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Lee model and source theory: A new method of calculation

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Schwinger's source theory is employed to present a new approach to the Lee model. Topics discussed include causal analysis and space-time extrapolation, relativistic and nonrelativistic V-particle propagation function (single-spectral form), and V-particle decay. The technical advantages of source methods are indicated; unlike previous authors, we derive a completely finite theory, i.e., the conventional mass- and charge-renormalization procedure becomes obsolete.

I. INTRODUCTION

This article presents a new approach to the Lee model with the aid of Schwinger's source theory.[1] Although Schwinger himself and his collaborators have applied source methods to reconstruct most of what is currently known in the realm of particles and fields, in particular in quantum electrodynamics[2] (QED), we consider it also meaningful to revisit a soluble model which, among others, served as a guide to understanding mass and charge renormalization some years ago.[3] Furthermore, the present renaissance in field theory invites a second look at a model which was originally set up to give some insight into the dynamics of strong interactions. However, the conventional approach, using field operators or pure S-matrix formalism, will be replaced by the extremely useful source techniques which Schwinger has advocated over the past several years. We emphasize that the source approach yields a completely finite theory. There will be no divergent expressions nor is there any necessity to introduce renormalization constants. Our approach, which follows closely that presented by Schwinger in Refs. 1 and 2 on QED, will emerge solely from the principles of causality and space-time uniformity.

Not only formal elegance, but also technical advantages will be exhibited in the following sections, which are divided into the construction of the relativistic modified V-particle propagation function (Sec. II), extraction of the contents of the original Lee model (Sec. III), and, finally, the V-particle decay process (Sec. IV).

II. CAUSAL ANALYSIS, MODIFIED V-PARTICLE PROPAGATION FUNCTION

The fundamental quantity in source theory is the vacuum amplitude (VA)

$$\langle 0_+ | 0_- \rangle = e^{iW},$$

where the action W in the relativistic Lee model is given by

$$W(\eta_V, \bar\eta_V; \eta_N, \bar\eta_N; J, J^*) = \int (dx)(\bar\psi_V \eta_V + \bar\eta_V \psi_V + \bar\psi_N \eta_N + \bar\eta_N \psi_N + \phi^* J + J^* \phi + \mathcal{L}) . \tag{1}$$

The Lagrange function consists of the free part $\mathcal{L}_0$ and the interaction term $\mathcal{L}'$. Thus,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}' ,$$

where

$$\mathcal{L}_0 = -\bar\psi_V(-i\gamma\cdot\partial + m_V)\psi_V - \bar\psi_N(-i\gamma\cdot\partial + m_N)\psi_N - (\partial\phi^*\partial\phi + \mu^2\phi^*\phi) \tag{2}$$

and m_V, m_N, and μ are the "observed" masses of the V, N, and Θ particle, respectively.

The dynamical content of the Lee model is specified by the primitive interaction

$$\mathcal{L}' = -g(\bar\psi_N \phi^* \psi_V + \bar\psi_V \phi \psi_N) , \tag{3}$$

which uses the definition $\bar\psi(x) = \psi^*(x)\gamma^0$.

Since the local interaction (3) will alter the propagation function of the freely moving particles, we are first of all interested in the modified propagation functions, especially that of the V particle. This can be obtained by the extended- and effective-source scheme and the following causal analysis: An extended V-particle source creates an N and Θ particle by emitting the timelike momentum P. The effective source in emission is then given by comparing the VA,

$$\langle 0_+ | 0_- \rangle = -ig \int (dx) \bar\psi_N(x) \phi^*(x) \psi_V(x) , \tag{4}$$

with an equivalent noninteracting two-particle N-Θ source:

$$\langle 0_+ | 0_- \rangle = i^2 \int (dx)(d\xi) \bar\psi_N(x) \eta_N(x) \phi^*(\xi) J(\xi) . \tag{5}$$

