$\pi\pi$ scattering in the energy region 0.6 to 1.42 GeV

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$\pi\pi$ scattering amplitudes have been determined in the energy range 0.6 to 1.42 GeV. The amplitudes have been extracted in the framework of an absorption-modified one-pion-exchange model with certain energy-dependent assumptions.

I. INTRODUCTION

The determination of $\pi\pi$ scattering amplitudes, particularly for dipion masses up to 1 GeV,[1] has been the subject of numerous experimental and theoretical investigations. Considerable controversy concerning the $I=0$ S-wave $\pi\pi$ phase shift, δ_0^0, had existed for some time. The ambiguity in δ_0^0 below the ρ mass region had been removed by Baton *et al.*[2] and by Baillon *et al.*[3] but there remained the "up-down" ambiguity in δ_0^0 above the ρ mass region.

It now appears that this remaining ambiguity in δ_0^0 has been removed as a result of the recent high-statistics experiments of Protopopescu *et al.*[4] and Hyams *et al.*[5] In both experiments an anomaly in the $\pi\pi$ system near the $K\bar{K}$ threshold was observed. This anomaly can be understood if δ_0^0 follows the "down" solution to a dipion mass of 0.9 GeV and then rises sharply to about 200° between 0.9 and 1.0 GeV. The corresponding inelasticity, η_0^0, is observed to drop sharply below unity around the $K\bar{K}$ threshold. In the analysis by the Berkeley group,[4] extrapolation to the pion pole was used to extract amplitudes, while in the analysis of the CERN-Munich data,[5] the $\pi\pi$ amplitudes were obtained from the density matrix elements integrated over a small four-momentum transfer (t) range near $t=0$. Studies of the reaction $\pi^- p \to n\pi^0\pi^0$ by Apel *et al.*[6] and by Skuja *et al.*[7] give results in agreement with those of Protopopescu *et al.* and Hyams *et al.*

Above 1 GeV, information concerning δ_0^0 comes mainly from the experiments of Carroll *et al.*[8] and Hyams *et al.*[5] Carroll *et al.* used an absorption-modified one-pion-exchange model (AOPE) to extract the $\pi\pi$ amplitudes. One of the main points of interest in the region of high dipion mass is the possibility of an S-wave resonance around the f^0 mass. This possibility was first pointed out by Carroll *et al.*[8] who found that δ_0^0 passes through 90° (270°) around 1.24 GeV. This result was obtained also by the CERN-Munich group,[5] but a resonance interpretation is not ascribed to the results by the authors.

We have obtained results on $\pi\pi$ scattering amplitudes for the dipion mass interval 0.6 to 1.42 GeV using the same AOPE model as Carroll *et al.*[8] to extract the amplitudes. However, we have introduced certain energy-dependent assumptions (Sec. V), and have included updated absorption and form-factor parameters (Sec. IV).

The main features of our results are as follows: (a) δ_0^0 increases rapidly at $K\bar{K}$ threshold; (b) η_0^0 drops from unity to about 0.6 near $K\bar{K}$ threshold and then returns to unity at about 1.1 GeV; (c) η_2^0 is less than unity in the region of the f^0 meson; (d) the total cross section in the ρ region agrees to within 5% of the AOPE model prediction but the predicted cross section in the f^0 region is about 25% higher than the experimental value.

In Sec. II we discuss the experimental procedure and in Sec. III we discuss the general features of the data. Section IV contains a brief description of the AOPE model, and in Sec. V we present the $\pi\pi$ scattering amplitudes.

II. EXPERIMENTAL METHOD

The 30-in. ANL-MURA deuterium-filled bubble chamber was exposed to a separated 6-GeV/c π^+ beam at the ZGS. Approximately 9.3×10^5 pictures with about 16 tracks per picture were obtained. The reaction of interest for elastic $\pi\pi$ scattering is

$$\pi^+ d \to \pi^+\pi^- p p_s, \tag{1}$$

where p_s denotes the spectator proton.

The film was scanned for events having an odd number of outgoing prongs or an even number of outgoing prongs with at least one positive track which stops in the chamber.

Approximately 180 000 three- or four-prong events were measured and processed through standard reconstruction and kinematic programs. Ionization estimates for outgoing tracks were independently obtained during the scanning and measuring stages. Any inconsistencies were subsequently resolved by physicists and experienced scanners. For those events having an invisible proton, a dummy track was constructed in the standard manner. Reaction (1) was thus subjected to a four-constraint fit whether or not the spectator proton was visible. Events were categorized as belonging to reaction (1) if (a) the χ^2 probability $\geq 1\%$ for that hypothesis; (b) $|MM^2| < 0.06$ (GeV)^2, where MM is the missing mass of any neutral system in the final state; (c) the ionization estimates were consistent with the results of the fit to reaction (1); and (d) the spectator-proton momentum was <250 MeV/c. Events attributed to the reaction $\pi^+ d \rightarrow K^+ K^- p p_s$ have been removed and will be discussed in a separate paper; the number of these K-pair events is less than 10% of the π-pair events. Approximately 1% of events which fit reaction (1) have a kinematic ambiguity between the outgoing proton and π^+. For these events, the hypothesis with higher χ^2 probability was accepted. We find 17 261 events which fit reaction (1) and satisfy our selection criteria.

The 17 261 events which fit reaction (1) correspond to a cross section of $\sigma = (1.09 \pm 0.11)$ mb. This cross section includes corrections for scanning and measuring efficiency, beam purity, spectator-proton momentum cut, and Glauber screening.[9] However, it does not include a correction for the Pauli principle effect on the final-state protons[10]; that correction ranges from 3% for pure spin-flip amplitude to 15% for pure non-spin-flip amplitude. Gordon *et al.*[11] have reported $\sigma = 1.11 \pm 0.07$ mb for the charge-symmetric reaction $\pi^- p \rightarrow \pi^- \pi^+ n$ at 6 GeV/c.

