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PHYSICAL REVIEW D VOLUME 10, NUMBER 7 1 OCTOBER 1974

Finite unitary theory of pure leptonic weak interaction

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Based on the $V-A$ interaction, a finite, unitary theory of pure leptonic weak interaction is formulated in the framework of the finite quantum field theory with shadow states. A perturbative series, originated from the conventional expansion, is obtained. This new perturbative series converges in the high-energy region where the original one does not. The difference between the prediction of this theory and the universal $V-A$ theory is negligible in low-energy regions but becomes significant in high-energy regions.

The universal $V-A$ theory of weak interaction[1] is well established for low-energy processes as long as one treats the interaction Lagrangian as a phenomenological interaction to be used only in the lowest order. It is also well known that the matrix elements calculated from the lowest-order perturbation violate unitarity at high energies. One might take this as an indication that the higher-order calculation should be included. However, the $V-A$ theory as it stands is not renormalizable; i.e., to renormalize all divergent amplitudes involves an infinite number of arbitrary constants. It is therefore, if not meaningless, at least aesthetically unappealing. The modification of the universal $V-A$ theory has been discussed quite extensively in the past.[2] Recently great progress in the experiments on high-energy neutrino weak interactions has been made.[3] The construction of a workable theory of weak interaction is therefore becoming more urgent. There has been some progress in the effort to unify the weak and the electromagnetic interactions such that the weak-interaction amplitudes become renormalizable.[4]

While the general approaches to modifying the $V-A$ theory have been concentrated in the direction of introducing new heavy particles following certain symmetry schemes, one might adopt a quite different approach by, instead of introducing new "physical particles," changing the dynamical law of the theory. This is the main idea of the theory of shadow states.[5] In this note we present a finite unitary theory[6] of pure leptonic weak interaction within the framework of the theory of shadow states.

In the $V-A$ theory, the interaction Lagrangian for pure leptonic interaction is given by

$$\mathcal{L}_I=\frac{G}{\sqrt{2}}J_\lambda J^{\lambda\dagger}\,, \tag{1}$$

where J_λ is the leptonic current,

$$J_\lambda=[\bar\psi_{\nu_e}\gamma_\lambda(1-\gamma_5)\psi_e+\bar\psi_{\nu_\mu}\gamma_\lambda(1-\gamma_5)\psi_\mu]\,. \tag{2}$$

Following the idea of shadow-state theory, we introduce two shadow fields for each of the leptonic fields,[7] and rewrite the current as follows:

$$J^s_\lambda=\left[\sum_{i=1}^{3}c_i^{\nu_e}\bar\psi^i_{\nu_e}\gamma_\lambda(1-\gamma_5)\sum_{j=1}^{3}c_j^e\psi_e^j+\sum_{i=1}^{3}c_i^{\nu_\mu}\bar\psi^i_{\nu_\mu}\gamma_\lambda(1-\gamma_5)\sum_{j=1}^{3}c_j^\mu\psi_\mu^j\right]. \tag{3}$$

Here ψ^1_α is the field of the physical lepton α with mass m_1^α and ψ^2_α and ψ^3_α are the corresponding shadow fields with masses m_2^α and m_3^α. The shadow fields are quantized with the "wrong" sign for the commutation relations. In other words, in contrast to the physical fields ψ^1_α which satisfy the commutation relations

$$[\psi^1_\alpha(x),\,\psi^1_\alpha(x')]=-iS(x'-x)\,, \tag{4}$$

the shadow fields ψ^2_α and ψ^3_α satisfy the commutation relations

$$[\psi^2_\alpha(x),\,\psi^2_\alpha(x')]=iS(x'-x)\,, \tag{5}$$

$$[\psi_\alpha^3(x), \psi_\alpha^3(x')] = iS(x'-x)\,. \tag{6}$$

The c_i^α are weight factors to be chosen in such a way that the theory is finite. We may choose the c_i^α as follows:

$$C_1^\alpha = |c_1^\alpha|^2 = 1\,,$$
$$C_2^\alpha = |c_2^\alpha|^2 = \frac{m_1^\alpha - m_3^\alpha}{m_3^\alpha - m_2^\alpha}\,, \tag{7}$$
$$C_3^\alpha = |c_3^\alpha|^2 = \frac{m_1^\alpha - m_2^\alpha}{m_2^\alpha - m_3^\alpha}\,.$$

The interaction Lagrangian in terms of the current J_λ^s has the same form as (1).