The VA is evidently derived by expanding

$$\langle 0_+|0_-\rangle = \exp\left(i\int \bar{\eta}_N G_+^N \eta_N\right)\exp\left(i\int J^*\Delta_+ J\right)$$

$$= \exp\left(i\int \bar{\psi}_N \eta_N\right)\exp\left(i\int \phi^* J\right).$$

The various sources and fields are related as in Ref. 1, e.g.,

$$\psi_V(x) = \int (dx')G_+^V(x-x')\eta_V(x'),$$

$$\phi(x) = \int (dx')\Delta_+(x-x')J(x'),$$

and the free propagation functions satisfy

$$(-i\gamma\cdot\partial + m_V)G_+^V(x-x') = \delta(x-x'),$$

$$(-\partial^2+\mu^2)\Delta_+(x-x') = \delta(x-x').$$

Comparing the two expressions (4) and (5) gives the effective source in emission

$$iJ(\xi)\eta_N(x)|_{\text{eff. em.}} = -g\delta(x-\xi)\psi_V(x). \tag{6}$$

Going to the momentum description, i.e.,

$$\eta(p) = \int (dx)e^{-ipx}\eta(x),$$

we obtain

$$iJ(k)\eta_N(p)|_{\text{eff. em.}} = -g\psi_V(P), \tag{7}$$

where $P = p + k$ represents the total momentum liberated by the source: $-p^2 = M^2 > 0$.

Likewise, we need the effective source in absorption for an N and Θ particle by an extended V-particle source. Again, using the VA, we have

$$\langle 0_+|0_-\rangle = -ig\int (dx)\bar{\psi}_V(x)\phi(x)\psi_N(x), \tag{8}$$

which is to be compared with

$$\langle 0_+|0_-\rangle = i^2\int (dx)(d\xi)J^*(\xi)\bar{\eta}_N(x)\phi(\xi)\psi_N(x), \tag{9}$$

which yields, when identified with (8), the effective detection source

$$iJ^*(\xi)\bar{\eta}_N(x)|_{\text{eff. abs.}} = -g\delta(x-\xi)\bar{\psi}_V(x) \tag{10}$$

or, in momentum space,

$$iJ^*(-k)\bar{\eta}_N(-p)|_{\text{eff. abs.}} = -g\bar{\psi}_V(-P). \tag{11}$$

The VA of interest for the specific causal arrangement $x_0 > x_0'$, $\xi_0 > \xi_0'$ is then given by

$$\langle 0_+|0_-\rangle = \left(i\int \bar{\eta}_N G_+^N \eta_N\right)\left(i\int J^*\Delta_+ J\right)$$

$$= \int [iJ^*(\xi)\bar{\eta}_N(x)]G_+^N(x-x')\Delta_+(\xi-\xi')$$

$$\times [iJ(\xi')\eta_N(x')],$$

where

$$G_+^N(x-x') = i\int d\omega_{p_N} e^{ip_N(x-x')}(m_N - \gamma\cdot p_N),$$

$$d\omega_p = \frac{d\vec{p}}{(2\pi)^3}\frac{1}{2p^0}$$

and

$$\Delta_+(\xi-\xi') = i\int d\omega_k e^{ik(\xi-\xi')},$$

$$k^0 = (\vec{k}^2+\mu^2)^{1/2}.$$

Using the effective-source expressions (6) and (10) we obtain

$$\langle 0_+|0_-\rangle = i^2g^2\int d\omega_{p_N}d\omega_k$$

$$\times\int (dx)(dx')\psi_V^\dagger(x)\gamma^0$$

$$\times e^{i(p_N+k)(x-x')}$$

$$\times(m_N-\gamma\cdot p_N)\psi_V(x'). \tag{12}$$

Insertion of a unit factor

$$1 = \int (2\pi)^3\delta(P-p_N-k)\frac{(dP)}{(2\pi)^3}$$

$$= \int d\omega_P dM^2(2\pi)^3\delta(P-p_N-k)$$

and employing the relation

$$\frac{(dP)}{(2\pi)^3} = \frac{d\vec{P}}{(2\pi)^3}dP^0$$

$$= \frac{d\vec{P}}{(2\pi)^3}\frac{dM^2}{2P^0}$$

$$= d\omega_P dM^2, \quad -P^2 = M^2$$

the VA now takes the form

$$\langle 0_+|0_-\rangle = ig^2\int (dx)(dx')\psi_V^\dagger(x)\gamma^0\left[(2\pi)^3\int d\omega_{p_N}d\omega_k(m_N-\gamma\cdot p_N)\delta(P-p_N-k)dM^2 i d\omega_P e^{iP(x-x')}\right]\psi_V(x') \tag{13}$$

