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Implications of anomalous Lorentz structure in neutral weak processes*

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Effects of possible anomalous Lorentz structures in the neutral weak interaction in neutrino-electron scattering, deep-inelastic neutrino-nucleon scattering, and pion production in the N^* region are discussed.

Evidence[1] has been accumulating that the weak interaction contains a neutral current which couples to neutrinos. While current theoretical thought heavily favors the proposition that the neutral current consists of a linear combination of vector and axial-vector currents, the ultimate verdict should be left to experiment. In this note we would like to work out the form of experimentally measurable distributions while assuming a general Lorentz-invariant structure for the neutral current. This question has already been commented upon by Rosen[2] and by Sakurai[3] for various special cases. Indeed, even the cherished $V-A$ structure of the charged current should be put to a rigorous experimental test, as discussed in detail by Cheng and Tung.[4]

Let us first consider the processes[5] $\nu_\mu(k)+e^-(p)\to\nu_\mu(k')+e^-(p')$ and $\bar\nu_\mu e^-\to\bar\nu_\mu e^-$. We write down the most general local Lagrangian without derivative coupling[6]:

$$\begin{aligned}\mathcal{L}_I &= g_{VV}\bar\nu\gamma_\alpha\nu\bar e\gamma^\alpha e+g_{AA}\bar\nu\gamma_\alpha\gamma_5\nu\bar e\gamma^\alpha\gamma_5 e\\ &\quad+g_{AV}\bar\nu\gamma_\alpha\gamma_5\nu\bar e\gamma^\alpha e+g_{VA}\bar\nu\gamma_\alpha\nu\bar e\gamma^\alpha\gamma_5 e\\ &\quad+g_{SS}\bar\nu\nu\bar e e+g_{PP}\bar\nu\gamma_5\nu\bar e\gamma_5 e\\ &\quad+ig_{PS}\bar\nu\gamma_5\nu\bar e e+ig_{SP}\bar\nu\nu\bar e\gamma_5 e\\ &\quad+g_{TT}\bar\nu\sigma_{\alpha\beta}\nu\bar e\sigma^{\alpha\beta}e+ig_{\tilde T T}\bar\nu\sigma_{\alpha\beta}\gamma^5\nu\bar e\sigma^{\alpha\beta}e\,. \end{aligned}\tag{1}$$

Hermiticity implies that all the g's are real. Note that CP invariance would imply $g_{PS}=g_{SP}=g_{\tilde T T}=0$. We do *not* assume CP invariance in what follows. Experimentally, the incoming ν_μ is left-handed and so the amplitude for $\nu_\mu e^-\to\nu_\mu e^-$ becomes

$$\begin{aligned}\mathfrak{M} &= \bar\nu\gamma_\alpha(1-\gamma_5)\nu\bar e\gamma^\alpha(g_V-g_A\gamma_5)e\\ &\quad+\bar\nu(1-\gamma_5)\nu\bar e(g_S-g_P\gamma_5)e\\ &\quad+g_T\bar\nu\sigma_{\alpha\beta}(1-\gamma_5)\nu\bar e\sigma^{\alpha\beta}e\,,\end{aligned}\tag{2}$$

where

$$\begin{aligned}g_V&=\tfrac12(g_{VV}-g_{AV})\,,\\ g_A&=\tfrac12(g_{AA}-g_{VA})\,,\\ g_S&=\tfrac12(g_{SS}-ig_{PS})\,,\\ g_P&=\tfrac12(g_{PP}-ig_{SP})\,,\\ g_T&=\tfrac12(g_{TT}-ig_{\tilde T T})\,.\end{aligned}\tag{3}$$

Hence, CP invariance would correspond to reality of all the quantities in Eq. (3). Note that the only terms which may violate P invariance without also violating CP invariance are the V and A terms.