III. GENERAL FEATURES OF THE FINAL STATE $\pi^+\pi^- p$

The dipion invariant mass spectrum for events fitting reaction (1) is shown in Fig. 1. Copious ρ^0 and f^0 meson production is evident along with a definite signal in the g-meson mass region. We observe no structure in the f^0 region; our mass resolution in the f^0 region is approximately 18 MeV (FWHM). At the high-mass end of the ρ region (about 860 MeV) we observe a shoulder followed by a sharp drop in intensity. This effect was first reported by Alston-Garnjost *et al.*[12] and later confirmed by others.

In Fig. 2 we show the $p\pi^+$ and $p\pi^-$ mass spectra. Examination of the Dalitz plots (not shown) reveals no significant N^* or Δ enhancements. The general features observed can be adequately described by a diffractive dissociation background (Sec. IV). For the $\pi\pi$ amplitude analysis (Sec. V) we select only events which have a four-momentum transfer (t) to the dipion system of $|t| \leq 0.3$ (GeV/c)^2. Thus it appears that the dipion system up to a mass of 1.45 GeV is free from serious distortions generated by baryon resonances produced in reaction (1).

To obtain cross sections for ρ^0, f^0, and g^0 meson production,[13] we have fitted the dipion mass spec-

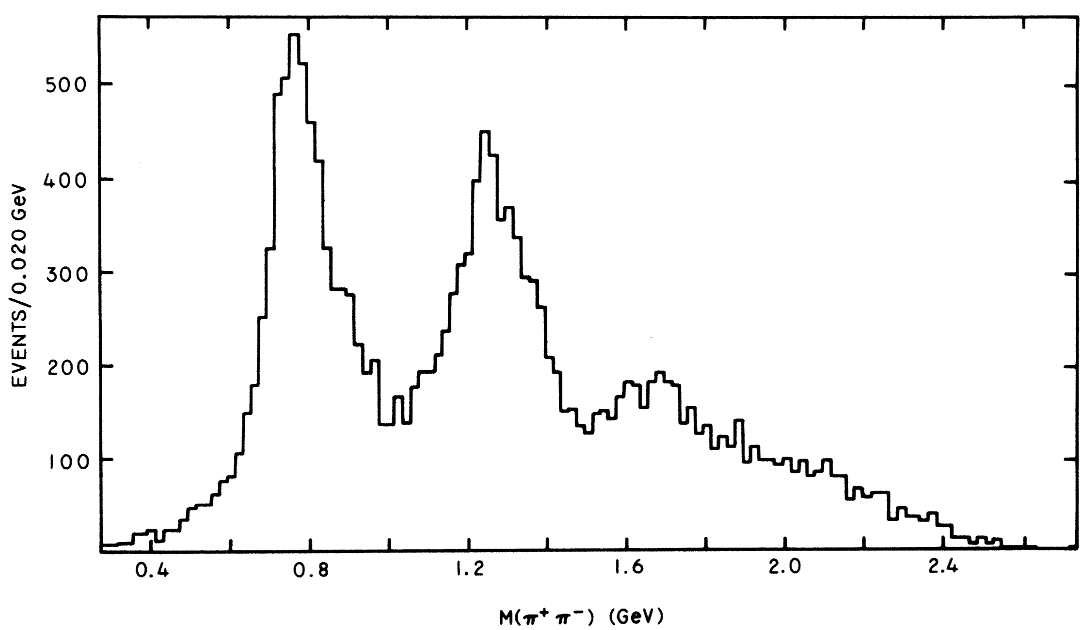

FIG. 1. $\pi^+\pi^-$ invariant mass distribution from the reaction $\pi^+ d \rightarrow \pi^+\pi^- p p_s$ at 6 GeV/c.

trum (Fig. 1) to an incoherent superposition of Breit-Wigner resonance shapes plus polynomial background. Explicitly, we fit to

$$F(m)=\sum_i \alpha_i B_i(m)+\beta P(m),$$

where $P(m)$ represents the polynomial background and α_i, β are variable coefficients. For each Breit-Wigner shape we take