Here we see that the parameters involved in the theory are the coupling constant and the eight shadow masses. From the theoretical point of view, there is no *a priori* reason that the shadow masses for different leptons have to be the same. However, since the shadow-mass parameters are considered as, instead of masses of "physical particles," dynamical parameters similar to the coupling constant, we may follow the principle of simplicity and choose the same masses, say, m_2 and m_3, for all four different leptons. The nine parameters in the theory are then reduced to three parameters. With this choice of shadow mass, the weight factors c_i^α become more simply

$$C_1^I = 1,\ C_2^I = \frac{m^I - m_3}{m_3 - m_2}\,,\ C_3^I = \frac{m^I - m_2}{m_2 - m_3}\,, \tag{8}$$

and

$$C_1'^I = 1,\ C_2'^I = \frac{-m_3}{m_3 - m_2}\,,\ C_3'^I = \frac{-m_2}{m_2 - m_3}\,, \tag{9}$$

where the C_i^I are the weight factors for the massive lepton fields and the $C_i'^I$ those for the massless lepton fields. The superscript $I = 1$ is for e or ν_e, and $I = 2$ is for μ or ν_μ.

The space defined by the state vectors of this system is a vector space with indefinite metric. The physical states are defined as all the states without any excitation of shadow quanta. The S-matrix operator describing physical processes is defined in the physical subspace formed by the physical states. In order to make this S-matrix operator unitary, the boundary conditions have to be properly altered. These boundary conditions are different from the Feynman-Dyson theory. In the interaction representation, the S-matrix operator for the finite quantum field theory with shadow states can be written in the following simple form[8]:

$$S = \sum_{n=0}^{\infty} G^n S_n\,, \tag{10}$$

with

$$S_n = \left(\frac{i}{G}\right)^n \int_{-\infty}^{\infty} \pi^p \mathcal{L}_I(t')[\theta(t'-t'')\pi^p + \tfrac{1}{2}\epsilon(t'-t'')\pi^s]\mathcal{L}_I(t'')\cdots[\theta(t^{n-1}-t^n)\pi^p + \tfrac{1}{2}\epsilon(t^{n-1}-t^n)\pi^s]\mathcal{L}_I(t^n)\pi^p dt'\cdots dt^n\,, \tag{11}$$

where π^p and π^s are respectively the physical and the shadow-state projection operators.

With the S matrix given above, one may proceed to compute the scattering amplitudes in the perturbative manner. The scattering amplitudes so computed are all expected to be finite. The main question here is how we obtain the better-approximated amplitude. In the $V-A$ theory, even though in low-energy processes the predictions of the lowest-order term agree with experiment so well, it is apparently not a good approximation in the high-energy region where higher-order terms are expected to be important. However, as we will show later, the perturbative series with the interaction Lagrangian in Eq. (1) does not converge, since part of the series, corresponding to those in Fig. 1(a), diverges there. In order to avoid this difficulty, we will rearrange the perturbative series in the energy region where it is convergent. The resulting new series is then analytically continued into the region where the original series diverges. The first-order diagram in the new series is the sum of all the diagrams in Fig. 1(a). In this note, we shall concentrate on $e\bar{\nu}$ scattering and muon decay.

