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$2S_{1/2} \rightarrow 1S_{1/2}$ + one-photon decay of muonic atoms and parity-violating neutral-current interactions*

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The decay $2S_{1/2} \rightarrow 1S_{1/2} + 1$ photon, in muonic atoms, is considered as a probe of parity-violating interactions between muons and nucleons. It is shown that a parity-violating interaction, arising from neutral currents, with a strength comparable to the Fermi interaction, would induce a small admixture of $2P_{1/2}$ state into the $2S_{1/2}$ state of muonic atoms, with a magnitude of approximately 10^{-7}. Such an admixture would allow the $2S_{1/2}$ to decay to $1S_{1/2} + 1$ photon by an $E1$ transition, as well as by the known $M1$ transition. Since the $E1$ matrix element is much larger than the $M1$ matrix element, the small state admixture will be enhanced in the decay matrix element. In particular, the circular polarization of the emitted photon, or the photon angular asymmetry relative to the muon polarization, would be of order 10^{-4} in nuclei near $Z = 30$, and so are perhaps measurable. The branching ratio of the $2S_{1/2} \rightarrow 1S_{1/2}$ +1-photon decay to other transitions from the $2S_{1/2}$ state is considered, and found to be around 10^{-3} in nuclei of interest. In an appendix, similar considerations are given for the $2S_{1/2} \rightarrow 1S_{1/2}$ +1-photon decay of electron hydrogenic ions, where it is found that the circular polarizations are much smaller, but the branching ratios can be substantially larger.

I. INTRODUCTION

Empirical evidence exists[1] that the muon in various muonic atoms occasionally cascades into the $2S_{1/2}$ state from whatever state it is originally captured into. This fact raises the possibility of measuring the one-photon transitions from the $2S_{1/2}$ state to the $1S_{1/2}$ ground state, a measurement that could furnish quite interesting information on a variety of questions. In particular, a measurement of the rate for the transition $2S_{1/2} \rightarrow 1S_{1/2} + 1$ photon could cast some light on a possible discrepancy between experiment and theory for magnetic dipole transitions that has recently been reported in high-Z heliumlike ions.[2] Looking further ahead, if this transition can be reliably detected and studied, it may eventually be possible to detect a small circular polarization of the emitted photon, or equivalently, a small asymmetry of the photon direction with respect to a residual muon polarization. This measurement would furnish direct information about the existence of a parity-violating interaction between neutral currents involving muons and hadrons, whose existence is entailed by some recent unified theories of the weak and electromagnetic interactions.[3] It could also give a value for the sign of the coupling constant of this weak interaction.

In this paper, we present some calculations of the decay ratios and photon distributions in the muonic decay $2S_{1/2} \rightarrow 1S_{1/2} + 1$ photon. We discuss the competition of this decay with other modes, such as $2S_{1/2} \rightarrow 2P_{1/2}$, and $2S_{1/2} \rightarrow 1S_{1/2} + 2$ photons. The outline of the paper is as follows. In Sec. II, we give some qualitative discussion of the relevant levels of muonic atoms, and their decays. In Sec. III, we discuss briefly the form and magnitude of a possible parity-violating interaction of muons with hadrons and the parity mixing of atomic energy levels induced by this interaction. In Sec. IV, we describe detailed calculations of the energy levels of a variety of decay rates and of the circular polarization in the decay $2S_{1/2} \rightarrow 1S_{1/2} + 1$ photon. In Sec. V, we discuss possible measurability of some of these transitions, and what information such measurements could provide. Finally, in a short appendix, we discuss parity mixing in electron hydrogenic ions.

II. A QUALITATIVE DESCRIPTION OF LOW-LYING MUONIC ATOM LEVELS AND DECAYS

The ground state of muonic atoms is of course always the $1S_{1/2}$ state. The next three higher states are $2S_{1/2}$, $2P_{1/2}$, and $2P_{3/2}$, which are all raised above the ground state by the Bohr energy difference, which is $\sim (Z\alpha)^2 m$. The precise arrangement and energy difference among these states is the result of a number of different interactions. However, for all nuclei with $Z \gtrsim 5$, the lowest state is

$2P_{1/2}$, the next lowest is $2P_{3/2}$, and the highest is $2S_{1/2}$. This is because the dominant perturbation is the finite nuclear size, which raises the $2S$ state and has small effect on the $2P$ states. The size of the splittings between $2S_{1/2}$ and the $2P$ states is therefore approximately that of the finite-size effect, whose order of magnitude is $C(Z\alpha)^4 m$, where C is a numerical factor typically around $\frac{1}{10}$. In muonic atoms of very low Z, the vacuum polarization becomes comparable to the finite-size effect, and opposite in sign, so that the magnitude of the splittings is qualitatively different from this. However, such light nuclei are not useful to us, for reasons we discuss below.

As a result of this arrangement of levels, the $2S_{1/2}$ state can decay electromagnetically to the $2P_{1/2}$ and $2P_{3/2}$ states, as well as to the $1S_{1/2}$ ground state. The branching ratio for various transitions depends on several factors, including the multipolarity of the transitions, the energy difference of the states, and in some cases, the magnitude of parity mixing between the states. We consider next the different transitions possible, and an estimate of their partial rates.

A. $2S_{1/2} \rightarrow 1S_{1/2} + 1$ photon

In the absence of any parity mixing of the states, or of parity violation in the electromagnetic interaction, this is a magnetic dipole transition. Because of the change in the principal quantum number, the matrix element is substantially suppressed compared to ordinary $M1$ matrix elements. It has been calculated for point nuclei,[4] in the context of hydrogenic ions, and found to be

$$\langle 2S_{1/2} | \vec{\alpha} \cdot \hat{\epsilon} e^{i\vec{k}\cdot\vec{r}} | 1S_{1/2} \rangle = + i\vec{\sigma} \cdot \hat{\epsilon} \times \hat{k} \left[\frac{(Z\alpha)^4}{27\sqrt{2}} + O((Z\alpha)^6) \right], \tag{2.1}$$

which is smaller by a factor $(Z\alpha)^2$ than ordinary $M1$ matrix elements.

This would correspond to a rate, in muonic atoms, of

$$R_{2S \rightarrow 1S + 1\gamma} \simeq 5 \times 10^{-4} Z^{10}\,\text{sec}^{-1}. \tag{2.2}$$

We shall see that while this is a correct order of magnitude, the finite-size corrections reduce this rate substantially.

We can get some idea of the suppression represented by this rate by comparing it with the $E1$ decay of the $2P_{1/2} \rightarrow 1S_{1/2} + 1$ photon, with approximately the same energy release:

$$\langle 2P_{1/2} | \vec{\alpha} \cdot \hat{\epsilon} e^{i\vec{k}\cdot\vec{r}} | 1S_{1/2} \rangle = i u^{+} \vec{\sigma} \cdot \hat{\epsilon} u \left[\tfrac{16}{243}\sqrt{6}\, Z\alpha + O((Z\alpha)^3) \right], \tag{2.3}$$

calculated again for a point nucleus. The ratio of matrix elements is therefore given by

$$\left| \frac{M_{E1}}{M_{M1}} \right| \approx \frac{1.58 \times 10^7}{Z^3}. \tag{2.4}$$

Since the $M1$ decay is so highly suppressed, especially in light elements, it suggests that a relatively small parity mixing in the $2S_{1/2}$ (or $1S_{1/2}$) states might be detectable, since it would allow the much larger $E1$ matrix element to contribute. In particular, a measurement of either the circular polarization of the photon, or of the photon asymmetry relative to a muon polarization, both of which involve an interference between the $E1$ and $M1$ matrix elements, would appear to be likely candidates for detecting such a parity mixing. Since the $2S_{1/2}$ state has a nearby state of opposite parity and equal J, it is plausible to assume that the mixing would be greatest there. We write

$$|2S_{1/2}\rangle' \cong |2S_{1/2}\rangle + \delta\, |2P_{1/2}\rangle \tag{2.5}$$

to represent the true $2S_{1/2}$ state as a parity mixture, where we have assumed δ to be small and δ is imaginary if time-reversal invariance is satisfied. We can then write the total matrix element for the decay of this state by one photon to the ground state as

$$M_{2S \rightarrow 1S + 1\text{photon}} = M_{M1} + \delta M_{E1}. \tag{2.6}$$

The corresponding circular polarization is given by

$$|P| \cong 2\,|\text{Im}\,\delta| \times \left| \frac{M_{E1}}{M_{M1}} \right|. \tag{2.7}$$

It follows that, just as for certain nuclear transitions, the suppression of M_{M1} relative to M_{E1} makes a measurement of circular polarization a quite sensitive test of any small parity admixture δ. We shall see below that a typical expected value of δ might be

$$\delta \sim 10^{-7}$$

so that the circular polarization can be $\sim 10^{-4}$ for $Z \sim 30$. The prospect of measuring this polarization is complicated by the fact that several other transitions can depopulate the $2S_{1/2}$ state, and the branching ratio for the decay $2S_{1/2} \rightarrow 1S_{1/2} + 1\gamma$ is generally less than 1% for all Z. We next discuss some of these other transitions, before turning to more detailed calculations of the $2S_{1/2} \rightarrow 1S_{1/2} + 1\gamma$ transition.

B. $2S_{1/2}\to 1S_{1/2}+2$ photons

In electron hydrogenic ions, it is known that at least for light elements, the dominant decay mode of the $2S_{1/2}$ state is $2S_{1/2}\to 1S_{1/2}+2(E1)$ photons. The rate, calculated nonrelativistically, and for a point nucleus, in a muonic atom for this transition, is given by[5]

$$R_{2S\to 1S+2\gamma}\simeq 1.7\times 10^3 Z^6\,\text{sec}^{-1}.$$

If we assume that this remains approximately correct for atoms with finite nuclei, we find that

$$\frac{R_{2S\to 1S+1\gamma}}{R_{2S\to 1S+2\gamma}}\approx 3.0\times 10^{-7}Z^4 < 1, \quad \text{for } Z<40. \tag{2.8}$$

For example, at $Z=20$, the 1-photon/2-photon branching ratio is about 5%.