$$B(m)=\frac{m_0\Gamma/2}{(m_0^2-m^2)^2+m_0^2\Gamma^2},$$

$$\Gamma(m)=\Gamma_0\left(\frac{q}{q_0}\right)^{2J+1},$$

where m_0, Γ_0 are the mass and width of a resonance

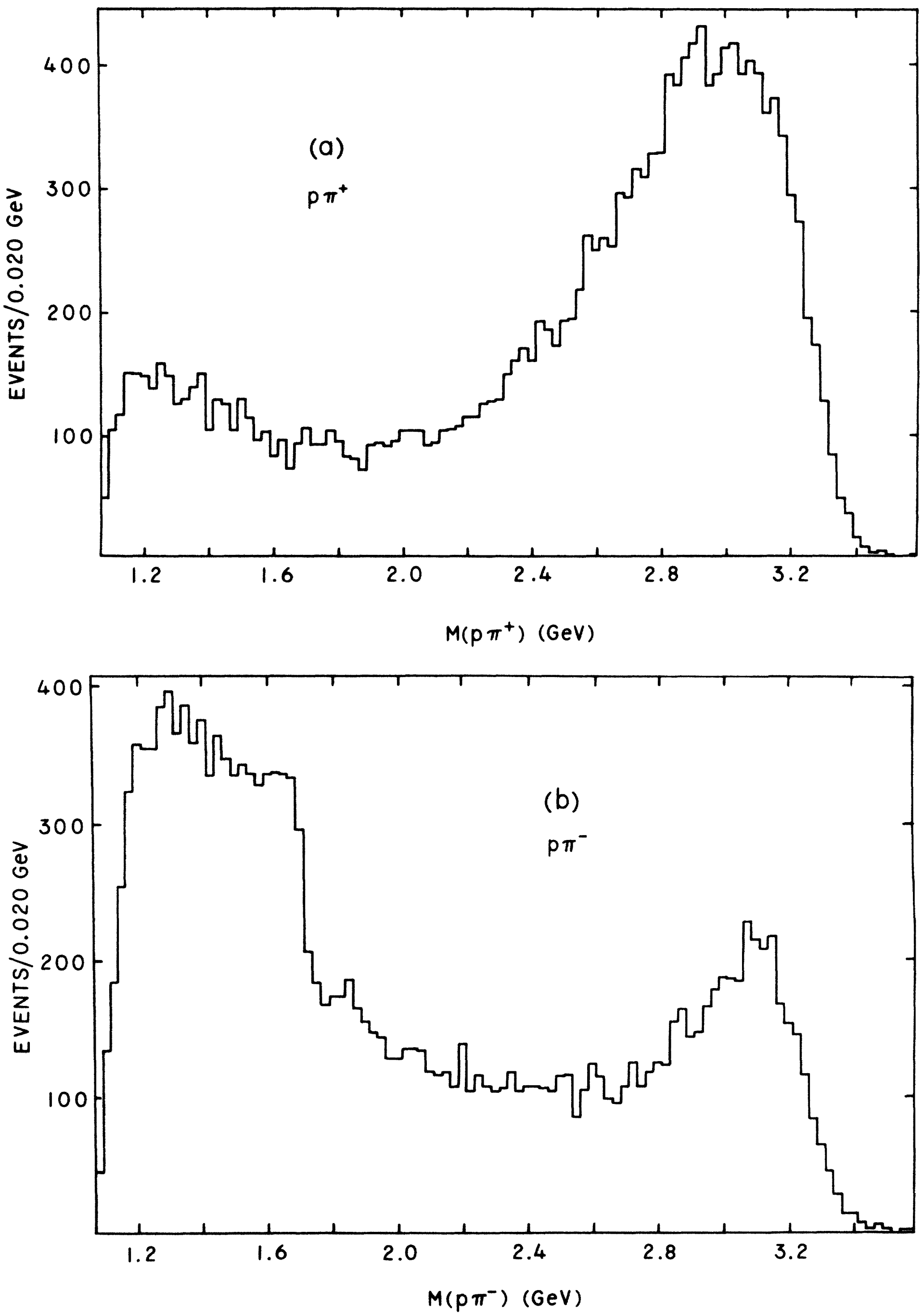

FIG. 2. Invariant mass distributions for (a) $p\pi^+$ and (b) $p\pi^-$ from the reaction $\pi^+d\rightarrow\pi^+\pi^-pp_s$ at 6 GeV/c.

of spin J and q (q_0) is the pion momentum associated with m (m_0).

For $P(m)$ we have tried polynomials of third, fourth, and fifth order and find that the central values (m_0) of the resonances are not sensitive to the specific form of the polynomial background. On the other hand, cross sections and resonance widths (Γ_0) do depend on the form of $P(m)$. Furthermore, because of the shoulder at the high-mass side of the ρ region, fits to the $\pi\pi$ mass spectrum generally have low confidence levels and yield rather high values for the mass and width of the ρ^0 meson. To minimize this effect, we have made a separate fit for the ρ^0 region for $410 < m_{\pi\pi} \leq 870$ MeV using quadratic background and the appropriate resonance shape function. We find $m_0(\rho^0) = 766 \pm 3$ MeV, $\Gamma_0(\rho^0) = 146 \pm 12$ MeV. For the f^0 and g^0 mesons we have extracted resonance parameters using $F(m)$ above. We find $m_0(f^0) = 1272 \pm 4$ MeV, $\Gamma_0(f^0) = 192 \pm 16$ MeV, $m_0(g^0) = 1690 \pm 5$ MeV, $\Gamma_0(g^0) = 167 \pm 40$ MeV.

The resonance cross sections we obtain are ($\pi^+\pi^-$ decay mode only)

$$\sigma(\rho^0) = 325 \pm 33\ \mu\text{b},$$

$$\sigma(f^0) = 290 \pm 30\ \mu\text{b},$$

$$\sigma(g^0) = 38.5 \pm 16.5\ \mu\text{b}.$$

The Pauli principle correction has been applied to each cross section with pure spin flip assumed at the nucleon vertex. For ρ^0 production this assumption can be justified from vector-dominance arguments, but the assumption is less certain for f^0 and g^0 production. However, the correction decreases in importance with increasing dipion mass.

TABLE I. Differential cross section slope parameter, b, for ρ^0, f^0, and g^0 production.

	$\lvert t\rvert$ interval $(\text{GeV}/c)^2$	b $(\text{GeV}/c)^{-2}$
ρ^0	0.02–0.30	10.5 ± 0.5
	0.30–0.80	3.3 ± 0.7
f^0	0.04–0.26	11.4 ± 0.8
	0.26–0.80	5.5 ± 0.5
g^0	0.10–0.80	6.6 ± 0.7

The differential cross sections, $d\sigma/dt$, for the ρ^0, f^0, g^0 mesons are shown in Fig. 3. For the ρ^0 and f^0 differential cross sections we observe a change in slope which occurs approximately at $\lvert t\rvert = 0.3$ $(\text{GeV}/c)^2$. This effect has been observed by Gordon *et al.*[11] at 6 GeV/c and by Oh *et al.*[14] at 7 GeV/c. We have fitted the ρ^0 and f^0 differential cross sections to an expression of the form $Ae^{-b\lvert t\rvert}$ in the intervals $\lvert t\rvert \leq 0.3$ $(\text{GeV}/c)^2$ and $\lvert t\rvert > 0.3$ $(\text{GeV}/c)^2$. The results are given in Table I. The slope change is consistent with the assumption that pion exchange dominates dipion production up to $\lvert t\rvert = 0.3$ $(\text{GeV}/c)^2$ and that other exchange processes become im-

