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Photon amplitudes predicted by the transformation between current and constituent quarks*

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The transformation between current and constituent quarks is discussed as it applies to real photon transitions. The general algebraic structure of such transitions is presented, and a resulting set of selection rules is derived. Many specific amplitudes for both mesons and baryons are worked out, and both their magnitudes and signs are compared with available experimental data.

I. INTRODUCTION

A complete knowledge of the nature of the transformation from constituent to current quark states, together with the identification of the observed hadrons with simple (constituent) quark-model states, would permit one to calculate all current-induced transitions between hadrons. A major step in this direction has been taken by Melosh,[1] who was able to formulate and explicitly calculate such a transformation in the free quark model. While the details of such a transformation certainly depend on strong-interaction dynamics, it is possible that certain general algebraic properties of the transformation abstracted from the free quark model may hold in nature.

We shall assume that such a transformation does indeed exist, and that some of its algebraic properties can be abstracted from the free quark model. For the case of the axial-vector charge, the many consequences of this for pion transitions have already been extensively worked out and compared with experiment.[2,3] Here we report the results for real ($q^2=0$) photon transitions.

In the next section we present the origin and the basic properties of the theory along with the assumptions involved in applying it to actual hadrons. The general algebraic structure of photon amplitudes is discussed, as well as the method of calculating specific matrix elements. We derive a set of selection rules which include, and generalize, the old result[4] that the transition from the nucleon to 3-3 resonance should be magnetic dipole in character. This general discussion of the theory is completed by a comparison with other theories with a related algebraic structure.

In Sec. III the photon transitions between meson states are detailed, along with a comparison of the predictions with the available experimental data. Then we turn to a detailed exposition of baryon electromagnetic transition amplitudes in Sec. IV. A comparison of the predicted amplitudes with experiment, in both their magnitude and sign, is found in Sec. V. The signs are testable through a multipole analysis of pion photoproduction $\gamma N \to N^* \to \pi N$, where the signs of the previously calculated pion decay amplitudes[2,3] also come into play. As reported in a previous paper,[5] consistency of experiment and theory is found, including agreement with the relative signs of pion decay amplitudes obtained from analyzing $\pi N \to N^* \to \pi\Delta$. A summary and some conclusions are found in Sec. VI. The general outlook is very good, encouraging further study of the underlying dynamics and the extension to the $q^2 \neq 0$ region.

II. PHOTON AMPLITUDES AND THE TRANSFORMATION FROM CURRENT TO CONSTITUENT QUARKS

As we shall be concerned with current-induced transitions between hadrons, let us first consider the algebra formed by the 16 vector and axial-vector charges, $Q^\alpha(t)$ and $Q_5^\alpha(t)$, which are simply integrals over all space of the time components of the corresponding currents measurable in weak and electromagnetic interactions:

$$Q^\alpha(t) = \int d^3x\, V_0^\alpha(\vec{x}, t)\,, \tag{1a}$$

$$Q_5^\alpha(t) = \int d^3x\, A_0^\alpha(\vec{x}, t)\,. \tag{1b}$$

Here α is an SU(3) index which runs from 1 to 8. At equal times these charges commute to form the algebra abstracted from the quark model by Gell-Mann[6]:

$$[\,Q^\alpha(t), Q^\beta(t)] = if^{\alpha\beta\gamma}Q^\gamma(t)\,, \tag{2a}$$

$$[\,Q^\alpha(t), Q_5^\beta(t)] = if^{\alpha\beta\gamma}Q_5^\gamma(t)\,, \tag{2b}$$

$$[\,Q_5^\alpha(t), Q_5^\beta(t)] = if^{\alpha\beta\gamma}Q^\gamma(t)\,. \tag{2c}$$

This is the algebra of chiral SU(3)×SU(3), for it can be easily shown that Eqs. (2) are equivalent to the statement that the right-handed charges, $Q^\alpha + Q_5^\alpha$, and the left-handed charges, $Q^\alpha - Q_5^\alpha$, each form an SU(3), and that they commute with each other—hence, chiral SU(3)×SU(3). For $\alpha = 1, 2, 3$ the Q^α's are the generators of isospin rotations; for $\alpha = 1, \ldots, 8$, they are the generators of SU(3). The last of Eqs. (2), sandwiched between nucleon states moving at infinite momentum in the z direction, yields the Adler-Weisberger sum rule.[7]

Taken between states at infinite momentum,[8] the Q^α's and Q_5^α's are "good" operators, i.e., they have finite (generally nonvanishing) values as $p_z \to \infty$. These values are the same as those of space integrals over the z components of the respective currents. If we adjoin to the integrals of the time component of the vector currents and the z component of the axial-vector currents[8] (which commute like $\frac{1}{2}\lambda^\alpha$ and $\frac{1}{2}\lambda^\alpha\sigma_z$), integrals over certain "good" tensor current densities, the SU(3)×SU(3) algebra between states at infinite momentum can be enlarged still further. Letting the index α correspond to an SU(3) singlet when $\alpha = 0$, we have 36 charges which commute like the products of SU(3) and Dirac matrices: $\frac{1}{2}\lambda^\alpha$, $\frac{1}{2}\lambda^\alpha\beta\sigma_x$, $\frac{1}{2}\lambda^\alpha\beta\sigma_y$, and $\frac{1}{2}\lambda^\alpha\sigma_z$, where $\alpha = 0, 1, \ldots, 8$. These act as an identity operator plus an $\mathrm{SU}(6)_W$ algebra of 35 generators. We refer to this algebra, introduced by Dashen and Gell-Mann[9] in 1965, as the $\mathrm{SU}(6)_W$ of currents. We denote these generators collectively by F^i, and use them to label the transformation properties of our states and operators. Note that $\beta\sigma_x$, $\beta\sigma_y$, and σ_z, which commute with z boosts and are "good" operators, are *not* the same as the spin components σ_x, σ_y, and σ_z. The appropriate algebra to use is that of $\mathrm{SU}(6)_W$ and not SU(6). For quarks, $\beta = +1$ and quark spin and "W spin" are the same; but for antiquarks, $\beta = -1$, we have $-\sigma_x$, $-\sigma_y$, and σ_z instead of the antiquark spin components σ_x, σ_y, and σ_z.

In what follows we will label states and operators by their transformation properties under this $\mathrm{SU}(6)_W$ algebra of currents. For this purpose we shall often use just the SU(3)×SU(3) subalgebra of the whole $\mathrm{SU}(6)_W$ algebra of currents, as this subalgebra has elements which are known to be directly measurable in weak and electromagnetic interactions. The over-all $\mathrm{SU}(6)_W$ representation will either be obvious or be made explicit. We will write

$$\{(A, B)_{S_z}, L_z\},$$

where A is the SU(3) representation under $Q^\alpha + Q_5^\alpha$, B is the representation under $Q^\alpha - Q_5^\alpha$, and S_z is the eigenvalue of Q_5^0, the singlet axial-vector charge.[8] The quantity L_z is then *defined* in terms of the z component of the total angular momentum J, as $L_z = J_z - S_z$. The "ordinary" SU(3) content (under Q^α) of such a representation is just that of the direct product $A \times B$.

All representations of chiral SU(3)×SU(3) can be built up from $(3,1)_{1/2}$, $(1,3)_{-1/2}$, $(1,\overline{3})_{1/2}$, and $(\overline{3},1)_{-1/2}$ which we *define* to be the *current quark* and *current antiquark* states with spin projection $\pm\frac{1}{2}$ in the z direction. The quarks form a $\underline{6}$ and the antiquarks form a $\underline{\overline{6}}$ in the full $\mathrm{SU}(6)_W$ of currents.

Consider next combining three current quarks to form a baryon. If we take $L_z = 0$ and a symmetrical quark wave function, then we find that the states with net spin $S = \frac{1}{2}$ and $S = \frac{3}{2}$ transform as

$$
\begin{aligned}
S=\tfrac{3}{2},\ S_z=\tfrac{3}{2}&:\ \{(10,1)_{3/2},0\},\\
S=\tfrac{3}{2},\ S_z=\tfrac{1}{2}&:\ \{(6,3)_{1/2},0\},\\
S=\tfrac{1}{2},\ S_z=\tfrac{1}{2}&:\ \{(6,3)_{1/2},0\},\\
S=\tfrac{1}{2},\ S_z=-\tfrac{1}{2}&:\ \{(3,6)_{-1/2},0\},\\
S=\tfrac{3}{2},\ S_z=-\tfrac{1}{2}&:\ \{(3,6)_{-1/2},0\},\\
S=\tfrac{3}{2},\ S_z=-\tfrac{3}{2}&:\ \{(1,10)_{-3/2},0\},
\end{aligned}
$$

and they all lie in a $\underline{56}$ of the full $\mathrm{SU}(6)_W$ of currents. In particular, if a nucleon at infinite momentum with $J_z = \frac{1}{2}$ acted under the algebra of currents as if it were simply composed of two current quarks with $S_z = \frac{1}{2}$ and one current quark with $S_z = -\frac{1}{2}$ in a symmetrical wave function, we would have

$$|N\rangle = |\underline{56},\ \{(6,3)_{1/2},0\}\rangle\,. \tag{3}$$

However, the SU(3) content of $(6,3)_{1/2}$ is just that of an octet (including the nucleon) and decuplet [including the $\Delta(1236)$]. Since Q_5^α is a generator of SU(3)×SU(3), it can only connect this representation to itself, i.e., for $\alpha = 1, 2, 3$ it can only connect the nucleon to the nucleon or to the $\Delta(1236)$. Furthermore, such a classification of the nucleon gives $g_A = \frac{5}{3}$. Both these results are in glaring contradiction with experiment. The nucleon *cannot* be in such a simple representation. This is already apparent from the Adler-Weisberger sum rule[7] itself, for it shows that the nucleon is connected by a generator of the algebra of currents, the axial-vector charge Q_5^α (in the guise of the pion field) to many higher mass N^*'s. Thus the nucleon and these N^*'s must be in the same representation of SU(3)×SU(3). Conversely, the nucleon state must span many different representations[10] of the SU(3)×SU(3) and $\mathrm{SU}(6)_W$ of currents.

Therefore physical hadron states like the nucleon are not simple in terms of current quarks, i.e., they are not in the irreducible representations of the $\mathrm{SU}(6)_W$ of currents prescribed by the naive construction of baryons out of three *current* quarks (or out of quark-antiquark for mesons). As the next simplest possibility, let us assume instead that there exists a unitary operator, V, which transforms an irreducible representation (I.R.) of the algebra of currents into the physical state:

$$|\text{hadron}\rangle = V|\text{I.R., currents}\rangle . \tag{4}$$

The state $|\text{I.R., currents}\rangle$ corresponds to baryons being built from just three current quarks and mesons from quark-antiquark. Thus, for example, the complicated nucleon state is written as

$$|N\rangle = V|\underline{56}, \{(6,3)_{1/2}, 0\}\rangle . \tag{5}$$

All the complicated mixing of the real hadron states has been subsumed in the operator V.

In the following we will be interested in evaluating the hadronic matrix elements of charge or current operators, $\mathcal{O}^\alpha$. Using Eq. (4) we have

$$\langle \text{hadron}'|\mathcal{O}^\alpha|\text{hadron}\rangle = \langle \text{I.R.}', \text{currents}|V^{-1}\mathcal{O}^\alpha V|\text{I.R., currents}\rangle . \tag{6}$$

The complexity of hadronic states under the algebra of currents has been transferred to the effective operator $V^{-1}\mathcal{O}^\alpha V$ which may be studied as an independent object. Moreover, if the operator $V^{-1}\mathcal{O}^\alpha V$ has definite and simple transformation properties under the algebra of currents, the way is open to systematically evaluate the matrix elements of $\mathcal{O}^\alpha$ between any two hadronic states.

The operator V can be viewed in another way. It is easy to see that if we define a new set of generators

$$W^i = VF^iV^{-1}, \tag{7}$$

then the W^i also form an $SU(6)_W$ algebra. Furthermore, from the definition of V in Eq. (4), hadron states transform under the W^i as those irreducible representations which correspond to the naive constituent quark model of hadrons. We therefore call the quark states of this new $SU(6)_W$ *constituent quarks*,[11] and identify this new algebra with that of the $SU(6)_W$ of strong interactions.[12] Equation (4) can therefore be rewritten as

$$|\text{hadron}\rangle = |\text{I.R., constituents}\rangle = V|\text{I.R., currents}\rangle , \tag{8}$$

while Eq. (6) becomes

$$\langle \text{hadron}'|\mathcal{O}^\alpha|\text{hadron}\rangle = \langle \text{I.R.}', \text{constituents}|\mathcal{O}^\alpha|\text{I.R., constituents}\rangle = \langle \text{I.R.}', \text{currents}|V^{-1}\mathcal{O}^\alpha V|\text{I.R., currents}\rangle . \tag{9}$$

From this standpoint the operator V just takes one from one set of basis states to another, or alternately, from one set of generators to another.