Because the 2-photon decay is essentially the result of two allowed $E1$ transitions, its matrix element is quite insensitive to parity admixtures in the atomic states, and so it is not a good candidate for detection of such effects. This suggests that experiments to detect the muonic $2S\to 1S+1\gamma$ decay are best performed in nuclei with $Z>15$. We shall see that other considerations confirm this, and also limit Z from above.

C. $2S_{1/2}\to 2P_{1/2}$ or $2P_{3/2}+1$ photon

Both of these decays can occur as $E1$ transitions, and the latter as an $M2$ transition as well. Since $kr\sim(Z\alpha)^3$ and is fairly small for these transitions even in heavy elements, the $M2$ term is small compared to the $E1$ term everywhere. In the nonrelativistic, no-retardation approximation, the ratio of the decay rates to the $2P_{1/2}$ and $2P_{3/2}$ states is given by $\frac{1}{2}[(E_{2S}-E_{2P_{1/2}})/(E_{2S}-E_{2P_{3/2}})]^3$, a number of order 1. Therefore, to get an idea of the magnitude of the decay rates, we consider the decay to the $2P_{1/2}$ state. This decay differs from ordinary $E1$ decays in that the energy difference is of order $(Z\alpha)^4m$ instead of $(Z\alpha)^2m$, and the corresponding matrix element is therefore of order $(Z\alpha)^3$ instead of $(Z\alpha)$. The decay rate for the transition is therefore of order $(Z\alpha)^{10}m$, compared to $(Z\alpha)^4m$ for an ordinary $E1$ transition. This makes it of the same order in $(Z\alpha)$ as the $M1$, $2S_{1/2}\to 1S_{1/2}$ transition considered above. Regrettably, the other numerical factors are such as to make the decay to the $2P_{1/2}$ state substantially more rapid than that to the $1S_{1/2}$ state. Detailed calculations, given in Sec. IV, indicate that the branching ratio decreases slowly with Z, and is given by

$$\frac{R_{2S\to 1S+1\gamma}}{R_{2S\to 2P_{1/2}+1\gamma}}\lesssim 10^{-2}, \quad \text{for } Z>17.$$

This result, combined with the previous ratio of the $M1$ transition to the 2γ transition, implies that the $M1$ transition is always less than 1% of the total rate.

A parity mixing in the atomic states would also affect these $E1$ decays. We can write as before

$$|2S_{1/2}\rangle' = |2S_{1/2}\rangle + \delta_1|2P_{1/2}\rangle\,, \qquad |2P_{1/2}\rangle' = |2P_{1/2}\rangle + \delta_2|2S_{1/2}\rangle\,, \tag{2.9}$$

where $\delta_1=-\delta_2^*\equiv\delta$, by requiring the states to be orthonormal. The radiative decay matrix element is now given by

$$\langle 2S|'H_{\text{rad}}|2P\rangle' = \langle 2S|H_{\text{rad}}|2P\rangle + \delta^*\langle 2S|H_{\text{rad}}|2S\rangle - \delta\langle 2P|H_{\text{rad}}|2P\rangle + O(\delta^2)\,. \tag{2.10}$$

The situation here is reversed because the terms multiplying δ are the $M1$ amplitudes, while the term of order 1 is the $E1$ amplitude, which is much larger. Hence the circular polarization or other parity-violating observables in the $2S\to 2P$ transition would be much smaller than in the $2S\to 1S$ transition, and almost certainly unobservable.

D. $2S_{1/2}$ decays via Auger emission

It is known that the decay of the highly excited states ($n\sim 15$), in which the muon is originally captured by the atom, to low-lying states proceeds mostly by Auger transitions, rather than by photon emission.[6] However, once the muon reaches low-lying states, it is believed that radiative transitions become the main decay modes. This is essentially because the radiative decay rates increase rapidly with energy, while the absolute Auger rates vary relatively slowly with energy.

However, a muon reaching the $2S_{1/2}$ state will be anomalous in this regard, because, as we have seen, the only available radiative decays either have relatively low energy ($2S\to 2P$), or are relatively suppressed by the multipolarity ($2S\to 1S$). This would suggest that, at least in light nuclei, the Auger transitions could still compete favorably with radiative decays from the 2S state. We have briefly examined this question, with the following results. The Auger transitions that should be considered are $2S_{1/2}\to 2P_{1/2}$, $2S_{1/2}\to 2P_{3/2}$, and $2S_{1/2}\to 1S_{1/2}$. The former two go by $E1$ Auger transitions, while the last goes by $M1$, and also by the $E0$ monopole transition allowed whenever initial and final states have the same spin.[7] The $2S\to 2P$ Auger conversion coefficients should be quite similar to the internal conversion coefficients for a hypothetical $E1$ nuclear transition of the same energy, in a nucleus with $Z'=Z-1$, at least for the K-shell coefficients, which only involve the electronic S-state wave functions near the origin. This is because a muon in the $2S_{1/2}$ state is approximately part of the nucleus, insofar as the electrons are concerned. Furthermore, the K-shell internal conversion coefficients (ICC's) depend very little on nuclear structure effects. Therefore, we can use calculated ICC's for nuclear transitions to estimate the branching ratio of Auger to $2S \rightarrow 2P$ radiative transitions.[8] Upon doing this, we find that for $10 < Z < 30$, the Auger rate is much larger than the rate for $2S \rightarrow 2P$ radiative transitions. For $Z > 30$, the Auger rate becomes less than the radiative decay rates, while for $Z < 10$, the calculation becomes substantially more complicated because the transition energies are comparable to the electron binding energies in the atom. Consequently, we can say that for $Z > 30$, the $2S \rightarrow 2P$ Auger transitions do not greatly alter the branching ratio of interest to us, which is

$$\frac{R_{2S \rightarrow 1S + 1\gamma}}{R_{2S \rightarrow \text{anything}}}.$$

On the other hand, in the region $10 < Z < 30$, this ratio will be reduced substantially by the Auger transitions, and the $2S \rightarrow 1S + 1\gamma$ decay will be correspondingly harder to detect. Finally, in the region $Z < 10$, the effect of the Auger transitions will depend on more detailed calculations, which will be presented elsewhere. In any case, in this region the branching ratio is already small because of the large $2S \rightarrow 1S + 2\gamma$ rate.

We next consider the effect of Auger transitions in the $2S \rightarrow 1S$ decay itself. Since the energies here are much larger than in $2S \rightarrow 2P$, we expect the ICC to be much smaller. Indeed, for $Z \sim 30$ the transition energy is 2 MeV, and the $M1$ ICC is 10^{-4}, which is negligible. A more important contribution comes from the $E0$ decay. This has been calculated in the nonrelativistic approximation,[9] with the result that the $E0$ rate depends only slightly on Z, and is given approximately by

$$R_{2S \rightarrow 1S}^{\text{Auger}, E0} \sim 10^9 \sec^{-1}.$$

In the region $Z \gtrsim 40$, where relativistic effects become important, this rate can be substantially increased, essentially because of the large value of the wave function for the outgoing electron at the nuclear radius. However even for $Z = 82$, $R_{2S \rightarrow 1S}^{\text{Auger}, E0}$ is probably $\lesssim 10^{+12} \sec^{-1}$. These values are still a small fraction of the dominant $2S \rightarrow 2P$ decays for large Z. Therefore, it seems safe to neglect the $2S \rightarrow 1S$ Auger transitions in calculating the total decay rates in heavy elements ($Z \gtrsim 20$).

On the other hand, for light elements, the $2S \rightarrow 1S$ $E0$ Auger transition may be an important part of the total decay rate. That would raise the interesting possibility of detecting parity mixing in the $2S$ state through an interference between the $E0$ Auger transition and the $E1$ Auger transition that can occur through the parity mixing. This might be measurable through a detection of the polarization of the Auger electron, or the correlation between the electron direction and the muon polarization. These questions will also be discussed elsewhere.

We mention finally the possibility of Auger emissions in $2S \rightarrow 1S$, through conversion of the photons in the decay $2S \rightarrow 1S + 2\gamma$. Since these are both primarily $E1$ photons, and since the photon energy ranges between 0 and $E_{2S} - E_{1S}$, one might think that the ICC could be quite large for the low-energy photons, giving a large Auger rate. However, this does not happen, because the ICC is the ratio of two weighted integrals over all energies of each photon, and the low-energy photons contribute little to either integral. Consequently, the ICC is approximately that corresponding to an $E1$ photon with energy $\frac{1}{2}(E_{2S} - E_{1S})$, which is quite small ($< 10^{-2}$) except in very light elements. Therefore, we can also neglect the contribution of this Auger process to the total decay rate of the $2S_{1/2}$ state in elements with $Z \gtrsim 10$.

We should also compare the radiative decays with the rates for μ capture and μ decay. The latter is approximately $5 \times 10^5 \sec^{-1}$, and is always small compared to the radiative rates for $Z > 3$. The capture rate is approximately $35Z^4 \sec^{-1}$, and so is always small compared to the radiative rate. Hence these two effects are unimportant for the purpose of computing total branching ratios.

We conclude from this qualitative survey that the $2S_{1/2} \rightarrow 1S_{1/2} + 1\gamma$ decay is likely to be observable only in elements with $Z \gtrsim 30$. In Sec. IV we shall give some more precise quantitative estimates of the branching ratio for this decay.

III. PARITY MIXING IN MUONIC ATOMS

Several of the spontaneously broken gauge models of weak and electromagnetic interactions involve massive neutral mesons that interact with neutral baryonic and leptonic currents.[10] Some also involve massive neutral spinless mesons interacting with scalar or pseudoscalar combinations of baryon and lepton fields. The former couplings are of a strength such that when taken to second order, they generate approximately local interactions among baryons and leptons, whose strength is comparable to that of the usual weak Fermi interaction. The couplings generated by the neutral mesons differ from the Fermi couplings, however, in that they involve no charge transfer between the leptons or between the hadrons. We shall therefore refer to them as neutral-current interactions. For the moment, we put aside consideration of second-order couplings generated by massive scalar mesons.