or, in momentum space,

$$\langle 0_+|0_-\rangle = ig^2\int dM^2\, i d\omega_P\psi_V^\dagger(-P)\gamma^0\left[(2\pi)^3\int d\omega_{p_N}d\omega_k(m_N-\gamma\cdot p_N)\delta(P-p_N-k)\right]\psi_V(P). \tag{14}$$

The integral in the square brackets of Eq. (14) is most conveniently evaluated in the rest frame of P. One then finds the value

$$(2\pi)^3 \int d\omega_{p_N} d\omega_k (m_N - \gamma \cdot p_N)\delta(P - p_N - k) = \left(m_N - \frac{M^2 + m_N{}^2 - \mu^2}{2M^2}\gamma \cdot P\right) I(M, m_N, \mu),$$

where

$$I(M, m_N, \mu) = \frac{1}{(4\pi)^2}\left(1 - \frac{(m_N + \mu)^2}{M^2}\right)^{1/2} \times \left(1 - \frac{(m_N - \mu)^2}{M^2}\right)^{1/2}. \tag{15}$$

Hence Eq. (14) turns into

$$\langle 0_+ | 0_- \rangle = ig^2 \int dM^2 I(M, m_N, \mu) i d\omega_P \psi_V^*(-P) \times \gamma^0 \left(m_N - \frac{M^2 + m_N{}^2 - \mu^2}{2M^2}\gamma \cdot P\right) \psi_V(P). \tag{16}$$

Going back from the field to the source description via

$$\psi_V(P) = \frac{1}{\gamma \cdot P + m_V}\eta_V(P),$$

$$\psi_V^*(-P)\gamma^0 = \eta_V^*(-P)\gamma^0 \frac{1}{\gamma \cdot P + m_V}$$

then leads to

$$\langle 0_+ | 0_- \rangle = ig^2 \int dM^2 I(M, m_N, \mu) i d\omega_P \eta_V^*(-P)\gamma^0 \frac{m_N - [(M^2 + m_N{}^2 - \mu^2)/2M^2]\gamma \cdot P}{(\gamma \cdot P + m_V)^2}\eta_V(P). \tag{17}$$

At this point it is useful to recall the relation $(\gamma \cdot P)^2 = -P^2 = M^2$ or equivalently the eigenvalue equation $(\gamma \cdot P)' = \pm M$. This allows us to introduce the following decomposition in terms of the eigenvalues of $\gamma \cdot P$:

$$\frac{m_N - [(M^2 + m_N{}^2 - \mu^2)/2M^2]\gamma \cdot P}{(\gamma \cdot P + m_V)^2} = \frac{\gamma \cdot P + M}{2M}\frac{m_N - (M^2 + m_N{}^2 - \mu^2)/2M}{(M + m_V)^2} + \frac{\gamma \cdot P - M}{-2M}\frac{m_N + (M^2 + m_N{}^2 - \mu^2)/2M}{(M - m_V)^2}$$

$$= -\left\{\frac{\gamma \cdot P + M}{2M^2}\frac{(M - m_N)^2 - \mu^2}{2(M + m_V)^2} + \frac{\gamma \cdot P - M}{2M^2}\frac{(M + m_N)^2 - \mu^2}{2(M - m_V)^2}\right\} \equiv - \{\cdots\}.$$

So far all the calculations have been carried out under the special causal condition $x^0 > x'^0$. Now we must perform the space-time extrapolation. This means essentially the instruction to replace

$$i d\omega_P e^{iP(x - x')} \to \Delta_+(x - x'; M^2) = \int \frac{(dP)}{(2\pi)^4}\frac{e^{iP(x - x')}}{P^2 + M^2 - i\epsilon} + \text{contact terms}.$$