In the present paper we are interested in current-induced transitions between hadrons at $q^2=0$. For the axial-vector current, which is not conserved, the axial-vector charge induces nontrivial transitions and one wants to know the algebraic properties of the transformed charge, $V^{-1}Q_5^\alpha V$. For the vector current, however, the corresponding charges Q^α generate SU(3), which is taken as exact, so that V is an SU(3) singlet and

$$V^{-1}Q^\alpha V = Q^\alpha . \tag{10}$$

The first nontrivial operator involving the vector current is

$$D_\pm^\alpha = i\int d^3x \left[\frac{\mp(x\pm iy)}{\sqrt{2}}\right] V_0^\alpha(\vec{x}, t) . \tag{11}$$

Matrix elements of $D_\pm^\alpha$ between states at infinite momentum are proportional to vector current transition amplitudes where the $q^2=0$ current carries $J_z=\pm1$, and we will then want to know the algebraic properties of $V^{-1}D_\pm^\alpha V$. Taken between states at infinite momentum, commutators of Q_5^α lead to Adler-Weisberger sum rules,[7] while commutators of $D_\pm^\alpha$ lead to Cabibbo-Radicati sum rules.[13]

The algebraic properties of the untransformed operators are that

$$Q_5^\alpha \text{ transforms as } \{(8,1)_0-(1,8)_0, 0\} , \tag{12}$$

$$D_\pm^\alpha \text{ transforms as } \{(8,1)_0+(1,8)_0, \pm1\} . \tag{13}$$

For guidance on what might be the algebraic properties of $V^{-1}Q_5^\alpha V$ and $V^{-}D_\pm^\alpha V$, we turn to the free quark model where a nontrivial transformation is already needed. There Melosh[1] has been able to construct an explicit form of the operator V. In a free quark model, both $V^{-1}Q_5^\alpha$ and $V^{-1}D_\pm^\alpha V$ must connect only single quark states to single quark states; they thus have the general form

$$V^{-1}Q_5^\alpha V \text{ or } V^{-1}D_\pm^\alpha V = \int d^3x\, q^+(x)\mathfrak{F}(\partial, \gamma)q(x) , \tag{14}$$

where $\mathfrak{F}$ is some function of the derivatives (∂) and the gamma matrices (γ). An explicit form of $\mathfrak{F}$ was originally determined by Melosh,[1] while

Eichten *et al.*[14] argued that a large class of such functions exist. More recently, Melosh has restricted this class by imposing angular momentum transformation properties on the "rotated" currents and obtained a form of the transformation V which is just that found by Dashen and Gell-Mann[11] for free quarks in connection with attempts to find representations of local current algebra. It is from the free quark model that we abstract only the algebraic properties of $V^{-1}Q_5^\alpha V$ and $V^{-1}D_+^\alpha V$. What is most important for us in such a model is that these transformed operators are "single quark" operators; i.e., they depend only on the coordinates of a single quark and they do not create connected $q\bar{q}$ pairs.

It is this property that we abstract from the free quark model and assume to hold in nature. In general, we assume the following: *The operators* $V^{-1}Q_5^\alpha V$ *and* $V^{-1}D_\pm^\alpha V$ *have the algebraic transformation properties under the* $SU(6)_W$ *of currents of the most general linear combination of single quark operators consistent with SU(3) and Lorentz invariance.*

This is verified in the explicit free quark model calculations. The operator $V^{-1}Q_5^\alpha V$, with $J_z=0$, contains two terms which transform under SU(3) ×SU(3) as $\{(8,1)_0-(1,8)_0,0\}$ and $\{(3,\bar{3})_1,-1\}-\{(\bar{3},3)_{-1},1\}$ and behave as components of $\underline{35}$'s of the full $SU(6)_W$ of currents. To apply this to observed hadron transitions in the case of Q_5^α, as few axial-vector weak decays are measured, one needs to relate matrix elements of Q_5^α between states of infinite momentum to matrix elements of the pion field via the partially conserved axial-vector current[15] (PCAC) hypothesis. One then has an approximate theory of the algebraic structure of pion amplitudes.[16]

As matrix elements of $D_\pm^3+(1/\sqrt{3})D_\pm^8$ are directly proportional to photon amplitudes, no additional assumption is necessary. Furthermore, matrix elements of D_+^α are equal, up to a sign, to those of D_-^α via parity conservation. We need then only consider the properties of D_+^α. Algebraically, the operator $V^{-1}D_+^\alpha V$, with $J_z=1$, is slightly more complicated than $V^{-1}Q_5^\alpha V$. In general, as pointed out by Hey and Weyers,[17] there are four possible terms: $\{(8,1)_0+(1,8)_0,1\}$, $\{(3,\bar{3})_1,0\}$, $\{(\bar{3},3)_{-1},2\}$, and $\{(8,1)_0-(1,8)_0,1\}$. It appears that all four occur in the operator $V^{-1}D_+^\alpha V$ in the free quark model.[1,14] However, the last term, which corresponds to $q\bar{q}$ in a net quark spin $S=0$, unnatural spin-parity state, has no analog with any natural spin-parity (in particular, vector-meson) state of the quark model. Moreover, under a generalized parity transformation, $Pe^{-i\pi J_y}$, which takes $\{(A,B)_{S_z},L_z\}\to\{(B,A)_{-S_z},-L_z\}$, the first three terms do not change sign while the last one does. For the longitudinal ($J_z=0$) component of the current this would eliminate the possibility of such a term. Therefore the $\{(8,1)_0-(1,8)_0,1\}$ term in D_+^α has no correspondence with any natural spin-parity meson state and cannot occur in the longitudinal component of the vector current. In the past we have therefore neglected such a term.[2,3,5] While we will carry all four terms in the remainder of this paper, we will at various times indicate experimental limits on the size of the $\{(8,1)_0-(1,8)_0,1\}$ term's contribution to various transitions and indicate what situation ensues if it is totally absent.

For photon decays, we have directly that in the narrow-resonance approximation,

$$\Gamma(\text{hadron}'\to\text{hadron}+\gamma)=\frac{e^2}{\pi}\,\frac{p_\gamma{}^3}{2J'+1}\sum_\lambda\left|\left\langle\text{hadron}',\ \lambda\left|D_+^3+\left(\frac{1}{\sqrt{3}}\right)D_+^8\right|\text{hadron},\ \lambda-1\right\rangle\right|^2, \tag{15}$$

where e is the proton charge, p_γ is the photon momentum, and the sum extends over all possible helicities λ. Matrix elements of D_- have been eliminated from Eq. (15) by relating them to those of D_+ via parity. Note that although the definition of D_+^α in Eq. (11) involves only a first moment of the current, between states at infinite momentum, *all* multipole amplitudes consistent with the spin and parity of the states enter matrix elements of D_+^α. Equation (15) may also be obtained from consideration of the narrow-resonance approximation to the "hadron′" contribution to the Cabibbo-Radicati sum rule[13] on "hadron" states. We have no arbitrary phase-space factors.

For the present we shall use the narrow-resonance approximation expression, Eq. (15), for photon decay widths in order to make a comparison of the theory with experiment. For broad resonances in the initial and/or final state, or for decays of resonances where the physically available phase space is small, such an approximation introduces non-negligible errors. However, we view the present comparison as being sufficiently accurate as a first test of the theory, particularly in view of the experimental errors on values for photon (as well as pion) decay widths. When the situation eventually warrants it, the values of $|\langle\text{hadron}'|Q_5^\alpha|\text{hadron}\rangle|^2$ and $|\langle\text{hadron}'|D_+^\alpha|\text{hadron}\rangle|^2$ should be determined irrespective of any approximation in terms of contributions to Adler-Weisberger and Cabibbo-Radicati sum rules, respectively.

Thus, in spite of the enormous complication of V itself, we abstract simple algebraic properties of $V^{-1}Q_5^\alpha V$ and $V^{-1}D_+^\alpha V$ from the free quark model and postulate them to hold in the real world. Namely, we assume that in nature $V^{-1}Q_5^\alpha V$ transforms as $\{(8,1)_0-(1,8)_0, 0\}$ and $\{(3,\overline{3})_1,-1\}-\{(\overline{3},3)_{-1},1\}$ while $V^{-1}D_+^\alpha V$ transforms as $\{(8,1)_0+(1,8)_0,1\}$, $\{(3,\overline{3})_1,0\}$, $\{(\overline{3},3)_{-1},2\}$, and $\{(8,1)_0-(1,8)_0,1\}$, all components of $\underline{35}$'s of the full $SU(6)_W$ of currents.

We are now almost in a position to apply the theory to actual decays. Recalling that, for example,

$$\langle \text{I.R.}', \text{ constituents } |D_+^\alpha|\text{I.R., constituents}\rangle = \langle \text{I.R.}', \text{ currents } |V^{-1}D_+^\alpha V|\text{I.R., currents}\rangle, \tag{16}$$

we see that with the assumed algebraic properties of $V^{-1}D_+^\alpha V$ (as abstracted from the free quark model), we know the transformation properties under the $SU(6)_W$ of currents of all quantities in a given matrix element of D_+^α between quark-model states. To make contact with experiment we make one physical assumption: Namely, we assume that we can *identify* the *observed* (nonexotic) *hadrons with constituent quark states.* In other words, we assume that there is a portion of the physical Hilbert space which is well approximated by the single-particle states of the constituent quark model. For baryons, composed of qqq, we have candidates[18] which fit very well into the $SU(6)_W\times O(3)$ representations $\underline{56}$ $L=0$, $\underline{70}$ $L=1$, and $\underline{56}$ $L=2$. For mesons we have correspondingly the $q\overline{q}$ states $\underline{35}$ $L=0$, $\underline{1}$ $L=0$, $\underline{35}$ $L=1$, etc. As we assume that states with different values of the quark spin as well as L_z and S_z are related as in the constituent quark model, i.e., by the $SU(6)_W$ of strong interactions, we relate different helicity states of a given hadron to each other.

With this physical assumption, we know the algebraic properties (under the algebra of *currents*) of all terms of a transformed matrix element of the current operators taken between physically observed states. Therefore we may use the Wigner-Eckart theorem and tables of Clebsch-Gordan coefficients to carry out the calculation from this point onward. Note that $SU(6)_W$ *invariance* of the transition operator under either the algebra of currents or that of strong interactions *is not assumed*—only the transformation properties of the various terms are needed in the calculation.

More explicitly, for a given matrix element of D_+^α we write the initial and final hadron state with $J_z=\lambda-1$ and λ, respectively, in terms of states with definite quark L_z and S_z. This involves coupling internal L and S to form total J for each hadron. After transforming to an $SU(6)_W$ of currents basis using V, the matrix element of any particular term in $V^{-1}D_+^\alpha V$ can then be written, using the Wigner-Eckart theorem applied to representations of the $SU(6)_W$ of currents, as a reduced matrix element times the product of quark angular momentum, $SU(6)_W$, SU(3), and W-spin Clebsch-Gordan coefficients.[19,20] For example, suppose we were calculating the matrix element of the $\{(3,\overline{3})_1,0\}$ piece of $V^{-1}D_+^\alpha V$ between initial and final states with helicity $\lambda-1$ and λ, total angular momentum J and J', internal quark orbital angular momentum L and L', quark spin S and S', $SU(6)_W$ representation R and R', and SU(3) representation A and A', respectively. Then we have that

$$\begin{aligned}\langle R',A',L',S',J',\lambda,\text{ currents } |\{(3,\overline{3})_1,0\}|R,A,L,S,J,\lambda-1,\text{ currents}\rangle \\ = \sum_{L_z,L_z',S_z,S_z'} \underbrace{(LSL_zS_z|J\lambda-1)(L'S'L_z'S_z'|J'\lambda)}_{\substack{\text{quark angular momentum}\\ \text{Clebsch-Gordan coefficients}}} \underbrace{\langle R'|35|R\rangle}_{\substack{SU(6)_W\text{ Clebsch-}\\ \text{Gordan coefficient}}} \underbrace{(A'|8|A)}_{\substack{SU(3)\text{ Clebsch-}\\ \text{Gordan coefficient}}} \underbrace{(1W1W_z|W'W_z')}_{\substack{W\text{-spin Clebsch-}\\ \text{Gordan coefficient}}} \\ \times \underbrace{\langle R',L',L_z'\|\{(3,\overline{3})_1,0\}\|R,L,L_z\rangle}_{\text{reduced matrix element}}.\end{aligned} \tag{17}$$

The W-spin Clebsch-Gordan coefficient follows since the $(3,\overline{3})_1$ operator has $W=1$ and $W_z=1$. For any state, $W_z=S_z$. For baryons, $\vec{W}=\vec{S}$, while for mesons we have the conventional correspondence ($W-S$ flip),[12]

$$\begin{aligned} |W=1,\ W_z=1\rangle &= |S=1,\ S_z=1\rangle, \\ |W=1,\ W_z=0\rangle &= -|S=0,\ S_z=0\rangle, \\ |W=1,\ W_z=-1\rangle &= -|S=1, S_z=-1\rangle, \\ |W=0,\ W_z=0\rangle &= -|S=1,\ S_z=0\rangle. \end{aligned} \tag{18}$$

The signs which result from using Eq. (18) to convert states from quark spin to W spin are understood to be included in Eq. (17) in the $SU(6)_W$ Clebsch-Gordan coefficient.