Consider first the electron–anti-electron-neutrino scattering. The diagrams contributing to the first-order terms of the new perturbative series

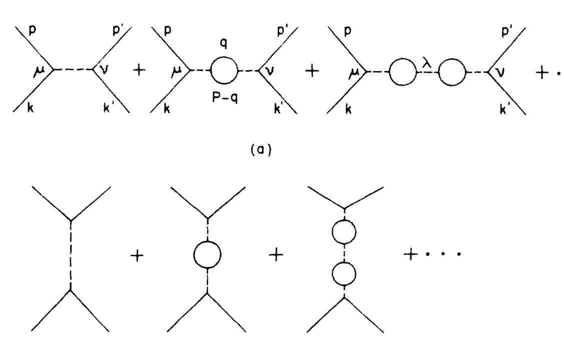

FIG. 1. Two sets of chain loop diagrams which are the two first-order diagrams in the new perturbative series.

to be defined are shown in Fig. 1(a). Let us denote the four-momenta of the incident (outgoing) electron and antineutrino by p (p') and k (k'). The scattering amplitude for the first diagrams is given, as usual, by

$$-i\frac{G}{\sqrt{2}}\,\bar{v}(k)\gamma_\mu(1-\gamma_5)u(p)\bar{u}(p')\,\gamma_\nu(1-\gamma_5)v(k')g^{\mu\nu} \equiv -iM_{\mu\nu}\overline{W}{}_0^{\mu\nu}\,, \quad (12)$$

where the usual conventions[9] have been adopted and $\overline{W}{}_0^{\mu\nu}=g^{\mu\nu}$. The tensor $M_{\mu\nu}$ represents the part of the four external lines of the first diagram which is in fact the common factor for all the diagrams in Fig. 1(a). In contrast, the tensor $\overline{W}{}_0^{\mu\nu}$ is the internal part of the first diagram varying from diagram to diagram. The total contribution $M^{(1)}$ of those diagrams in Fig. 1(a) to the amplitude can be summed:

$$M^{(1)}(p) = -iM_{\mu\nu}\left[\sum_{n=0}^{\infty}(-i)^n\overline{W}{}_n^{\mu\nu}\right], \quad (13)$$

where $\overline{W}{}_n^{\mu\nu}$ is from the n-loop chain diagram. Each loop involves an electronic or muonic line. In the present model, there are totally eighteen combinations, half of them for the electronic loops and the others for muonic loops, because of the presence of two shadow states accompanying each physical particle. The one-loop contribution $\overline{W}{}_1^{\mu\nu}$ can be immediately written down:

$$\overline{W}{}_1^{\mu\nu} = -\frac{G}{\sqrt{2}}\sum_{I=1}^{2}\int_{(S)}d^4q\,\mathrm{Tr}\left[\gamma^\mu(1-\gamma_5)\left(\sum_{i=1}^{3}\frac{C_i^I}{\not{q}-m_i}\right)\gamma^\nu(1-\gamma_5)\left(\sum_{j=1}^{3}\frac{C_j'^I}{\underline{\not{p}}-\not{q}-m_j'}\right)\right]$$

$$= -\frac{G}{\sqrt{2}}\sum_{I=1}^{2}\sum_{i,j}C_i^IC_j'^ID_{ij}\,, \quad (14a)$$

$$D_{ij} = \int_{(S)}d^4q\,\mathrm{Tr}\left[\gamma^\mu(1-\gamma_5)\frac{1}{\not{q}-m_i}\,\gamma^\nu(1-\gamma_5)\frac{1}{\underline{\not{p}}-\not{q}-m_j'}\right], \quad (14b)$$

where $\underline{p}$ $(=p+k=p'+k')$ is the total four-momentum of the system. For definiteness, we have taken the following assignments:

$$m_1=m^I,\quad m_1'=0,\quad m_2'=m_2,\quad m_3'=m_3\,. \quad (15)$$

The integral $\int_{(S)}$ means that its analytic structure is specified according to the shadow-state theory. In fact, its imaginary part vanishes if one or both of the two internal lines are shadow states. The integral D_{ij} obviously has quadratic divergence. However, this divergence in $\overline{W}{}_1^{\mu\nu}$ can be removed if the conditions in Eqs. (8) and (9) are satisfied. By standard technique[10] and by making use of Eqs. (8) and (9), one can calculate the one-loop part $\overline{W}{}_1^{\mu\nu}$. For convenience, we separate it into two parts, the s-wave and p-wave components:

$$\overline{W}{}^{\mu\nu} = i\left(g^{\mu\nu}-\frac{\underline{p}^\mu\underline{p}^\nu}{\underline{p}^2}\right)A(\underline{p}^2) - i\frac{\underline{p}^\mu\underline{p}^\nu}{\underline{p}^2}B(\underline{p}^2)\,, \quad (16a)$$

$$A(\underline{p}^2) = \sqrt{2}\,\pi^2G\sum_{I=1}^{2}\sum_{i,j}C_i^IC_j'^IA_{ij}(\underline{p}^2)\,, \quad (16b)$$

$$B(\underline{p}^2) = \sqrt{2}\,\pi^2G\sum_{I=1}^{2}\sum_{i,j}C_i^IC_j'^IB_{ij}(\underline{p}^2)\,. \quad (16c)$$

The p-wave and s-wave form factors, $A(\underline{p}^2)$ and $B(\underline{p}^2)$, depend on the combination of the masses. The zero-mass case, where one of the two internal lines in the loop is a neutrino, cannot be obtained by analytic continuation in mass from the cases involving no neutrinos. Therefore, we express them separately: One has, for the neutrino case,

$$A_{i1}(\underline{p}^2) = \left[-\tfrac{2}{3}\underline{p}^2\left(\frac{\underline{p}^2-m_i^2}{\underline{p}^2}\right)^3 + \tfrac{1}{2}(2\underline{p}^2-m_i^2)\left(\frac{\underline{p}^2-m_i^2}{\underline{p}^2}\right)^2\right]\ln\left(\frac{m_i^2-\underline{p}^2}{m_i^2}\right) + \tfrac{2}{3}\underline{p}^2\left(\frac{\underline{p}^2-m_i^2}{\underline{p}^2}\right)^2$$

$$-\tfrac{1}{6}(4\underline{p}^2-3m_i^2)\left(\frac{\underline{p}^2-m_i^2}{\underline{p}^2}\right) + \tfrac{1}{6}(2\underline{p}^2-3m_i^2)\ln m_i^2 + \tfrac{1}{2}m_i^2\,, \quad (17a)$$

$$B_{i1}(\underline{p}^2) = \tfrac{1}{2}m_i^2\left[\left(\frac{\underline{p}^2-m_i^2}{\underline{p}^2}\right)^2\ln\left(\frac{m_i^2-\underline{p}^2}{m_i^2}\right) - \left(\frac{\underline{p}^2-m_i^2}{\underline{p}^2}\right) + \ln m_i^2 - 1\right], \quad (17b)$$

in the domain $\underline{p}^2\leqslant m_i^2$, and for the other cases ($j\neq 1$),

$$A_{ij}(\underline{p}^2) = \left[\frac{1}{6}-\frac{m_i^2+m_j'^2}{12\underline{p}^2}-\frac{(m_i^2-m_j'^2)^2}{12\underline{p}^4}\right]U(\underline{p}^2,m_i^2,m_j'^2)\ln\left[\frac{\underline{p}^2-m_i^2-m_j'^2+U(\underline{p}^2,m_i^2,m_j'^2)}{\underline{p}^2-m_i^2-m_j'^2-U(\underline{p}^2,m_i^2,m_j'^2)}\right]$$

$$-\frac{(m_i^2-m_j'^2)^3}{3\underline{p}^4}\ln\frac{m_i^2}{m_j'^2} + \tfrac{1}{12}(2\underline{p}^2-3m_i^2-3m_j'^2)\ln(m_i^2m_j'^2) + \frac{(m_i^2-m_j'^2)^2}{6\underline{p}^2} + \tfrac{1}{3}(m_i^2+m_j'^2)\,, \quad (18a)$$