Hence the extrapolated VA (apart from contact terms) is given by

$$\langle 0_+ | 0_- \rangle = i \int \frac{(dp)}{(2\pi)^4}\eta_V^*(-p)\gamma^0 \left[g^2 \int dM^2 I(M, m_N, \mu)\frac{\{\cdots\}}{(\gamma p + M - i\epsilon)(\gamma p - M + i\epsilon)}\right]\eta_V(p)$$

$$= i \int \frac{(dp)}{(2\pi)^4}\bar{\eta}_V(-p)\left[g^2 \int \frac{dM^2}{2M^2} I(M, m_N, \mu)\left(\frac{(M + m_N)^2 - \mu^2}{2(M - m_V)^2}\frac{1}{\gamma p + M - i\epsilon} + \frac{(M - m_N)^2 - \mu^2}{2(M + m_V)^2}\frac{1}{\gamma p - M + i\epsilon}\right)\right]\eta_V(p). \tag{18}$$

If we add to this expression the one associated with single-V-particle exchange, we obtain the modified V-particle propagation function

$$\bar{G}_+^V(p) = \frac{1}{\gamma p + m_V - i\epsilon} + g^2 \int_{(m_N + \mu)}^{\infty} \frac{dM}{M} I(M, m_N, \mu)\left[\frac{(M + m_N)^2 - \mu^2}{2(M - m_V)^2}\frac{1}{\gamma p + M - i\epsilon} + \frac{(M - m_N)^2 - \mu^2}{2(M + m_V)^2}\frac{1}{\gamma p - M + i\epsilon}\right], \tag{19}$$

where the scalar quantity $I(M, m_N, \mu)$ is given by Eq. (15). Notice that $\bar{G}_+^V(p)$ is a completely finite quantity. There are no divergences nor is there any need for any conventional renormalization procedure.

Had we stopped at the field description, i.e., at Eq. (13), we would have received as contribution to the VA

$$\langle 0_+ | 0_- \rangle = -i \int (dx)(dx')\psi_V^*(x)\gamma^0 m(x - x')\psi_V(x'), \tag{20}$$

where in momentum description

$$m(p)=m(\gamma p)$$
$$=-g^2\int\frac{dM}{M}I(M,m_N,\mu)\times\frac{1}{2}\left[\frac{(M+m_N)^2-\mu^2}{\gamma p+M-i\epsilon}+\frac{(M-m_N)^2-\mu^2}{\gamma p-M+i\epsilon}\right], \qquad (21)$$

which represents an undesired change in the V-particle mass and in addition is divergent. The trouble stems from the creation of a double pole in the modified propagation function for the V particle:

$$\overline{G}^V_+(p)=\frac{1}{\gamma p+m_V-i\epsilon}-\frac{1}{\gamma p+m_V-i\epsilon}m(p)\frac{1}{\gamma p+m_V-i\epsilon}, \qquad (22)$$

which is obvious if we remove the fields in (20) in terms of sources.

The incorrect structure of $m(x-x')$ has to be changed in such a way that the mass of the free particle remains unchanged, i.e., the second term in (22) should not have a singularity in the neighborhood of $m_V+\gamma p=0$, as stated by the boundary condition

$$m(\gamma p=-m_V)=0\,,\quad \frac{d}{d(\gamma p)}m(\gamma p=-m_V)=0\,.$$

This can be achieved by the following choice of the contact terms:

$$\frac{1}{\gamma p+M-i\epsilon}\to\frac{1}{\gamma p+M-i\epsilon}-\frac{1}{M-m_V}+\frac{(\gamma p+m_V)}{(M-m_V)^2}=\frac{(\gamma p+m_V)^2}{(M-m_V)^2}\frac{1}{\gamma p+M-i\epsilon}\,.$$