A straightforward but somewhat tedious calculation gives, in the laboratory frame, that

$$\begin{aligned}\frac{2\pi}{mE}\frac{d\sigma}{dy}(\nu_\mu e^-\to\nu_\mu e^-) &= 2(g_V+g_A)^2+2(g_V-g_A)^2(1-y)^2\\ &\quad+(|g_S|^2+|g_P|^2)y^2\\ &\quad+32|g_T|^2(1-\tfrac12 y)^2\\ &\quad-8\,\mathrm{Re}[g_T(g_S^*+g_P^*)]\,y(1-\tfrac12 y)\,,\end{aligned}\tag{4}$$

where $y\equiv E(\text{recoil electron})/E(\text{incoming neutrino})$. Of course, one may also equivalently express this in terms of the center-of-mass angular distribution $d\sigma/d\Omega$. To obtain the corresponding differential cross section for $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$ we simply make the substitution $g_A \to -g_A$ and $g_T \to -g_T$ in Eq. (4).

To discuss the deep-inelastic experiments[7] $\nu_\mu(\bar{\nu}_\mu)+N \to \nu_\mu(\bar{\nu}_\mu)+X$ we need to know something about the strong interaction. We will simply rely on the parton model[8] since the present charged-current deep-inelastic experiments are in good agreement with it.[9] The data are consistent with a nearly complete absence of antiquarks in the nucleon.[9] The kinematics for these deep-inelastic neutrino processes is well known. It is most convenient to consider $d^2\sigma/dx\,dy$, where $x \equiv -q^2/2\nu$ and $y \equiv \nu/mE$ are variables which vary between 0 and 1.

For the neutral current experiments the outgoing neutrino is unobserved, and so the x distribution cannot be measured. The y distribution, on the other hand, can be determined by calorimetry of the outgoing hadronic spray X. Let us assume for the moment that the nucleon contains no antiquarks. The Lagrangian coupling neutrino to quark partons of type i will be assumed to have exactly the same form as Eq. (1) (with the substitution $g_V \to g_V^i$, etc.). We may now obtain $d\sigma/dy$ by simply copying Eq. (4). The result is

$$\frac{d\sigma^\nu}{dy} = \frac{s}{4\pi}\int_0^1 dx\, x \sum_i f_i(x)\{2[(g_V^i+g_A^i)^2+(g_V^i-g_A^i)^2(1-y)^2]+(|g_S^i|^2+|g_P^i|^2)y^2 + 32|g_T^i|^2(1-\tfrac{1}{2}y)^2 - 8\,\mathrm{Re}[g_T^i(g_S^i+g_P^i)^*]\,y(1-\tfrac{1}{2}y)\}\,. \tag{5}$$

Here $f_i(x)$ denotes the longitudinal momentum distribution function[8] of quark partons of type i in the nucleon. We may remind the reader that in the parton model the electroproduction structure function $F_2 = x\sum_i Q_i^2 f_i(x)$, with Q_i = charge of the quark parton of type i. [To obtain the contribution of antiquarks one simply substitutes $g_A^j \to -g_A^j$ and $g_T^j \to -g_T^j$. This has no effect on the y distribution to the extent that $\sum_i f_i(x)|g_V^i|^2$, etc., are unknown.]

Finally, $d\sigma/dy$ for $\bar{\nu}_\mu + N \to \bar{\nu}_\mu + X$ may be obtained from Eq. (5) by substituting $g_A^i \to -g_A^i$ and $g_T^i \to -g_T^i$.

Some special cases of Eq. (5) may be noteworthy. If the current is a linear combination of V and A,

$$\frac{1}{E}\frac{d\sigma^\nu}{dy} = a + b(1-y)^2\,,\qquad \frac{1}{E}\frac{d\sigma^{\bar{\nu}}}{dy} = a(1-y)^2 + b\,. \tag{6}$$

[a, b and c, d, e (to appear below) are independent of E and y.]