$$B_{ij}(\underline{p}^2)=\left[\frac{(m_i{}^2+m_j'^2)}{4\underline{p}^2}-\frac{(m_i{}^2-m_j'^2)^2}{4\underline{p}^4}\right]U(\underline{p}^2,m_i{}^2,m_j'^2)\ln\left[\frac{\underline{p}^2-m_i{}^2-m_j'^2+U(\underline{p}^2,m_i{}^2,m_j'^2)}{\underline{p}^2-m_i{}^2-m_j'^2-U(\underline{p}^2,m_i{}^2,m_j'^2)}\right]$$
$$-\left[\frac{(m_i{}^2-m_j'^2)^3}{4\underline{p}^4}-\frac{(m_i{}^4-m_j{}^4)}{2\underline{p}^2}\right]\ln\frac{m_i{}^2}{m_j'^2}+\frac{(m_i{}^2-m_j'^2)^2}{2\underline{p}^2}+\tfrac{1}{4}(m_i{}^2+m_j'^2)\ln(m_i{}^2m_j'^2)-(m_i{}^2+m_j'^2)\,, \qquad (18b)$$

$$U(\underline{p}^2,m_i{}^2,m_j'^2)=[(\underline{p}^2-m_i{}^2+m_j'^2)^2-4m_j'^2\underline{p}^2]^{1/2}\,, \qquad (18c)$$

in the domain $0\leq\underline{p}^2\leq(m_i-m_j'^2)^2$.

The form factors $A_{ij}(\underline{p}^2)$ and $B_{ij}(\underline{p}^2)$ are real in the aforementioned regions, respectively. Below the threshold involving shadow states, they are real analytic. Hence, once the cut structure of the form factors is specified, they are defined in other regions by analytic continuation, with the prescription in accordance with the shadow-state theory. The logarithmic cut of the form factors $A_{i1}(\underline{p}^2)$ and $B_{i1}(\underline{p}^2)$ is extended from $m_i{}^2$ to infinity. The cut structure for $A_{ij}(\underline{p}^2)$ and $B_{ij}(\underline{p}^2)$ is similar to that of the lowest-order self-energy diagram in quantum electrodynamics. Its square-root cut from $U(\underline{p}^2,m_i{}^2,m_j'^2)$ runs from $(m_i-m_j')^2$ to $(m_i-m_j')^2$, while the logarithmic cut runs from $(m_i-m_j')^2$ to infinity. Whenever one or both of the internal lines are shadow states, their imaginary parts (if any) must be discarded in the formulation of the shadow-state theory. This is explicitly indicated by the integration notation $\int_{(S)}$. The above analytic structure is valid for the equal-mass cases also. It is interesting to note that $A(\underline{p}^2)$ and $B(\underline{p}^2)$ have an unwanted pole at $\underline{p}^2=0$. However, we will later show that in the new perturbative series (to be defined), the pole at $\underline{p}^2=0$ disappears.

One can also calculate the n-loop contribution $\overline{W}_n^{\mu\nu}(\underline{p}^2)$ for $n>1$. It turns out to be the tensor product of N one-loop functions $\overline{W}_1^{\mu\nu}$:

$$\overline{W}_n^{\mu\nu}=\overline{W}_{1\lambda}^{\mu}\overline{W}_{1\delta}^{\lambda}\cdots\overline{W}_1^{\rho\nu} \qquad (19a)$$
$$=i^n\left(g^{\mu\nu}-\frac{\underline{p}^\mu\underline{p}^\nu}{\underline{p}^2}\right)A^n(\underline{p}^2)+(-i)^n\frac{\underline{p}^\mu\underline{p}^\nu}{\underline{p}^2}B^n(\underline{p}^2)\,, \qquad (19b)$$