Likewise

$$\frac{1}{\gamma p-M+i\epsilon}\to\frac{(\gamma p+m_V)^2}{(M+m_V)^2}\frac{1}{\gamma p-M+i\epsilon}\,.$$

Here, then, is the correct expression for $m(p)$:

$$m(\gamma p)=-(\gamma p+m_V)^2g^2\int_{(m_N+\mu)}^{\infty}\frac{dM}{M}I(M,m_N,\mu)\frac{1}{2}\left[\frac{(M+m_N)^2-\mu^2}{(M-m_V)^2}\frac{1}{\gamma p+M-i\epsilon}+\frac{(M-m_N)^2-\mu^2}{(M+m_V)^2}\frac{1}{\gamma p-M+i\epsilon}\right], \qquad (23)$$

which not only contains the right pole structure, but at the same time makes (23) a convergent expression. It is also evident that the V-particle propagation function (19) is obtained by substituting Eq. (23) into (22).

III. SOURCE-THEORETICAL APPROACH TO THE ORIGINAL LEE MODEL

After having derived a single spectral form for the relativistic V-particle propagator (19), we now want to make contact with the original nonrelativistic Lee model. This can be achieved most easily by mutilating the various free-particle propagation functions in the following way: First observe that the square brackets of Eq. (19) contain excitations of either parity $\gamma^0=\pm 1$. Therefore, a first step toward a nonrelativistic description will be the omission of, e.g., $\gamma^0=-1$. Furthermore, we restrict ourselves to the static limit for the free N and V particles, meaning $p^0_{V,N}=m_{V,N}$. The Θ particle is constrained to travel forward in time, $x^0>x'^0$, however, with a relativistic particle spectrum, i.e., $k^0=(\vec{k}^2+\mu^2)^{1/2}$. With these qualifications in mind the mutilated VA which now emerges is given by

$$\langle 0_+|0_-\rangle=-g^2\int(dx)(dx')\psi_V^*(x)G_N(x-x')\times\Delta_+(x-x')\psi_V(x')\,,$$

where

$$G_N(x-x')\Delta_+(x-x')=i^2\int\frac{d\vec{p}_N}{(2\pi)^3}\frac{d\vec{k}}{(2\pi)^3}\frac{1}{2k^0}\times\exp[i\vec{p}_N\cdot(\vec{x}-\vec{x}')-i(m_N+k^0)(x^0-x'^0)]$$
$$=-\int\frac{d\vec{p}_N}{(2\pi)^3}e^{i\vec{p}_N\cdot(\vec{x}-\vec{x}')}\frac{1}{(2\pi)^2}\times\int_{(m_N+\mu)}^{\infty}dW[(W-m_N)^2-\mu^2]^{1/2}e^{-iW(x^0-x'^0)}$$

and $W=m_N+k^0$. Introducing Fourier-transformed fields, we obtain

$$\langle 0_+|0_-\rangle=i\frac{g^2}{4\pi^2}\int_{(m_N+\mu)}^{\infty}dW[(W-m_N)^2-\mu^2]^{1/2}\int\frac{d\vec{p}}{(2\pi)^3}\int dx^0dx'^0\psi_V^*(\vec{p},x^0)\frac{1}{i}e^{-iW(x^0-x'^0)}\psi_V(\vec{p},x'^0)\,. \qquad (24)$$

Until now we restricted ourselves to $x^0 > x'^0$. However, the source picture continues to be meaningful also for noncausal arrangements. Therefore we time-extrapolate the expression for the propagation function of the particle with energy W, i.e., we introduce the spectral integral

$$G_V(W; x^0 - x'^0) = \int \frac{dE}{2\pi} \frac{e^{-iE(x^0 - x'^0)}}{E - W + i\epsilon} + \text{contact terms}. \quad (25)$$

The contact terms indicate that $G_V(W; x^0 - x'^0)$ is undetermined at $x^0 = x'^0$; we can add a finite polynomial in E which is equivalent to the Fourier transform of δ functions in time plus finite derivatives. In order to maintain the correct pole structure of the free V particle, we have to replace

$$\frac{1}{E + i\epsilon - W} \to \left(\frac{E - m_V}{W - m_V}\right)^2 \frac{1}{E + i\epsilon - W}.$$