The precise strength of the neutral-current interactions, as well as the particles involved in them, depends on the model chosen both through the gauge group involved and through its realization. In the present stage, where there is no convincing evidence about these questions, it is probably best to adopt a phenomenological approach, and to ask what information about neutral-current interactions can be obtained from a given experiment, rather than to attempt to test a specific model. However, we mention below the form of the coupling for one such model. Evidently, experiments in muonic atoms can only detect neutral-current interactions involving muons. Parity-conserving couplings of the expected strength would almost certainly be masked by the electromagnetic interaction, and not be readily detectable. Hence we restrict ourselves to parity-violating interactions of muons, with the nucleons present in ordinary nuclei. Furthermore, we assume for the present, as is the case in many spontaneously broken gauge theories, that CP invariance is satisfied to a good approximation. Then, there are 3 parity-violating neutral-current interactions to be considered:

$$H_1 = C_1 \frac{G}{\sqrt{2}} \bar{\psi}_\mu \gamma_\alpha \gamma_5 \psi_\mu \bar{\psi}_N \gamma_\alpha \psi_N , \quad (3.1)$$

$$H_2 = C_2 \frac{G}{\sqrt{2}} \bar{\psi}_\mu \gamma_\alpha \psi_\mu \bar{\psi}_N \gamma_\alpha \gamma_5 \psi_N , \quad (3.2)$$

$$H_3 = C_3 \frac{G}{\sqrt{2}} \bar{\psi}_\mu \gamma_\alpha \gamma_5 \psi_\mu \frac{\partial_\beta}{m_N} (\bar{\psi}_N \sigma_{\alpha\beta} \psi_N) . \quad (3.3)$$

In these formulas, ψ_N represents either a neutron or proton, G is the Fermi constant of μ decay, and C_1, C_2, and C_3 are dimensionless constants, whose magnitude measures the strength of the interaction.

The coupling H_3 may be thought of as an induced coupling, of the same character as the "weak magnetism" in the β-decay interaction. The other couplings H_1, H_2 are direct analogs of the VA cross terms in the β-decay interaction. To get some idea of the relative importance of these terms, let us take the nucleons, but not the muons, to be described nonrelativistically. Then only certain terms will survive in each of H_1, H_2, H_3, and we find that

$$H_1 \to \frac{C_1 G}{\sqrt{2}} \bar{\psi}_\mu \gamma_4 \gamma_5 \psi_\mu \psi_N^\dagger \psi_N + \text{"forbidden" terms} , \quad (3.4)$$

$$H_2 \to \frac{C_2 G}{\sqrt{2}} \bar{\psi}_\mu \gamma_i \psi_\mu \psi_N^\dagger \sigma_i \psi_N + \text{"forbidden" terms} , \quad (3.5)$$

$$H_3 \to \frac{C_3 G}{m_N \sqrt{2}} \bar{\psi}_\mu \gamma_i \gamma_5 \psi_\mu \frac{\partial [\bar{\psi}_N (\epsilon_{ijk} \sigma_k) \psi_N]}{\partial x_j} + \text{"doubly forbidden" terms} . \quad (3.6)$$

In these expressions, we have denoted by "forbidden" terms involving the small components of the nuclear wave functions, which are negligible for our purposes. We see that H_2 and H_3 contain only terms involving nucleon spin, and therefore, unless a single nuclear spin state is observed, will not contribute to any first-order interaction. This leaves only H_1 as a candidate for a parity-violating first-order interaction in muonic atoms. We shall therefore concentrate on the interaction H_1.

For a complex nucleus, it is necessary to sum H_1 over all the nucleons present. Since the constant C_1 may be different for proton and neutron, we write

$$H_1 \to \frac{G}{\sqrt{2}} \bar{\psi}_\mu \gamma_4 \gamma_5 \psi_\mu \left(C_p \sum_i \psi_{p_i}^\dagger \psi_{p_i} + C_n \sum_i \psi_{n_i}^\dagger \psi_{n_i} \right) . \quad (3.7)$$

The quantities $\sum_i \psi_{p_i}^\dagger \psi_{p_i}$ and $\sum_i \psi_{n_i}^\dagger \psi_{n_i}$ are just the number densities of protons and neutrons in the nucleus. In simple nuclear models, these are taken to be proportional to each other, with factors that are just Z and $(A-Z)$. So we can write[11]

$$H_1 = \frac{G}{\sqrt{2}} \psi_\mu^\dagger \gamma_5 \psi_\mu \frac{\rho}{4\pi} [C_p Z + C_n (A-Z)] , \quad (3.8)$$

where ρ is a nuclear matter density, normalized so that

$$\int \rho r^2 dr = 1 . \quad (3.9)$$

We shall only consider nuclei with spherically symmetric ρ.

We note that for $C_p = -C_n$, the interaction is proportional to $A - 2Z$, which is much smaller than A, particularly in light nuclei. This case corresponds to a purely isovector neutral hadron current, and therefore the muonic atom transitions we consider are not very sensitive to such a current. If the current is instead isoscalar, $C_p = C_n$, and the interaction is just proportional to the nucleon number A.

As an example of the values for C_p and C_n, we consider the SU(2)×U(1) model of weak interactions due to Weinberg.[3] In this model, a muon-nucleon interaction of the form H_1 is generated by exchange of a heavy neutral vector meson, the Z meson. The strength of the coupling depends only on a single parameter, the Weinberg angle, which essentially measures the ratio of the Z-meson mass to the W-meson mass. Specifically, we find that in this model

$$C_p = \tfrac{1}{2}(1 - 4\sin^2\theta_W),$$
$$C_n = -\tfrac{1}{2}, \tag{3.10}$$

where θ_W, the Weinberg angle, is given by $\cos\theta_W = m_W/m_Z$. This result implies $-\frac{3}{2} < C_p < \frac{1}{2}$, and so places some constraint on the size of the parity-violating muon-hadron interaction. From Eq. (3.8) we see that the effective coupling is

$$C_p Z + C_n(A - Z) = \tfrac{1}{2}(1 - 4\sin^2\theta_W)Z - \tfrac{1}{2}(A - Z)$$
$$= -(2\sin^2\theta_W)Z + \tfrac{1}{2}(2Z - A). \tag{3.11}$$

Thus in nuclei with $A \sim 2Z$, the effective coupling will be negative.

Some other unified gauge models give predictions for C_p and C_n, which are qualitatively similar, but quantitatively different, from Weinberg's model. Others have no interaction H_1. Thus the detection of H_1 would be a useful piece of evidence about such models.

The interaction H_1 will have matrix elements between any two muonic states of equal spin and opposite parity. In particular, it will have matrix elements between $2S_{1/2}$, and all $P_{1/2}$ states. In all but the heaviest nuclei, the $2S_{1/2}$-$2P_{1/2}$ splitting will be much smaller than the $2S_{1/2}$ splitting from any other $P_{1/2}$ state. On the other hand, the matrix elements of H_1 will probably not depend so much on which $P_{1/2}$ state is involved. Hence the parity mixing, which is proportional to the ratio

$$\delta_{2S_{1/2},\,nP_{1/2}} = \frac{\langle 2S_{1/2}|H_1|nP_{1/2}\rangle}{E_{2S_{1/2}} - E_{nP_{1/2}}}, \tag{3.12}$$

will be the greatest for the $2P_{1/2}$ state. For the moment we consider only that mixing.

The numerator factor is just the matrix element of H_1, evaluated between the two muonic states,

$$\langle 2S_{1/2}|H_1|2P_{1/2}\rangle = \frac{G}{\sqrt{2}}[C_p Z + C_n(A - Z)]\frac{1}{4\pi} \times \int \rho(r)\, d^3r\, \psi^\dagger_{2S_{1/2}}\gamma_5\psi_{2P_{1/2}}. \tag{3.13}$$

We write[12]

$$|\psi_{2S_{1/2},\,1/2}\rangle = \begin{bmatrix} g_{2S}Y_{0,0} \\ 0 \\ -i(\frac{1}{3})^{1/2}f_{2S}Y_{1,0} \\ -i(\frac{2}{3})^{1/2}f_{2S}Y_{1,1}\end{bmatrix};$$
$$|\psi_{2P_{1/2},\,1/2}\rangle = \begin{bmatrix} (\frac{1}{3})^{1/2}g_{2P_{1/2}}Y_{1,0} \\ (\frac{2}{3})^{1/2}g_{2P_{1/2}}Y_{1,1} \\ -if_{2P}Y_{0,0} \\ 0\end{bmatrix}. \tag{3.14}$$

If we assume a spherical nuclear charge distribution, we can do the angular integral in Eq. (3.13), giving

$$\langle 2S_{1/2}|H_1|2P_{1/2}\rangle = +\frac{iG}{\sqrt{2}}[C_p Z + C_n(A - Z)]\frac{1}{4\pi} \times \int r^2\, dr\, \rho(r) \times [-g_{2S}f_{2P_{1/2}} + f_{2S}g_{2P_{1/2}}], \tag{3.15}$$

where g and f are the large- and small-component wave functions in the field produced by the finite nucleus. The remaining radial integral has been evaluated numerically by the method described in Sec. IV, with results given in Table I.

We also give there, for comparison, the result in the approximation[11] in which the nuclear size is neglected, except that the functions f and g are evaluated at the actual nuclear radius rather than at $r = 0$, and only the first terms in their expansions in powers of $Z\alpha$ are kept. It can be seen from the table that the effect of the finite size is generally to decrease the radial integral by a significant extent for large Z. From the known value for G, we can then express the matrix element in terms of the constants C_p, C_n. This result can in turn be combined with the calculated or measured energy differences $E_{2P_{1/2}} - E_{2S_{1/2}}$ to give the parity admixtures $\delta_{2S_{1/2}}$, corresponding to mixing with the $2P_{1/2}$ state. These are given in the last column of Table I. The admixtures are quite small, but perhaps measurable through the considerations given above and below.

The order of magnitude of the parity mixing is determined by a rough estimate of the matrix element (3.15):

$$G|\psi_\mu(r=0)|^2 Z(Z\alpha) \sim \frac{Gm_\mu^{\,2}}{4\pi\alpha}(Z\alpha)^5 m_\mu.$$

This may be compared to the difference $E_{2S_{1/2}} - E_{2P_{1/2}} \approx (Z\alpha)^4 m_\mu$, to give an estimate for $\delta_{2S,\,2P} \sim [Gm_\mu^{\,2}/4\pi\alpha](Z\alpha) \simeq 10^{-6}Z\alpha$, in rough agreement with the calculated values of Table I.