where we have used the orthonormal relations for the s-wave and p-wave projection operators, $\underline{p}^\mu\underline{p}^\nu/\underline{p}^2$ and $[g^{\mu\nu}-(\underline{p}^\mu\underline{p}^\nu/\underline{p}^2)]$. From Eqs. (16b)–(18b), one can easily show that in the physical region where $\underline{p}^2$ is the total energy,

$$A(\underline{p}^2)\sim-\frac{i\sqrt{2}}{3}G\pi^3\underline{p}^2$$

and (20)

$$B(\underline{p}^2)\sim-i\frac{\sqrt{2}}{2}G\pi^3(m^2+\mu^2)\,,$$

as $\underline{p}^2\to\infty$, where m and μ are respectively the masses of electron and muon.

From Eqs. (13), (19b), and (20), one can easily see that the original perturbative series diverges when $\underline{p}^2$ is larger than some constant c. Since the coupling constant G is expected to be small, the infinite sum $\sum(-i)^n\overline{W}_n^{\mu\nu}$ with proper choices of shadow masses converges in some region of $\underline{p}^2$, particularly near the physical threshold. In this region, we shall express the sum in terms of a closed-form expression, so that the latter can be analytically continued into the region where the former is divergent. The closed form corresponds to one of the two first-order diagrams [see Figs. 2(a) and 2(b)] in a new perturbative series. This new perturbative series can be diagrammatically represented by all the Feynman diagrams with no chain loops, which we call dressed diagrams. It is due to the fact that any Feynman diagram with chain loops is contained in the corresponding dressed diagram in the new perturbative expansion obtained from the former by suppressing the chain loops.

From Eqs. (13) and (19b), the first-order amplitude $M^{(1)}$ in the new perturbative series is given by

$$M^{(1)}=-iM_{\mu\nu}\left[g^{\mu\nu}F_1(\underline{p}^2)+\frac{\underline{p}^\mu\underline{p}^\nu}{\underline{p}^2}F_2(\underline{p}^2)\right]\,, \qquad (21a)$$

$$F_1(\underline{p}^2)=\frac{1}{1-A(\underline{p}^2)}\,,$$
$$F_2(\underline{p}^2)=\frac{1}{1+B(\underline{p}^2)}-\frac{1}{1-A(\underline{p}^2)}\,. \qquad (21b)$$

The above expression is one of our main results. It means that the contribution of all the chain diagrams in Fig. 1(a) is represented by the two form factors, $F_1(\underline{p}^2)$ and $F_2(\underline{p}^2)$, depending only on the square of the four-momentum of the system. As a prescription, the scattering amplitude for any dressed diagram in the new perturbative series can be obtained by replacing $g^{\mu\nu}$ in the corresponding

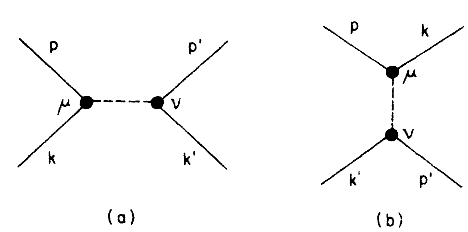

FIG. 2. Diagrammatical representations of the two lowest-order diagrams in the new perturbative series.

Feynman diagram in the original series by

$$g^{\mu\nu}F_1(p^2)+\frac{p^\mu p^\nu}{p^2}F_2(p^2).$$

The new amplitude $M^{(1)}$ vanishes at $p^2=0$, since $A(p^2)$ and $B(p^2)$ approach infinity there. As mentioned before, the amplitude $M^{(1)}$ does not have a pole at $p^2=0$ while each n-loop amplitude for $n \geq 1$ has. From Eqs. (20) and (21b), one can easily show that

$$F_1(p^2)\sim -\frac{3\sqrt{2}\,i}{2\pi^3 G p^2}\,,\quad F_2(p^2) \sim \frac{1}{1-i(\sqrt{2}/2)G\pi^3(m^2+\mu^2)}\,, \tag{22}$$

as $p^2\to\infty$. Hence, the amplitude $M^{(1)}$ can be continued into the region where the original infinite sum diverges.