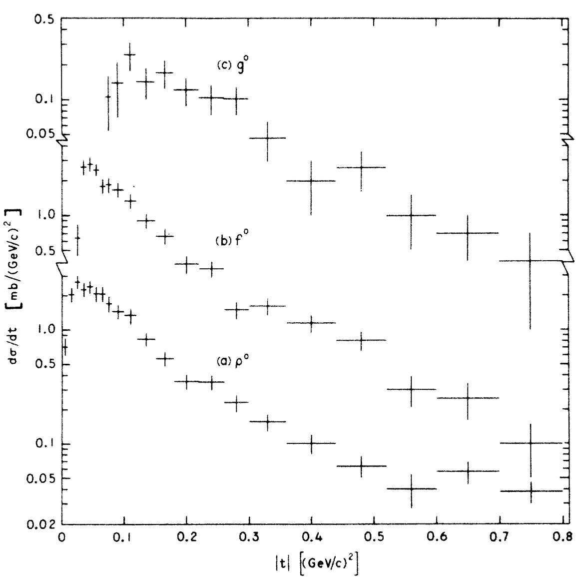

FIG. 3. Differential cross section, $d\sigma/dt$, for production of (a) ρ^0, (b) f^0, and (c) g^0 mesons.

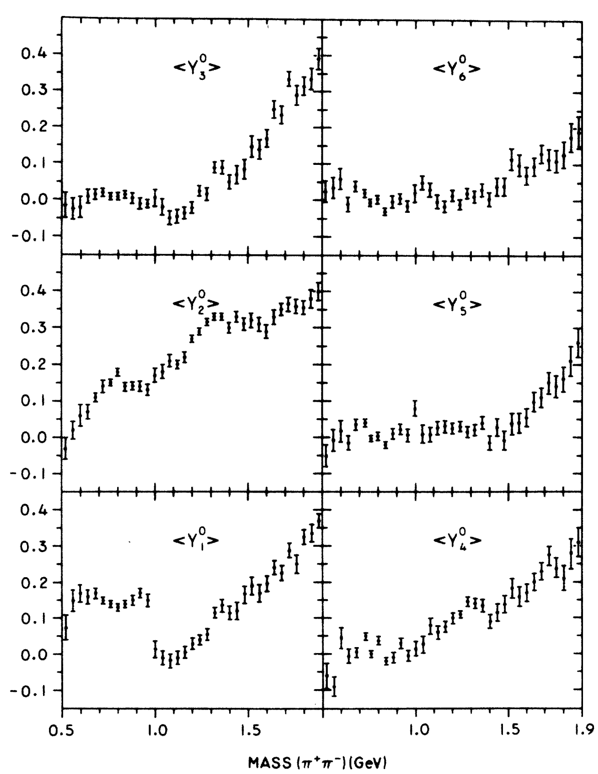

FIG. 4. Normalized moments, $\langle Y_L^0\rangle$, for $L = 1$ to $L = 6$ in the $\pi\pi$ mass region 0.5 to 1.9 GeV.

TABLE II. Energy-dependent assumptions for the π-π scattering amplitude analysis. [a]

Partial wave	ρ region $0.60 < m_{\pi\pi} \leq 0.90$ GeV	Intermediate region $0.90 < m_{\pi\pi} \leq 1.1$ GeV	f region $1.1 < m_{\pi\pi} \leq 1.42$ GeV
$I=0$, S	δ_0^0 variable	δ_0^0 variable	δ_0^0 variable
	$\eta_0^0 = 1$	η_0^0 variable	η_0^0 variable
$I=1$, P	$\delta_1^1 = \arctan[m_0\Gamma(s)/(m_0^2 - s)]$	δ_1^1 as in ρ region	$\delta_1^1 = a + b m_{\pi\pi}$, a,b variable
	$\eta_1^1 = 1$	η_1^1 variable	η_1^1 variable
	m_0, Γ_0 variable	m_0, Γ_0 fixed by ρ region	m_0, Γ_0 fixed by ρ region
$I=0$, D		$\delta_2^0 = \arctan[m_0\Gamma(s)/(m_0^2 - s)]$	
		η_2^0 variable	
		$m_0 = 1.27$ GeV, $\Gamma_0 = 0.20$ GeV	
$I=1$, F		$\delta_3^1 = \arctan[m_0\Gamma'(s)/(m_0^2 - s)]$	
		η_3^1 variable	
		$m_0 = 1.68$ GeV, $\Gamma_0 = 0.13$ GeV	

[a] $\Gamma(s) = [2m_0/(m_0 + \sqrt{s})](k/k_0)^{2L+1}[D_L(k_0)/D_L(k)]\Gamma_0$, $\Gamma'(s) = (m_0/\sqrt{s})(k/k_0)^7\Gamma_0$, where s = square of the dipion mass, L = spin of the resonance, k and k_0 are momenta of the π in the rest frame of the dipion system of mass $\sqrt{s}$, m_0, respectively:

$$D_1(k) = 1 + R_p^2 k^2,\quad R_p = 2.31\ \text{GeV}^{-2};\quad D_2(k) = 9 + 3R_d^2 k^2 + R_d^4 k^4,\ R_d = 3.23\ \text{GeV}^{-2}\ .$$

portant at higher values of four-momentum transfer. Examination of $\pi\pi$ density matrix elements indicates that pion exchange dominates reaction (1) for low t. In Table I we also give, for the g^0 meson, the parameter b taken over the interval $0.10 \leq |t| \leq 0.80$ (GeV/c)^2.