Note that the $\delta_{2S,\,2P}$ coming from the interaction H_1 is pure imaginary, in the approximation in which the decay widths of $2S_{1/2}$ and $2P_{1/2}$ are neglected. Otherwise, the quantity $E_{2S_{1/2}} - E_{2P_{1/2}}$ should be $E_{2S_{1/2}} - E_{2P_{1/2}} - \frac{1}{2}i(\Gamma_{2S_{1/2}} - \Gamma_{2P_{1/2}})$, where the Γ's are these widths. Actually, in most cases $\Gamma_{2P} \lesssim 10^{-2}(E_{2S} - E_{2P})$, and Γ_{2S} is even smaller, so that the approximation neglecting Γ is a good one, except perhaps in the lightest elements. However, the small change in phase of δ coming from the width could be of some value in placing a limit on a T-violating contribution to δ, which otherwise is quite hard to detect. See the discussions in

TABLE I. Parity mixing for various nuclear charges. [a]

Z	$I_{2S,2P}$ (F^{-3})	$I^{(0)}_{2S,2P}$ (F^{-3})	$E_{2S}-E_{2P_{1/2}}$ (MeV)	$\frac{i\delta_{2S,2P}}{C_p+C_n(A-Z)/Z}$
3	$+6.87\times10^{-9}$	7.634×10^{-9}	-1.8×10^{-7}	-5.77×10^{-7}
6	$+1.07\times10^{-7}$	1.221×10^{-7}	$+3.84\times10^{-5}$	$+0.844\times10^{-7}$
11	$+1.09\times10^{-6}$	1.380×10^{-6}	$+7.27\times10^{-4}$	$+0.831\times10^{-7}$
17	$+5.36\times10^{-6}$	7.872×10^{-6}	$+4.64\times10^{-3}$	$+0.991\times10^{-7}$
26	$+2.27\times10^{-5}$	4.307×10^{-5}	$+2.60\times10^{-2}$	$+1.14\times10^{-7}$
32	$+4.34\times10^{-5}$	9.883×10^{-5}	$+5.76\times10^{-2}$	$+1.22\times10^{-7}$
35	$+5.66\times10^{-5}$	1.414×10^{-4}	$+8.08\times10^{-2}$	$+1.24\times10^{-7}$
42	$+9.48\times10^{-5}$	2.933×10^{-4}	$+1.54\times10^{-1}$	$+1.30\times10^{-7}$
50	$+1.48\times10^{-4}$	5.891×10^{-4}	$+2.81\times10^{-1}$	$+1.32\times10^{-7}$
60	$+2.27\times10^{-4}$	1.221×10^{-3}	$+4.99\times10^{-1}$	$+1.37\times10^{-7}$
74	$+3.34\times10^{-4}$	2.826×10^{-3}	$+9.23\times10^{-1}$	$+1.35\times10^{-7}$
82	$+3.87\times10^{-4}$	4.261×10^{-3}	$+1.21$	$+1.32\times10^{-7}$

[a] $I_{2S,2P}=-\int r^2dr\,\rho(r)(f_{2S}g_{2P_{1/2}}-g_{2S}f_{2P_{1/2}})$, $\int r^2dr\,\rho(r)=1$. $I^{(0)}_{2S,2P}=\frac{1}{8}\sqrt{3}(Z\alpha)^4m_\mu$ is the small-nucleus, low-$Z\alpha$ approximation to $I_{2S,2P}$,

$$\frac{\delta_{2S,2P}}{C_p+C_n(A-Z)/Z}=\frac{iG}{\sqrt{2}}\frac{Z}{4\pi}\frac{I_{2S,2P}}{E_{2S}-E_{2P_{1/2}}}\,.$$

Sec. V below.

We next consider briefly the mixing of other states, both into the $2S_{1/2}$ state, and into the $1S_{1/2}$ final state of the $M1$ decay. As we mentioned above, these admixtures should be smaller because of the much larger energy differences. Let us calculate for example, in the point nucleus, small-$Z\alpha$ approximation, the mixing of the $2P_{1/2}$ state into the $1S_{1/2}$ state:

$$\delta_{1S_{1/2},2P_{1/2}}\simeq\frac{\langle 2P_{1/2}|H_1|1S_{1/2}\rangle}{E_{2P_{1/2}}-E_{1S_{1/2}}}\,,$$

$$E_{2P_{1/2}}-E_{1S_{1/2}}\simeq\tfrac{3}{8}(Z\alpha)^2m_\mu\,,$$

$$\langle 2P_{1/2}|H_1|1S_{1/2}\rangle\sim\frac{iG}{\sqrt{2}}\frac{[C_pZ+C_n(A-Z)]}{4\pi}\times(f_{2P}g_{1S}-f_{1S}g_{2P})_{r=R}\,,$$

$$f_{2P}\sim-\frac{Z\alpha}{\sqrt{8}}(Z\alpha m)^{3/2}(\tfrac{3}{4})^{1/2}\,,$$

$$g_{1S}\sim 2(Z\alpha m)^{3/2}\,.$$

Therefore

$$\langle 2P_{1/2}|H_1|1S_{1/2}\rangle\sim\frac{iGm_\mu^2}{\sqrt{2}}\frac{[C_p+C_n(A-Z)/Z]}{4\pi}\times Z(Z\alpha)^4(\tfrac{3}{8})^{1/2}m_\mu\,,\qquad(3.16)$$

$$\delta_{1S_{1/2},2P_{1/2}}\sim\frac{iGm_\mu^2}{\sqrt{2}}\frac{[C_p+C_n(A-Z)/Z]}{4\pi\alpha}\times(Z\alpha)^3(\tfrac{8}{3})^{1/2}\,.\qquad(3.17)$$

This may be compared to the corresponding expression for $2S_{1/2}$, computed in the same approximation:

$$\langle 2P_{1/2}|H_1|2S_{1/2}\rangle_0=-\frac{iGm_\mu^2}{\sqrt{2}}\frac{[C_p+C_n(A-Z)/Z]}{4\pi\alpha}\times(Z\alpha)^5\tfrac{1}{8}\sqrt{3}\,m_\mu\,,\qquad(3.18)$$

$$E_{2S_{1/2}}-E_{2P_{1/2}}\simeq\tfrac{1}{20}(Z\alpha)^4m_\mu(m_\mu R)^2\,,$$

$$\delta_{2S_{1/2},2P_{1/2}}\sim\frac{iGm_\mu^2}{\sqrt{2}}\frac{[C_p+C_n(A-Z)/Z]}{4\pi\alpha}\frac{Z\alpha\frac{5}{2}\sqrt{3}}{(m_\mu R_n)^2}\,,\qquad(3.19)$$

$$\frac{\delta_{1S_{1/2},2P_{1/2}}}{\delta_{2S_{1/2},2P_{1/2}}}\sim0.15(Z\alpha)^2A^{2/3}\,.\qquad(3.20)$$

For nuclei with $Z\sim30$, $A\sim60$, where these approximate expressions are expected to be reasonably accurate, we obtain

$$\frac{\delta_{1S_{1/2},2P_{1/2}}}{\delta_{2S_{1/2},2P_{1/2}}}\sim0.09\,.\qquad(3.21)$$

Furthermore, the $E1$ matrix element corresponding to the $2P_{1/2}$-$1S_{1/2}$ mixing, which is $\langle 2P_{1/2}|H_{\rm rad}|2S_{1/2}\rangle$ is also suppressed somewhat compared to $\langle 2P_{1/2}|H_{\rm rad}|1S_{1/2}\rangle$, so that this mixing should not alter our estimates significantly. It therefore seems justified to neglect it, and other mixings than $2S$-$2P$.

We consider next some other possible sources of parity mixing of muonic energy levels. One such source, which could occur if CP invariance is violated, would be a muonic electric dipole (ED) moment or a nuclear electric dipole moment. We concentrate on the former effect because less is known about possible muonic electric dipole moments. Such a moment corresponds to an interaction

$$H_{\rm ED} = \frac{\xi e}{2m_\mu}\gamma_4 \sigma \cdot E\,, \tag{3.22}$$

where E is the electric field of the nucleus, and ξ is a dimensionless parameter measuring the strength of the interaction. The parity mixing again involves the matrix element of this operator between opposite parity states, and we expect the major effect to be mixing of $2S_{1/2}$ and $2P_{1/2}$. This effect has been examined long ago, for electrons, by Salpeter[13] and by one of the authors,[14] with the result that insofar as effect on radiative decays is concerned, it is indeed true in the electronic case that the important mixing is between $2S_{1/2}$ and $2P_{1/2}$.

For muonic atoms, because the $2S_{1/2}$-$2P_{1/2}$ energy difference is a much larger fraction of the Bohr splittings, it is no longer true that the $2S_{1/2}$-$2P_{1/2}$ mixing will give the dominant effect. This is because the matrix element of $H_{\rm ED}$ between $2S_{1/2}$ and $2P_{1/2}$ is suppressed compared to its matrix element between $2S_{1/2}$ and other $P_{1/2}$ states, by about the same factor of $(Z\alpha)^2$ that gives the ratio $(E_{2S_{1/2}} - E_{2P_{1/2}})/(E_{2S_{1/2}} - E_{nP_{1/2}})$. Hence the admixture of all $P_{1/2}$ states into $2S_{1/2}$ will be comparable, and of order of magnitude

$$\delta_{2S_{1/2},\,nP_{1/2}} \sim \xi(Z\alpha)\,.$$

We have not computed these accurately, although it could be done easily from the results of Refs. 13 and 14. However, we note the order of magnitude, and also that this admixture is 90° out of phase with that due to H_1, a result following from their different transformations under T. The orders of magnitude of δ would be comparable for $\xi \sim G m_\mu^2/4\pi\alpha \sim 10^{-6}$, corresponding to a muon electric dipole moment of $10^{-19}\ e$ cm.