The reduction of the other terms in $V^{-1}D_+^\alpha V$ proceeds just as above, and we need only recall that $(8,1)_0+(1,8)_0$, $(8,1)_0-(1,8)_0$, and $(\overline{3},3)_{-1}$ transform as $W=W_z=0$; $W=1$, $W_z=0$; and $W=1$, $W_z=-1$ objects, respectively. Pion decays (matrix elements of Q_5^α) are handled in an analogous manner, except, of course, the initial and final states have $J_z = \lambda$. Note that since total J_z is conserved for either hadron$'\to$hadron$+\pi$ or hadron$'\to$hadron$+\gamma$ decays, and since the net value of $W_z = S_z$ must also be the same by the W-spin Clebsch-Gordan coefficient in Eq. (17) and its analogs, it follows that $L_z = J_z - S_z$ must also be additively conserved between the initial and final state (*including* the pion or photon operator).

The general structure of the results is now apparent. A matrix element of $D_+^\alpha(Q_5^\alpha)$ between hadron states will be equal to the sum of four (two) terms, each of which is a product of Clebsch-Gordan coefficients and a reduced matrix element which depends on the $SU(6)_W$ multiplet (and L_z values) of the external state components and the particular term in $V^{-1}D_+^\alpha V$ ($V^{-1}Q_5^\alpha V$) involved.

If L is zero, as is the case in essentially all cases of physical interest at the present time, then of course $L_z = 0$ and the L_z' dependence of the $SU(6)_W$ reduced matrix element becomes trivial due to conservation of L_z. In such a case ($L=0$), all photon decays from one $SU(6)_W$ multiplet to another are related to the same four reduced matrix elements (dropping the trivial L_z labels):

$$\langle R', L' \| (8,1)_0 + (1,8)_0 \| R, 0\rangle,$$

$$\langle R', L' \| (3,\bar{3})_1 \| R, 0\rangle,$$

$$\langle R', L' \| (\bar{3},3)_{-1} \| R, 0\rangle,$$

and

$$\langle R', L' \| (8,1)_0 - (1,8)_0 \| R, 0\rangle,$$

some of which may be zero or have zero coefficients due to selection rules. All pion decays (matrix elements of Q_5^α) similarly depend on two reduced matrix elements

$$\langle R', L' \| (8,1)_0 - (1,8)_0 \| R, 0\rangle,$$

$$\langle R', L' \| (3,\bar{3})_1 - (\bar{3},3)_{-1} \| R, 0\rangle$$

for given $SU(6)_W$ multiplets R', L' and $R, L=0$.

This algebraic structure of photon matrix elements already leads to interesting and powerful selection rules. Consider the $\{(8,1)_0 + (1,8)_0, 1\}$ term in $V^{-1}D_+^\alpha V$, which has W-spin zero. The W-spin Clebsch-Gordan coefficient in the analog of Eq. (17) implies

$$\vec{W}' = \vec{W}, \tag{19}$$

which is the same as

$$\vec{S}' = \vec{S}. \tag{20}$$

Now, for the hadron$'$ and hadron states we have

$$\vec{J}' = \vec{L}' + \vec{S}' \tag{21}$$

and

$$\vec{J} = \vec{L} + \vec{S}, \tag{22}$$

while angular momentum conservation for the total decay demands

$$\vec{J}' = \vec{J} + \vec{j}_\gamma, \tag{23}$$

where j_γ is the net angular momentum carried by the photon and determines the multipole character of the decay. Combining Eqs. (20)–(23) results in

$$|L - L'| \le j_\gamma \le |L + L'|, \tag{24}$$

and in the case $L=0$,

$$j_\gamma = L'. \tag{25}$$

Thus decays through the $\{(8,1)_0 + (1,8)_0, 1\}$ term in $V^{-1}D_+^\alpha V$ to $L=0$ baryons or mesons always have $j_\gamma = L'$ of the decaying hadron. As the parity change is $(-1)^{L'-L} = (-1)^{L'} = (-1)^{j_\gamma}$, this always corresponds to an electric $2L'$-pole transition in the usual multipole notation.

For the $\{(3,\bar{3})_1, 0\}$, $\{(\bar{3},3)_{-1}, +2\}$, and $\{(8,1)_0 - (1,8)_0, 1\}$ terms in $V^{-1}D_+^\alpha V$, all of which have W-spin one, Eq. (20) is modified to[21]

$$\vec{S}' = \vec{S} + \vec{1} \tag{26}$$

and as a result one finds in place of Eq. (24) that

$$\big||L - L'| - 1\big| \le j_\gamma \le \big||L + L'| + 1\big|. \tag{27}$$

For $L=0$ this simplifies to

$$|L' - 1| \le j_\gamma \le |L' + 1|, \tag{28}$$

so that

$$j_\gamma = L' - 1, L', L' + 1. \tag{29}$$

As the parity change is again $(-1)^{L'}$, these correspond to magnetic $2(L'-1)$-pole, electric $2L'$-pole, and magnetic $2(L'+1)$-pole transitions, respectively.

The actual correspondence between reduced matrix elements and a set of multipole amplitudes can also be proved using Racah coefficients to rewrite Eq. (17) and its analogs. For example, baryon transitions from $R, L=0$ to R', L' are describable in terms of multipole amplitudes

$$M(j_\gamma = L') = \langle R', L' \| (8,1)_0 + (1,8)_0 \| R, L=0\rangle \tag{30a}$$

and

$$\begin{aligned} M(j_\gamma = L'-1, L', L'+1) = {} & (1L'10|j_\gamma 1)\langle R', L' \| (3,\bar{3})_1 \| R, L=0\rangle \\ & + (1L'01|j_\gamma 1)\langle R', L' \| (8,1)_0 - (1,8)_0 \| R, L=0\rangle \\ & + (1L'-1\ 2|j_\gamma 1)\langle R', L' \| (\bar{3},3)_{-1} \| R, L=0\rangle . \end{aligned} \tag{30b}$$

Note, of course, that one only has $j_\gamma \geq 1$. Thus for $L'=0 \rightarrow L=0$, only $j_\gamma = 1$ is allowed. This is just the old result[4] that the nucleon to 3-3 resonance transition is magnetic dipole in character in the case of baryons.

For pion decays a parallel analysis[2,3] leads immediately to the rule

$$\big| |L-L'|-1 \big| \leq l \leq \big| |L+L'|+1 \big| , \tag{31}$$

where l is the angular momentum carried by the pion. For $L=0$, this reduces to

$$|L'-1| \leq l \leq |L'+1| , \tag{32}$$

and parity conservation forces the nontrivial result that

$$l = L'-1 \text{ or } L'+1 . \tag{33}$$

Note that for values of $L' \geq 3$, not only does the theory forbid values of j_γ or l larger than $L'+1$, but it also nontrivially forbids[22] values of j_γ or l less than $L'-1$ which are otherwise kinematically allowed, and even favored by angular momentum barrier arguments. The transition of a $J^P = \frac{3}{2}^-$ baryon resonance in a $\underline{70}$ $L'=3$ multiplet into a nucleon plus a photon with $j_\gamma = 1$ is forbidden, for example, even though this is the lowest allowed multipole on spin-parity grounds.

The algebraic structure of the theory of photon transitions presented above is closely related to various quark-model calculations, both nonrelativistic[23] and relativistic,[24] done in the past. They may be put into one to one correspondence if the $(8,1)_0+(1,8)_0$ term in $V^{-1}D_+^\alpha V$ is identified with the photon interacting with the quark convection current, and the $(3,\overline{3})_1$ term identified with the photon interacting with the quark magnetic moments. The $(\overline{3},3)_{-1}$ and $(8,1)_0-(1,8)_0$ terms in $V^{-1}D_+^\alpha V$ do not appear in these quark models.[23,24] Therefore one can make a complete algebraic correspondence with the identification of certain combinations of parameters there with the reduced matrix elements discussed here. However, the assumption of a "potential" and the resulting wave functions for the bound states in the quark-model calculations yield definite predictions for the reduced matrix elements themselves as they depend on masses and other parameters of the model. This is something we do not obtain, since we consider only the algebraic structure.

A similar correspondence occurs for pion decays. The results of the nonrelativistic quark model[25] (no recoil) correspond to keeping only the $(8,1)_0-(1,8)_0$ term in $V^{-1}Q_5^\alpha V$ while the relativistic quark model[24] yields amplitudes corresponding to the presence of both the $(8,1)_0-(1,8)_0$ and $(3,\overline{3})_1-(\overline{3},3)_{-1}$ reduced matrix elements discussed here.

Closely related to the quark-model results are those following from various versions of $SU(6)_W$ (of *strong interactions*) invariance.[12] The results of assuming $SU(6)_W$ conservation for pion transitions are reproduced in the present theory by retaining only the $\{(8,1)_0-(1,8)_0, 0\}$ term in $V^{-1}Q_5^\alpha V$ and using PCAC. The assumption of $SU(6)_W$ conservation plus vector dominance is equivalent to keeping only the $\{(3,\overline{3})_1, 0\}$ term in $V^{-1}D_+^\alpha V$.

As we will soon see, this is totally contradicted by the data. As a result, various broken $SU(6)_W$ schemes were developed.[26] Some of these are very similar to the present theory in algebraic structure, particularly for decays to $L=0$ hadrons.

The relation of such schemes for pion decays, and in particular l-broken $SU(6)_W$, to the present theory is discussed in detail in Ref. 27. For vector-meson decays, and via vector dominance for photon decays, one such scheme[28] corresponds in algebraic structure to the one presented here if the reduced matrix element of the $\{(\overline{3},3)_{-1}, 2\}$ term in $V^{-1}D_+^\alpha V$ vanishes and those of the $\{(8,1)_0+(1,8)_0, 1\}$ and $\{(8,1)_0-(1,8)_0, 1\}$ terms are equal.

III. PHOTON TRANSITIONS OF MESONS

Now that the basic properties of the theory and the manner of its application to actual hadrons have been spelled out, we begin the discussion of detailed predictions with radiative meson decays. We limit our listing of amplitudes to those corresponding to nonstrange mesons; the extension to transitions involving strange mesons is easily accomplished using SU(3).

Let us begin with the photon transitions from $L'=0$ to $L=0$ mesons, i.e., among the members of the $SU(6)_W$ $\underline{35}$ and $\underline{1}$, whose nonstrange members are the ρ, ω, ϕ, π, η, and (presumably) X^0. As $L_z'=L_z=0$ for the external states follows from $L=L'=0$, only the term with $L_z=0$ and transforming as $\{(3,\overline{3})_1, 0\}$ in $V^{-1}D_+^\alpha V$ can contribute. The selection rule in Eq. (28) immediately gives the result that $j_\gamma=1$ only. This is already nontrivial, as $j_\gamma=2$ transitions are possible from $\rho^\pm$ to $\rho^\pm$ in general, and the theory then predicts zero electric quadrupole moment for the ρ meson.

Since W-spin zero octets and singlets belong to the $\underline{35}$ and $\underline{1}$ representations of $SU(6)_W$, respectively, decays involving meson states which are mixtures of $W=0$ SU(3) octets and singlets may be used to fix the ratio of the reduced matrix elements,

$$\langle \underline{1}\ L'=0 \| (3,\overline{3})_1 \| \underline{35}\ L=0 \rangle$$

and

$$\langle \underline{35}\ L'=0 \| (3,\overline{3})_1 \| \underline{35}\ L=0 \rangle .$$

In particular, for this purpose we use Zweig's rule[29] to forbid the decay $\phi\to\gamma\pi$, where the ϕ is assumed to be the usual ideal mixture of singlet and octet so as to be composed of purely strange quarks. All amplitudes are then multiples of a single magnetic dipole amplitude, or alternatively, are proportional to the single reduced matrix element,

$$\langle \underline{35}\ L'=0\,\|(3,\bar{3})_1\|\,\underline{35}\ L=0\rangle\,.$$

One observed transition then fixes all the other decay rates.[30] The results of the computation of transition matrix elements are given in Table I where the η and X^0 are assumed to be SU(3) octet and singlet, respectively, while the ω and ϕ are ideal mixtures of octet and singlet:

$$\begin{aligned}\omega &= \cos\theta\,\omega^{(1)}+\sin\theta\,\omega^{(8)}\,,\\ \phi &= -\sin\theta\,\omega^{(1)}+\cos\theta\,\omega^{(8)}\,,\end{aligned}\tag{34}$$

where

$$\sin\theta=+(\tfrac{1}{3})^{1/2}.$$

Table II contains the corresponding predictions for all the $L'=0\to L=0$ radiative decay widths using $\Gamma(\omega\to\gamma\pi)=890$ keV as input.[31] The sparse experimental data[31,32] are also given. Note that the predictions in the first column are for unmixed pseudoscalar mesons. Taking a mixing angle[33] $\theta_p=-10.5°$, as suggested by a quadratic mass formula, gives the second column. The predicted width for $\phi\to\gamma\eta$ is reduced to 170 keV, agreeing with experiment within errors.[34] The corresponding prediction in this case for $\Gamma(X^0\to\gamma\rho)$ is 120 keV. Assuming that $X^0\to\gamma\pi^+\pi^-$ is dominated by $X^0\to\gamma\rho$, and taking the branching ratio[31] for this mode to be 26%, we find a total X^0 width of 460 keV. This is also consistent with the X^0 width obtained from the branching ratio[31] for $X^0\to\gamma\gamma$ plus SU(3) and the new value[35] of $\Gamma(\eta\to\gamma\gamma)$. The overall situation for $L'=0\to L=0$ decays is thus quite satisfactory, although many pieces of information are absent in comparing theory and experiment.