This replacement is necessary if we stay within the field description. If one prefers to work with sources, the modified VA is immediately given by

$$\langle 0_+ | 0_- \rangle = -\frac{g^2}{4\pi^2} \int_{(m_N+\mu)}^{\infty} dW \frac{[(W - m_N)^2 - \mu^2]^{1/2}}{(W - m_V)^2} \times \int \frac{d\vec{p}}{(2\pi)^3} \frac{dE}{2\pi} \eta_V^*(\vec{p}, E) \times \frac{1}{i} \frac{1}{E + i\epsilon - W} \eta_V(\vec{p}, E). \quad (26)$$

Together with the original nonrelativistic amplitude

$$\langle 0_+ | 0_- \rangle = -\int \frac{d\vec{p}}{(2\pi)^3} \frac{dE}{2\pi} \eta_V^*(\vec{p}, E) \times \frac{1}{i} \frac{1}{E + i\epsilon - m_V} \eta_V(\vec{p}, E),$$

we then find the modified V-particle propagation function:

$$\overline{G}_V(E) = \frac{1}{E - m_V + i\epsilon} + \frac{g^2}{4\pi^2} \int_{(m_N+\mu)>m_V}^{\infty} dW \frac{[(W - m_N)^2 - \mu^2]^{1/2}}{(W - m_V)^2} \times \frac{1}{E - W + i\epsilon}, \quad (27)$$

where the instruction on the lower limit of the spectral integral, i.e., $W = m_N + \mu > m_V$, is introduced to avoid spontaneous decay: $V \to N + \Theta$.

If we return to the field description, we obtain for the modified action of the V particle (in momentum space $\int \equiv [d\vec{p}/(2\pi)^3](dE/2\pi)$)

$$W_V = \int -(\eta^*\psi + \psi^*\eta)|_V + \int \psi_V^*(E - m_V)\psi_V - \int \psi_V^* \frac{g^2}{4\pi^2} dW [(W - m_N)^2 - \mu^2]^{1/2} \frac{(E - m_V)^2}{(W - m_V)^2} \times \frac{1}{E + i\epsilon - W} \psi_V. \quad (28)$$

From here the modified propagation function can be read off to give

$$\left[E - m_V - \frac{g^2}{4\pi^2} \int_{(m_N+\mu)}^{\infty} dW \frac{[(W - m_N)^2 - \mu^2]^{1/2}}{(W - m_V)^2} \frac{1}{E + i\epsilon - W} \times (E - m_V)^2\right] \overline{G}_V(E) = 1. \quad (29)$$

Introducing the positive weight function

$$a(W) = \frac{g^2}{4\pi^2} \frac{[(W - m_N)^2 - \mu^2]^{1/2}}{(W - m_V)^2}, \quad (30)$$

we can rewrite Eq. (29) in the form

$$\overline{G}_V(E) = \frac{1}{E - m_V + i\epsilon} \times \left[1 - (E - m_V) \int_{(m_N+\mu)}^{\infty} dW \frac{a(W)}{E - W + i\epsilon}\right]^{-1}. \quad (31)$$

If $a(W)$ is sufficiently small, we are allowed to expand the second factor. Keeping only the first terms in the expansion of (31) we obtain

$$\overline{G}_V(E) \cong \frac{1}{E - m_V + i\epsilon} + \int_{(m_N+\mu)}^{\infty} dW \frac{a(W)}{E - W + i\epsilon},$$

which is precisely the expression (27).