Still another source of parity mixing could be an interaction of muons and hadrons with the neutral scalar mesons introduced in some gauge theories. Such interactions ordinarily are taken to be T-invariant and then necessarily are also P-invariant. However, in one model of spontaneous T violation,[15] the scalar-meson interactions are not T-invariant, and hence not P-invariant either. In that case, a muon-nucleon interaction of the form

$$H_4 = iC_4 \frac{G}{\sqrt{2}} \bar{\psi}_\mu \gamma_5 \psi_\mu \bar{\psi}_N \psi_N$$

can occur, where the i has been inserted to make C_4 a real dimensionless constant. The interaction H_4 will induce parity mixing similar to that of H_1. One important difference is that because of the extra factor of i, this mixing will again be 90° out of phase with that due to H_1. The other difference is that C_4 is probably $\ll 1$, so that the effect is correspondingly reduced in size. In Sec. V, we consider the effect of such a possible imaginary parity admixture, as well as the real parity mixing coming from H_1, on various observable quantities in the $2S_{1/2} \to 1S_{1/2} + 1$-photon decay.

IV. CALCULATION OF DECAY RATES FOR THE $2S_{1/2}$ STATE

In this section, we describe our calculations of various radiative decays of the $2S_{1/2}$ state. The decays we have calculated are the following:

(a) $2S_{1/2} \to 1S_{1/2} + 1\gamma$ ($M1$),
(b) $2S_{1/2} \to 1S_{1/2} + 1\gamma$ ($E1$ through parity mixing),
(c) $2S_{1/2} \to 2P_{1/2} + 1\gamma$,
(d) $2S_{1/2} \to 2P_{3/2} + 1\gamma$.

We label the relevant matrix elements for these decays M_a, M_b, M_c, M_d, i.e.,

$$M_a = \langle 2S_{1/2} | \vec{\alpha} \cdot \hat{\epsilon} \exp(i\vec{k}_a \cdot \vec{r}) | 1S_{1/2} \rangle\,, \tag{4.1}$$

$$M_b = \langle 2P_{1/2} | \vec{\alpha} \cdot \hat{\epsilon} \exp(i\vec{k}_b \cdot \vec{r}) | 1S_{1/2} \rangle\, \delta_{2S_{1/2},\,2P_{1/2}}\,, \tag{4.2}$$

$$M_c = \langle 2S_{1/2} | \vec{\alpha} \cdot \hat{\epsilon} \exp(i\vec{k}_c \cdot \vec{r}) | 2P_{1/2} \rangle\,, \tag{4.3}$$

$$M_d = \langle 2S_{1/2} | \vec{\alpha} \cdot \hat{\epsilon} \exp(i\vec{k}_d \cdot \vec{r}) | 2P_{3/2} \rangle\,. \tag{4.4}$$

In these formulas, we have

$$|\vec{k}_a| = |\vec{k}_b| = E_{2S_{1/2}} - E_{1S_{1/2}}\,,$$

$$|\vec{k}_c| = E_{2S_{1/2}} - E_{2P_{1/2}}\,,$$

$$|\vec{k}_d| = E_{2S_{1/2}} - E_{2P_{3/2}}\,,$$

and $\delta_{2S_{1/2},\,2P_{1/2}}$ is the quantity defined in Sec. III.

The matrix elements M_a, M_b, M_c, M_d can easily be reduced to integrals over the muonic radial wave functions. In doing this, we have not neglected retardation effects, i.e., we have not expanded the exponentials, because the quantities kr, especially $k_a r$, are comparable to 1 in heavy atoms.

As an example of our procedure, consider M_a. We can write[12]

$$|1S_{1/2}\rangle = \frac{1}{(4\pi)^{1/2}} \begin{pmatrix} g_{1S}u \\ -if_{1S}\sigma \cdot \hat{r}u \end{pmatrix},$$
$$|2S_{1/2}\rangle = \frac{1}{(4\pi)^{1/2}} \begin{pmatrix} g_{2S}u \\ -if_{2S}\sigma \cdot \hat{r}u \end{pmatrix}, \tag{4.5}$$

where u is the two-component spinor giving the spin content of the state, while g and f are the large and small radial wave functions, including the effects of finite size and of vacuum polarization. Then

$$M_a = \frac{1}{4\pi}\int r^2 dr\, d\Omega \exp(i\vec{k}_a\cdot\vec{r})u^\dagger\sigma\cdot\hat{\epsilon}\times\hat{r}u \times(g_{2S}f_{1S}+f_{2S}g_{1S}), \quad (4.6)$$

$$\frac{1}{4\pi}\int r_i \exp(i\vec{k}\cdot\vec{r})d\Omega = i\hat{k}_i r j_1(kr) = -i\hat{k}_i\left(\frac{\cos kr}{k}-\frac{\sin kr}{k^2 r}\right). \quad (4.7)$$

Therefore

$$M_a = iu^\dagger\sigma\cdot\hat{\epsilon}\times\hat{k}u\int j_1(k_a r)(g_{2S}f_{1S}+f_{2S}g_{1S})r^2 dr \equiv iu^\dagger\sigma\cdot\hat{k}\times\hat{\epsilon}u\alpha \quad (4.8)$$

so that

$$\alpha = -\int j_1(k_a r)(g_{2S}f_{1S}+f_{2S}g_{1S})r^2 dr. \quad (4.9)$$

Similarly, we can evaluate M_c and M_b by using

$$|2P_{1/2}\rangle = \frac{1}{(4\pi)^{1/2}}\begin{pmatrix}\sigma\cdot\hat{r}u g_{2P_{1/2}}\\ -if_{2P_{1/2}}u\end{pmatrix}.$$

We get

$$M_c = -iu^\dagger\sigma\cdot\hat{\epsilon}u\int r^2 dr\left[\frac{\sin k_c r}{k_c r}(g_{2S}f_{2P_{1/2}}+f_{2S}g_{2P_{1/2}})+2\left(\frac{\cos k_c r}{k_c^2 r^2}-\frac{\sin k_c r}{k_c^3 r^3}\right)g_{2P_{1/2}}f_{2S}\right] \equiv -iu^\dagger\sigma\cdot\hat{\epsilon}u\gamma \quad (4.10)$$

so that

$$\gamma = \int r^2 dr\left[\frac{\sin k_c r}{k_c r}(g_{2S}f_{2P_{1/2}}+f_{2S}g_{2P_{1/2}})+2\left(\frac{\cos k_c r}{k_c^2 r^2}-\frac{\sin k_c r}{k_c^3 r^3}\right)g_{2P_{1/2}}f_{2S}\right],$$

$$M_b = iu^\dagger\sigma\cdot\hat{\epsilon}u\int r^2 dr\left[\frac{\sin k_b r}{k_b r}(g_{1S}f_{2P_{1/2}}+f_{1S}g_{2P_{1/2}})+2\left(\frac{\cos k_b r}{k_b^2 r^2}-\frac{\sin k_b r}{k_b^3 r^3}\right)g_{2P_{1/2}}f_{1S}\right]\delta_{2S_{1/2},2P_{1/2}} \equiv \beta u^\dagger\sigma\cdot\hat{\epsilon}u \quad (4.11)$$

so that

$$\beta = \delta_{2S_{1/2},2P_{1/2}}i\int r^2 dr\left[\frac{\sin k_b r}{k_b r}(g_{1S}f_{2P_{1/2}}+f_{1S}g_{2P_{1/2}})+2\left(\frac{\cos k_b r}{k_b^2 r^2}-\frac{\sin k_b r}{k_b^3 r^3}\right)g_{2P_{1/2}}f_{1S}\right]. \quad (4.12)$$

Note that since $\delta_{2S_{1/2},2P_{1/2}}$ is imaginary if T invariance is satisfied and we neglect Γ_{2S} and Γ_{2P}, it follows that β is a real quantity under these conditions.

The remaining matrix element M_d is a bit more complicated to express, since in general it has both $E1$ and $M2$ contributions. Furthermore, there are four possible final $2P_{3/2}$ states, rather than two as for the $S_{1/2}$ or $P_{1/2}$ states. The calculation of M can be reduced to the evaluation of three radial integrals:

$$I_d^{(1)} = \int r^2 dr\, j_0(k_d r)g_{2P_{3/2}}f_{2S_{1/2}},$$

$$I_d^{(2)} = \int r^2 dr\, j_2(k_d r)g_{2P_{3/2}}f_{2S_{1/2}},$$

$$I_d^{(3)} = \int r^2 dr\, j_2(k_d r)f_{2P_{3/2}}g_{2S_{1/2}}.$$

The seven radial integrals appearing in $\delta_{2S_{1/2},2P_{1/2}}$, α, β, M_a, $I_d^{(1)}$, $I_d^{(2)}$, and $I_d^{(3)}$, as well as the muonic transition energies, were computed numerically in the following way. The charge distribution of all nuclei except ^{6}Li are assumed to be of the following form:

$$\rho \propto \left[1+\exp\left(\frac{r-C}{\lambda}\right)\right]^{-1},$$

where $C = 1.12\ \text{F}\times A^{1/3}$ and $\lambda = 0.51$ F. The charge distribution of ^{6}Li is assumed to be of the form

$$\rho \propto \left(1+\frac{r}{\alpha_1}\right)e^{-r/\alpha_1},$$

where α_1 is so chosen that the root-mean-square radius is 2.78 F. The Dirac equation is then solved numerically in the electrostatic potential of the charge distribution, corrected for the lowest-order vacuum polarization effect. The detailed numerical procedure is given in Ref. 16 by Barrett, the author of the computer program. The same program has been used extensively in the analysis of high-precision experimental data[17]; therefore, its accuracy is certainly adequate for our purpose.

The decay rates are obtained from M_a–M_d by squaring, multiplying by suitable factors, and summing over all unobserved parameters. We find

$$R_a(2S_{1/2} \to 1S_{1/2} + 1\gamma) = 4\left(\frac{e^2}{4\pi}\right) k_a |\alpha|^2 , \tag{4.13}$$

$$R_c(2S_{1/2} \to 2P_{1/2} + 1\gamma) = 4\left(\frac{e^2}{4\pi}\right) k_c |\gamma|^2 , \tag{4.14}$$

$$R_d(2S_{1/2} \to 2P_{3/2} + 1\gamma) = 2\left(\frac{e^2}{4\pi}\right) k_d \tfrac{16}{9}[|I_d^{(1)}|^2 + \tfrac{1}{2} I_d^{(1)}(3I_d^{(2)} - I_d^{(3)}) + O(|I_d^{(2)}|^2, |I_d^{(3)}|^2, I_d^{(2)} I_d^{(3)})] . \tag{4.15}$$

These rates are given for various values of Z, in Table II.