When we go to $L'=1$ to $L=0$ decays, there is no experimental information available, although there are both many amplitudes and many predictions. Of the four terms generally present in $V^{-1}D_+^\alpha V$, only $\{(\bar{3},3)_{-1},2\}$ cannot contribute (since it changes L_z by two units). The selection rules of Sec. II show that the $\{(8,1)_0+(1,8)_0,1\}$ term in $V^{-1}D_+^\alpha V$ leads to purely electric dipole ($j_\gamma=1$) transitions, and only $j_\gamma=1$ and 2 can arise from the $\{(3,\bar{3})_1,0\}$ $\{(8,1)_0-(1,8)_0,1\}$ terms. In fact it is possible to express linear combinations of their reduced matrix elements as electric dipole and magnetic quadrupole amplitudes, multiples of which occur in all decays from $L'=1$ to $L=0$ mesons.

All possible radiative decay amplitudes for nonstrange $L'=1$ mesons[36] to $L=0$ mesons are given in Table III in terms of the reduced matrix elements

TABLE I. Matrix elements for photon transitions among $\underline{35}$ and $\underline{1}$ $L=0$ states. The ω and ϕ are assumed to be ideally mixed, while the η and X^0 are taken as the SU(3) octet and singlet pseudoscalar mesons (see text).

Transition	Coefficient of $\langle\underline{35}\,L'=0\|(3,\bar{3})_1\|\underline{35}\,L=0\rangle$
$\omega\to\gamma\pi$	$\frac{1}{6}\sqrt{3}$
$\rho\to\gamma\pi$	$\frac{1}{18}\sqrt{3}$
$\phi\to\gamma\pi$	0
$\rho\to\gamma\eta$	$\frac{1}{6}$
$\omega\to\gamma\eta$	$\frac{1}{18}$
$\phi\to\gamma\eta$	$-\frac{1}{9}\sqrt{2}$
$X^0\to\gamma\rho$	$\frac{1}{6}\sqrt{2}$
$X^0\to\gamma\omega$	$\frac{1}{18}\sqrt{2}$
$\phi\to\gamma X^0$	$\frac{1}{9}$

TABLE II. Predicted and experimental widths for radiative transitions among $\underline{35}$ and $\underline{1}$ $L=0$ mesons.

Decay	Predicted width (keV) (no mixing)	Predicted width (keV) ($\theta_P=-10.5°$)	Experimental width[a] (keV)
$\omega\to\gamma\pi$	890 (input)	890 (input)	890 ± 90
$\rho\to\gamma\pi$	94	94	<730
$\phi\to\gamma\pi$	0	0	<14
$\rho\to\gamma\eta$	37	57	<160
$\omega\to\gamma\eta$	5	7	<49
$\phi\to\gamma\eta$	230	170	126 ± 46
$X^0\to\gamma\rho$	160	120	$0.26\,\Gamma(X^0\to\text{all})$
$X^0\to\gamma\omega$	15	11	
$\phi\to\gamma X^0$	0.5	0.6	

[a] See Refs. 31 and 32.

$$\langle \underline{35}\ L'=1 \| (8,1)_0+(1,8)_0 \| \underline{35}\ L=0\rangle ,$$

$$\langle \underline{35}\ L'=1 \| (3,\bar{3})_1 \| 35\ L=0\rangle ,$$

and

$$\langle 35\ L'=1 \| (8,1)_0-(1,8)_0 \| \underline{35}\ L=0\rangle .$$

Matrix elements of $SU(6)_W$ singlet states are related to those in the $\underline{35}$ by using Zweig's rule,[29] as was done above for $L'=0$ to $L=0$ decays. The η and H are assumed to be purely octet members, while the f, D, σ, and ω are all taken to be ideal mixtures of singlets and octets, so as to be composed of only nonstrange quarks. Note that in the decay $2^+ \to \gamma 1^-$, e.g., $A_2 \to \gamma\rho$, an electric octupole amplitude could be present in principle, as well as electric dipole and magnetic quadrupole amplitudes. However, the selection rule limiting j_γ to 1 or 2 eliminates the octupole amplitude and results in the linear relation

$$A_{\lambda=2}(A_2 \to \gamma\rho)=2\sqrt{2}A_{\lambda=1}(A_2\to\gamma\rho) - \sqrt{6}A_{\lambda=0}(A_2\to\gamma\rho) , \qquad (35)$$

among the three helicity amplitudes for $2^+ \to \gamma 1^-$. Almost any experimental information on these decays would be helpful in sorting out the relative importance of the various (3) possible amplitudes, and in testing the theory.

TABLE III. Photon transition amplitudes from nonstrange mesons (Ref. 36) with $L'=1$ and $J_z=\lambda$ to those with $L=0$ and $J_z=\lambda-1$. The ω, f, D, and σ are assumed to be ideal mixtures of singlets and octets, so as to be composed purely of nonstrange quarks; the η and H are purely octet, and the X^0 a pure singlet. Zweig's rule (Ref. 29) is used to relate $SU(6)_W$ $\underline{35}$ and $\underline{1}$ reduced matrix elements (see text), and forbids decays like A_2, A_1, δ, f, D, $\sigma \to \gamma\phi$, and $f' \to \gamma\rho$ or $\gamma\omega$. (a) transition, (b) coefficient of $\langle \underline{35}\ L'=1 \| (8,1)_0+(1,8)_0 \| \underline{35}\ L=0\rangle$, (c) coefficient of $\langle \underline{35}\ L'=1 \| (3,\bar{3})_1 \| \underline{35}\ L=0\rangle$, (d) coefficient of $\langle \underline{35}\ L'=1 \| (8,1)_0-(1,8)_0 \| \underline{35}\ L=0\rangle$.

(a)		(b)	(c)	(d)
$A_2^+ \to \gamma\pi^+$,	$\lambda=1$	0	$\frac{1}{8}\sqrt{3}$	$\frac{1}{12}\sqrt{6}$
$A_1^+ \to \gamma\pi^+$,	$\lambda=1$	0	$-\frac{1}{8}\sqrt{3}$	$\frac{1}{12}\sqrt{6}$
$B \to \gamma\pi$,	$\lambda=1$	$\frac{1}{24}\sqrt{6}$	0	0
$B \to \gamma\eta$,	$\lambda=1$	$\frac{1}{8}\sqrt{2}$	0	0
$B \to \gamma X^0$,	$\lambda=1$	$\frac{1}{4}$	0	0
$H \to \gamma\pi$,	$\lambda=1$	$\frac{1}{8}\sqrt{2}$	0	0
$H \to \gamma\eta$,	$\lambda=1$	$-\frac{1}{24}\sqrt{6}$	0	0
$H \to \gamma X^0$,	$\lambda=1$	$\frac{1}{12}\sqrt{3}$	0	0
$A_2 \to \gamma\rho$,	$\lambda=0$	$\frac{1}{24}$	$-\frac{1}{12}$	$-\frac{1}{36}\sqrt{2}$
	$\lambda=1$	$\frac{1}{24}\sqrt{3}$	$-\frac{1}{24}\sqrt{3}$	0
	$\lambda=2$	$\frac{1}{24}\sqrt{6}$	0	$\frac{1}{18}\sqrt{3}$
$A_1 \to \gamma\rho$,	$\lambda=0$	$\frac{1}{24}\sqrt{3}$	0	$-\frac{1}{36}\sqrt{6}$
	$\lambda=1$	$\frac{1}{24}\sqrt{3}$	$\frac{1}{24}\sqrt{3}$	0
$\delta \to \gamma\rho$,	$\lambda=0$	$\frac{1}{24}\sqrt{2}$	$\frac{1}{24}\sqrt{2}$	$-\frac{1}{18}$
$B \to \gamma\rho$,	$\lambda=0$	0	$-\frac{1}{24}\sqrt{6}$	0
	$\lambda=1$	0	0	$\frac{1}{6}\sqrt{3}$

$A_\lambda[A_2 \to \gamma\omega]=3A_\lambda[A_2\to\gamma\rho]$ $\quad$ $A_\lambda[A_1 \to \gamma\omega]=3A_\lambda[A_1\to\gamma\rho]$

$A_\lambda[f \to \gamma\rho]=3A_\lambda[A_2\to\gamma\rho]$ $\quad$ $A_\lambda[D \to \gamma\rho]=3A_\lambda[A_1\to\gamma\rho]$

$A_\lambda[f \to \gamma\omega]=A_\lambda[A_2\to\gamma\rho]$ $\quad$ $A_\lambda[D \to \gamma\omega]=A_\lambda[A_1\to\gamma\rho]$

$A_\lambda[f' \to \gamma\phi]=-2A_\lambda[A_2\to\gamma\rho]$ $\quad$ $A_\lambda[\delta \to \gamma\omega]=3A_\lambda[\delta\to\gamma\rho]$

$A_\lambda[\sigma \to \gamma\rho]=3A_\lambda[\delta\to\gamma\rho]$

$A_\lambda[\sigma \to \gamma\omega]=A_\lambda[\delta\to\gamma\rho]$

IV. PHOTON TRANSITIONS BETWEEN BARYONS

The electromagnetic transitions of baryons provide a second and very rich area of predictions for the theory. As before, we restrict our attention primarily to nonstrange baryons decaying into $L=0$ states, this being by far the main area for experimental comparison. In this section we will enumerate the possible decay amplitudes, deferring an experimental comparison to the next section.

The case of transitions from $\underline{56}$ $L'=0$ to $\underline{56}$ $L=0$, i.e., within the $L=0$ baryon multiplet, is particularly simple. As for mesons, only magnetic dipole transitions are allowed by the theory and all amplitudes are proportional to a single reduced matrix element, that of the term transforming as $\{(3,\bar{3})_1,0\}$ in $V^{-1}D_+^\alpha V$. The results are presented in Table IV for the three possible transitions $N\to N$, $N\to\Delta$, and $\Delta\to\Delta$. It can be explicitly checked that all the transitions are magnetic dipole in character, as demanded by the selection rule [Eq. (28)], including those for $\Delta\to\Delta$ where both electric quadrupole and magnetic octupole transitions are also possible in principle.

For decays from the next identified baryon multiplet, the $\underline{70}$ $L'=1$, to the ground state $\underline{56}$ $L=0$ we have the three possible reduced matrix elements

$$\langle \underline{70}\ L'=1 \| (8,1)_0+(1,8)_0 \| \underline{56}\ L=0\rangle ,$$

$$\langle \underline{70}\ L'=1 \| (3,\bar{3})_1 \| \underline{56}\ L=0\rangle ,$$

and

$$\langle \underline{70}\ L'=1 \| (8,1)_0-(1,8)_0 \| \underline{56}\ L=0\rangle .$$

TABLE IV. Photon amplitudes for transitions from $\underline{56}$ $L'=0$ states with $J_z=\lambda$ to $\underline{56}$ $L=0$ states with $J_z=\lambda-1$.

Transition		Coefficient of $\langle\underline{56}\ L'=0\Vert(3,\bar{3})_1\Vert\underline{56}\ L=0\rangle$
$N^+(\frac{1}{2}\ )\to\gamma N^+$,	$\lambda=\frac{1}{2}$	$-\frac{2}{15}\sqrt{5}$
$N^0(\frac{1}{2}\ )\to\gamma N^0$,	$\lambda=\frac{1}{2}$	$\frac{4}{45}\sqrt{5}$
$\Delta^+(\frac{3}{2}^+)\to\gamma N^+$,	$\lambda=\frac{1}{2}$	$-\frac{2}{45}\sqrt{10}$
	$\lambda=\frac{3}{2}$	$-\frac{2}{45}\sqrt{30}$
$A_\lambda[\Delta^+\to\gamma N^+]=A_\lambda[\Delta^0\to\gamma N^0]$		
$\Delta^{++}(\frac{3}{2}^+)\to\gamma\Delta^{++}$,	$\lambda=-\frac{1}{2}$	$-\frac{4}{45}\sqrt{15}$
	$\lambda=\frac{1}{2}$	$-\frac{8}{45}\sqrt{5}$
	$\lambda=\frac{3}{2}$	$-\frac{4}{45}\sqrt{15}$
$A_\lambda[\Delta^+\to\gamma\Delta^+]=\frac{1}{2}A_\lambda[\Delta^{++}\to\gamma\Delta^{++}]$		
$A_\lambda[\Delta^0\to\gamma\Delta^0]=0$		
$A_\lambda[\Delta^-\to\gamma\Delta^-]=-\frac{1}{2}A_\lambda[\Delta^{++}\to\gamma\Delta^{++}]$		

The matrix elements of D_+^α for decays into both γN and $\gamma\Delta$ are enumerated[37] in Table V in terms of these reduced matrix elements.

