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Correlation expansion in scalar electrodynamics

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The recoil to the eikonal model in scalar electrodynamics is derived. We then make use of our generalization of Low's theorem, which in an earlier paper was derived for soft recoil due to real-photon emission to infinite order in the coupling constant. Here, we generalize this further to yield hard recoil due to virtual exchanged photons as well. Recoil is defined relative to the straight-line-path approximation which we know gives the eikonal model. The recoil appears in terms of a correlation expansion where the first term, which is noncorrelated, is the usual eikonal model. A second result is that this gives a better and more explicit alternative to a Bethe-Salpeter ladder amplitude, since our ladder includes all crisscrosses.

I. INTRODUCTION

In a previous work[1] we derived the covariant recoil to a generalized infinite ϕ^3 ladder subjected to the well-known eikonal approximation. The usual eikonal result[2-5] could be identified as the first term in this recoil expansion and the recoil appeared in terms of a correlation expansion, which was obtained by a rearrangement of the ordinary perturbation expansion. The recoil is here defined relative to the straight-line-path approximation for the two throughgoing particles, which gives the eikonal model. It should be mentioned that this generalized ladder amplitude includes all types of crisscrosses in contrast to a Bethe-Salpeter ladder, and the result is an explicit closed form. The infrared (IR) factorization was then performed without introducing any cutoff, and therefore full relativistic invariance was guaranteed.

We now apply the same machinery in a true IR-divergent theory and will therefore derive a similar correlation expansion for a generalized ladder within the framework of scalar electrodynamics. As we know, in such a theory of charged spin-zero particles interacting via photon exchanges, momentum dependence then also appears in the vertices. This situation is different from that in a scalar ϕ^3 model, where this dependence occurs only in the denominators. By experience from our generalized soft-recoil theorem[6] we further know that this will contribute to the pair-correlation current and give off-shell effects in the hard core. The latter is here represented by the two vertices and the propagator for the photon, through which we eliminate the four-momentum-conserving δ function. Furthermore we get an additional contribution from seagull terms.

We here solve these problems, and as in the scalar ϕ^3 theory we find that one part of the amplitude is totally factorizable and eikonalized. We further derive the corresponding unique generalized pair-correlation tensor. All quantities are conserved and the resulting amplitude is therefore gauge-invariant. As in the previous work, we assume that the correlation expansion will decrease with increasing order of correlation. In most cases it should therefore be sufficient to include pair effects and neglect higher-order correlations. However, in principle we can derive this correlation expansion to an arbitrarily high order. For example, the triple correlations are fairly easy to calculate.[1] The method allows for inclusion of self-energy corrections and vacuum-polarization effects.

We first derive the result for massless photons, thereby retaining a fictitious photon mass to circumvent IR divergences in the intermediate finite-order calculations. Finally we generalize to the neutral massive vector-meson case. The derivation goes via a soft-recoil expansion, and the result is then generalized to the corresponding hard recoil, for virtual exchanged photons. As we will see, this requires a further generalization of Low's theorem.[6,7] In Sec. V we generalize this a third time to include rather general core functions with distorted propagators, spin effects, and arbitrary off-shell effects which occur for all physical sources which are not c numbers.

II. SOFT RECOIL

The interacting Hamiltonian is here given by

$$\mathcal{H}_I = ieA_\nu\left(\phi^*\frac{\partial\phi}{\partial x_\nu}-\frac{\partial\phi^*}{\partial x_\nu}\phi\right)-e^2\phi^*\phi A_\nu A^\nu\,, \quad (2.1)$$

where ϕ is a charged spin-zero field with which we associate the two throughgoing particles of mass m. The electromagnetic field denoted A is first associated with massless spin-one photons, with a fictitious mass to circumvent the IR problem in the intermediate finite-order calculations. In the next step A_ν represents a neutral vector-meson field with quanta of mass μ. Following the notations of our previous work[1] the $(n+1)$th-order exchange amplitude (Fig. 1), with exchanged momenta denoted $k_1, \ldots, k_{n+1}$, is defined by

$$-iM_{n+1}(s;t)=\frac{(-ie)^{2n+2}}{(n+1)!}\times\int\prod_{l=1}^{n+1}\left(\frac{d^4k_l}{(2\pi)^4}\tilde{\Delta}_F(k_l)\right)\times I(2\pi)^4\delta^4\Big(q-\sum_{l=1}^{n+1}k_l\Big), \quad (2.2)$$

where $\tilde{\Delta}_F(k)=-i(k^2-\lambda^2+i\epsilon)^{-1}$ is the photon propagator. As usual we define

$$q=p_a-p_{a'}=p_{b'}-p_b\,, \quad (2.3)$$

$$t=q^2, \quad s=(p_a+p_b)^2\,, \quad (2.4)$$

where p_a and p_b are the momenta of the two ingoing charged spin-zero particles and the corresponding primed subscripts denote momenta of the outgoing particles. The quantity I is a product of charged-particle propagators.

We eliminate the δ function by integrating over the rth photon momentum, thereby factorizing out its two vertices $I=I_rV_r$, where V_r is given by

$$V_r=-e^2(P_a+P_{a'})\,(P_b+P_{b'})\,, \quad (2.5)$$

$$P_a=p_a-\sum_{l=j_1}^{j_{r-1}}k_l, \quad P_{a'}=p_{a'}+\sum_{l=j_{r+1}}^{j_{n+1}}k_l\,, \quad (2.6)$$

$$P_b=p_b+\sum_{l=j_1}^{j_{s-1}}k_l, \quad P_{b'}=p_{b'}-\sum_{l=j_{s+1}}^{j_{n+1}}k_l\,. \quad (2.7)$$

For simplicity we first choose to work in the Feynman gauge, and then, once we have checked that derived currents are properly conserved, we can move to an arbitrary gauge. The I_r quantity is a product of propagators and vertices, where we now also include the coupling constants

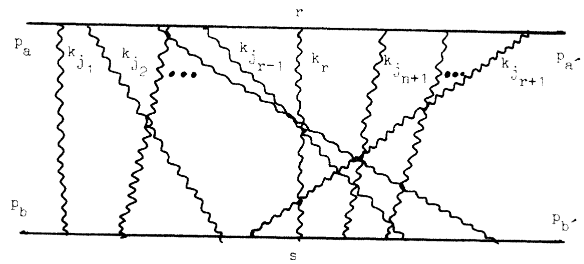

FIG. 1. A typical s-ladder diagram.