The other radiative decay that is relevant is the $2S_{1/2} \to 2S_{1/2} + 2\gamma$. We have not calculated that in detail, as it involves a sum over states. However, it is likely that for Z not too large the rate is given approximately by that for a point nucleus, corrected by the energy-dependent factor $[(E_{2S} - E_{1S})/(E_{2S} - E_{1S})_{PN}]^7$, where $(E_{2S} - E_{1S})_{PN}$ is the energy difference for a point nucleus. The rate is then[18]

$$R_{2S \to 1S + 2\gamma} \simeq 1.7 \times 10^3 Z^6 \left[\frac{E_{2S} - E_{1S}}{(E_{2S} - E_{1S})_{PN}} \right]^7 .$$

This rate is also given in Table II. We also give the branching ratio of the decay $2S \to 1S + 1\gamma$ to all the above radiative decays. For $Z \gtrsim 30$, this will approximate the over-all branching ratio, since Auger emission becomes improbable compared to radiative decays. For lower Z the over-all branching ratio is substantially reduced by Auger transitions.

Finally, in Table III we give the ratio β/α of the $E1$ to $M1$ matrix element in $2S_{1/2} \to 1S_{1/2} + 1\gamma$, calculated for a nominal value $C_p = 1$, $C_n = 0$ of the muon-nucleon parity-violating coupling constants. We shall see in Sec. V that the ratio is equal to $\frac{1}{2}$ the photon polarization, or to $\frac{1}{2}$ the angular asymmetry relative to the muon polarization direction. Thus these quantities should be on the order of 10^{-4} in the region around $Z = 30$, which might be experimentally accessible.

V. MEASURABILITY OF $2S_{1/2} \to 1S_{1/2} + 1$-PHOTON DECAY

We consider first the spin analysis of the $2S_{1/2} \to 1S_{1/2} + 1$-photon decay. Since the initial muon may retain a substantial polarization, the final-muon spin might be measurable, and the photon polarization can sometimes be measured, we shall initially not average over any of these quantities. The analysis of Sec. IV shows that the matrix element for the transitions has the general form

$$M = u_f^\dagger (i\alpha\sigma \cdot \hat{k} \times \hat{\epsilon} + \beta\sigma \cdot \hat{\epsilon}) u_i . \tag{5.1}$$

Here $u_f^\dagger$, u_i are the final and initial two-component spinors for the muon, $\hat{k}$, $\hat{\epsilon}$ are unit momentum and polarization vectors for the photon, and α, β are the $M1$ and $E1$ matrix element coefficients, given in Eqs. (4.9)–(4.11) and in Table III. As we have seen, if time-reversal invariance is satisfied, α and β are relatively real, otherwise not. For example, if the only P-violating interaction were also T-violating, then α and β would be relatively imaginary, except for a small effect due to the width of the $2P_{1/2}$ state.

The decay rate corresponding to (5.1) is given by

TABLE II. Decay rates for various transitions.[a]

Z	$R_{2S\to 1S+1\gamma}$	$R_{2S\to 2P_{1/2}+1\gamma}$	$R_{2S\to 2P_{3/2}+1\gamma}$	$R_{2S\to 1S+2\gamma}$	$R_{2S\to 1S+1\gamma}/R_{2S\to \text{all radiative}}$	$R_{2P_{1/2}\to 1S+1\gamma}$
3	3.01×10^{1}	0	0	1.2×10^{6}	2.5×10^{-5}	1.03×10^{13}
6	3.20×10^{4}	3.05×10^{5}	2.30×10^{5}	7.9×10^{7}	4.0×10^{-4}	1.66×10^{14}
11	1.20×10^{7}	7.15×10^{8}	7.61×10^{8}	2.6×10^{9}	2.9×10^{-3}	1.87×10^{15}
17	7.56×10^{8}	7.64×10^{10}	8.89×10^{10}	3.2×10^{10}	3.8×10^{-3}	1.04×10^{16}
26	3.41×10^{10}	5.68×10^{12}	6.85×10^{12}	2.5×10^{11}	2.7×10^{-3}	5.30×10^{16}
32	1.92×10^{11}	3.98×10^{13}	4.89×10^{13}	5.3×10^{11}	2.2×10^{-3}	1.14×10^{17}
35	3.89×10^{11}	9.09×10^{13}	1.12×10^{14}	7.8×10^{11}	1.9×10^{-3}	1.57×10^{17}
42	1.57×10^{12}	4.32×10^{14}	5.32×10^{14}	1.4×10^{12}	1.6×10^{-3}	2.94×10^{17}
50	5.34×10^{12}	1.76×10^{15}	2.20×10^{15}	2.2×10^{12}	1.3×10^{-3}	5.13×10^{17}
60	1.80×10^{13}	6.55×10^{15}	8.33×10^{15}	3.2×10^{12}	1.2×10^{-3}	8.75×10^{17}
74	6.18×10^{13}	2.57×10^{16}	3.41×10^{16}	3.7×10^{12}	1.0×10^{-3}	1.46×10^{18}
82	1.06×10^{14}	4.68×10^{16}	6.33×10^{16}	3.7×10^{12}	1.0×10^{-3}	1.79×10^{18}

[a] All rates are in $\sec^{-1}$. The branching ratio in column 6 is of the decay $2S \to 1S + 1\gamma$ to all radiative decays of the 2S state. The rate in column 7, that for the $2P_{1/2}$ state, is given for comparison.

$$dR_{2S\to 1S+1\gamma}=e^2|M|^2\frac{1}{4\pi^2}\tfrac{1}{2}k\,d\Omega_k\,. \tag{5.2}$$

This can be evaluated with the usual projection operator method:

$$|M|^2=\mathrm{Tr}\{[\tfrac{1}{2}(1+\sigma\cdot\hat{n}_i)](-i\alpha^*\sigma\cdot\hat{k}\times\hat{\epsilon}^*+\beta^*\sigma\cdot\hat{\epsilon}^*)\times[\tfrac{1}{2}(1+\sigma\cdot\hat{n}_f)](i\alpha\sigma\cdot\hat{k}\times\hat{\epsilon}+\beta\sigma\cdot\hat{\epsilon})\}. \tag{5.3}$$

Here $\hat{n}_i, \hat{n}_f$ are the spin directions of the initial and final muon. We have taken $\hat{\epsilon}$ to be complex, to allow for the possibility of measuring the circular polarization of the photon.

If we define a vector $\vec{A}$ by

$$\vec{A}=-i\alpha^*\hat{k}\times\hat{\epsilon}^*+\beta^*\hat{\epsilon}^*,$$

then the trace gives

TABLE III. Ratios of $E1$ to $M1$ matrix elements.[a]

Z	$\left\vert\frac{M_{E1}}{M_{M1}}\right\vert$	$-\beta/\alpha$
3	5.85×10^5	-0.338
6	7.40×10^4	$+6.25\times10^{-3}$
11	1.25×10^4	$+1.04\times10^{-3}$
17	3.73×10^3	$+3.70\times10^{-4}$
26	1.26×10^3	$+1.44\times10^{-4}$
32	7.85×10^2	$+0.958\times10^{-4}$
35	6.48×10^2	$+0.803\times10^{-4}$
42	4.45×10^2	$+0.579\times10^{-4}$
50	3.23×10^2	$+0.426\times10^{-4}$
60	2.33×10^2	$+0.319\times10^{-4}$
74	1.67×10^2	$+0.225\times10^{-4}$
82	1.44×10^2	$+0.190\times10^{-4}$

[a] The value of β/α is calculated for $C_p=1$, $C_n=0$. The actual value of β/α depends on C_p and C_n by $\beta/\alpha \propto C_p+C_n(A-Z)/Z$. The photon circular polarization, or photon angular asymmetry for a polarized muon, in the decay $2S_{1/2}\to 1S_{1/2}+1\gamma$ is given by $2\beta/\alpha\,(1+\beta^2/\alpha^2)^{-1}$.

$$\begin{aligned}|M|^2=&\tfrac{1}{2}[\vec{A}\cdot\vec{A}^*(1-\hat{n}_i\cdot\hat{n}_f)+i(\hat{n}_i-\hat{n}_f)\cdot\vec{A}\times\vec{A}^*+\hat{n}_i\cdot\vec{A}\,\hat{n}_f\cdot\vec{A}^*+\hat{n}_i\cdot\vec{A}^*\hat{n}_f\cdot A]\\
=&\tfrac{1}{2}[(1-\hat{n}_i\cdot\hat{n}_f)(|\alpha|^2+|\beta|^2)\hat{\epsilon}\cdot\hat{\epsilon}^*+(1-\hat{n}_i\cdot\hat{n}_f)\hat{k}\cdot\hat{\epsilon}\times\hat{\epsilon}^*(-i)(\alpha^*\beta+\alpha\beta^*)\\
&+i(\hat{n}_i-\hat{n}_f)\cdot\hat{\epsilon}^*\times\hat{\epsilon}\,|\alpha|^2-i(\hat{n}_i-\hat{n}_f)\cdot\hat{k}\hat{k}\cdot\hat{\epsilon}^*\times\hat{\epsilon}\,|\beta|^2\\
&+(\hat{n}_i-\hat{n}_f)\cdot\hat{k}\hat{\epsilon}\cdot\hat{\epsilon}^*(\alpha\beta^*+\beta^*\alpha)+(\hat{n}_i\cdot\hat{\epsilon}\hat{n}_f\cdot\hat{\epsilon}^*+\hat{n}_i\cdot\hat{\epsilon}^*\hat{n}_f\cdot\hat{\epsilon})|\alpha|^2\\
&+(\hat{n}_i\cdot\hat{k}\times\hat{\epsilon}\hat{n}_f\cdot\hat{k}\times\hat{\epsilon}^*+\hat{n}_i\cdot\hat{k}\times\hat{\epsilon}^*\hat{n}_f\cdot\hat{k}\times\hat{\epsilon})|\beta|^2\\
&-i\hat{n}_i\cdot\hat{\epsilon}^*\hat{n}_f\cdot\hat{k}\times\hat{\epsilon}\alpha^*\beta-i\hat{n}_i\cdot\hat{k}\times\hat{\epsilon}\hat{n}_f\cdot\hat{\epsilon}^*\alpha^*\beta\\
&+i\hat{n}_i\cdot\hat{k}\times\hat{\epsilon}^*\hat{n}_f\cdot\hat{\epsilon}\alpha\beta^*+i\hat{n}_i\cdot\hat{\epsilon}\hat{n}_f\cdot\hat{k}\times\hat{\epsilon}^*\alpha\beta^*].\end{aligned} \tag{5.4}$$

