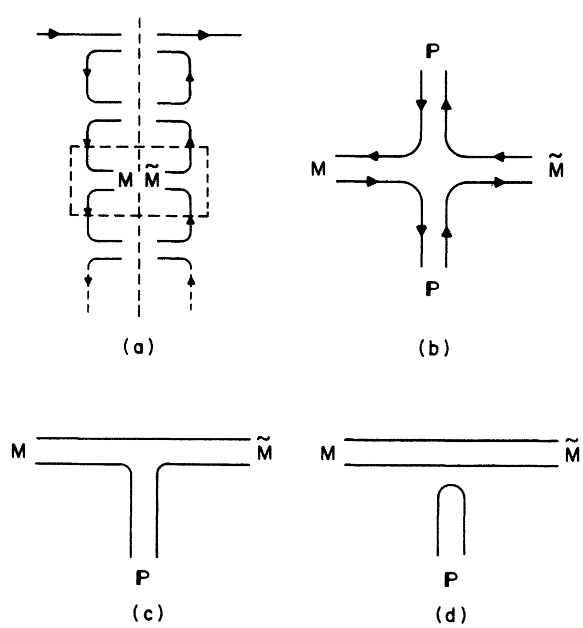

FIG. 2. (a) Mueller-Regge description of single meson production. (b) Expanded version of contents of dashed rectangle in (a). (c) "Connected" meson-meson-Pomeron vertex (allowed). (d) "Disconnected" meson-meson-Pomeron vertex (forbidden).

Eq. (4) also holds, so that $y/x = P(s)/P(u)$. Hence we can write, adopting an overall normalization,

$$\mathsf{P} = \begin{bmatrix} P(u) & 0 & 0 \\ 0 & P(d)=P(u) & 0 \\ 0 & 0 & P(s) \end{bmatrix} . \qquad (9)$$

The couplings in Eq. (7) imply strong constraints between ratios of particle types. For η belonging to an unmixed octet, for example,

$$\frac{W(\eta)}{W(\pi)} = \tfrac{1}{3} + \tfrac{2}{3}r^2 , \qquad (10)$$

while for an η' belonging to a unitary singlet but coupling via Eq. (7),

$$\frac{W(\eta')}{W(\pi)} = \tfrac{2}{3} + \tfrac{1}{3}r^2 . \qquad (11)$$

If the present model makes sense, Eqs. (10) and (11) should be testable in any hadron-hadron collision, not just in deep-inelastic lepton scattering.

An estimate of r independent of Eq. (8) can be obtained by assuming that the meson-meson-Pomeron coupling consists of "connected" quark graphs [as in Fig. 2(c) rather than Fig. 2(d)].[10] The assumption of connected meson-meson-Pomeron graphs [Fig. 2(c); Zweig's rule] goes beyond that of Fig. 2(b). Theories with such structure have been discussed by several authors. Then one finds

$$\frac{\sigma_T^{\mathsf{P}}(KN)}{\sigma_T^{\mathsf{P}}(\pi N)} = \frac{P(u)+P(s)}{2P(u)} = \frac{1+r}{2} . \qquad (12)$$

Since this ratio is about 17 mb/21 mb $\simeq 80\%$, one expects $r \simeq 0.6$. The experimental K/π ratio appears to be lower than this number. However, in order to apply the model to nature, states besides the pseudoscalars probably must be included in the set $\{M_i\}$, as in Ref. 11.

Resonance decays tend to lead to many more pions than kaons or η's. As an example of this effect, we compute the observed particle ratios, based on Eqs. (8), (10), and (11), for a set $\{M_i\}$ consisting of vector and pseudoscalar mesons, produced in the ratio of statistical weights (3 : 1). The ω and ϕ are assumed to be ideal nonet members. Then

$$W(\omega)/W(\rho^+) = 1 , \quad W(\phi)/W(\rho^+) = r^2 , \quad W(K^{*+})/W(\rho^+) = r , \qquad (13)$$

and, by assumption,

$$W(\rho^+)/W(\pi^+) = 3 . \qquad (14)$$

For definiteness we shall count all electromagnetic or strong decay products in the *total* hadron production probability $\tilde{W}(M_i)$. Then, for example,

$$\begin{aligned} \tilde{W}(\pi^+) &= W(\pi^+) + 0.3W(\eta) + 0.9W(\eta') \\ &\quad + W(\rho^+) + W(\rho^0) + 0.9W(\omega) + \tfrac{2}{3}W(K^{*+}) \\ &\quad + \tfrac{2}{3}W(\bar{K}^{*0}) + 0.2W(\phi) \\ &\cong W(\pi^+)(10.4 + 4r + r^2) . \end{aligned} \qquad (15)$$