$$A_{\mu_{j(l)}}=ie\frac{2(p_a-k_{j_1}-\cdots-k_{j_{l-1}})_{\mu_{j(l)}}}{-2p_a\cdot K_{j_l}+K_{j_l}{}^2+i\epsilon}\,,$$

$$K_{j_l}=\sum_{t=j_1}^{j_l}k_t, \quad l=1,\ldots,r-1 \quad (2.8)$$

$$A'_{\mu_{j(l)}}=ie\frac{2(p_{a'}+k_{j_{r+1}}+\cdots+k_{j_{l-1}})_{\mu_{j(l)}}}{2p_{a'}\cdot K_{j_l}+K_{j_l}{}^2+i\epsilon}\,,$$

$$K_{j_l}=\sum_{t=j_{r+1}}^{j_l}k_t, \quad l=r+1,\ldots,n+1 \quad (2.9)$$

$$B_{\mu_{j(l)}}=ie\frac{2(p_b+k_{j_1}+\cdots+k_{j_{l-1}})_{\mu_{j(l)}}}{2p_b\cdot K_{j_l}+K_{j_l}{}^2+i\epsilon}\,,$$

$$K_{j_l}=\sum_{t=j_1}^{j_l}k_t, \quad l=1,\ldots,s-1 \quad (2.10)$$

$$B'_{\mu_{j(l)}}=ie\frac{2(p_{b'}-k_{j_{s+1}}-\cdots-k_{j_{l-1}})_{\mu_{j(l)}}}{-2p_{b'}\cdot K_{j_l}+K_{j_l}{}^2+i\epsilon}\,,$$

$$K_{j_l}=\sum_{t=j_{s+1}}^{j_l}k_t, \quad l=s+1,\ldots,n+1. \quad (2.11)$$

The kernel is defined by

$$I_{r\,\mathrm{sym}}=\sum_{\substack{s_1s_2\\ D_{\mathrm{I}}^AD_{\mathrm{II}}D_{\mathrm{III}}}}\;\sum_{D_aD_{a'}}\prod_{l=1}^{r-1}A_{\mu_{j(l)}}\prod_{l=r+1}^{n+1}A'_{\mu_{j(l)}}\times\sum_{D_bD_{b'}}\prod_{l=1}^{s-1}B_{\mu_{j(l)}}\prod_{l=s+1}^{n+1}B'_{\mu_{j(l)}}\,. \quad (2.12)$$

To this we must add the contributions from seagull diagrams (Fig. 2) due to the last term in (2.1):

$$\delta I^s_{r\,\mathrm{sym}}=-e^2\sum_{t=1}^{r-2}\;\sum_{\substack{s_1s_2\\ D_{\mathrm{I}}^AD_{\mathrm{II}}D_{\mathrm{III}}}}\;\sum_{D_aD_{a'}}\prod_{l=1}^{t-1}A_{\mu_{j(l)}}\frac{g_{\mu_{j(t)}\mu_{j(t+1)}}}{-2p_a\cdot K_{j_{t+1}}{}^2+K_{j_{t+1}}{}^2+i\epsilon}\prod_{l=t+2}^{r-1}A_{\mu_{j(l)}}\prod_{l=r+1}^{n+1}A'_{\mu_{j(l)}}\sum_{D_bD_{b'}}\prod_{l=1}^{s-1}B_{\mu_{j(l)}}\prod_{l=s+1}^{n+1}B'_{\mu_{j(l)}}$$

$$+\text{(seagulls in the other prongs)}. \quad (2.13)$$

Seagulls with the rth photon involved are treated, together with the off-shell vertex effects, later. Here s_1 is the number of quanta going from a to b without crossing the rth line (Fig. 3). Similarly s_2 is the number of quanta going from a' to b thereby crossing the rth line. These two numbers are restricted through $s_1+s_2=s-1$, where s is that bb' vertex where the rth quantum is absorbed. The set D_i $(i=a,a',b,b')$ is the set of all internal permutations among all quanta attached to the ith

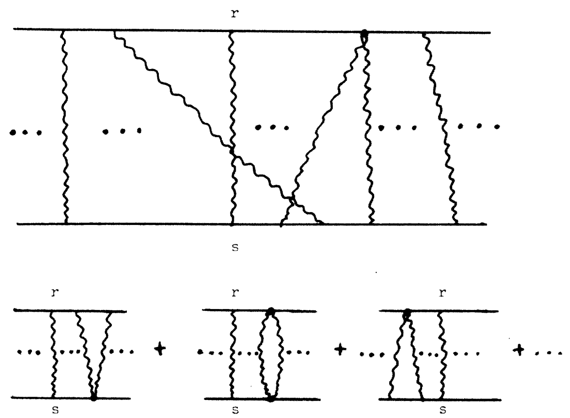

FIG. 2. Ladder with a seagull term.

prong and D_{I}^A is the factor set $D_n/(D_a\otimes D_{a'})$, where D_n denotes all permutations on, for example, the $A=\{a,a'\}$ side. A similar summation is, of course, required on the B side, but since the A quanta are in fact the same individuals as the B quanta, we have already summed over some of them (cf. Fig. 3 and our previous work[1]). We therefore should sum only over $D_{\mathrm{II}}=D_a/(D_{ab}\otimes D_{ab'})$ and $D_{\mathrm{III}}=D_{a'}/(D_{a'b}\otimes D_{a'b'})$. The meaning of the subsets D_{ij} follows from Fig. 3.

Following the lines of our previous work,[1] we first drop all k_l^2 terms, since those are easily factorized and included afterwards anyway. We also leave off-shell effects in the vertex function (2.5) and longitudinal components $k_{l\mu_l}$ in the numerators for a separate treatment later in this work.

We thus first work with the on-shell vertex

$$V_0=-e^2(p_a+p_{a'})\cdot(p_b+p_{b'})\,. \tag{2.14}$$

Following Ref. 1 we expand (2.12) and (2.13) in a soft-recoil expansion where terms of essential order $e^n k^{-n}$ and $e^n k^{-n+1}$ are retained in the different prongs. A direct application of the generalized soft-recoil theorem,[6] by summation over D_i, where $i=a$, a', b, or b', shows that the ith prong contributes with

$$\Delta I^i_{r\,\mathrm{sym}}=\prod_{l=\alpha(i)}^{\beta(i)} f^{0i}_{\mu_{j(l)}}+\sum_{\substack{p<q\\ \alpha(i)}}^{\beta(i)}\chi^{0i}_{\mu_{j(p)}\mu_{j(q)}}\prod_{\substack{l\neq pq\\ \alpha(i)}}^{\beta(i)} f^{0i}_{\mu_{j(l)}}\,. \tag{2.15}$$

The noncorrelated soft currents are given by

$$f^{0i}_\mu=ie\frac{p_{i\mu}}{\epsilon_i p_i\cdot k+i\epsilon}\,,\quad J^{0Q}_\mu=\sum_{i\in Q} f^{0i}_\mu\,,\quad Q=A,B \tag{2.16}$$

and the partially factorized soft pair-correlation current reads

$$\chi^{0i}_{\mu_p\mu_q}=(ie)^2\frac{1}{\epsilon_i p_i\cdot(k_p+k_q)}\times\left(\frac{p_{i\mu_p}k_{p\mu_q}}{p_i\cdot k_p}+\frac{p_{i\mu_q}k_{q\mu_p}}{p_i\cdot k_q}-\frac{p_{i\mu_p}}{p_i\cdot k_p}\frac{p_{i\mu_q}}{p_i\cdot k_q}k_p\cdot k_q-g_{\mu_p\mu_q}\right). \tag{2.17}$$

The form (2.15) is easily proved by induction and both J^{0Q}_μ and $\chi^{0i}_{\mu\nu}$ are conserved currents. In the full amplitude the total pair-correlation tensor

$$\chi^{0Q}_{\mu_p\mu_q}=\sum_{i\in Q}\chi^{0i}_{\mu_p\mu_q}\,,\quad Q=A,B \tag{2.18}$$

will appear. Inserting (2.15) in the $(n+1)$th-order generalized ladder (2.12) and (2.13) and summing over D_{I}^A, D_{II}, D_{III}, s_1, s_2, and r, after some straightforward algebra[1,6] we find