It can be seen that only the last four terms involve the combination $\alpha^*\beta-\alpha\beta^*$, which is sensitive to an imaginary relative phase of α and β. Hence, aside from the small effect due to the $2P_{1/2}$ width, only these terms are sensitive to a T-violating interaction. Unfortunately, these terms disappear upon summing over any polarization or spin, as can be seen from the formulas below:

$$\begin{aligned}\tfrac{1}{2}\sum_{\hat{n}_i}|M|^2=&\tfrac{1}{2}[(|\alpha|^2+|\beta|^2)\hat{\epsilon}\cdot\hat{\epsilon}^*-i\hat{k}\cdot\hat{\epsilon}\times\hat{\epsilon}^*(\alpha^*\beta+\alpha\beta^*)-i\hat{n}_f\cdot\hat{\epsilon}^*\times\hat{\epsilon}\,|\alpha|^2+i\hat{n}_f\cdot\hat{k}\hat{k}\cdot\hat{\epsilon}^*\times\hat{\epsilon}\,|\beta|^2\\
&-\hat{n}_f\cdot\hat{k}\hat{\epsilon}\cdot\hat{\epsilon}^*(\alpha\beta^*+\alpha^*\beta)],\end{aligned} \tag{5.5}$$

$$\begin{aligned}\sum_{\hat{n}_f}|M|^2=&[(|\alpha|^2+|\beta|^2)\hat{\epsilon}\cdot\hat{\epsilon}^*-i\hat{k}\cdot\hat{\epsilon}\times\hat{\epsilon}^*(\alpha^*\beta+\alpha\beta^*)+i\hat{n}_i\cdot\hat{\epsilon}^*\times\hat{\epsilon}\,|\alpha|^2-i\hat{n}_i\cdot\hat{k}\hat{k}\cdot\hat{\epsilon}^*\times\hat{\epsilon}\,|\beta|^2\\
&+\hat{n}_i\cdot\hat{k}\hat{\epsilon}\cdot\hat{\epsilon}^*(\alpha\beta^*+\alpha^*\beta)],\end{aligned} \tag{5.6}$$

$$\sum_{\hat{\epsilon}}|M|^2=[(1-\hat{n}_i\cdot\hat{n}_f)(|\alpha|^2+|\beta|^2)+(\hat{n}_i-\hat{n}_f)\cdot\hat{k}(\alpha\beta^*+\beta\alpha^*)+(\hat{n}_i\cdot\hat{n}_f-\hat{n}_i\cdot\hat{k}\hat{n}_f\cdot\hat{k})(|\alpha|^2+|\beta|^2)], \tag{5.7}$$

$$\sum_{\hat{\epsilon},\hat{n}_f}|M|^2=2[|\alpha|^2+|\beta|^2+\hat{n}_i\cdot\hat{k}(\alpha\beta^*+\beta^*\alpha)], \tag{5.8}$$

$$\tfrac{1}{2}\sum_{\hat{n}_i,\hat{n}_f}|M|^2=(|\alpha|^2+|\beta|^2)\hat{\epsilon}\cdot\hat{\epsilon}^*-i\hat{k}\cdot\hat{\epsilon}\times\hat{\epsilon}^*(\alpha\beta^*+\beta^*\alpha). \tag{5.9}$$

If we identify $i\hat{\epsilon} \times \hat{\epsilon}^* = \vec{n}_\gamma$ as the circular polarization vector of the photon, then we see from (5.8) and (5.9) that a measurement of either the photon asymmetry relative to the muon polarization or of the photon circular polarization determines the quantity

$$r = \frac{\alpha\beta^* + \alpha^*\beta}{|\alpha|^2 + |\beta|^2} \simeq 2\left|\frac{\beta}{\alpha}\right| \cos\phi\,, \qquad (5.10)$$

where ϕ is the phase angle between α and β. It seems probable that providing that the muons reaching the $2S_{1/2}$ state retain a significant amount of polarization, it would be easier to measure the angular asymmetry than the circular polarization. However, to answer this question requires a detailed study of possible experiments, and is therefore beyond the scope of this paper.

Some values of r are given in Table III. We note that r is proportional to the coupling constants of the muon-nucleon interaction, so that the sign of the polarization, or asymmetry, is determined by the sign of these couplings. This sign, as well as the magnitude of the coupling, is of direct theoretical interest.

It may appear surprising that no T-violating terms occur if the photon polarization is not measured, since it is known that terms such as $\hat{n}_i \cdot \hat{n}_f \times \hat{k}$ involving T violation occur in the spin analysis of a decay such as $\Lambda^0 \rightarrow p + \pi^-$, in which a spin-zero particle is emitted. One might naively expect that such terms would remain in the decay we are considering here after summing over photon polarization. That this is not the case follows from the transverseness of the photon polarization. It can be seen from the fact that the matrix element (5.1) is invariant under the substitutions

$$\begin{aligned} &\hat{\epsilon} \rightarrow \hat{k} \times \hat{\epsilon}\,, \quad \alpha \rightarrow -i\beta\,, \\ &\hat{k} \times \hat{\epsilon} \rightarrow \hat{\epsilon}\,, \quad \beta \rightarrow i\alpha\,, \\ &\hat{k} \rightarrow -\hat{k}\,, \end{aligned} \qquad (5.11)$$

so that

$$\mathrm{Re}(\alpha^*\beta) \rightarrow -\mathrm{Re}(\alpha^*\beta)\,,$$

$$\mathrm{Im}(\alpha^*\beta) \rightarrow +\mathrm{Im}(\alpha^*\beta)\,.$$

Consequently, the decay rate summed over polarization cannot contain a term like

$$\hat{n}_i \cdot \hat{n}_f \times \hat{k}\, \mathrm{Im}(\alpha^*\beta)\,,$$

which changes sign under the substitution. On the other hand, a quantity like $\mathrm{Re}(\alpha^*\beta)\hat{n}_i \cdot \hat{n}_f \times \hat{k}$ cannot occur by time-reversal considerations. Hence all terms proportional to $\hat{n}_i \cdot \hat{n}_f \times \hat{k}$ vanish after summing over photon polarization. This conclusion may not be true for a quantity that gets a contribution from the longitudinal field, such as electron asymmetry in Auger emission. For our case, the small value of $|\beta/\alpha|$ makes it very doubtful that a T violation can be detected through a measurement of a correlation that requires the measurement of all the possible polarizations.

An examination of Table II indicates that the branching ratio of $2S_{1/2} \rightarrow 1S_{1/2} + 1\gamma$ to all $2S_{1/2}$ decays has a maximum value of several times 10^{-3} at $Z \sim 30$. The actual number of such decays seen, per stopped muon, will be this branching ratio multiplied by the fraction of muons that reach the $2S_{1/2}$ state. Estimates of this fraction vary from element to element, and give numbers of the order of 5%. Therefore, the expected value for the fraction of all stopped muons decaying by $2S_{1/2} \rightarrow 1S_{1/2} + 1\gamma$ will be something like 10^{-4}, for elements with $Z \sim 30$. This low branching ratio would not by itself rule out the detection of the transition in a high-statistics experiment.

In elements with $Z \sim 30$, the $2S$-$1S$ energy difference is about 2 MeV, while the $2S$-$2P$ energy difference is about 50 keV. Hence, to distinguish the $2S$-$1S$ transition from the $2P$-$1S$ transition requires an energy resolution of about 1%, which seems easily attainable.[17]

Another source of background for the $2S \rightarrow 1S + 1\gamma$ transition is the tail of the $2S \rightarrow 1S + 2\gamma$ transition, in which one photon takes almost all the available energy. Since the number of photons in the tail goes as $(E_{2S} - E_{1S} - E_\gamma)^4/(E_{2S} - E_{1S})^4$, we see that the number of photons with $E_\gamma = E_{2S} - E_{1S} - \Delta E$, where $\Delta E < 50$ keV, will be only 10^{-6} of all photons emitted in $2S \rightarrow 1S + 2\gamma$ decay, which provides a negligible background for $Z \simeq 30$.

Next we consider as a background the "mixed" $2S \rightarrow 1S + 1\gamma +$ "Auger" transition described by Ruderman.[19] This involves a mixing of the muonic $2S_{1/2}$ and $2P_{1/2}$ states, combined with a mixing of opposite parity states of the atomic electrons through their Coulomb interaction with the muon, and leads to a transition in which an $E1$ photon is emitted with about the same energy as the $2S \rightarrow 1S$ transition, the small difference being taken up by an electron rearrangement. Of course, this type of mixing does not give any circular polarization or other parity-violating effects, since it is the result of the parity-conserving Coulomb interaction. Indeed, the change of electronic state involved implies that there is no interference between the $E1$ and $M1$ amplitudes. However, their effect must be considered as a source of photons with about the energy $E_{2S} - E_{1S}$. Ruderman's estimate is not applicable to atoms with $Z \sim 30$, because he took the $2S$-$2P$ energy difference as small compared to the electronic energy differences. In the case of interest to us, his estimate of the "mixed" decay rate must be reduced by a factor of about

$$\left[\frac{m_e}{m_\mu}\frac{1}{(Z\alpha)^2}\right]^2 \sim \frac{10^4}{Z^4},$$

or about a factor of 10^{-2} for $Z \sim 30$. Combined with his result, this gives a value for the mixed radiative decay rate of about 3×10^8 for $Z=30$, which is several orders of magnitude less than the $M1$ rate.