By the selection rules of Sec. II, the $(8,1)_0+(1,8)_0$ term in $V^{-1}D_+^\alpha V$ acts as an electric dipole transition operator, while the two remaining terms act as a combination of electric dipole ($j_\gamma=1$) and magnetic quadrupole ($j_\gamma=2$). According to the discussion around Eq. (30) in Sec. II we can in fact write amplitudes,

$$
\begin{aligned}
E1' &= \langle\underline{70}\ L'=1\Vert(8,1)_0+(1,8)_0\Vert\underline{56}\ L=0\rangle\,,\\
E1 &= (\tfrac{1}{2})^{1/2}\langle\underline{70}\ L'=1\Vert(3,\bar{3})_1\Vert\underline{56}\ L=0\rangle\\
&\quad-(\tfrac{1}{2})^{1/2}\langle\underline{70}\ L'=1\Vert(8,1)_0-(1,8)_0\Vert\underline{56}\ L=0\rangle\,,
\end{aligned}
\tag{36}
$$

$$
\begin{aligned}
M2 &= (\tfrac{1}{2})^{1/2}\langle\underline{70}\ L'=1\Vert(3,\bar{3})_1\Vert\underline{56}\ L=0\rangle\,,\\
&\quad+(\tfrac{1}{2})^{1/2}\langle\underline{70}\ L'=1\Vert(8,1)_0-(1,8)_0\Vert\underline{56}\ L=0\rangle\,,
\end{aligned}
$$

which are electric dipole and magnetic quadrupole amplitudes in terms of which all the helicity decay amplitudes given in Table V may be alternately expressed. Note that $N^*(J^P=\frac{5}{2}^-)\to\gamma N$, for example, could in general go via $j_\gamma=2$ or 3, but only $j_\gamma=2$ (magnetic quadrupole) is allowed by the theory. Similarly, $N^*(\frac{5}{2}^-)\to\gamma\Delta$ could proceed with $j_\gamma=1$, 2, 3, or 4 in general, but only $j_\gamma=1$ and 2 are allowed by the theory. Note also that the Moorhouse quark-model selection rule[38] forbidding $\gamma p\to N^{*+}$, where the N^* has quark spin $S=\frac{3}{2}$, is reflected in Table V.

For $\underline{56}$ $L'=2$ decays to $\underline{56}$ $L=0$ we have reached a high enough value of L' that all four terms in $V^{-1}D_+^\alpha V$ can contribute to the decay amplitudes. In this case the $(8,1)_0+(1,8)_0$ term is electric quadrupole ($j_\gamma=2$) in character, while linear combina-

TABLE V. Photon amplitudes for transitions from $\underline{70}$ $L'=1$ states with $J_z=\lambda$ to nucleon and Δ states in the $\underline{56}$ $L=0$. States are labeled by J^P and $[\text{SU(3) multiplet}]^{2S+1}$ where S is the quark spin. (a) Transitions; (b) coefficient of $\langle\underline{70}\ L'=1\Vert(8,1)_0+(1,8)_0\Vert\underline{56}\ L=0\rangle$; (c) coefficient of $\langle\underline{70}\ L'=1\Vert(3,\bar{3})_1\Vert\underline{56}\ L=0\rangle$; (d) coefficient of $\langle\underline{70}\ L'=1\Vert(8,1)_0-(1,8)_0\Vert\underline{56}\ L=0\rangle$.

(a)			(b)	(c)	(d)
$N^*(\frac{3}{2}^-)$	$\to\gamma N^+$,	$\lambda=\frac{1}{2}$	$-\frac{1}{12}\sqrt{2}$	$\frac{1}{6}\sqrt{2}$	$\frac{1}{12}\sqrt{2}$
$[8]^2$		$\lambda=\frac{3}{2}$	$-\frac{1}{12}\sqrt{6}$	0	$-\frac{1}{12}\sqrt{6}$
	$\to\gamma N^0$,	$\lambda=\frac{1}{2}$	$\frac{1}{12}\sqrt{2}$	$-\frac{1}{18}\sqrt{2}$	$-\frac{1}{36}\sqrt{2}$
		$\lambda=\frac{3}{2}$	$\frac{1}{12}\sqrt{6}$	0	$\frac{1}{36}\sqrt{6}$
	$\to\gamma\Delta^+$,	$\lambda=-\frac{1}{2}$	0	$\frac{1}{9}\sqrt{3}$	0
		$\lambda=\frac{1}{2}$	0	$\frac{1}{9}$	$-\frac{1}{9}$
		$\lambda=\frac{3}{2}$	0	0	$-\frac{1}{9}\sqrt{3}$
$N^*(\frac{1}{2}^-)$	$\to\gamma N^+$,	$\lambda=\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$+\frac{1}{6}$
$[8]^2$	$\to\gamma N^0$,	$\lambda=\frac{1}{2}$	$\frac{1}{6}$	$+\frac{1}{18}$	$-\frac{1}{18}$
	$\to\gamma\Delta^+$,	$\lambda=-\frac{1}{2}$	0	$\frac{1}{18}\sqrt{6}$	0
		$\lambda=\frac{1}{2}$	0	$-\frac{1}{18}\sqrt{2}$	$-\frac{1}{9}\sqrt{2}$
$\Delta^*(\frac{1}{2}^-)$	$\to\gamma N^+$,	$\lambda=\frac{1}{2}$	$-\frac{1}{6}$	$+\frac{1}{18}$	$-\frac{1}{18}$
$[10]^2$	$\to\gamma\Delta^+$,	$\lambda=-\frac{1}{2}$	0	$-\frac{1}{18}\sqrt{6}$	0
		$\lambda=\frac{1}{2}$	0	$\frac{1}{18}\sqrt{2}$	$\frac{1}{9}\sqrt{2}$
$\Delta^*(\frac{3}{2}^-)$	$\to\gamma N^+$,	$\lambda=\frac{1}{2}$	$-\frac{1}{12}\sqrt{2}$	$-\frac{1}{18}\sqrt{2}$	$-\frac{1}{36}\sqrt{2}$
$[10]^2$		$\lambda=\frac{3}{2}$	$-\frac{1}{12}\sqrt{6}$	0	$\frac{1}{36}\sqrt{6}$
	$\to\gamma\Delta^+$,	$\lambda=-\frac{1}{2}$	0	$-\frac{1}{9}\sqrt{3}$	0
		$\lambda=\frac{1}{2}$	0	$-\frac{1}{9}$	$\frac{1}{9}$
		$\lambda=\frac{3}{2}$	0	0	$\frac{1}{9}\sqrt{3}$
$N^*(\frac{5}{2}^-)$	$\to\gamma N^+$,	$\lambda=\frac{1}{2}$	0	0	0
$[8]^4$		$\lambda=\frac{3}{2}$	0	0	0
	$\to\gamma N^0$,	$\lambda=\frac{1}{2}$	0	$\frac{1}{30}\sqrt{5}$	$\frac{1}{30}\sqrt{5}$
		$\lambda=\frac{3}{2}$	0	$\frac{1}{30}\sqrt{10}$	$\frac{1}{30}\sqrt{10}$
	$\to\gamma\Delta^+$,	$\lambda=-\frac{1}{2}$	$-\frac{1}{60}\sqrt{30}$	$\frac{1}{30}\sqrt{30}$	$\frac{1}{60}\sqrt{30}$
		$\lambda=\frac{1}{2}$	$-\frac{1}{20}\sqrt{10}$	$\frac{1}{15}\sqrt{10}$	$\frac{1}{60}\sqrt{10}$
		$\lambda=\frac{3}{2}$	$-\frac{1}{10}\sqrt{5}$	$\frac{1}{15}\sqrt{5}$	$-\frac{1}{30}\sqrt{5}$
		$\lambda=\frac{5}{2}$	$-\frac{1}{6}\sqrt{3}$	0	$-\frac{1}{6}\sqrt{3}$
$N^*(\frac{3}{2}^-)$	$\to\gamma N^+$,	$\lambda=\frac{1}{2}$	0	0	0
$[8]^4$		$\lambda=\frac{3}{2}$	0	0	0
	$\to\gamma N^0$,	$\lambda=\frac{1}{2}$	0	$-\frac{1}{90}\sqrt{5}$	$\frac{2}{45}\sqrt{5}$
		$\lambda=\frac{3}{2}$	0	$-\frac{1}{30}\sqrt{15}$	$+\frac{1}{45}\sqrt{15}$
	$\to\gamma\Delta^+$,	$\lambda=-\frac{1}{2}$	$-\frac{1}{30}\sqrt{30}$	$\frac{1}{90}\sqrt{30}$	$\frac{1}{30}\sqrt{30}$
		$\lambda=\frac{1}{2}$	$-\frac{1}{15}\sqrt{10}$	$-\frac{1}{45}\sqrt{10}$	$\frac{1}{45}\sqrt{10}$
		$\lambda=\frac{3}{2}$	$-\frac{1}{30}\sqrt{30}$	$-\frac{1}{30}\sqrt{30}$	$-\frac{1}{90}\sqrt{30}$
$N^*(\frac{1}{2}^-)$	$\to\gamma N^+$,	$\lambda=\frac{1}{2}$	0	0	0
$[8]^4$	$\to\gamma N^0$,	$\lambda=\frac{1}{2}$	0	$-\frac{1}{18}$	$+\frac{1}{18}$
	$\to\gamma\Delta^+$,	$\lambda=-\frac{1}{2}$	$-\frac{1}{12}\sqrt{6}$	$-\frac{1}{18}\sqrt{6}$	$\frac{1}{12}\sqrt{6}$
		$\lambda=\frac{1}{2}$	$-\frac{1}{12}\sqrt{2}$	$-\frac{1}{9}\sqrt{2}$	$\frac{1}{36}\sqrt{2}$

$A_\lambda(\Delta^{*+}\to\gamma N^+)=A_\lambda(\Delta^{*0}\to\gamma N^0)$ $\quad A_\lambda(\Delta^{*0}\to\gamma\Delta^0)=0$

$A_\lambda(N^{*+}\to\gamma\Delta^+)=A_\lambda(N^{*0}\to\gamma\Delta^0)$ $\quad A_\lambda(\Delta^{*-}\to\gamma\Delta^-)=-A_\lambda(\Delta^{*+}\to\gamma\Delta^+)$

$A_\lambda(\Delta^{*++}\to\gamma\Delta^{++})=2A_\lambda(\Delta^{*+}\to\gamma\Delta^+)$

tions of the other three terms act as $j_\gamma = 1$, 2, and 3 transitions:

$$
\begin{aligned}
E2' &= \langle \underline{56}\ L'=2 \,\|(8,1)_0 + (1,8)_0 \|\, \underline{56}\ L=0\rangle ,\\
M1 &= (\tfrac{1}{10})^{1/2} \langle \underline{56}\ L'=2 \,\|(3,\bar{3})_1 \|\, \underline{56}\ L=0\rangle \\
&\quad - (\tfrac{3}{10})^{1/2} \langle \underline{56}\ L'=2 \,\|(8,1)_0 - (1,8)_0 \|\, \underline{56}\ L=0\rangle \\
&\quad + (\tfrac{3}{5})^{1/2} \langle \underline{56}\ L'=2 \,\|(\bar{3},3)_{-1} \|\, \underline{56}\ L=0\rangle ,\\
E2 &= (\tfrac{1}{2})^{1/2} \langle \underline{56}\ L'=2 \,\|(3,\bar{3})_1 \|\, \underline{56}\ L=0\rangle \qquad (37)\\
&\quad - (\tfrac{1}{6})^{1/2} \langle \underline{56}\ L'=2 \,\|(8,1)_0 - (1,8)_0 \|\, \underline{56}\ L=0\rangle \\
&\quad - (\tfrac{1}{3})^{1/2} \langle \underline{56}\ L'=2 \,\|(\bar{3},3)_{-1} \|\, \underline{56}\ L=0\rangle ,\\
M3 &= (\tfrac{6}{15})^{1/2} \langle \underline{56}\ L'=2 \,\|(3,\bar{3})_1 \|\, \underline{56}\ L=0\rangle \\
&\quad + (\tfrac{8}{15})^{1/2} \langle \underline{56}\ L'=2 \,\|(8,1)_0 - (1,8)_0 \|\, \underline{56}\ L=0\rangle \\
&\quad + (\tfrac{1}{15})^{1/2} \langle \underline{56}\ L'=2 \,\|(\bar{3},3)_{-1} \|\, \underline{56}\ L=0\rangle .
\end{aligned}
$$

The various amplitudes for resonances in the $\underline{56}$ $L'=2$ to decay into γN are listed[37] in Table VI. The $\gamma\Delta$ amplitudes are straightforward to work out,[39] but at present add little of interest. Again, the selection rules derived in Sec. II have clear and direct consequences: $\Delta(\frac{7}{2}^+) \to \gamma N$, for example, which could go by $j_\gamma = 3$ or 4 is restricted to be purely magnetic octupole ($j_\gamma = 3$).