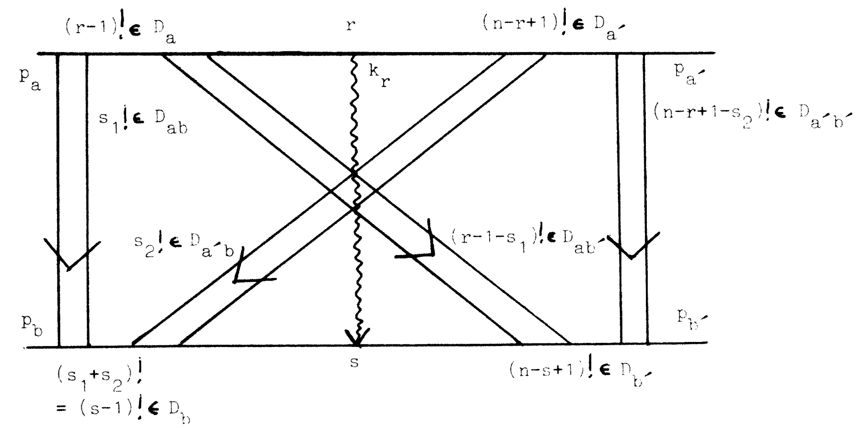

FIG. 3. Schematic picture of the permutation sets relative to the rth photon.

$$I_{\rm sym}=\prod_{l=1}^{n} J^{0A}_{\mu_l} J^{0B*\mu_l} + \sum_{\substack{p<q\\1}}^{n} (\chi^{0A}_{\mu_p\mu_q} J^{0B*\mu_p} J^{0B*\mu_q} + \chi^{0B}_{\mu_p\mu_q} J^{0A\mu_p} J^{0A\mu_q}) \prod_{\substack{l\neq pq\\1}}^{n} J^{0A}_{\mu_l} J^{0B*\mu_l}\,. \tag{2.19}$$

By insertion in (2.2) this simplifies further, since under the integral sign the pair effects can be expressed in a form-invariant functional of an arbitrary pair of two correlating quanta. The sum over p and q then reduces to a multiplicative factor $\binom{n}{2}$, since this is the number of ways we can select two quanta out of n, if we do not bother about their internal order. We then get

$$-iM_{n+1}(s;t)=\frac{1}{(n+1)!}\times\int d^4x\, e^{iq\cdot x}\Delta_F(x)\times\left[(iU)^n+\binom{n}{2}i^2P(iU)^{n-2}\right]V_0\,, \tag{2.20}$$

where U is a scalar product

$$U(x;s;t)=i\int\frac{d^4k}{(2\pi)^4}\tilde{\Delta}_F(k)e^{-ik\cdot x}J^{0A}_{\mu}(k)J^{0B*\mu}(k)\,, \tag{2.21}$$

and P is a pair-correlation functional

$$P(x;s;t)=i^2\int\frac{d^4k_p}{(2\pi)^4}\frac{d^4k_q}{(2\pi)^4}\tilde{\Delta}_F(k_p)\tilde{\Delta}_F(k_q)e^{-i(k_p+k_q)\cdot x}\times(\chi^{0A}_{\mu_p\mu_q}J^{0B*\mu_p}J^{0B*\mu_q}+\chi^{0B}_{\mu_p\mu_q}J^{0A\mu_p}J^{0A\mu_q})\,. \tag{2.22}$$

Thanks to total decoupling of the one-particle scalar products and partial factorization of a form-invariant pair functional, (2.20) is now on a totally summable form. However, first we generalize to the corresponding hard-recoil amplitude.

III. HARD RECOIL

We here follow the scheme of Ref. 1 and approach the problem via an infinite-order recoil expansion, discarding all induced currents of higher-order correlation than pair correlations. The simplest part is to include uncorrelated recoil to infinite order, e.g., the total factorization of k^2 terms, which is easily proved by induction. This modifies the soft uncorrelated currents (2.16) according to

$$f'^{\,i}_{\mu}=ie\frac{2p_{i\mu}}{2\epsilon_ip_i\cdot k+k^2+i\epsilon}\,,\qquad J'^{\,Q}_{\mu}=\sum_{i\in Q}f'^{\,i}_{\mu}\,,\quad Q=A,B \tag{3.1}$$

with $\epsilon_i=+1,-1$ for out- and ingoing particles, respectively, on the A side. On the B side ϵ_i is replaced by $-\epsilon_i$.

The pair-correlation tensor (2.17) is still kept to first order in the recoil expansion, although somewhat modified by inclusion of the $k_l^{\,2}$ terms.

$$\chi'^{\,i}_{\mu_p\mu_q}=f'^{\,i}_{\mu_p}ie\frac{2\epsilon_ik_{p\mu_q}}{y^i_{pq}}+f'^{\,i}_{\mu_q}ie\frac{2\epsilon_ik_{q\mu_p}}{y^i_{pq}}-f'^{\,i}_{\mu_p}f'^{\,i}_{\mu_q}x^i_{pq}-(ie)^2\frac{2g_{\mu_p\mu_q}}{y^i_{pq}}\,, \tag{3.2}$$

where x and y are defined by

$$x^i_{pq}=\frac{2k_p\cdot k_q}{y^i_{pq}}\,,\qquad y^i_{pq}=2\epsilon_ip_i\cdot(k_p+k_q)+k_p^{\,2}+k_q^{\,2}\,. \tag{3.3}$$

We notice here that the currents f' in (3.1) and χ' in (3.2) are conserved quantities in the long-wavelength limit where $k_l^{\,2}$ terms vanish, but nonconserved in the short-wavelength limit. As is shown in the Appendix, this is cured by inclusion of longitudinal modes, which contribute to the totally factorizable noncorrelated currents according to

$$f^i_{\mu}=ie\frac{2p_{i\mu}+\epsilon_ik_{\mu}}{2\epsilon_ip_i\cdot k+k^2+i\epsilon}\,,\quad J^Q_{\mu}=\sum_{i\in Q}f^i_{\mu}\,,\quad Q=A,B \tag{3.4}$$

and to the pair currents through the new f's in (3.4),

$$\bar{\chi}^i_{\mu_p\mu_q}=f^i_{\mu_p}ie\frac{2\epsilon_ik_{p\mu_q}}{y^i_{pq}}+f^i_{\mu_q}ie\frac{2\epsilon_ik_{q\mu_p}}{y^i_{pq}}-f^i_{\mu_p}f^i_{\mu_q}x^i_{pq}-(ie)^2\frac{2g_{\mu_p\mu_q}}{y^i_{pq}}\,. \tag{3.5}$$

Using the relation

$$k\cdot f^i=ie\epsilon_i\,, \tag{3.6}$$

it is then easily verified that both J^Q in (3.4) and χ in (3.5) are conserved.