From Eq. (21a), the differential cross section for $e\bar{\nu}$ scattering is given by

$$\frac{d\sigma}{d\Omega}=\frac{G^2(s-m^2)^2}{16\pi^2 s^3}\{4[(s+m^2)^2-(s-m^2)\cos\theta]^2|F_1(s)|^2-2m^4(1-\cos\theta)\,\mathrm{Re}[F_1(s)F_2^*(s)]-m^4|F_2(s)|^2\} \tag{23a}$$

$$\sim\frac{1}{s}\left[\frac{9}{2\pi^8}(1-\cos\theta)^2-\frac{G^2}{16\pi^2[1+\frac{1}{2}G^2\pi^6(m^2+\mu^2)^2]}\right], \tag{23b}$$

as $s\to\infty$, where θ is the c.m. scattering angle and $s=p^2=(p+k)^2$. Partial-wave unitarity puts a bound on $d\sigma/d\Omega$:

$$\frac{d\sigma}{d\Omega}\to |f_0+3f_1\cos\theta|^2/s\,, \tag{24}$$

where f_0 and f_1 are the s-wave and p-wave amplitudes with $|f_0|$, $|f_1|\leq 1$. Since the weak coupling constant G is expected to be small ($\sim 10^{-11}$ MeV^{-2}), we can see from Eqs. (23b) and (24) that the differential cross section in Eq. (23a) satisfies partial-wave unitarity irrespective of what the shadow masses are. In a similar manner, the electron energy spectrum of polarized muon decay can be calculated.

We have shown explicitly that the first-order amplitude $M^{(1)}$ of $e\bar{\nu}$ scattering satisfies unitarity and has no unwanted pole at $p^2=0$. These behaviors are independent of the choice of shadow masses. However, further properties of the amplitude depend on the choice of the coupling constant and the shadow masses. For presently available data,[11,12] this choice might not be unique. With $G=1.0129\, G^{V-A}$, $m_2=785.41$ MeV, and $m_3=224.00$ MeV, the deviation of the present model from the universal $V-A$ theory is within several thousandths, in the low c.m. energy region up to 10^3 MeV. Hence, the electron energy spectrum of polarized muon decay and its decay rate are in good agreement with the experiments within experimental error. The elastic cross section for $e\bar{\nu}$ scattering in the energy region from 3.6 to 5 MeV is about 2% larger than that from the universal $V-A$ theory. In these low-energy regions, the difference between the two theories is negligible. In high-energy regions the former deviates from the latter significantly, as is manifested by the fact that one violates partial-wave unitarity while the other does not. When the c.m. energy of incoming neutrinos increases to 10^5 MeV, the differential cross section for $e\bar{\nu}$ scattering drops to one quarter of that calculated with the $V-A$ theory. It is interesting to see whether the high-energy behavior of $e\bar{\nu}$ scattering predicted by the present model can be confirmed by experiments.

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PHYSICAL REVIEW D VOLUME 10, NUMBER 7 1 OCTOBER 1974

Implications of anomalous Lorentz structure in neutral weak processes*

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Effects of possible anomalous Lorentz structures in the neutral weak interaction in neutrino-electron scattering, deep-inelastic neutrino-nucleon scattering, and pion production in the N^* region are discussed.