In Fig. 4 we show the normalized moments of the spherical harmonics, $\langle Y_L^0(\cos\theta)\rangle$, as a function of dipion mass and averaged over the region $|t| \leq 0.3$ (GeV/c)^2. Here, θ is the $\pi\pi$ scattering angle in the Jackson coordinate frame. The most striking feature is the rapid decrease in $\langle Y_1^0\rangle$ at a dipion

TABLE III. Results of the π-π scattering amplitude analysis as a function of dipion mass.

Mass (GeV)	δ_0^0 (deg)	η_0^0	η_1^1	δ_1^1 (deg)	η_2^0	η_3^1
0.64	59 ± 18					
0.68	66 ± 15					
0.72	74 ± 18					
0.76	60 ± 10					
0.80	66 ± 9					
0.84	72 ± 9					
0.88	80 ± 9					
0.92	112 ± 10	1.00 ± 0.12	1.00 ± 0.11		0.79 ± 0.08	0.97 ± 0.07
0.96	99 ± 8	0.93 ± 0.10	0.83 ± 0.06		1.00 ± 0.09	1.00 ± 0.06
1.00	186 ± 8	1.00 ± 0.11	0.72 ± 0.09		0.87 ± 0.09	0.94 ± 0.06
1.04	231 ± 10	0.65 ± 0.15	0.97 ± 0.06		0.89 ± 0.08	0.97 ± 0.06
1.08	255 ± 14	0.33 ± 0.11	0.96 ± 0.06		0.92 ± 0.08	0.99 ± 0.04
1.12	279 ± 13	0.90 ± 0.12	0.96 ± 0.07	154 ± 13	1.00 ± 0.10	1.00 ± 0.11
1.16	285 ± 10	0.99 ± 0.11	0.99 ± 0.10	156 ± 13	0.90 ± 0.10	1.00 ± 0.11
1.20	279 ± 10	1.00 ± 0.10	0.99 ± 0.11	159 ± 13	0.90 ± 0.11	1.00 ± 0.10
1.24	279 ± 11	0.99 ± 0.11	1.00 ± 0.10	161 ± 15	0.91 ± 0.11	1.00 ± 0.10
1.28	291 ± 17	1.00 ± 0.12	1.00 ± 0.09	164 ± 15	0.71 ± 0.09	1.00 ± 0.10
1.32	279 ± 15	1.00 ± 0.12	0.90 ± 0.10	166 ± 15	0.70 ± 0.10	0.99 ± 0.11
1.36	285 ± 10	0.99 ± 0.09	1.00 ± 0.11	168 ± 17	0.51 ± 0.11	0.90 ± 0.10
1.40	308 ± 14	0.96 ± 0.08	1.00 ± 0.09	171 ± 17	0.52 ± 0.11	0.92 ± 0.11

mass of about 1.0 GeV. It was this feature which enabled Protopopescu *et al.*[4] to resolve the up-down ambiguity above the ρ mass region. This experimental feature plays a crucial role in our analysis of $\pi\pi$ amplitudes (Sec. V). All moments $\langle Y_L^m \rangle$ with $m \neq 0$ are consistent with zero over the mass range 0.6 to 1.42 GeV with the exception of $\langle \mathrm{Re}\, Y_1^1 \rangle$ which is $\simeq -0.05$ up to 0.9 GeV dipion mass and consistent with zero at higher masses. Thus, the data are consistent with a one-pion-exchange mechanism.

IV. METHOD

To extract the $\pi\pi$ scattering amplitudes from our experimental data we have made use of the AOPE model of Durand and Chiu,[15] with the modifications applied by Oh *et al.*[14] and by Carroll *et al.*[8] We assume that for $|t| \leq 0.3$ (GeV/c)^2, the dipion system is produced via one-pion exchange with background described by diffractive dissociation of the target neutron (nucleon pole term, NPT). Details of the model are given in Refs. 14 and 16; an outline is presented here. The total amplitude for reaction (1) is written

$$\langle \mu, \lambda' | T | \lambda \rangle = \langle \mu, \lambda' | T_\pi | \lambda \rangle + \langle \lambda' | T_{\mathrm{NPT}} | \lambda \rangle ,$$

where λ, λ' are the helicities of the incident and outgoing nucleon, respectively, and μ is the helicity of the dipion system. The one-pion-exchange term is

$$\langle \mu\lambda' | T_\pi | \lambda \rangle = \frac{m_{\pi\pi}}{k} \sum_{L,I} C^N C^I [\eta_L^I \exp(2i\delta_L^I - 1) f_{\mu\lambda\lambda'}(s,t) \times d_{\mu 0}^L(-\psi) Y_L^\mu(\theta,\phi)] ,$$

where L, I indicate the relative orbital angular momentum and isotopic spin of the dipion system, C^N and C^I are the Clebsch-Gordan isospin coefficients for the nucleon and $\pi\pi$ vertices, respectively; k is the pion momentum in the dipion system, $\sqrt{s} = m_{\pi\pi}$, η_L^I and δ_L^I are the inelasticities and phase shifts of the dipion system; (θ, ϕ) are the pion scattering angles in the Jackson coordinate frame, ψ is the rotation angle which relates the helicity frame to the Jackson frame, and $d_{\mu 0}^L(-\psi)$ is the Wigner rotation function. The factors $f_{\mu\lambda'\lambda}(s,t)$ contain the amplitude s and t dependence [for pure OPE we have $\sum |f_{0\lambda'\lambda}|^2$ proportional to $-t/(t-m_\pi^{\,2})^2$]. Thus, $f_{\mu\lambda'\lambda}$ contains absorption corrections and off-mass-shell corrections for the