In order to extract all pair-correlation effects from (2.12) and (2.13) we must study the infinite-order recoil expansion. We then notice that higher-order recoil due to k dependence in the numerators necessarily induces higher-order correlations and this is therefore discarded. Similarly, higher-order seagull contributions are higher than pair correlated ones. Therefore, as far as higher-order recoil is concerned, since we are just interested in pair effects, we have reduced the problem to the same as that in the scalar case.[1] Thus, only recoil from denominators in powers of x^i_{pq} contributes to higher-order pair-correlated recoil. As in Ref. 1 we can put the mth order pair-correlating recoil in the form

$$R^{i(m)}_{n\mu_1\cdots\mu_n}=\sum_{\substack{k<l\\1}}^{n}\bar{\chi}^i_{\mu_k\mu_l}(-x^i_{kl})^{m-1}\prod_{\substack{t\neq kl\\1}}^{n}f^i_{\mu_t}$$

$$+\text{higher correlations}\,, \qquad (3.7)$$

which after summation in m gives the total pair-correlation currents to all orders in the recoil expansion:

$$\chi^i_{\mu_p\mu_q}=\bar{\chi}^i_{\mu_p\mu_q}\frac{1}{1+x^i_{pq}}\,,$$
$$\chi^Q_{\mu_p\mu_q}=\sum_{i\in Q}\chi^i_{\mu_p\mu_q}\,,\quad Q=A,B\,. \qquad (3.8)$$

The expanded form (3.7) is, of course, valid just for $|x|<1$ values, but, as we showed in Appendix C of Ref. 1, we can derive the form (3.8) for arbitrary $|x|$ values by successively neglecting induced terms of higher-order correlation. In other words, we should keep the original form of the total pair-correlation tensor as it appears already for two quanta, in which case we can treat everything exactly. *Starting from the exact three-quantum form, we could then also define the exact triple-correlation tensor*, etc. This defines the full correlation expansion. However, here we content ourselves with pair effects, the first correction term to the eikonal model.

The inclusion of hard recoil, and longitudinal k components as well, modifies the scalar products U and the pair-correlation functional P as compared to (2.21) and (2.22):

$$U(x;s;t)=i\int\frac{d^4k}{(2\pi)^4}\,\Delta_F(k)e^{-ik\cdot x}\,J^A_\mu J^{B\mu}\,, \qquad (3.9)$$

$$P(x;s;t)=i^2\int\frac{d^4k_p}{(2\pi)^4}\frac{d^4k_q}{(2\pi)^4}\tilde{\Delta}_F(k_p)\tilde{\Delta}_F(k_q)e^{-i(k_p+k_q)\cdot x}$$
$$\times(\chi^A_{\mu_p\mu_q}J^{B\mu_p}J^{B\mu_q}+\chi^B_{\mu_p\mu_q}J^{A\mu_p}J^{A\mu_q}+\chi^A_{\mu_p\mu_q}\chi^{B\mu_p\mu_q})\,. \qquad (3.10)$$

The third term in (3.10) is of course not present in (2.22), since there we kept only first-order recoil terms. It is interesting and important to notice here that it is possible to rearrange the conventional perturbation expansion in a consistent and unique way with respect to the order of correlation and simultaneously retain gauge invariance. In fact, gauge invariance is directly necessary, as we will see in Sec. IV, to solve the off-shell vertex problem.

IV. OFF-SHELL VERTEX

We first observe that the off-shell vertex (2.5) is invariant under D_a, $D_{a'}$, D_b, and $D_{b'}$. Therefore the factorization machinery (2.12)–(2.15) works even for this case. The kernel can thus be put on the convenient form

$$I^{\text{tot}}_{r\,\text{sym}}V_r=\sum_{\substack{s_1 s_2\\ D^A_{\text{I}}D_{\text{II}}D_{\text{III}}}}\;\prod_{i=aa'bb'}\left(\prod_{l=\alpha(i)}^{\beta(i)}f^i_{\mu_{j(l)}}+\sum_{\substack{p<q\\ \alpha(i)}}^{\beta(i)}\chi^i_{\mu_p\mu_q}\prod_{\substack{l\neq pq\\ \alpha(i)}}^{\beta(i)}f^i_{\mu_{j(l)}}\right)V_r(P_a;P_{a'};P_b;P_{b'})\,, \qquad (4.1)$$

where $\alpha(a)=1$, $\beta(a)=r-1$, $\alpha(a')=r+1$, $\beta(a')=n+1$, $\alpha(b')=1$, $\beta(b)=s-1$, $\alpha(b')=s+1$, and $\beta(b')=n+1$. In the general case we assume that V_r is a smooth function in the momenta $P_i=p_i+\epsilon_iK_{\beta(i)}$, with $K_{\beta(i)}=k_{\alpha(i)}+\cdots+k_{\beta(i)}$, and can be expanded around $p_i^{\,2}=m_i^{\,2}$:

$$V_r(P_i)=\left[1+\sum_{s=1}^{\infty}\frac{1}{s!}(\hat{K}^i_{\beta(i)})^s\right]V_0(p_i)\,, \qquad (4.2)$$

where

$$\hat{K}^i_{\beta(i)}=\epsilon_iK_{\beta(i)}\cdot\frac{\partial}{\partial p_i}\,. \qquad (4.3)$$

Factorization properties are most conveniently carried out in the formal exponential form of (4.2):

$$V_r(P_i)=e^{\hat{K}^i_{\beta(i)}}V_0(p_i)$$
$$=\prod_{l=\alpha(i)}^{\beta(i)}e^{\hat{k}^i_l}V_0(p_i)\,, \qquad (4.4)$$

where the one-photon operator $\hat{k}^i_l$ is defined by

$$\hat{k}^i_l=\epsilon_ik_l\cdot\frac{\partial}{\partial p_i}\,. \qquad (4.5)$$

The off-shell effects in (4.1) are now easily included and related to the corresponding on-shell vertex amplitude through the replacements in (4.1),

$$f^i_{\mu_s}\to\hat{f}'^i_{\mu_s}\equiv f^i_{\mu_s}e^{\hat{k}^i_s}\,, \qquad (4.6)$$

$$\chi^i_{\mu_p\mu_q}\to\hat{\chi}'^i_{\mu_p\mu_q}\equiv\chi^i_{\mu_p\mu_q}e^{(\hat{k}^i_p+\hat{k}^i_q)}\,, \qquad (4.7)$$

$$V_r\to V_0\equiv-e^2(p_a+p_{a'})\cdot(p_b+p_{b'})\,. \qquad (4.8)$$

As is seen in (4.8), we are still in the Feynman gauge, but in Sec. V we demonstrate the explicit gauge invariance. The derivation of (4.1) is now reduced to the corresponding on-shell problem and can be performed by direct application of the machinery (2.15)–(2.19). The $(n+1)$th-order ladder kernel is then given by

$$I_{\rm sym} = \left[\prod_{l=1}^{n} \hat{J}'^{A}_{\mu_l} \hat{J}'^{B\mu_l} + \sum_{\substack{p<q \\ 1}}^{n} (\hat{\chi}'^{A}_{\mu_p\mu_q} \hat{J}'^{B\mu_p} \hat{J}'^{B\mu_q} + \hat{\chi}'^{B}_{\mu_p\mu_q} \hat{J}'^{A\mu_p} \hat{J}'^{A\mu_q} + \hat{\chi}'^{A}_{\mu_p\mu_q} \hat{\chi}'^{B\mu_p\mu_q}) \prod_{l\neq pq}^{n} J'^{A}_{\mu_l} J'^{B\mu_l} \right] V_0 , \tag{4.9}$$

where the total operator currents are defined by

$$J'^{Q}_{\mu} = \sum_{i\in Q} \hat{f}'^{i}_{\mu} , \quad Q = A, B \tag{4.10}$$

$$\hat{\chi}'^{Q}_{\mu\nu} = \sum_{i\in Q} \hat{\chi}'^{i}_{\mu\nu} . \tag{4.11}$$

The explicit evaluation of (4.9) inserted in the amplitude (2.2), with a general vertex function, could be quite complicated. However, here it is given by (4.8), and only first-order derivations with respect to a particular prong and second-order derivatives which are mixed with respect to A and B particles are nonzero. Therefore the exponential forms in (4.6) and (4.7) can be replaced by the first-order expanded forms

$$\hat{f}'^{i}_{\mu_s} = f^{i}_{\mu_s} + f^{i}_{\mu_s} \hat{k}^{i}_{s} , \tag{4.12}$$

$$\hat{\chi}'^{i}_{\mu_s\mu_t} = \chi^{i}_{\mu_s\mu_t} + \chi^{i}_{\mu_s\mu_t} (\hat{k}^{i}_{s} + \hat{k}^{i}_{t}) , \tag{4.13}$$

which yield exact results except for seagull terms with the rth photon involved. As we remember from (2.12) and (2.13) these had to be omitted in order to eliminate the δ function via integration over the rth momentum. We thereby lost gauge invariance. However, as we shall see, this can also be cured.