Similar expressions can be written for other hadrons, leading to

$$\begin{aligned} \tilde{W}(\pi^0) &\cong W(\pi^+)(11.3 + 4r + 1.8r^2) , \\ \tilde{W}(K^+) &\cong W(\pi^+)(4r + 1.4r^2) , \\ \tilde{W}(\eta \to \gamma\gamma) &\cong W(\pi^+)(0.3 + 0.4r^2) , \\ \tilde{W}(\eta') &= W(\eta') = W(\pi^+)(\tfrac{2}{3} + \tfrac{1}{3}r^2) . \end{aligned} \qquad (16)$$

The consequences of Eqs. (15) and (16) may be summarized as follows:

(a) For $0 \le r \le 1$, $\tilde{W}(K^+)/\tilde{W}(\pi^+) \simeq \frac{1}{3}[W(K^+)/W(\pi^+)] \simeq r/3$. The value $r = 0.6$ deduced from total cross sections [Eq. (12), above] leads to the prediction

$$\tilde{W}(K^+)/\tilde{W}(\pi^+) \simeq 0.2 . \qquad (17)$$

(b) For $0 \le r \le 1$, $\tilde{W}(\pi^0)/\tilde{W}(\pi^+) \simeq 1.1$.

(c) $\tilde{W}(\eta[\to\gamma\gamma])/\tilde{W}(\pi^0)$ rises from about 3% at $r = 0$ to about 4% at $r = 1$.

(d) For $0 \le r \le 1$, $\tilde{W}(\eta')/\tilde{W}(\pi^+) \simeq \frac{1}{15}$.

(e) As many η's are produced from $\eta' \to \eta\pi\pi$ as are produced "directly."

Clearly the inclusion of resonance production has led to a drastic dilution of kaons, η's, and η''s. One might expect a further dilution if still higher resonances were included (see, e.g., Ref. 11). This means that r can be a good deal closer to 1 than one might suspect on the basis of observed K/π ratios.

Taking $r \simeq 0.6$ from Eq. (12), Eq. (5) gives

$$\mathcal{Q} \cong 0.05 , \qquad (18)$$

a small number compared to the average quark charges to be measured. *In reasonable models we expect a small (but not strictly zero) quark charge to leak through the hadronic plateau.*

To summarize, we have constructed a model in which quark charge is not retained in the fragmentation region. Instead, when the particles in the plateau in rapidity space are not produced with SU(3)-invariant ratios, quark charge "leaks" back into the remnants of the struck hadron. Our model is expected to be only one of a number of such cases. The assumption of *independent* quark pair creation probabilities as the sole mechanism for breaking SU(3) may not be valid. We might (for example) expect direct η production to be *even weaker* than indicated in Eq. (10). Nonetheless, the simple example shown above indicates that resonance production could swamp the η's and K's with pions. While particle-production ratios thus lose some of their appeal for measuring the average quark-charge leakage $\mathfrak{L}$, we have suggested more general properties of the Pomeron that may be related to such leakage.

One helpful experiment would be to measure the production of resonances directly (for example, via their leptonic decays.) While one could not check whether these resonances were produced "directly" or as decay products of some other resonances, one would at least expect the production of (say) ϕ and ρ to be much more comparable to one another than that of (say) η' and π if the ideas presented here are valid.

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Factorization and shielding of hard Regge surfaces*

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Hard branch-point trajectories require shielding cuts in order to be compatible with elastic unitarity. For coupled channels like $\pi\bar{\pi}$ and $N\bar{N}$, shielding of the two-pion threshold is required for all amplitudes if the hard branch point factorizes. Explicit examples are given, for the shielding of the lowest as well as the higher thresholds.

I. INTRODUCTION

In previous papers,[1] one of us has shown how a Regge trajectory $j=\alpha(t)$ can be made compatible with two-particle t-channel unitarity in case it is not a simple pole surface with the appropriate branch point at the threshold $t=t_0$. Shielding cuts were introduced, which make the limit $j\rightarrow\alpha(t)$ and the continuation of the partial-wave amplitude $F(t,j)$ around the branch point $t=t_0$ noninterchangeable. These shielding cuts may well be important phenomenologically at medium high energies. In Ref. 1 the discussion was restricted to elastic amplitudes like $\pi\bar{\pi}\rightarrow\pi\bar{\pi}$, but it is of interest to extend the shielding mechanism to sets of coupled amplitudes.

It is the purpose of this paper to discuss the shielding problem in cases with two or more coupled channels. In order to restrict our considerations to the essential points, we assume that the same singular surface $j=\alpha(t)$ is present in all of the coupled amplitudes, and that it has a t-independent character. We are mainly interested in the lowest threshold (e.g., $t_0=4m_\pi^2$) of a set of