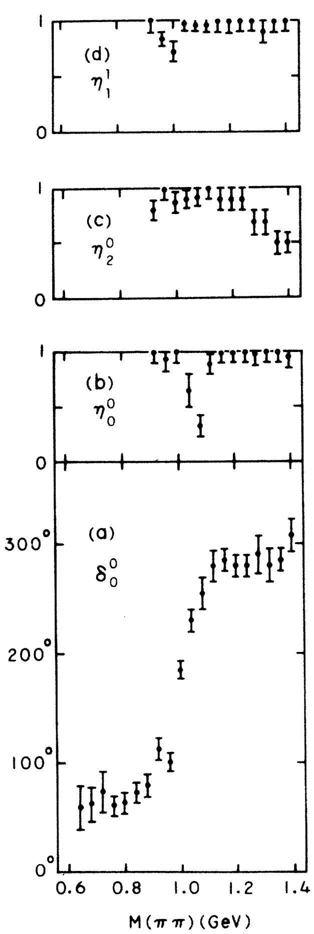

FIG. 5. (a) S-wave $\pi\pi$ phase shift, δ_0^0, as determined in the energy range 0.6 to 1.42 GeV; (b) S-wave inelasticity, η_0^0; (c) D-wave inelasticity, η_2^0; (d) P-wave inelasticity, η_1^1.

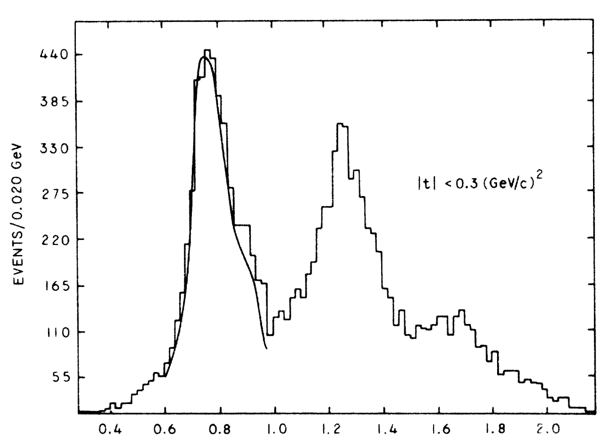

FIG. 6. $\pi^+\pi^-$ invariant mass distribution for $|t| \leq 0.3$ (GeV/c)^2 from the reaction $\pi^+ d \to \pi^+\pi^- p p_s$ at 6 GeV/c. The solid curve is the AOPE model prediction from 0.6 to 0.98 GeV.

$\pi\pi$ vertex. Some numerical factors entering $f_{\mu\lambda',\lambda}$ and the nucleon-pole term are different from those chosen by Oh *et al.*[14] For example, the off-shell correction factor contains the $\pi\pi$ vertex radius factors suggested by Wolf,[17] and the nucleon vertex amplitude is assigned a real part which is 0.2 times the imaginary part. Also the diffractive nucleon vertex is modified to agree with the experimental t dependence at 6 GeV/c.[18] The differential cross section is then written

$$\frac{\partial^4\sigma}{\partial m_{\pi\pi}\partial t\,\partial\cos\theta\,\partial\phi}=\frac{1}{1024\pi^4}\,\frac{k}{k_L}\,\frac{1}{M^2}\sum_{\mu\lambda'\lambda}|\langle\mu,\lambda'|T|\lambda\rangle|^2, \tag{2}$$

where k_L is the incident pion laboratory momentum and M is the mass of the neutron target.

V. RESULTS

For the $\pi\pi$ phase-shift analysis we have selected events with $|t|\leq 0.3$ (GeV/c)^2. This cut reduces the sample to 12 192 events. Furthermore, because of limited statistics at high dipion mass, we analyze only the mass interval 0.60 to 1.42 GeV. This interval contains 9368 events.

To extract $\pi\pi$ scattering amplitudes the experimental angular distributions and four-momentum distributions as a function of $m_{\pi\pi}$ were fitted to Eq. (2), with η_L^I and δ_L^I as free parameters. Since the number of parameters to be determined is large and the data are limited, some reasonable assumptions were introduced to reduce the number of free parameters to a manageable size.

The dipion mass distribution was divided into three regions which were analyzed separately. These mass regions are (a) the ρ region, 0.60–0.90 GeV; (b) the intermediate region, 0.90–1.1 GeV; and (c) the f region, 1.1–1.42 GeV. The assumptions used in each of the three regions are summarized in Table II and discussed in detail below. For the $I=2$ S- and D-wave amplitudes we have taken the results of Baton *et al.*[2] and Carroll *et al.*[8] as input to our analysis.

The experimental distributions in the three regions were separately fitted in $m_{\pi\pi}$ intervals of 40 MeV; the program MINUIT was utilized to minimize χ^2. Preliminary results based on a smaller data sample and employing the maximum-likelihood method have been reported.[19] The two methods yield consistent results.

A. The ρ^0 region (0.60–0.90 GeV)

Evidence from other experiments[4,5] indicates that D- and F-wave amplitudes are small in this region. We therefore assume only elastic S and P waves. Furthermore, we take the P wave to be resonating (ρ meson) and have parameterized it according to the form given in Table II, with m_0 and Γ_0 as free parameters. The number of parameters to be determined then is three: δ_0^0, $m_0(\rho)$, $\Gamma_0(\rho)$. A large number of starting points were tried in the χ^2 search and a satisfactory fit having χ^2/D.F. = 1.1 was obtained. For the mass and width of the ρ meson we find $m_0=(767\pm4)$ MeV and $\Gamma_0=146\pm14$ MeV, which agree with the simple Breit-Wigner fit discussed in Sec. III. The S-wave phase shift, δ_0^0, is given in Table III and shown in Fig. 5. We find only the "down" solution for δ_0^0.[20] Similar results for δ_0^0 in this mass region have been found by Carroll *et al.*[8] in the framework of the AOPE model.