The first term on the right-hand side of (4.12) adds up through (4.10) to a conserved current, whereas the second term does not. In the spirit of the Low theorem[7] we must therefore add a leakage current $l^{i}_{\mu_s}$ to restore the conservation properties. In fact, this $l^{i}_{\mu_s}$ corresponds to the seagull contribution where the rth photon is one of the two involved quanta, since in this case the rth photon plays the role of the "hard core." Conservation of currents then gives

$$k_s \cdot f^{i} \hat{k}^{i}_{s} + k_s \cdot l^{i} = 0 , \tag{4.14}$$

which relates $l^{i}_{\mu_s}$ to a known quantity. We notice here that there are two different ways to extract k_s from the first term, but since $l^{i}_{\mu_s}$ corresponds to the seagull, (4.12) must correctly contain the pole in $(p_i + \epsilon_i k_s)^2 = m_i^2$, giving

$$l^{i}_{\mu_s} = -ie \frac{\partial}{\partial p_i^{\mu_s}} . \tag{4.15}$$

The uncorrelated total currents are then given by

$$\hat{f}^{i}_{\mu_s} = f^{i}_{\mu_s} + D^{i}_{\mu_s} , \quad \hat{J}^{Q}_{\mu_s} = \sum_{i\in Q} \hat{f}^{i}_{\mu_s} , \tag{4.16}$$

where the differential operators $D^{i}_{\mu_s}$ are defined by

$$D^{i}_{\mu_s} = f^{i}_{\mu_s} \hat{k}^{i}_{s} - ie \frac{\partial}{\partial p^{i}_{\mu_s}} , \quad D^{Q}_{\mu_s} = \sum_{i\in Q} D^{i}_{\mu_s} . \tag{4.17}$$

When this result is included in the ladder kernel (4.9), care should be taken before insertion under the integral in (2.2). Namely, in the proper expansion of the first factor term of (4.9), with the currents (4.16) inserted instead of (4.10), we get

$$\prod_{l=1}^{n} \hat{J}^{A}_{\mu_l} \hat{J}^{B\mu_l} = \prod_{l=1}^{n} J^{A}_{\mu_l} J^{B\mu_l} + \sum_{s=1}^{n} (D^{A}_{\mu_l} J^{B\mu_l} + J^{A}_{\mu_l} D^{B\mu_l} + D^{A}_{\mu_l} D^{B\mu_l}) \prod_{\substack{l\neq s \\ 1}}^{n} J^{A}_{\mu_l} J^{B\mu_l}$$
$$+ \sum_{\substack{s<t \\ 1}}^{n} (J^{A}_{\mu_s} D^{A}_{\mu_t} D^{B\mu_s} J^{B\mu_t} + D^{A}_{\mu_s} J^{A}_{\mu_t} J^{B\mu_s} D^{B\mu_t}) \prod_{\substack{l\neq st \\ 1}}^{n} J^{A}_{\mu_l} J^{B\mu_l} . \tag{4.18}$$

[It is easy to forget the double sum which generates terms of type $k_s \cdot k_t$ in (2.5) with $s \neq t$.] The last term in the single sum of (4.8) generates k_l^2 terms in (2.5) since both operators depend on k_l. This problem is not present in the first-order derivative terms, nor in the pair-correlation sum in (4.9), since such nonzero terms would give nonzero contribution in the triple correlations. However, here we restrict ourselves to just pair correlations.

A graphical interpretation of (4.18) is straightforward. The leakage currents in the first two terms, linear in D_μ, display seagull terms with one of the end points of the rth photon involved [Figs. 4(a)–(4d)]. The quadratic D_μ term in the single sum generates "double seagulls," where both ends of the rth photon and an arbitrary second photon are contracted [Figs. 5(a) and 5(b)]. Correspondingly, the second-order derivative terms in the double sum generate "seagulls," where again both ends of the rth photon are involved, but now connected with end points of two different photons [Figs. 5(c)–5(f)].

From the point of view of conservation there is no need to add a leakage current to the off-shell part of the pair-correlation tensor defined by (4.13). Furthermore, if we take one of the two pair-correlated photons apart and let it form a

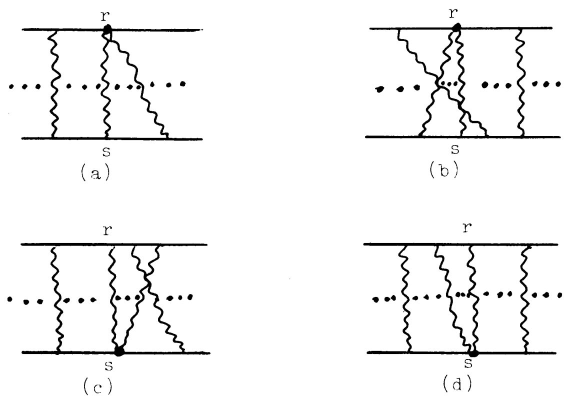

FIG. 4. Possible seagull formations with the rth photon.

seagull term with the rth photon, then clearly the single remaining photon from the original pair cannot be pair correlated any more. *Therefore a pair-leakage current must be zero.* On the other hand, if we start with the two-photon amplitude and pick out that part from (3.5) and (3.8) which corresponds to recoil from the denominator (which easily compares with the ϕ^3 case[1]), we get

$$A'^{i}_{\mu_1\mu_2} = \hat{f}'^{i}_{\mu_1}\hat{f}'^{i}_{\mu_2} + \hat{f}'^{i}_{\mu_1}\hat{f}'^{i}_{\mu_2}\frac{(-x^i_{12})}{1+x^i_{12}}$$
$$= \hat{f}'^{i}_{\mu_1}\hat{f}'^{i}_{\mu_2}\frac{1}{1+x^i_{12}} \,. \qquad (4.19)$$

Thus it appears that we should make the replacement $\hat{f}'^i \to \hat{f}^i$ (4.16) both in the uncorrelated and pair-correlated parts of (4.19) in order to get a smooth connection. In other words, we should apply the Low theorem on each separate current, irrespective of the correlation mechanism $(1+x^i_{12})^{-1}$, which in fact implies additional leakage currents. This paradox is easily solved if we work it out for the rest of the pair-correlation tensor.