Decays from higher L' multiplets are easily computable, but little in the way of experimental tests is available at present. For the $\underline{56}$ $L'=0$, $\underline{70}$ $L'=1$, and $\underline{56}$ $L'=2$ photon transition amplitudes to $\underline{56}$ $L=0$ which we have enumerated, however, photoproduction data permit many direct experimental comparisons. To these we now turn.

V. EXPERIMENTAL TESTS OF BARYON AMPLITUDES

The predictions for transitions within the $\underline{56}$ $L=0$ multiplet are already testable using the magnetic moments of the neutron and proton, for a direct evaluation of D_+^α between nucleon states at infinite momentum gives

$$\mu_A = +\left(\frac{1}{\sqrt{2}}\right) \langle N, \lambda = \tfrac{1}{2} \,|D_+^\alpha|\, N, \lambda = -\tfrac{1}{2}\rangle , \qquad (38)$$

where μ_A is the *anomalous magnetic moment* of the nucleon. However, a careful calculation of $V^{-1} D_+^\alpha V$ between one-nucleon states at infinite momentum gives a result which has the transformation properties of the four terms discussed in Sec. II minus a term proportional to the charge. Melosh[1] claims that term is exactly the *Dirac moment*. While this has been challenged by Osborn,[14] we note that if it were the Dirac moment, then the ratio of matrix elements in Table IV should be interpreted as predicting for the *total moments* of the nucleon

$$\mu_T(n)/\mu_T(p) = -\tfrac{2}{3} , \qquad (39)$$

TABLE VI. Photon amplitudes for transitions from $\underline{56}$ $L'=2$ states with $J_z = \lambda$ to nucleon states in the $\underline{56}$ $L=0$ with $J_z = \lambda - 1$. States in the $\underline{56}$ $L'=2$ are labeled by J^P and $[\mathrm{SU}(3)\ \text{multiplet}]^{2S+1}$ where S is the quark spin. (a) transition, (b) coefficient of $\langle \underline{56}\ L'=2 \| (8,1)_0 + (1,8)_0 \| \underline{56}\ L=0\rangle$, (c) coefficient of $\langle \underline{56}\ L'=2 \| (3,\bar{3})_1 \| \underline{56}\ L=0\rangle$, (d) coefficient of $\langle \underline{56}\ L'=2 \| (\bar{3},3)_{-1} \| \underline{56}\ L=0\rangle$, (e) coefficient of $\langle \underline{56}\ L'=2 \| (8,1)_0 - (1,8)_0 \| \underline{56}\ L=0\rangle$.

(a)		(b)	(c)	(d)	(e)
$N^*(\frac{5}{2}^+)$ $[8]^2$	$\to \gamma N^+$, $\lambda = \frac{1}{2}$	$\frac{2}{15}$	$-\frac{2}{15}\sqrt{3}$	0	$-\frac{2}{15}$
	$\lambda = \frac{3}{2}$	$\frac{2}{15}\sqrt{2}$	0	$\frac{2}{15}$	$\frac{2}{15}\sqrt{2}$
	$\to \gamma N^0$, $\lambda = \frac{1}{2}$	0	$\frac{4}{45}\sqrt{3}$	0	$\frac{4}{45}$
	$\lambda = \frac{3}{2}$	0	0	$-\frac{4}{45}$	$-\frac{4}{45}\sqrt{2}$
$N^*(\frac{3}{2}^+)$ $[8]^2$	$\to \gamma N^+$, $\lambda = \frac{1}{2}$	$\frac{1}{15}\sqrt{6}$	$\frac{2}{15}\sqrt{2}$	0	$-\frac{1}{15}\sqrt{6}$
	$\lambda = \frac{3}{2}$	$-\frac{1}{15}\sqrt{2}$	0	$\frac{4}{15}$	$-\frac{1}{15}\sqrt{2}$
	$\to \gamma N^0$, $\lambda = \frac{1}{2}$	0	$-\frac{4}{45}\sqrt{2}$	0	$\frac{2}{45}\sqrt{6}$
	$\lambda = \frac{3}{2}$	0	0	$-\frac{8}{45}$	$\frac{2}{45}\sqrt{2}$
$\Delta^*(\frac{7}{2}^+)$ $[10]^4$	$\to \gamma N^+$, $\lambda = \frac{1}{2}$	0	$-\frac{4}{105}\sqrt{7}$	$-\frac{2}{315}\sqrt{42}$	$-\frac{8}{315}\sqrt{21}$
	$\lambda = \frac{3}{2}$	0	$-\frac{4}{315}\sqrt{105}$	$-\frac{2}{315}\sqrt{70}$	$-\frac{8}{315}\sqrt{35}$
$\Delta^*(\frac{5}{2}^+)$ $[10]^4$	$\to \gamma N^+$, $\lambda = \frac{1}{2}$	0	$\frac{2}{315}\sqrt{42}$	$-\frac{4}{105}\sqrt{7}$	$-\frac{2}{63}\sqrt{14}$
	$\lambda = \frac{3}{2}$	0	$\frac{4}{105}\sqrt{21}$	$-\frac{8}{315}\sqrt{14}$	$-\frac{4}{315}\sqrt{7}$
$\Delta^*(\frac{3}{2}^+)$ $[10]^4$	$\to \gamma N^+$, $\lambda = \frac{1}{2}$	0	$\frac{2}{45}\sqrt{2}$	$-\frac{4}{45}\sqrt{3}$	0
	$\lambda = \frac{3}{2}$	0	$-\frac{2}{45}\sqrt{6}$	$-\frac{4}{45}$	$\frac{4}{45}\sqrt{2}$
$\Delta^*(\frac{1}{2}^+)$ $[10]^4$	$\to \gamma N^+$, $\lambda = \frac{1}{2}$	0	$-\frac{2}{45}\sqrt{2}$	$-\frac{4}{45}\sqrt{3}$	$\frac{2}{45}\sqrt{6}$

$A_\lambda[\Delta^{*+} \to \gamma N^+] = A_\lambda[\Delta^{*0} \to \gamma N^0]$

the SU(6) result,[40] which is within 5% of the experimental value of $-1.91/2.73 = -0.70$. Thus we find from experiment that the extra term is quite close to the Dirac moment.

For the transition from Δ to N the ratio of $\sqrt{3}$ between the $\lambda = \frac{3}{2}$ and $\lambda = \frac{1}{2}$ matrix elements corresponds to a pure magnetic dipole transition, as we already know must occur from the discussion in the last section. All photoproduction analyses[41] agree that the electric quadrupole amplitude is at most a few percent of the magnetic dipole amplitude for excitation of the 3-3 resonance. The strength of this transition, μ^*, is conventionally defined so that

$$\mu^* = \sqrt{2}\,\langle \Delta, \lambda = \tfrac{1}{2} \,|D_+^\alpha|\, N, \lambda = -\tfrac{1}{2}\rangle . \qquad (40)$$

The results in Table IV then translate to

$$\mu^*/\mu_T(p) = +\tfrac{2}{3}\sqrt{2} . \qquad (41)$$

An older phenomenological analysis[42] of the data for pion photoproduction gave a result for $\mu^*/\mu_T(p)$ which is 1.28 ± 0.03 times the right-hand side of Eq. (41) by finding the residue at the Δ pole in $\gamma N \to \pi N$. By considering the contribution[43] of the Δ to the Cabibbo-Radicati sum rule we find a value of $\mu^*/\mu_T(p)$ which is 0.9 ± 0.1 times the right-hand side of Eq. (41), and in quite satisfactory agreement with the theory. While the sign of $\mu^*/\mu_T(p)$ cannot be measured, the product of the γN and πN couplings of the nucleon can be compared with that of the 3-3 resonance in pion photoproduction. As the theory[2,3] also predicts the relative sign of the πN couplings, it makes an unambiguous prediction of the sign of the resonance excitation amplitude relative to the nucleon Born terms. This sign is correctly given by the theory.[44]

For the transition from Δ to Δ, which is also purely magnetic in character, we should again interpret the results in Table IV as being for the total moment. The relation between matrix elements of D_+ and the conventional anomalous magnetic moment of the Δ, μ_A^{**}, is

$$\mu_A^{**} = -(\tfrac{3}{2})^{1/2}\langle \Delta, \lambda=\tfrac{3}{2}|D_+^\alpha|\Delta, \lambda=\tfrac{1}{2}\rangle . \qquad (42)$$

From this we see that we have from Table IV,

$$\begin{aligned} \mu_T^{**}(\Delta^{++})/\mu_T(p) &= 2 ,\\ \mu_T^{**}(\Delta^{+})/\mu_T(p) &= 1 ,\\ \mu_T^{**}(\Delta^{0})/\mu_T(p) &= 0 ,\\ \mu_T^{**}(\Delta^{-})/\mu_T(p) &= -1 . \end{aligned} \qquad (43)$$

As with Eqs. (39) and (41), all these are standard SU(6) results,[40] as is to be expected since the $\{(3,\bar{3})_1, 0\}$ term in $V^{-1}D_+^\alpha V$ has the same transformation properties as the magnetic moment operator used in SU(6).

The transitions from the $\underline{70}$ $L'=1$ to the ground state $\underline{56}$ $L=0$ provide a much richer set of amplitudes for comparison of theory and experiment. Rather than carry out a statistical "best fit" to all the data, in Table VII we have fixed the possible reduced matrix elements allowed by the theory in

TABLE VII. Comparison of matrix elements of $D_+^3 + (1/\sqrt{3})D_+^8$ for $\underline{70}$ $L'=1 \to \underline{56}$ $L=0$ photon transitions with experiment (see Ref. 48). Nucleon resonances are identified as in Ref. 18 with the quark-model states, which are labeled by their quantum numbers J^P and $[\mathrm{SU}(3)\ \text{multiplet}]^{2S+1}$, where S is the quark spin. The signs of amplitudes are those in $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$, with the S and D amplitudes at the πNN^* vertex taken to have opposite sign (see text).

Transition		$\langle N^*,\lambda\|D_+\|N,\lambda-1\rangle$ experiment (Ref. 48) (1/GeV)	$\langle N^*,\lambda\|D_+\|N,\lambda-1\rangle$ predicted with $\langle\underline{70}\|(8,1)_0-(1,8)_0\|\underline{56}\rangle=0$	$\langle N^*,\lambda\|D_+\|N,\lambda-1\rangle$ predicted with $\langle\underline{70}\|(8,1)_0-(1,8)_0\|\underline{56}\rangle\neq0$
$D_{13}(1520)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	-0.10 ± 0.04	-0.10 (input)	-0.10 (input)
$\frac{3}{2}^-$, $[8]^2$	$\lambda=\frac{3}{2}$	$+0.91\pm0.06$	$+0.91$ (input)	$+0.91$ (input)
	$\to\gamma n$, $\lambda=\frac{1}{2}$	$+0.41\pm0.03$	$+0.32$	$+0.23$
	$\lambda=\frac{3}{2}$	$+0.64\pm0.05$	$+0.91$	$+0.64$ (input)
$S_{11}(1535)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	$+0.30\pm0.10$	$+1.18$	$+0.07$
$\frac{1}{2}^-$, $[8]^2$	$\to\gamma n$, $\lambda=\frac{1}{2}$	$+0.27\pm0.03$	$+0.89$	$+0.30$
$S_{31}(1650)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	$+0.16\pm0.07$	$+0.59$	$+0.53$
$\frac{1}{2}^-$, $[10]^2$				
$D_{33}(1670)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	$+0.36\pm0.04$	$+0.73$	$+0.36$
$\frac{3}{2}^-$, $[10]^2$	$\lambda=\frac{3}{2}$	$+0.32\pm0.04$	$+0.91$	$+0.38$
$D_{15}(1670)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	$+0.06\pm0.07$	0	0
$\frac{5}{2}^-$, $[8]^4$	$\lambda=\frac{3}{2}$	$+0.07\pm0.04$	0	0
	$\to\gamma n$, $\lambda=\frac{1}{2}$	$+0.20\pm0.03$	$+0.20$	$+0.13$
	$\lambda=\frac{3}{2}$	$+0.33\pm0.14$	$+0.28$	$+0.19$
$D_{13}(1700)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	-0.07 ± 0.18	0	0
$\frac{3}{2}^-$, $[8]^4$	$\lambda=\frac{3}{2}$	$+0.14\pm0.18$	0	0
	$\to\gamma n$, $\lambda=\frac{1}{2}$	$+0.16\pm0.18$	$+0.07$	-0.19
	$\lambda=\frac{3}{2}$	-0.11 ± 0.11	$+0.34$	-0.19
$S_{11}(1700)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	$+0.26\pm0.08$	0	0
$\frac{1}{2}^-$, $[8]^4$	$\to\gamma n$, $\lambda=\frac{1}{2}$	$+0.07\pm0.16$	-0.15	$+0.12$

terms of some relatively well-determined amplitudes for the process $\gamma N \to D_{13}(1520) \to \pi N$.