Figure 6 shows the dipion mass spectrum for events with $|t|\leq 0.3$ (GeV/c)^2. The curve shown is the absolute yield predicted by the model in the ρ

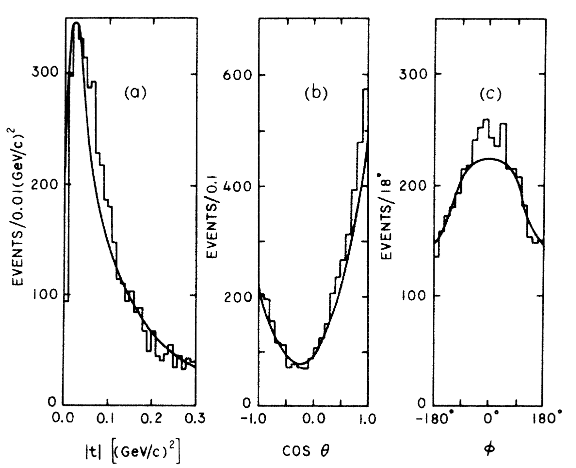

FIG. 7. (a) dN/dt, (b) $dN/d\cos\theta$, and (c) $dN/d\phi$ for the ρ^0 region (0.60–0.90 GeV). The solid curves are the AOPE model predictions.

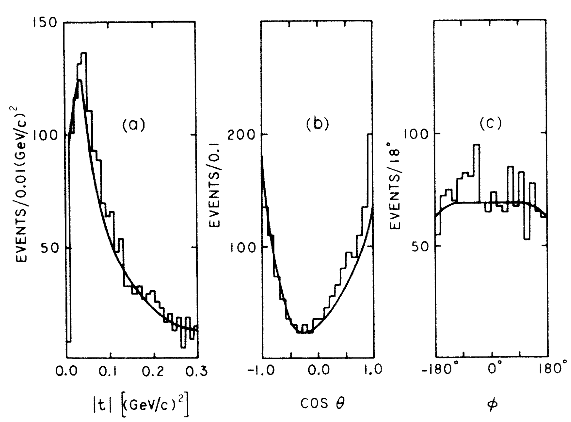

FIG. 8. (a) dN/dt, (b) $dN/d\cos\theta$, and (c) $dN/d\phi$ for the intermediate region (0.90–1.1 GeV). The solid curves are the AOPE model predictions.

region. The total cross section for the ρ region agrees to within 5% of the model prediction even though this agreement was not imposed as a constraint. Omission of the nucleon-pole term in the overall reaction amplitude results in a somewhat poorer fit to the mass spectrum, but changes δ_0^0 by less than 0.5σ. The four-momentum transfer distribution is shown in Fig. 7(a) and the angular distributions are shown in Figs. 7(b) and 7(c). The curves display the AOPE model predictions incorporating the best-fit values for $m_0(\rho)$, $\Gamma_0(\rho)$, and δ_0^0.

B. The intermediate region (0.90–1.1 GeV)

In this mass region partial waves up to and including $L=3$ are necessary. Thus eight parameters must in principle be determined. To reduce the number of fitted parameters, we assume that the phases δ_1^1, δ_2^0, and δ_3^1 are given by appropriate Breit-Wigner formulas (Table II) for the ρ, f, and g meson, respectively. For $m_0(\rho)$ and $\Gamma_0(\rho)$ we take the values determined in the ρ region, while for the corresponding m_0 and Γ_0 for the f and g mesons we take the values given in Table II.[21] Thus, there remain five parameters to be determined in the mass region 0.90 to 1.1 GeV: δ_0^0, η_0^0, η_1^1, η_2^0, and η_3^1.

We find a satisfactory fit with $\chi^2/\text{D.F.} \simeq 1$ and, as was the case for the ρ region, the fit is somewhat poorer if the nucleon-pole term is omitted, but the effect on the free parameters is negligible. In the fitting procedure, continuity with the neighboring regions was not a constraint. We find that a change of 25 MeV in the width of the f^0 does not affect the results, and we also find the fit to be insensitive to the assumed mass or width of the g meson.

The results for this mass region are presented in Table III and in Fig. 5. The S-wave phase shift, δ_0^0, exhibits the sharp rise in the vicinity of the $K\bar{K}$ threshold that was first observed by Protopopescu *et al.*[4] A dip in η_0^0 is apparent from our results, but the precise location in dipion mass where η_0^0 begins to drop is difficult to establish. The inelasticities η_1^1, η_2^0, and η_3^1 are consistent with unity. The four-momentum transfer distribution is shown in Fig. 8(a) and the angular distributions in Figs. 8(b) and 8(c). Again, the curves display the AOPE model predictions incorporating the best-fit values of the free parameters. As in the ρ region, agreement between the data and the model predictions is good.

C. The f^0 region (1.10–1.42 GeV)

Partial waves up to and including $L=3$ are required in this mass interval. The energy-dependent assumptions are identical to those of the intermediate region except for the phase δ_1^1. A linear variation with mass ($\delta_1^1 = a + b m_{\pi\pi}$) was assumed; two free parameters then describe the P-wave phase shift. In all, there are seven parameters to be determined in the f^0 region: δ_0^0, a, b, η_0^0, η_1^1, η_2^0, η_3^1.

The overall χ^2 fit in the f^0 region is poorer than that for either the ρ or intermediate regions. We obtain $\chi^2/\text{D.F.} \simeq 2$. Omission of the nucleon-pole term has no effect in this high-mass region. The total cross section predicted by the AOPE model is about 25% higher than experimentally observed.[22]

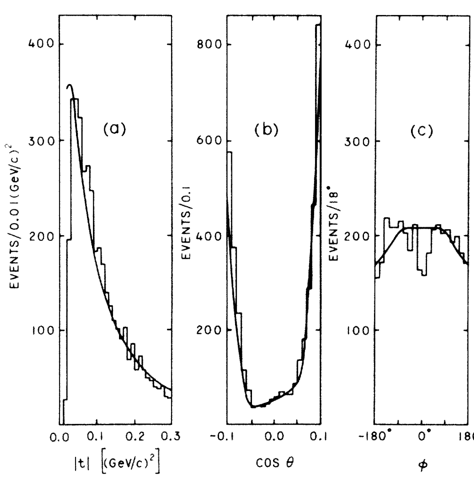

FIG. 9. (a) dN/dt, (b) $dN/d\cos\theta$, and (c) $dN/d\phi$ for the f^0 region (1.10–1.42 GeV). The solid curves are the AOPE model predictions.