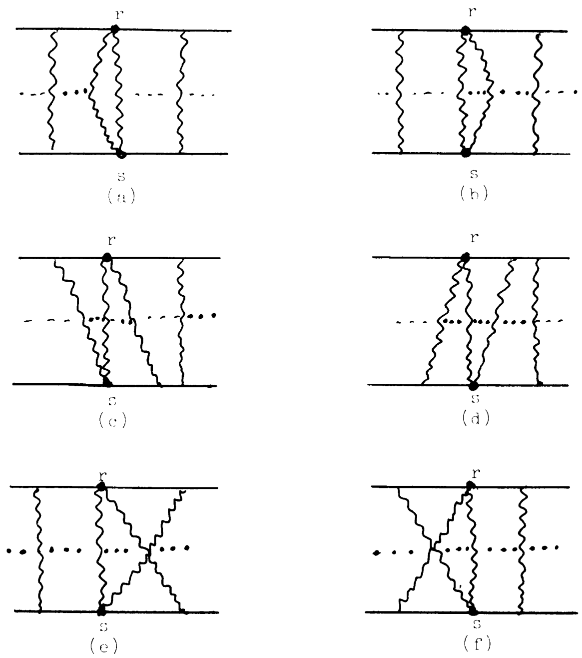

FIG. 5. Possible formations where both ends of the rth photon are of a seagull type.

Adding leakage current everywhere in (4.13), disregarding the correlation mechanism, and performing the scheme (4.14)–(4.17), we find that the leakage currents so obtained cancel completely. Thus (4.13) gives the full pair-correlation tensor:

$$\hat{\chi}^i_{\mu_s\mu_t} = \hat{\chi}'^i_{\mu_s\mu_t}$$
$$= \chi^i_{\mu_s\mu_t}(1+\hat{k}^i_s+\hat{k}^i_t)\,, \qquad (4.20)$$

where $\chi^i_{\mu_s\mu_t}$ is given in (3.5) and (3.8). The total pair-correlation tensor is defined by

$$\hat{\chi}^Q_{\mu_1\mu_2} = \sum_{i\in Q}\hat{\chi}^i_{\mu_1\mu_2}\,, \quad Q=A,B\,. \qquad (4.21)$$

Insertion in the integral (2.2) then gives

$$-iM_{n+1}(s;t) = \frac{1}{(n+1)!}\int d^4x\, e^{iq\cdot x}\Delta_F(x)\left[(iU)^n + \binom{n}{1}(i\hat{S})(iU)^{n-1} + \binom{n}{2}i^2\hat{P}(iU)^{n-2}\right]V_0\,, \qquad (4.22)$$

where U is again defined by (3.9), $\hat{S}$ is a "single"-correlated recoil functional given by

$$\hat{S}(x;s;t) = i\int\frac{d^4k}{(2\pi)^4}\Delta_F(k)e^{-ik\cdot x}(D^A_\mu J^{B\mu} + J^A_\mu D^{B\mu} + D^A_\mu D^{B\mu})\,, \qquad (4.23)$$

and $\hat{P}$ is a pair-correlation functional which follows from (4.9), (4.18), and (4.21):

$$\hat{P}(x;s;t) = i^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\tilde{\Delta}_F(k_1)\tilde{\Delta}_F(k_2)e^{-i(k_1+k_2)\cdot x}$$
$$\times(J^A_{\mu_1}D^A_{\mu_2}D^{B\mu_1}J^{B\mu_2} + D^A_{\mu_1}J^A_{\mu_2}J^{B\mu_1}D^{B\mu_2} + \hat{\chi}^A_{\mu_1\mu_2}\hat{J}^{B\mu_1}\hat{J}^{B\mu_2} + \hat{\chi}^B_{\mu_1\mu_2}\hat{J}^{A\mu_1}\hat{J}^{A\mu_2} + \hat{\chi}^A_{\mu_1\mu_2}\hat{\chi}^{B\mu_1\mu_2})\,, \qquad (4.24)$$

where again all nonmixed second-order derivatives vanish due to the form of V_0.

Summing on n in (4.22) we obtain the closed form

$$M(s;t)=i\int d^4x\, e^{iq\cdot x}\Delta_F(x)\left\{\frac{e^{iU}-1}{iU}+\frac{i\hat{S}}{iU}\left(\frac{iUe^{iU}-e^{iU}+1}{iU}\right)+\frac{i^2\hat{P}}{2!(iU)^2}\left[(iU-2)e^{iU}+2\frac{e^{iU}-1}{iU}\right]\right\}V_0. \tag{4.25}$$

Correspondingly, if the rth quantum is not an object identical to the other n exchanged quanta in (2.2), we should divide by $n!$ instead of $(n+1)!$ in (2.2) and (4.22), since then the rth quantum should not be permuted over. Then the summed-up closed form reads

$$M(s;t)=\int d^4x\, e^{iq\cdot x}\Delta_F(x)e^{iU}\left(1+i\hat{S}+\frac{i^2}{2!}\hat{P}\right)V_0. \tag{4.26}$$

This summation to a closed form became possible through the factorization and partial factorization, respectively, and the fact that we could define form-invariant correlation functionals.

V. GAUGE INVARIANCE AND MASSIVE NEUTRAL VECTOR MESONS

We have shown that it is possible to rearrange the generalized ladder expansion into a correlation expansion also in scalar electrodynamics. By addition of leakage currents l_μ^i, given by (4.15), all currents (4.16) are conserved. In passing, we remember that these leakage currents display seagull diagrams which were earlier omitted. We now demonstrate that our result is gauge-invariant by proving invariance under the replacement

$$\frac{-g^{\mu\nu}}{k^2+i\epsilon}\to\frac{-1}{k^2+i\epsilon}\left(g^{\mu\nu}-c\frac{k^\mu k^\nu}{k^2}\right). \tag{5.1}$$

This replacement can be made because it does not affect the factorization scheme. Obviously the functionals (3.9) and (4.23) and (4.24) are invariant, since all involved currents are conserved. All we need to study then is the vertex function (2.5) under the same transformation. For the sake of completeness let us here include the rth propagator. Thus, instead of (2.5)–(2.7) we write

$$V_G=-e^2(P_a+P_{a'})_\mu\frac{G^{\mu\nu}}{k_r^{\,2}+i\epsilon}(P_b+P_{b'})_\nu, \tag{5.2}$$

where G is defined by

$$G_{\mu\nu}=g_{\mu\nu}-c\frac{k_{r\mu}k_{r\nu}}{k_r^{\,2}}. \tag{5.3}$$

Using four-momentum conservation we have

$$\begin{aligned}k_r&=q-\sum_{t\neq r}^{n+1}k_t\\&=\left(p_a-\sum_{t=1}^{r-1}k_t\right)-\left(p_{a'}+\sum_{t=r+1}^{n+1}k_t\right)\\&=\left(p_{b'}-\sum_{t=1}^{s-1}k_t\right)-\left(p_b+\sum_{t=s+1}^{n+1}k_t\right),\end{aligned} \tag{5.4}$$

from which we see that all functions of k_r are functions of q and invariant under all $n!$ permutations among the n other photons. The rest of (5.2), which is just the "pure" vertex (2.5), is invariant under the subsets D_a, $D_{a'}$, D_b, and $D_{b'}$. Inserting V_G instead of V_r we can again exploit the invariance under D_i and obtain (4.1). As before, we then expand in a series

$$V_G(P_i)=e^{\hat{K}_{\beta(i)}}V_{G_0}(p_i)=\prod_{l=\alpha(i)}^{\beta(i)}e^{\hat{k}_l^i}V_{G_0}(p_i), \tag{5.5}$$

where the on-shell vertex is now given by

$$V_{G_0}=-e^2(p_a+p_{a'})_\mu\frac{g^{\mu\nu}-cq^\mu q^\nu/q^2}{q^2+i\epsilon}(p_b+p_{b'})_\nu. \tag{5.6}$$

Using the on-shell restriction for the external particles

$$\begin{aligned}(p_a+p_{a'})\cdot q&=q\cdot(p_{b'}+p_b)\\&=m^2-m^2=0,\end{aligned} \tag{5.7}$$

we see that the term proportional to c in (5.6) does not contribute. This proves the gauge invariance.