The quantities in the table are the matrix elements of $D_+^3 + (1/\sqrt{3})D_+^8$ taken between identified resonant states[18] in the $\underline{70}$ with $J_z = \lambda$ and nucleon states with $J_z = \lambda - 1$. The signs are those found in the specific processes $\gamma p \to N^{*+} \to \pi^+ n$ and $\gamma n \to N^{*0} \to \pi^- p$. To make a theoretical prediction of these signs we need a theory of both the γNN^* and πNN^* vertices. The γNN^* couplings are taken from Table V while for $\underline{70}$ $L'=1 \to 56$ $L=0$ pion transitions we may express the reduced matrix elements of the two terms in $V^{-1}Q_5^\alpha V$ as linear combinations of amplitudes S and D, corresponding to $l=0$ and 2[45]:

$$\langle \underline{70},\ L'=1 \| (8,1)_0 - (1,8)_0 \| \underline{56},\ L=0\rangle = \tfrac{1}{3}(S+2D)\,,$$
$$\langle \underline{70},\ L'=1 \| (3,\bar{3})_1 - (\bar{3},3)_{-1} \| \underline{56},\ L=0\rangle = \tfrac{1}{3}(S-D)\,. \tag{44}$$

Then $S=+D$ if only the $(8,1)_0-(1,8)_0$ term in $V^{-1}Q_5^\alpha V$ is present, while $S=-2D$ if only $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ is present. While an earlier phase-shift solution[46] to the $\pi N \to \pi\Delta$ data disagreed with the signs predicted for pion transitions, a new solution agrees completely[47] and shows that the signs of S and D are opposite, i.e., it appears the $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ reduced matrix element is dominant for $\underline{70}$ $L'=1 \to \underline{56}$ $L=0$ pion decays. In constructing Table VII we have taken the πNN^* couplings from Table V of Ref. 3 and have assumed that the signs of S and D are opposite in calculating the πNN^* vertex sign. Mixing between the two S_{11} or two D_{13} states in the $\underline{70}$ has been neglected in calculating the predicted amplitudes.

The "data" are taken from a very recent analysis[48] of electromagnetic couplings of N^* resonances from single pion photoproduction data. In terms of amplitudes A_λ for $\gamma N \to N^*$ of that analysis,[48] matrix elements of $D_+^3 + (1\sqrt{3})D_+^8$ are related by

$$\langle N^*, \lambda | D_+^3 + (1\sqrt{3})D_+^8 | N, \lambda-1\rangle = \left(\frac{M_N}{2\pi\alpha M_{N^*} p_\gamma}\right)^{1/2} A_\lambda\,, \tag{45}$$

where p_γ is the photon momentum in the N^* rest frame, and λ can take the values $\frac{1}{2}$ and $\frac{3}{2}$. The results of Ref. 48 generally agree well with those of another recent[49] analysis, although the "errors" on the amplitudes quoted in the latter are much larger. Judging from the differences between successive or independent analyses, we would opt for larger "errors" than those of Ref. 48, which are reproduced in Table VII.

As a first comparison, we set the reduced matrix element $\langle \underline{70}\ L'=1 \| (8,1)_0 - (1,8)_0 \| \underline{56}\ L=0\rangle$ equal to zero, so that we are left with only the two terms in $V^{-1}D_+^\alpha V$ which are present in quark-model calculations.[23,24] The well-determined amplitude for $\gamma p \to D_{13}^+(1520)$ with $\lambda = \frac{3}{2}$ then determines $\langle \underline{70}\ L'=1 \| (8,1)_0 + (1,8)_0 \| \underline{56}\ L=0\rangle$ directly and fixes an over-all free sign. The $\lambda=\frac{1}{2}$ transition to the same resonance then fixes

$$\langle \underline{70}\ L'=1 \| (3,\bar{3})_1 \| \underline{56}\ L=0\rangle\,.$$

In fact, the smallness of the $\lambda=\frac{1}{2}$ amplitude means that

$$\langle \underline{70}\ L'=1 \| (8,1)_0 + (1,8)_0 \| \underline{56}\ L=0\rangle \simeq 2\langle \underline{70}\ L'=1 \| (3,\bar{3})_1 \| \underline{56}\ L=0\rangle\,. \tag{46}$$

The signs of the resulting amplitudes are exactly those discussed by us previously.[5] *All* the well-determined ones agree in sign with experiment (nine in addition to the input). However, the magnitudes of a number of the predicted amplitudes are not in such great agreement with experiment. The $\lambda=\frac{3}{2}$ amplitude for $\gamma n \to D_{13}^0(1520)$ is too large. Mixing, at least with the small mixing angles otherwise suggested,[50] will not cure this, although it could well help improve the situation with regard to the poorly known $D_{13}(1700)$ amplitudes.

For the two S_{11} states, a fairly large mixing angle is known to be necessary from other considerations,[50] and would give $S_{11}(1700)$ amplitudes which agree with experiment in sign. The predicted $S_{11}(1535)$ amplitudes would still be much too large, however. The amplitudes predicted for the S_{31} and D_{33} also are all too large, and no mixing (within the $\underline{70}$) is possible in these cases. A fit to all the data would of course scale down the reduced matrix elements, making the agreement better for the magnitudes of the S_{31}, D_{33}, and S_{11} amplitudes, at some cost to those of the $D_{13}(1520)$.

A second comparison of the theory with experiment is also found in Table VII where all three possible reduced matrix elements are allowed to be nonzero, and fixed by the transitions $\gamma p \to D_{13}^+(1520)$ with $\lambda=\frac{1}{2}$ and $\frac{3}{2}$, and $\gamma n \to D_{13}^0(1520)$ with $\lambda=\frac{3}{2}$. Again, all the well-determined signs agree with experiment, although the predicted (and poorly determined experimentally) signs for the $D_{13}(1700)$ and $S_{11}(1700)$ are opposite to those discussed above.

There is still trouble in this case with the magnitudes of various amplitudes. The $\lambda=\frac{1}{2}$, $\gamma n \to D_{13}^0$ amplitude is too small, as is the amplitude for $\gamma p \to S_{11}^+(1535)$. Mixing only hurts here, as the γp transition to the other S_{11} is forbidden, resulting in an even smaller prediction for $\gamma p \to S_{11}^+(1535)$ and too small a result as well for $\gamma p \to S_{11}^+(1700)$. Although the D_{33} amplitude predictions now agree well with experiment, that for the S_{31} is still much too large.

It is interesting to note that for this second fit we have

$$\langle \underline{70}\ L'=1 \| (8,1)_0 + (1,8)_0 \| \underline{56}\ L=0\rangle \simeq \langle \underline{70}\ L'=1 \| (8,1)_0 - (1,8)_0 \| \underline{56}\ L=0\rangle\,, \quad \langle \underline{70}\ L'=1 \| (3,\bar{3})_1 \| \underline{56}\ L=0\rangle \simeq 0. \tag{47}$$

Equality of the first two reduced matrix elements is exactly what is forced by vector dominance plus the scheme of Petersen and Rosner[28] for vector-meson decays. The reason why

$$\langle \underline{70}\ L=1 \| (3,\bar{3})_1 \| \underline{56}\ L=0\rangle$$

should be so small, which in the fit is forced by the smallness of the amplitude for $\gamma p \to D_{13}^{+}(1520)$ with $\lambda=\frac{1}{2}$, is possibly an interesting theoretical problem.

At the present time, given the uncertainties we feel exist in the electromagnetic couplings of the N^*'s, either set of predictions should be regarded as in fair agreement with experiment as far as magnitudes are concerned. The signs in either case are a triumph of the theory for both photon and pion transitions and verify that the S and D amplitudes have opposite sign.

For transitions from the $\underline{56}$ $L'=2$ to the ground state $\underline{56}$ $L=0$ we also have in principle a large set of amplitudes for comparison with experiment. In practice the amplitudes are less well known, as seen in Table VIII. The quantities in the table, as in the previous one, are matrix elements of $D_+^3 + (1/\sqrt{3})D_+^8$ with signs appropriate to $\gamma p \to N^{*+} \to \pi^+ n$ and $\gamma n \to N^{*0} \to \pi^- p$. For the πNN^* vertex we express the two reduced matrix elements for $\underline{56}$ $L'=2 \to \underline{56}$ $L=0$ pion decays as[45]

$$\langle \underline{56}\ L'=2 \| (8,1)_0 - (1,8)_0 \| \underline{56}\ L=0\rangle = \tfrac{1}{5}(2P+3F)\,, \quad \langle \underline{56}\ L'=2 \| (3,\bar{3})_1 - (\bar{3},3)_{-1} \| \underline{56}\ L=0\rangle = \tfrac{1}{5}\sqrt{3}(P-F)\,, \tag{48}$$

where the amplitudes P and F correspond to $l=1$ and 3 pion orbital angular momenta, respectively. The relative signs of P and F are the same (opposite) if the $(8,1)_0-(1,8)_0$ $((3,\bar{3})_1-(\bar{3},3)_{-1})$ matrix element dominates. The reaction[46,47] $\pi N \to \pi\Delta$ indi-

TABLE VIII. Comparison of matrix elements of $D_+^3 + (1/\sqrt{3})D_+^8$ for $\underline{56}$ $L'=2 \to \underline{56}$ $L=0$ photon transitions with experiment (see Ref. 48). Nucleon resonances are identified as in Ref. 18 with the quark-model states, which are labeled by their quantum numbers J^P and [SU(3) multiplet]$^{2S+1}$, where S is the quark spin. The signs of amplitudes are those in $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$, with the P and F amplitudes at the πNN^* vertex taken to have the same sign (see text).

Transition		$\langle N^*,\lambda \vert D_+ \vert N,\lambda-1\rangle$ experiment (Ref. 48) (1/GeV)	$\langle N^*,\lambda \vert D_+ \vert N,\lambda-1\rangle$ predicted with $\langle \underline{56} \Vert (\bar{3},3)_{-1} \Vert \underline{56}\rangle$ and $\langle \underline{56} \Vert (8,1)_0-(1,8)_0 \Vert \underline{56}\rangle=0$
$F_{15}(1688)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	-0.07 ± 0.06	-0.07 (input)
$\frac{5}{2}^+$, $[8]^2$	$\lambda=\frac{3}{2}$	$+0.44\pm0.03$	$+0.44$ (input)
	$\to\gamma n$, $\lambda=\frac{1}{2}$	-0.11 ± 0.02	-0.26
	$\lambda=\frac{3}{2}$	0 ± 0.08	0
$P_{13}(1770)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	-0.02 ± 0.14	-0.70
$\frac{3}{2}^+$, $[8]^2$	$\lambda=\frac{3}{2}$	-0.03 ± 0.13	$+0.22$
	$\to\gamma n$, $\lambda=\frac{1}{2}$	-0.06 ± 0.06	-0.21
	$\lambda=\frac{3}{2}$	$+0.03\pm0.11$	0
$F_{37}(1920)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	-0.27 ± 0.05	-0.17
$\frac{7}{2}^+$, $[10]^4$	$\lambda=\frac{3}{2}$	-0.30 ± 0.04	-0.22
$F_{35}(1860)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	$+0.17\pm0.06$	-0.07
$\frac{5}{2}^+$, $[10]^4$	$\lambda=\frac{3}{2}$	-0.09 ± 0.08	-0.30
$P_{33}(2000)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	-0.12 ± 0.07	-0.11
$\frac{3}{2}^+$, $[10]^4$	$\lambda=\frac{3}{2}$	$+0.05\pm0.03$	$+0.18$
$P_{31}(1860)$	$\to\gamma p$, $\lambda=\frac{1}{2}$	$+0.04\pm0.05$	-0.11
$\frac{1}{2}^+$, $[10]^4$			

cates that P and F have the same sign, and we use this together with Table VI of Ref. 3 in constructing Table VIII. The "data" are again from Ref. 48.

To compare theory and experiment, we simplify the situation for the photon vertex by setting both the $\langle \underline{56}\ L'=2 \| (8,1)_0-(1,8)_0 \| \underline{56}\ L=0\rangle$ and $\langle \underline{56}\ L'=2 \| (\bar{3},3)_{-1} \| \underline{56}\ L=0\rangle$ reduced matrix elements to zero. This leaves only

$$\langle \underline{56}\ L'=2 \| (8,1)_0+(1,8)_0 \| \underline{56}\ L=0\rangle$$

and

$$\langle \underline{56}\ L'=2 \| (3,\bar{3})_1 \| \underline{56}\ L=0\rangle ,$$

as would be the case in most quark-model calculations.[23,24] Rather than making a fit to all the amplitudes, we use the well-measured $\gamma p \to F_{15}^+$ amplitudes to fix the two reduced matrix elements, and then calculate the remaining amplitudes.