In particular, for cases[10] of pure V or pure A, $a=b$ and

$$\frac{1}{E}\frac{d\sigma^\nu}{dy} = \frac{1}{E}\frac{d\sigma^{\bar{\nu}}}{dy} = 2a(1-y+\tfrac{1}{2}y^2)\,. \tag{7}$$

(This includes a world in which the neutral current couples to some quarks via pure V and others via pure A.)

If the current is a linear combination of S and P

$$\frac{1}{E}\frac{d\sigma^\nu}{dy} = \frac{1}{E}\frac{d\sigma^{\bar{\nu}}}{dy} = cy^2\,. \tag{8}$$

This equation has already been pointed out by Sakurai.[3]

If the current is pure T

$$\frac{1}{E}\frac{d\sigma^\nu}{dy} = \frac{1}{E}\frac{d\sigma^{\bar{\nu}}}{dy} = d(1-\tfrac{1}{2}y)^2\,. \tag{9}$$

Notice that the y distribution for the difference of neutrino and antineutrino cross sections has the same shape whatever interactions are present, viz.,

$$\frac{1}{E}\left(\frac{d\sigma^\nu}{dy} - \frac{d\sigma^{\bar{\nu}}}{dy}\right) \propto y(1-\tfrac{1}{2}y) \tag{10}$$

in deep-inelastic scattering. Thus, failure of Eq. (10) in deep-inelastic scattering would imply either that the point-coupling Lagrangian [Eq. (1)] (as generalized to cover quarks) is inadequate or that scaling breaks down (or that the data are being contaminated by heavy-lepton production). Violation of Eq. (10) in neutrino-electron scattering would be a sure signal to us to revise our present thinking about the weak interaction.

We may note that, not surprisingly, these two types of experiments cannot tell us if the neutral current violates CP invariance.

One may also ask what one may learn about the Lorentz structure of the neutral weak interaction from exclusive processes. In particular, the production process $\nu_\mu + N \to \nu_\mu + N + \pi$ in the N^* region is now being investigated at several laboratories[11] and may be expected to yield valuable information about the neutral current. With the point-coupling Lagrangian Eq. (1) (suitably extended to cover quarks) the differential cross section (spin averaged) would be the sum of products of certain leptonic tensors with certain hadronic tensors. It is easy to see that the leptonic tensor will be bilinear in leptonic momenta. One can readily convince oneself that contraction with the hadronic matrix elements squared would yield at most nine invariants. These are

$$1,\ (p_1\cdot k),\ (p_2\cdot k),\ (p_1\cdot k)^2,\ (p_2\cdot k)^2,$$
$$(p_1\cdot k)(p_2\cdot k),\ \epsilon^{\alpha\beta\gamma\delta}k_\alpha q_\beta p_{1\gamma}p_{2\delta}, \tag{11}$$
$$\epsilon^{\alpha\beta\gamma\delta}k_\alpha q_\beta p_{1\gamma}p_{2\delta}(p_1\cdot k),\ \epsilon^{\alpha\beta\gamma\delta}k_\alpha q_\beta p_{1\gamma}p_{2\delta}(p_2\cdot k),$$

where p_1, p_2 are the momenta of the final hadrons, k is the momentum of the incoming neutrino, and q is the momentum transferred to the hadrons.

It is already known that if the neutral current consists of a linear combination of V and A, all nine invariants in Eq. (11) will be allowed.[12] Thus, in general, a study of exclusive processes will not lead to any conclusion about the possible presence of the exotic S, P, and T terms, if the V and A terms are already present. On the other hand, one may rule out the simple case in which only S and P are present if other terms than the unit term in (11) are detected.

In conclusion, we see that pion production in the N^* region cannot provide a signal that exotic terms S, P, and T are present unless one is willing to introduce specific dynamical assumptions.

Note added in proof. We have been informed that some similar work has been done by E. Fischbach, B. Kayser, G. P. Garvey, and S. P. Rosen (Phys. Lett. B, to be published).

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