In fact, this form still leads to an infinite-order expansion because of the form of the propagator. We therefore split up the "core" function (5.2) in the pure vertex part (2.5) and the propagator of the rth photon. The off-shell effects of the latter are then included as before:

$$\begin{aligned}\frac{1}{k_r^{\,2}+i\epsilon}&=\int d^4x\,\Delta_F(x)e^{ik_r\cdot x}\\&=\int d^4x\,\Delta_F(x)\exp\left(iq\cdot x-i\sum_{\substack{t\neq r\\1}}^{n+1}k_t\cdot x\right),\end{aligned} \tag{5.8}$$

where again we have made use of (5.4). The k_t dependence is then, as earlier, included in the correlation functionals. The remaining operator part in (5.5) is now linearized as in (4.12) and (4.13) and easily included as before. This separation of the off-shell effects is possible because of the invariance of the propagator and G [see (5.3)] under all $n!$ permutations.

These effects could also be derived in an alternative formulation. With $q^2=(p_a-p_{a'})^2$ we have

$$\frac{1}{k_r^{\,2}+i\epsilon}=\prod_{\substack{t\neq r\\1}}^{n+1}e^{\hat{k}_t^A}\frac{1}{q^2+i\epsilon}, \tag{5.9}$$

where $\hat{k}_t^A$ is defined by

$$\hat{k}_t^A = \hat{k}_t^a + \hat{k}_t^{a'}\ , \quad \hat{k}_t^i = \epsilon_i k_t \cdot \frac{\partial}{\partial p_i}\ . \tag{5.10}$$

In passing, we notice that we could also use $q^2 = (p_{b'} - p_b)^2$ if A is replaced by B in (5.10). In order to prove that (5.9) also gives the form (5.8), we use the transformation

$$\frac{1}{q^2 + i\epsilon} = \int d^4x\, \Delta_F(x) e^{iq\cdot x}\ , \tag{5.11}$$

which we insert in (5.9). Then by means of the operator relation

$$e^{\hat{k}_t^A} e^{iq\cdot x} = \exp(\hat{k}_t^A + iq\cdot x + \tfrac{1}{2}[\hat{k}_t^A, iq\cdot x]) \tag{5.12}$$

and

$$\tfrac{1}{2}[\hat{k}_t^A, iq\cdot x] = -ik_t \cdot x\ , \tag{5.13}$$

we again obtain (5.8). Thus we can safely work with this technique, which is particularly useful when we want to demonstrate the explicit gauge invariance (5.5)–(5.7).

Therefore all "off shell" effects in the variable q for any arbitrarily distorted but smooth propagator-vertex function can be managed by this technique. With a corresponding transform defined by

$$F_{\mu\nu}(q) = \int d^4x\, \mathfrak{F}_{\mu\nu}(x) e^{iq\cdot x}\ , \tag{5.14}$$

we can write

$$F_{\mu\nu}(k_r) = \prod_{\substack{t\neq r\\1}}^{n+1} e^{\hat{k}_t^A} F_{\mu\nu}(q)\ . \tag{5.15}$$

Inserting (5.14) in (5.15) and by the use of (5.12) and (5.13) we then get

$$\begin{aligned} F_{\mu\nu}(k_r) &= \int d^4x\, \mathfrak{F}(x) e^{ik_r\cdot x} \\ &= \int d^4x\, \mathfrak{F}_{\mu\nu}(x) \exp\left(iq\cdot x - \sum_{\substack{t\neq r\\1}}^{n+1} ik_t\cdot x\right)\ . \end{aligned} \tag{5.16}$$

The k_t dependence is then, as before, included in the uncorrelated and correlation functionals. This provides an extra little piece to the generalization of Low's theorem.[6,7] In passing, we notice that the same technique applies for real quantum emission as, for example, by bremsstrahlung.

We can now also permit scattering into resonant states $p_i^2 = m_i^{*2}$ (diffractive excitation). This problem is here reduced to that of finding the transform of the second term in (5.6) of form

$$F'(q^2) = \Delta m_a^2 \Delta m_b^2 \frac{c}{(q^2)^2}\ . \tag{5.17}$$

Clearly (4.22) shares the property of renormalizability with the original amplitude, provided that it permits an extension to include all radiative and vacuum corrections on this form. From Refs. 8–10 we can see that self-energies are as simple as the exchanges. Further, we notice that a regularization of the photon propagator

$$\frac{1}{k^2} - \frac{1}{k^2 - \Lambda^2} = \frac{\sigma(k^2;\Lambda^2)}{k^2} \tag{5.18}$$

does not affect the factorization scheme but just gives a distorted propagator of the above-mentioned type.

A similar function enters by the insertion of vacuum-polarization diagrams in each individual photon line. All such photon self-energies can therefore easily be included now since neither of these functions disturb the factorization. Similarly, the two-photon kernel contributes to the pair correlation, and together with (4.24) this gives the complete pair-correlation tensor. Higher-order insertions would give rise to higher-order correlations. To get an exact correspondence to a certain order with the ordinary perturbation expansion, since we want to simultaneously reproduce these results exactly, we must also include, for example, the three-two-one photon kernel, etc. Including just photon self-energies to the fourth order pair correlations without the two-photon kernel, and dropping triple and higher correlations, we can exactly reproduce the ordinary sixth-order result and the rest to an infinite order within the eikonal approximation. We close this section by considering massive neutral vector mesons, in which case (5.1) is replaced by

$$\frac{1}{k^2 - \mu^2 + i\epsilon}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{\mu^2}\right)\ . \tag{5.19}$$

In case of nonexcited external particles we get the same result as in the photon case, but for the presence of the vector-meson mass $\mu \neq 0$. If we permit excitation of the external particles, e.g., scattering into higher mass states (fragmentation), in (5.19) we get a contribution from the second term of type

$$F''(q^2) = \frac{\Delta m_a^2 \Delta m_b^2}{\mu^2} \frac{1}{q^2 - \mu^2 + i\epsilon}\ , \tag{5.20}$$

which is just as simple as the first term in (5.19).

VI. SUMMARY

We have demonstrated that also in scalar electrodynamics a generalized infinite ladder, including all crisscrosses, in contrast to a Bethe-Salpeter ladder, can be rearranged into a dynamical correlation expansion. By kinematical correlations we mean those due to four-momentum conservation, which is superimposed everywhere. Also, here the infinite summation into a closed explicit form (4.25) became possible because of the factorization and partial factorization, respectively, and the form invariance of the correlation functionals, irrespective of which individual quanta were involved.