All the predicted signs agree with our previous results,[5] and, with the possible exception of the F_{35} amplitude with $\lambda=\frac{3}{2}$, the experimentally well-determined signs agree with the theory. In a previous analysis,[51] both the F_{35} amplitudes also agreed. The signs of the $P_{33}(2000)$ amplitudes, among the p-wave πN resonances, provide some (marginal) support for the P and F amplitudes at the pion vertex having the same sign, as the $\pi N \to \pi\Delta$ analysis[41,47] shows much more definitely.

The magnitudes of the predicted amplitudes are in fair agreement with what is observed. There is no need to allow $\langle \underline{56}\ L'=2 \| (8,1)_0-(1,8)_0 \| \underline{56}\ L=0\rangle$ and $\langle \underline{56}\ L'=2 \| (\bar{3},3)_{-1} \| \underline{56}\ L=0\rangle$ to be nonzero. In fact, fitting all four reduced matrix elements to $\gamma p \to F_{15}^+$ with $\lambda=\frac{1}{2}$ and $\frac{3}{2}$, $\gamma n \to F_{15}^0$ with $\lambda=\frac{3}{2}$, and $\gamma p \to P_{33}^+$ with $\lambda=\frac{1}{2}$ results in essentially the same predictions; the two additional reduced matrix elements have values more than an order of magnitude smaller than either $\langle \underline{56}\ L'=2 \| (8,1)_0+(1,8)_0 \| \underline{56}\ L=0\rangle$ or $\langle \underline{56}\ L'=2 \| (3,\bar{3})_1 \| \underline{56}\ L=0\rangle$. The smallness of the $\lambda=\frac{3}{2}$ amplitude for $\gamma n \to F_{15}^0$ by itself assures the strong constraint on the two additional reduced matrix elements

$$-\tfrac{4}{45}\langle \underline{56}\ L'=2 \| (\bar{3},3)_{-1} \| \underline{56}\ L=0\rangle \simeq +\tfrac{4}{45}\sqrt{2}\langle \underline{56}\ L'=2 \| (8,1)_0-(1,8)_0 \| \underline{56}\ L=0\rangle . \quad (49)$$

There is thus fairly good evidence in this case that *only* the two reduced matrix elements found in the quark model are present at a significant strength, and, in particular, that equality of

$$\langle \underline{56}\ L'=2 \| (8,1)_0+(1,8)_0 \| \underline{56}\ L=0\rangle$$

and

$$\langle \underline{56}\ L'=2 \| (8,1)_0-(1,8)_0 \| \underline{56}\ L=0\rangle$$

is ruled out.

Finally, we examine the transitions from a "radially excited" $\underline{56}\ L'=0$ back to the ground state $\underline{56}\ L=0$. The $\underline{56}\ L'=0$ includes the Roper resonance, $P_{11}(1470)$, and the $P_{33}(1718)$. We fit the one possible reduced matrix element,

$$\langle \underline{56}\ L'=0 \| (3,\bar{3})_1 \| \underline{56}\ L=0\rangle ,$$

to the amplitude for $\gamma p \to P_{11}^+(1470)$, and predict the other amplitudes in Table IX using the $\underline{56}\ L'=0 \to \underline{56}\ L=0$ matrix elements from Table IV. Again the signs are those in $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$. The experimental results of both the Berkeley[48] and Lancaster[49] analyses are shown, there being some discrepancy between the two. Note that this is a case where the explicit quark-model results of Feynman *et al.*[24] fail by predicting the wrong sign[48,51] for the $P_{11}(1470)$ excitation.

VI. SUMMARY AND CONCLUSION

The operator V, which by definition takes us from a current to constituent quark basis, contains in principle all the information about matrix elements of the weak and electromagnetic currents when taken between hadron states, assuming that the hadrons can be treated as if constructed out of (constituent) quarks. Lacking a complete knowledge of V, we have abstracted only certain of its algebraic properties from the free quark model and assumed them to hold in the real world. In particular, in this paper we have abstracted properties of the operators $V^{-1}D_\pm^\alpha V$, which correspond to those which induce real photon transitions between hadrons.

In our case, abstraction from the free quark model leads to $V^{-1}D_+^\alpha V$ being assumed to be the sum of four terms which transform as $\{(8,1)_0+(1,8)_0, 1\}$, $\{(3,\bar{3})_1, 0\}$, $\{(8,1)_0-(1,8)_0, 1\}$ and $\{(\bar{3},3)_{-1}, 2\}$, all of which belong to $\underline{35}$'s of the full $SU(6)_W$ of currents. In Sec. II we have shown how matrix elements of $D_+^3+(1/\sqrt{3})D_+^8$ are related to real photon amplitudes and how they may be related to a sum of $[SU(6)_W]$ Clebsch-Gordan coefficients

TABLE IX. Comparison with experiment of matrix elements of $D_+^3+(1/\sqrt{3})D_+^8$ for photon transitions from resonances in a radially excited $\underline{56}\ L'=0$ multiplet to the nucleon in $\underline{56}\ L=0$. Amplitude signs are those in $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$.

Transition	Predicted matrix element	Experimental matrix element Ref. 48	Ref. 49
$P_{11}(1470)\to\gamma p,\ \lambda=\frac{1}{2}$	−0.37 (input)	−0.37 ± 0.04	−0.55 ± 0.13
$\to\gamma n,\ \lambda=\frac{1}{2}$	−0.25	0 ± 0.07	−0.51 ± 0.32
$P_{33}(1718)\to\gamma p,\ \lambda=\frac{1}{2}$	−0.18	+0.01 ± 0.07	+0.07 ± 0.25
$\lambda=\frac{3}{2}$	−0.31	−0.15 ± 0.10	+0.33 ± 0.29

times at most four reduced matrix elements for photon transitions from one hadronic $SU(6)_W$ multiplet to another. We have also shown that the theory leads to multipole selection rules, a particular example of which is the old SU(6) result[4] that the transition from the nucleon to 3-3 resonance should be magnetic dipole in character. In fact, we may generally express the four reduced matrix elements for transitions between two given multiplets in terms of four multipole amplitudes, two electric (of the same multipolarity) and two magnetic. These selection rules yield very interesting predictions, which may be subject to a qualitative experimental test in that low values of j_γ (or l for pions) are forbidden for $L' \geq 3 \to L=0$ transitions, even though they are otherwise allowed by spin-parity considerations and favored by angular momentum barrier arguments.

When applied to mesons there are many amplitudes which are related, but little to compare with experiment besides the transitions between the vector and pseudoscalar mesons, both of which lie in the $\underline{35}$ and $\underline{1}$ with $L=0$. The available data are consistent with the theory, but little else can be said at the moment.

For baryons, on the other hand, we have years of experimental effort that has been devoted to pion photoproduction in the resonance region, from which baryon electromagnetic couplings may be extracted by phase-shift analysis. For the $\underline{70}$ $L=1$ baryon states, not only do we find agreement of all the experimentally well-determined signs with the theory, but the photopion matrix elements, which contain information on both the γNN^* and πNN^* vertices, indicate that the S and D wave amplitudes at the pion vertex have opposite sign. This is in agreement with the results[46,47] from the reaction $\pi N \to \pi\Delta$. For the $\underline{56}$ $L=2$ baryon resonances, again all signs agree with the theory, except for possibly one of the $\gamma N \to F_{35}^+$ amplitudes. There is also an indication from $\gamma N \to \pi N$ that the P and F amplitudes at the πNN^* vertex have the same sign, in agreement with results[46,47] from $\pi N \to \pi\Delta$. While the signs are in good shape, the magnitudes, particularly for $\underline{70}$ $L=1 \to \underline{56}$ $L=0$ transitions, leave something to be desired. Given the uncertainties in the experimental analyses, however, we feel the present situation is fairly satisfactory.

The general outlook then is extremely good. Between the phase-shift analyses of $\pi N \to \pi\Delta$ and $\gamma N \to \pi N$, more than 25 signs predicted by the theory agree with experiment. For the first time we have some good evidence that not only is the multiplet structure of the quark model found in nature, but further that the wave functions of the states resemble those of the constituent quark model, in that the relative signs (and more roughly, magnitudes) of amplitudes are correctly predicted. However, neither the results at the πNN^* nor γNN^* vertices corresponds to the hypothesis of $SU(6)_W$ conservation, the most direct and powerful evidence being the signs and magnitudes of amplitudes for $\underline{70}$ $L'=1$ baryon resonances to decay into πN, $\pi\Delta$, and γN. The predictions resulting from the quark model,[23,24] where the reduced matrix elements are explicitly calculable, are wrong in places also—in particular in the signs of pion transition amplitudes for $\underline{56}$ $L'=2$ to $\underline{56}$ $L=0$ baryons and in the signs of photoproduction amplitudes for $\gamma N \to P_{11}(1470) \to \pi N$.

With the success of the theory, it may now be used as a tool to help in classifying new resonances into multiplets by using information on their signs in $\pi N \to \pi\Delta$ and $\gamma N \to \pi N$. What is still needed is a dynamics, or possibly an even higher symmetry, which will correctly give the magnitude and sign of the reduced matrix elements. This, and the extension to $q^2 \neq 0$, remain as important problems for the future.

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Finite unitary theory of pure leptonic weak interaction

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Based on the $V-A$ interaction, a finite, unitary theory of pure leptonic weak interaction is formulated in the framework of the finite quantum field theory with shadow states. A perturbative series, originated from the conventional expansion, is obtained. This new perturbative series converges in the high-energy region where the original one does not. The difference between the prediction of this theory and the universal $V-A$ theory is negligible in low-energy regions but becomes significant in high-energy regions.

The universal $V-A$ theory of weak interaction[1] is well established for low-energy processes as long as one treats the interaction Lagrangian as a phenomenological interaction to be used only in the lowest order. It is also well known that the matrix elements calculated from the lowest-order perturbation violate unitarity at high energies. One might take this as an indication that the higher-order calculation should be included. However, the $V-A$ theory as it stands is not renormalizable; i.e., to renormalize all divergent amplitudes involves an infinite number of arbitrary constants. It is therefore, if not meaningless, at least aesthetically unappealing. The modification of the universal $V-A$ theory has been discussed quite extensively in the past.[2] Recently great progress in the experiments on high-energy neutrino weak interactions has been made.[3] The construction of a workable theory of weak interaction is therefore becoming more urgent. There has been some progress in the effort to unify the weak and the electromagnetic interactions such that the weak-interaction amplitudes become renormalizable.[4]

While the general approaches to modifying the $V-A$ theory have been concentrated in the direction of introducing new heavy particles following certain symmetry schemes, one might adopt a quite different approach by, instead of introducing new "physical particles," changing the dynamical law of the theory. This is the main idea of the theory of shadow states.[5] In this note we present a finite unitary theory[6] of pure leptonic weak interaction within the framework of the theory of shadow states.

In the $V-A$ theory, the interaction Lagrangian for pure leptonic interaction is given by

$$\mathcal{L}_I = \frac{G}{\sqrt{2}} J_\lambda J^{\lambda\dagger}, \tag{1}$$

where J_λ is the leptonic current,

$$J_\lambda = [\bar{\psi}_{\nu_e}\gamma_\lambda(1-\gamma_5)\psi_e + \bar{\psi}_{\nu_\mu}\gamma_\lambda(1-\gamma_5)\psi_\mu]. \tag{2}$$

Following the idea of shadow-state theory, we introduce two shadow fields for each of the leptonic fields,[7] and rewrite the current as follows:

$$J_\lambda^s = \left[\sum_{i=1}^{3} c_i^{\nu_e}\bar{\psi}^i_{\nu_e}\gamma_\lambda(1-\gamma_5)\sum_{j=1}^{3} c_j^e\psi_e^j + \sum_{i=1}^{3} c_i^{\nu_\mu}\bar{\psi}^i_{\nu_\mu}\gamma_\lambda(1-\gamma_5)\sum_{j=1}^{3} c_j^\mu\psi_\mu^j\right]. \tag{3}$$

Here ψ_α^1 is the field of the physical lepton α with mass m_1^α and ψ_α^2 and ψ_α^3 are the corresponding shadow fields with masses m_2^α and m_3^α. The shadow fields are quantized with the "wrong" sign for the commutation relations. In other words, in contrast to the physical fields ψ_α^1 which satisfy the commutation relations

$$[\psi_\alpha^1(x), \psi_\alpha^1(x')] = -iS(x'-x), \tag{4}$$

the shadow fields ψ_α^2 and ψ_α^3 satisfy the commutation relations

$$[\psi_\alpha^2(x), \psi_\alpha^2(x')] = iS(x'-x), \tag{5}$$