Evidence[1] has been accumulating that the weak interaction contains a neutral current which couples to neutrinos. While current theoretical thought heavily favors the proposition that the neutral current consists of a linear combination of vector and axial-vector currents, the ultimate verdict should be left to experiment. In this note we would like to work out the form of experimentally measurable distributions while assuming a general Lorentz-invariant structure for the neutral current. This question has already been commented upon by Rosen[2] and by Sakurai[3] for various special cases. Indeed, even the cherished $V-A$ structure of the charged current should be put to a rigorous experimental test, as discussed in detail by Cheng and Tung.[4]

Let us first consider the processes[5] $\nu_\mu(k)+e^-(p)\to\nu_\mu(k')+e^-(p')$ and $\bar\nu_\mu e^-\to\bar\nu_\mu e^-$. We write down the most general local Lagrangian without derivative coupling[6]:

$$\begin{aligned}\mathcal{L}_I = {}& g_{VV}\bar\nu\gamma_\alpha\nu\bar e\gamma^\alpha e + g_{AA}\bar\nu\gamma_\alpha\gamma_5\nu\bar e\gamma^\alpha\gamma_5 e \\ &+ g_{AV}\bar\nu\gamma_\alpha\gamma_5\nu\bar e\gamma^\alpha e + g_{VA}\bar\nu\gamma_\alpha\nu\bar e\gamma^\alpha\gamma_5 e \\ &+ g_{SS}\bar\nu\nu\bar e e + g_{PP}\bar\nu\gamma_5\nu\bar e\gamma_5 e \\ &+ ig_{PS}\bar\nu\gamma_5\nu\bar e e + ig_{SP}\bar\nu\nu\bar e\gamma_5 e \\ &+ g_{TT}\bar\nu\sigma_{\alpha\beta}\nu\bar e\sigma^{\alpha\beta}e + ig_{\tilde T T}\bar\nu\sigma_{\alpha\beta}\gamma^5\nu\bar e\sigma^{\alpha\beta}e\,. \end{aligned} \tag{1}$$

Hermiticity implies that all the g's are real. Note that CP invariance would imply $g_{PS}=g_{SP}=g_{\tilde T T}=0$. We do *not* assume CP invariance in what follows. Experimentally, the incoming ν_μ is left-handed and so the amplitude for $\nu_\mu e^-\to\nu_\mu e^-$ becomes

$$\begin{aligned}\mathfrak{M} = {}& \bar\nu\gamma_\alpha(1-\gamma_5)\nu\bar e\gamma^\alpha(g_V-g_A\gamma_5)e \\ &+\bar\nu(1-\gamma_5)\nu\bar e(g_S-g_P\gamma_5)e \\ &+g_T\bar\nu\sigma_{\alpha\beta}(1-\gamma_5)\nu\bar e\sigma^{\alpha\beta}e\,, \end{aligned} \tag{2}$$

where

$$\begin{aligned} g_V &= \tfrac{1}{2}(g_{VV}-g_{AV})\,, \\ g_A &= \tfrac{1}{2}(g_{AA}-g_{VA})\,, \\ g_S &= \tfrac{1}{2}(g_{SS}-ig_{PS})\,, \\ g_P &= \tfrac{1}{2}(g_{PP}-ig_{SP})\,, \\ g_T &= \tfrac{1}{2}(g_{TT}-ig_{\tilde T T})\,. \end{aligned} \tag{3}$$

Hence, CP invariance would correspond to reality of all the quantities in Eq. (3). Note that the only terms which may violate P invariance without also violating CP invariance are the V and A terms.

A straightforward but somewhat tedious calculation gives, in the laboratory frame, that

$$\begin{aligned}\frac{2\pi}{mE}\frac{d\sigma}{dy}(\nu_\mu e^-\to\nu_\mu e^-) = {}& 2(g_V+g_A)^2+2(g_V-g_A)^2(1-y)^2 \\ &+(|g_S|^2+|g_P|^2)y^2 \\ &+32|g_T|^2(1-\tfrac{1}{2}y)^2 \\ &-8\,\mathrm{Re}[g_T(g_S^*+g_P^*)]\,y(1-\tfrac{1}{2}y)\,, \end{aligned} \tag{4}$$

where $y\equiv E(\text{recoil electron})/E(\text{incoming neutrino})$. Of course, one may also equivalently express this