The first term in this correlation expansion is the usual eikonal result, which involves no other than kinematical correlations. The rest of the expansion expresses dynamical correlations and defines the recoil to the eikonal= straight-line-path approximation. However, here we are not so crucially dependent on a straight-line-path, infinite-momentum, or high-energy limit, but obtain our result from a direct rearrangement of the ordinary perturbation expansion. Although we have here just derived the expansion for exchanged photons, as discussed in Sec. V, the radiative and vacuum corrections could also be included in this form. As in the scalar ϕ^3 model[1] we have here assumed that the correlation expansion decreases with increasing order of correlation, and for some restricted domain of applicability it should then be sufficient to include just pair correlations, e.g., the correction term to the usual eikonal result. The loss of information by neglect of higher-order correlations naturally requires a separate thorough investigation. However, concerning the simple case of exchanges, we notice that there is no information loss in our result for three arbitrarily hard exchanged photons, whereas the rest of the indefinite number of photons are accounted for by the eikonal= soft current. This distribution of momenta with three "hard" and an arbitrary number of "soft" quanta is very likely for some limited domain of momentum transfer $q \leq q_c$. In passing, we notice that the usual eikonal model for exchanges of photons, without dynamical pair correlations, reproduces just the one-photon exchange amplitude exactly and the rest in the eikonal approximation. If we want to reproduce the four-photon amplitude exactly and the rest in the eikonal approximation, which would enlarge the above q_c domain where this corresponds to the actual physical situation, we must then also include all triple correlations.[1]

The recoil-correlation expansion was derived via a soft-recoil theorem for real quanta,[6] which is generalized here to yield hard virtual quanta as well. This generalization eliminates the need for any type of cutoff as long as the photon mass is different from zero. Thus, in our approach there is no difference between soft and hard quanta in the traditional individual meaning, since all quanta are present in both parts of the spectrum. This is more realistic, since from the beginning all photons are integrated over the whole of each four-momentum subspace. All photons have got a totally factorizable and eikonalizable tail, and their hard residual effects appear in terms of the correlation expansion. Nevertheless, we could still speak of "soft" and "hard" quanta, not in the strict individual meaning but in an effective interacting sense. This is so because of the fact that in the uncorrelated current (3.4) there is a covariant cutoff automatically built in through the k^2 term in the denominator and a corresponding infrared suppression in the correlation terms. The current (3.4) has been known since 1961 as Yennie's form.[11,12] However, because of technical complications in computation, the residual hard spectrum was not so easy to evaluate. A strict mathematical formulation of the IR problem, can be read about in Refs. 13–16, and in Refs. 15 and 16 quantum-dynamical recoil-correlation effects are also indicated.

One reason for an infinite summation is the infrared divergence, which in fact requires a noncovariant cutoff. As we will see in a subsequent work, this changes this result just slightly. However, it seems there is a second reason. The tower insertions, as shown by Cheng and Wu,[2] lead to a growth with ln$s$ of α^2. Thus, for asymptotic energies the effective coupling should tend to infinity, and only the summed-up result could make sense. (This argument holds to the extent that the tower estimations are correct.) Now, slowly increasing the bare coupling constant implies that the ln$s$ influence already becomes important at lower energies, and in a strong-coupling theory we must therefore stick to the summed-up form for all energies. Then, including the exact two-photon kernels, which could possibly be approximated with the towers, it might then appear that we should sum over pair, double-pair, triple-pair correlations, etc., together with the iteration of towers. However, at this level with simple exchanges of photons, there is no apparent need to consider these higher-order correlations, as was the case for infinite towers, which must be iterated not to violate the Froissart bound. It will be highly interesting to see if this correlation of a simple pair of exchanged photons affects the linear high s dependence. This will be considered in a subsequent work.

A compromised presentation of the results of this paper and those of Ref. 1 was made in Ref. 17. We also suggested there a new multiperipheral model.[18] Namely, to be consequent with the idea of identical clusters we must coherently sum over all crisscross graphs. This was performed there for just scalar particles; however, from the result of this paper, we are now able to treat clusters with spin and more general off-shell effects. This paper further shows that the correlations in (4.25) and (4.26) are proportional to α^{c+1}, where c is the order of correlation. Therefore, in a weak coupling theory, our working hypothesis is probably correct and explains why the eikonal approximation is rather good.

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APPENDIX: LONGITUDINAL MODES

For simplicity we first derive the formula for two emitted photons from the ith prong. The generalization to arbitrary many photons is proved by induction.[6] The exact two-photon amplitude is given by

$$M_{\mu_1\mu_2}=\sum_{\text{perm}}\frac{(2p_i+\epsilon_i k_1)_{\mu_1}(2p_i+\epsilon_i 2k_1+\epsilon_i k_2)_{\mu_2}}{(2\epsilon_i p_i\cdot k_1+k_1^2)[2\epsilon_i p_i\cdot(k_1+k_2)+k_1^2+k_2^2]}\left(1-\frac{x_{12}^i}{1+x_{12}^i}\right), \tag{A1}$$

where x_{12}^i is given by

$$x_{12}^i=\frac{2k_1\cdot k_2}{y_{12}^i},\qquad y_{12}^i=2\epsilon_i p_i\cdot(k_1+k_2)+k_1^2+k_2^2. \tag{A2}$$

For the uncorrelated one-photon currents we use the notation

$$f_{\mu_l}^i=\frac{(2p_i+\epsilon_i k_l)_{\mu_l}}{2\epsilon_i p_i\cdot k_l+k_l^2}. \tag{A3}$$

The last factor in (A1) is invariant under permutations and is therefore omitted for the moment. The rest of it we split in two parts:

$$M^{(1)}_{\mu_1\mu_2}=\sum_{\text{perm}}\frac{(2p_i+\epsilon_i k_1)_{\mu_1}(2p_i+\epsilon_i k_2)_{\mu_2}}{(2\epsilon_i p_i\cdot k_1+k_1^2)y_{12}^i},$$

$$M^{(2)}_{\mu_1\mu_2}=\sum_{\text{perm}}\frac{(2p_i+\epsilon_i k_1)_{\mu_1}2\epsilon_i k_{1\mu_2}}{(2\epsilon_i p_i\cdot k_1+k_1^2)y_{12}^i}.$$

After summation over permutations, multiplication by the dropped factor $(1+x_{12}^i)^{-1}$, and addition of seagull terms, the amplitude reads

$$\begin{aligned}M^{\text{tot}}_{\mu_1\mu_2}&=f^i_{\mu_1}f^i_{\mu_2}+\left(f^i_{\mu_1}\frac{2\epsilon_i k_{1\mu_2}}{y_{12}^i}+f^i_{\mu_2}\frac{2\epsilon_i k_{2\mu_1}}{y_{12}^i}-f^i_{\mu_1}f^i_{\mu_2}x^i_{12}-\frac{g_{\mu_1\mu_2}}{y^i_{12}}\right)\frac{1}{1+x^i_{12}}\\&=f^i_{\mu_1}f^i_{\mu_2}+\chi^i_{\mu_1\mu_2}.\end{aligned} \tag{A4}$$

Repeating this for 3, 4 quanta, etc., dropping all higher than pair correlations we are led to the formula[1,6]

$$M_{\mu_1\cdots\mu_n}=\prod_{l=1}^n f^i_{\mu_l}+\sum_{\substack{s<t\\1}}^n\chi^i_{\mu_s\mu_t}\prod_{\substack{l\neq st\\1}}^n f^i_{\mu_l}, \tag{A5}$$

which is easily proved by induction.

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