If we choose to represent the propagation function (31) in a form similar to (27) we have to introduce a different weight factor, i.e.,

$$\overline{G}_V(E) = \frac{1}{E - m_V + i\epsilon} + \int dW \frac{A(W)}{E - W + i\epsilon}.$$

The weight function $A(W)$ can be related to $a(W)$ by comparing the imaginary parts for $E = W$ and using

$$\frac{1}{W' - W + i\epsilon} = \mathrm{P}\frac{1}{W' - W} - i\pi\delta(W' - W).$$

This yields

$$A(W) = \frac{a(W)}{\left[1 - (W - m_V)\mathrm{P}\int dW' \frac{a(W')}{(W' - W)}\right]^2 + [\pi(W - m_V)a(W)]^2},$$

where $a(W)$ is defined by Eq. (30).

IV. V-PARTICLE DECAY: $V \to N+\Theta$

Here we want to concentrate on the spontaneous decay of the V particle. Since source theory can accommodate stable as well as unstable particles, we can use the extended source picture to describe the instability of the V particle. In particular, we shall utilize the form of the propagation function as presented in Eq. (29). There the W range of integration excludes the V-particle mass. Now we remove the restriction $E=W\neq m_V$. We can check that the mass m_V is not displaced by this operation, provided the double singularity of $1/(W-m_V)^2$ is interpreted as the Cauchy principal value:

$$\frac{1}{(W-m_V)^2} \to \frac{d}{dm_V} \mathrm{P} \frac{1}{W-m_V}.$$

However, for the physical situation of interest, $m_V > m_N + \mu$, one can show that the value of the real part of the spectral integral in (29), which is represented by

$$(E-m_V)^2 \frac{d}{dm_V} \mathrm{P} \int_{-\infty}^{+\infty} dW \frac{[(W-m_N)^2-\mu^2]^{1/2}}{W-m_V} \frac{1}{E-W},$$

is zero. Therefore the correct pole structure of the free V particle is preserved.

The imaginary part which describes the instability of the V particle is given by

$$\mathrm{Im}\overline{G}_V{}^{-1}(E)|_{E=W} = \pi \frac{g^2}{4\pi^2}[(W-m_N)^2-\mu^2]^{1/2}$$
$$= \pi a(W)(W-m_V)^2 .$$

Sufficiently close to resonance, i.e., $E \simeq m_V$, we obtain

$$\overline{G}_V(E) \underset{E\sim m_V}{\sim} \frac{1}{E-m_V+i\frac{1}{2}\Gamma}, \tag{32}$$

where

$$\tfrac{1}{2}\Gamma = \pi \frac{g^2}{4\pi^2}[(m_V-m_N)^2-\mu^2]^{1/2}.$$

Hence the time behavior of the V particle is determined by

$$e^{-im_V t}e^{-\Gamma t/2}.$$

Taking the absolute square we arrive at the exponential decay law $e^{-\Gamma t}$, with

$$\tau = \frac{1}{\Gamma} = \frac{2\pi}{g^2[(m_V-m_N)^2-\mu^2]^{1/2}}$$

the lifetime of the decaying V particle. Consequently, experimental knowledge of the decay width Γ is sufficient to determine the coupling constant, a feature which is also shared by other more realistic models.

V. CONCLUSION

We used Schwinger's source theory in the context of the Lee model. The calculation for the V-particle propagation function was first performed for a specific causal arrangement and thereafter space-time extrapolated. Contact was then made with the original nonrelativistic Lee model, which was obtained by a specific choice for the various free-particle propagation functions. Also discussed are two versions of the single spectral form for the V-particle propagator and their respective weight functions. Finally, the V-particle decay was investigated and its time-behavior determined.

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[1]J. Schwinger, *Particles, Sources, and Fields* (Addison-Wesley, Reading, Mass.), Vol. I (1970); Vol. II (1973); *Particles and Sources* (Gordon and Breach, New York, 1969); Phys. Rev. 152, 1219 (1966); L. L. DeRaad, Jr., R. J. Ivanetich, K. A. Milton, and W.-y. Tsai, Phys. Rev. D 5, 358 (1972).

[2]J. Schwinger, Phys. Rev. 158, 1391 (1967).

[3]T. D. Lee, Phys. Rev. 95, 1329 (1954); H. M. Fried, J. Math. Phys. 7, 583 (1966); W. Dittrich, Nuovo Cimento 63A, 85 (1969).