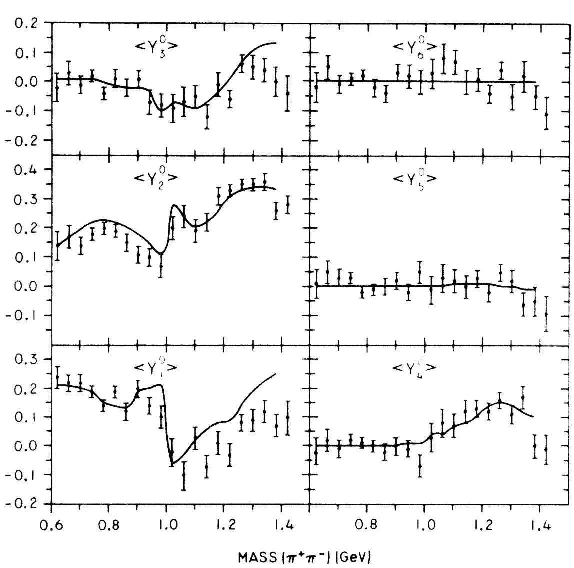

FIG. 10. Extrapolated moments for $L=1$ to $L=6$ in the $\pi\pi$ mass region 0.6 to 1.42 GeV. The solid curve has been calculated using the parameters of Table III.

Since absolute normalization was not imposed as a constraint in the fit, we do not believe this cross section discrepancy affects the results for phases and inelasticities.

Results are presented in Table III and in Fig. 5. δ_0^0 increases slowly from about 260° to 300° in this region; η_0^0 returns to unity at 1.14 GeV and remains so throughout. Whether this behavior of the *S* wave is consistent with the existence of a resonance near the f^0 mass is still an open question.[5] The *P* wave remains elastic and δ_1^1 increases slowly toward 170°. The *D* wave becomes inelastic around 1.28 GeV; our value of $\eta_2^0 \approx 0.7 \pm 0.1$ is consistent with the known branching ratios of the f^0 meson into inelastic channels. The *F*-wave inelasticity, η_3^1, is consistent with unity throughout.

In Fig. 9 the experimental distributions for the f^0 region are shown. Again, the curves are the AOPE model predictions calculated for the best-fit values of the free parameters. Agreement between the model and the azimuthal (Treiman-Yang) angle distribution is poor, but the predicted distributions in $\cos\theta$ and $|t|$ are in excellent agreement with the data.

In Fig. 10 we show the normalized moments $\langle Y_L^0 \rangle$ extrapolated to the pion pole. The moments, for a fixed $m_{\pi\pi}$, were fitted to linear functions of t for $|t| \leq 0.3$ $(\text{GeV}/c)^2$. The confidence levels of the fits were 25% on the average. The curves on Fig. 10 were calculated from parameters of Table III. Even though a linear extrapolation to the pion pole of the normalized moments may be an oversimplified approach,[1] it is interesting that the extrapolated moments agree so well with the results of the AOPE model fit to the data in the physical region.

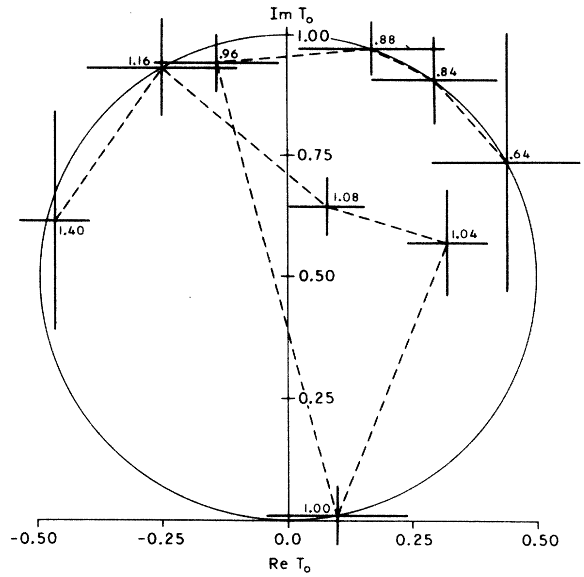

FIG. 11. The Argand diagram for the *S*-wave $I=0$ $\pi\pi$ amplitude from 0.64 to 1.40 GeV.

VI. CONCLUSIONS

In the framework of an absorption-modified OPE model and certain energy-dependent assumptions, we have extracted $\pi\pi$ amplitudes in the dipion mass interval 0.60–1.42 GeV. We find that the $I=0$ *S*-wave phase, δ_0^0, follows the "down" solution from 0.6 GeV until about 0.9 GeV. Then δ_0^0 increase rapidly from 90° near 0.9 GeV to 270° near 1.1 GeV. This behavior of δ_0^0 near the $K\bar{K}$ threshold is accompanied by a sharp drop in η_0^0. These features agree well with those first observed by Protopopescu *et al.*[4] and by the CERN-Munich group analyses.[5] In the higher-mass region (above 1.1 GeV) we find the *D* wave to be inelastic near the mass of the *f* meson, and we find the *F* wave consistent with being completely elastic up to 1.42 GeV. In Fig. 11 we show the Argand diagram for the $I=0$ *S*-wave $\pi\pi$ amplitude from 0.64 to 1.40 GeV.

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