Other kinds of background for the $2S \to 1S + 1\gamma$ transitions will depend on the details of the experimental setup, and cannot be usefully discussed here. It would appear to us that a reasonable program would involve first trying to detect the transition, in a variety of elements with $Z \sim 30$. If this is accomplished, an experiment with much higher statistics could be envisaged in an effort to detect the circular polarization or angular asymmetry of the photon. In view of the great interest in the neutral-current interactions, we believe that such a program is well worth the consideration of experimentalists.

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APPENDIX

We discuss briefly in this appendix the parity mixing in electronic hydrogenlike ions of high Z. The interaction H_1 will also occur between electrons and nucleons in some models in which it occurs for muons. However, there exist models in which H_1 occurs for electrons, but not muons, or conversely, or for both, but with different coupling constants.[20] For that reason, we denote the electron coupling constants by $C^{\text{el.}}$, to distinguish them from the muon couplings previously treated. The interaction will cause a parity mixing in the levels of hydrogenlike ions along the lines we have calculated for muonic atoms. The effect will be much smaller in electronic ions because the probability of finding the electron at the nucleus is much smaller than that for the muon. To some extent, this is compensated for by the smaller energy difference between $2S_{1/2}$ and $2P_{1/2}$ in electronic ions.

To estimate the effect, we can make the approximations leading to Eq. (3.18), i.e., a point nucleus, and the leading term in powers of $Z\alpha$. We then obtain for the $2S_{1/2}$ and $2P_{1/2}$ state of electron hydrogenic ions[21]

$$\langle 2S_{1/2}|H_1|2P_{1/2}\rangle_{\text{el.}} \cong \frac{iGm_e^2}{\sqrt{2}}\frac{[C_p^{\text{el.}}+C_n^{\text{el.}}(A-Z)/Z]}{4\pi\alpha} \times (Z\alpha)^5\tfrac{1}{8}\sqrt{3}\,m_e. \qquad \text{(A1)}$$

The energy difference is now the Lamb shift. We have estimated this in hydrogenic ions by taking the numerical values for the Lamb shift of high-Z ions given by Erickson.[22] These give an answer of the form

$$E_{2S_{1/2}} - E_{2P_{1/2}} \simeq \frac{\alpha(Z\alpha)^4}{6\pi} m_e F(Z), \qquad \text{(A2)}$$

where $F(Z)$ is a function decreasing from 7.7 for $Z=1$ to about 1.5 for $Z=40$, and then decreasing slowly beyond $Z=40$.

Furthermore, in electron hydrogenic ions, the $2P_{1/2}$ width is a substantial fraction of the $2S_{1/2} - 2P_{1/2}$ energy difference. Therefore, we must write

$$\delta^{\text{el.}}_{2S_{1/2},2P_{1/2}} = \frac{\langle 2S_{1/2}|H_1|2P_{1/2}\rangle}{E_{2S_{1/2}} - E_{2P_{1/2}} + \frac{1}{2}i(\Gamma_{2P_{1/2}} - \Gamma_{2S_{1/2}})}\,. \qquad \text{(A3)}$$

Here $\Gamma_{2P_{1/2}} \sim Z^4(6\times 10^8\ \text{sec}^{-1})$, which is about $\frac{1}{10}$ of $E_{2S_{1/2}} - E_{2P_{1/2}}$, while $\Gamma_{2S_{1/2}}$ is negligible. We can therefore write approximately

$$\delta^{\text{el.}}_{2S_{1/2},2P_{1/2}} \simeq \frac{iGm_e^2}{\sqrt{2}}\frac{3\pi\sqrt{3}}{2}\frac{[C_p^{\text{el.}}+C_n^{\text{el.}}(A-Z)/Z]}{4\pi\alpha} \times \frac{Z}{F(Z)}, \qquad \text{(A4)}$$

where $Z/F(Z)$ is given in Table IV. The quantity $3\sqrt{3}\,Gm_e^2/8\alpha\sqrt{2}$, which sets the scale of the effect, is a quite small number:

$$\frac{3\sqrt{3}}{8\alpha\sqrt{2}}Gm_e^2 \simeq 2\times 10^{-10}.$$

TABLE IV. Parity mixing and polarization in electronic ions.[a]

Z	$Z/F(Z)$	$i\delta^{\text{el}}_{2S,2P}$	Circular polarization	$\frac{R_{2S\to 1S+1\gamma}}{R_{2S\to 1S+2\gamma}}$
1	0.129	2.58×10^{-11}	4.08×10^{-4}	$3\ \times 10^{-7}$
5	1.04	2.08×10^{-10}	2.63×10^{-5}	1.9×10^{-4}
10	2.66	5.32×10^{-10}	8.40×10^{-6}	$3\ \times 10^{-3}$
15	4.76	9.52×10^{-10}	4.46×10^{-6}	1.5×10^{-2}
20	7.38	1.47×10^{-9}	2.93×10^{-6}	4.8×10^{-2}
25	10.3	2.07×10^{-9}	2.08×10^{-6}	1.2×10^{-1}
30	13.6	2.72×10^{-9}	1.59×10^{-6}	2.4×10^{-1}
35	17.5	3.50×10^{-9}	1.29×10^{-6}	4.5×10^{-1}
40	21.3	4.26×10^{-9}	1.05×10^{-6}	7.7×10^{-1}
45	25.7	5.14×10^{-9}	0.89×10^{-7}	1.2
50	31.1	6.22×10^{-9}	0.77×10^{-7}	1.9

[a] Both $\delta^{\text{el}}_{2S,2P}$ and the circular polarization are calculated for $C_p^{\text{el.}}=\frac{1}{2}$, $C_n^{\text{el.}}=0$. For other values, the result can be obtained from Eq. (A4).

From Table IV, we see that the factor $Z/F(Z)$ ranges from 0.13 to about 30 over the periodic table. Consequently, the admixture of $2P_{1/2}$ into $2S_{1/2}$ is probably $<10^{-8}$ for electronic hydrogenic ions.

We can combine these results with the point-nucleus estimates of the $E1$ and $M1$ matrix elements for $2S_{1/2} \to 1S_{1/2} + 1\gamma$, given in Sec. II, to obtain an estimate of the photon circular polarization. This gives

$$\text{circular polarization} \simeq \frac{6\times10^{-3}}{Z^2F(Z)}, \tag{A5}$$

which is always substantially smaller than in muonic atoms. However, this must be balanced against the fact that for electronic hydrogenic ions, the $2S_{1/2} \to 1S_{1/2} + 1\gamma$ decay has a much larger branching ratio because the $2S \to 2P$ decays have a proportionately much lower energy release and do not compete with it. In such ions, the other important decay mode is $2S_{1/2} \to 1S_{1/2} + 2\gamma$, and the branching ratio of these is approximately

$$\frac{R_{2S\to1S+1\gamma}}{R_{2S\to1S+2\gamma}} \sim 3\times10^{-7}Z^4, \tag{A6}$$

which becomes equal to 1 at $Z=42$. This suggests that the small circular polarization of the photon in the $2S_{1/2} \to 1S_{1/2} + 1\gamma$ decay of hydrogenic ions may be detectable if ions of such high Z can be produced.

It is also worth noting that because of the substantial imaginary term in the denominator of (A3), a parity mixing due to a P- and T-violating electron-nucleon interaction might also be detectable through a measurement of circular polarization.

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[1]See D. A. Jenkins *et al.*, Nucl. Phys. A175, 73 (1971); Phys. Lett. 32B, 267 (1970). Also H. L. Anderson *et al.*, Phys. Rev. Lett. 22, 221 (1969).

[2]R. Marrus and R. W. Schmieder, Phys. Rev. A 5, 1160 (1972).

[3]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967), and many others afterwards.

[4]Originally by G. Breit and E. Teller, Astrophys. J. 91, 215 (1940); later, with a slight correction, by G. W. F. Drake, Phys. Rev. A 3, 908 (1971), and by G. Feinberg and J. Sucher, Phys. Rev. Lett. 26, 681 (1971).

[5]We have taken the well-known rate for electron hydrogenic ions, which is $8.2Z^6$ sec^{-1}, and scaled it by the mass ratio to obtain this result. See Ref. 18.

[6]See for example the discussions by C. S. Wu and L. Wilets, Annu. Rev. Nucl. Sci., 19, 527 (1969).

[7]E. L. Church and J. Weneser, Phys. Rev. 103, 1035 (1956).

[8]We have taken the results given by E. H. S. Burhop, in *The Auger Effect* (Cambridge Univ. Press, Cambridge, England, 1952), Sec. 5.7.

[9]G. R. Burbridge and H. A. de Borde, Phys. Rev. 89, 189 (1953). Their result is in approximate agreement in light nuclei with an adaptation of the calculation of Ref. 7.

[10]For example, the model given in Ref. 3. Other models, such as one given by H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972), do not have such currents.

[11]The effect of an interaction of this type between electrons and protons, arising from an electron "pseudocharge" distribution, was considered some time ago by B. Sakitt and G. Feinberg, Phys. Rev. 151, 1341 (1966). The considerations of the present paper may also be used to relate the parity mixing in muonic atoms to a muonic "pseudocharge" distribution. The relation is such that the muonic pseudocharge radius $(\lambda/m_\mu^2)^{1/2}$ is equivalent to $[(G/\sqrt{2})(C_p/4\pi\alpha)]^{1/2}$. With that substitution, the analysis of this paper may be used together with experiments, to detect a muonic pseudocharge parameter λ as small as 10^{-6}.

[12]We use the notation and convention of H. A. Bethe and E. Salpeter, in *Handbuch der Physik*, edited by S. Flügge (Springer, Berlin, 1957), Vol. 35, p. 88.

[13]E. Salpeter, Phys. Rev. 112, 1642 (1958).

[14]G. Feinberg, Phys. Rev. 112, 1637 (1958).

[15]T. D. Lee, Phys. Rev. D 8, 1226 (1973); also Phys. Rep. 9C, 143 (1974).

[16]R. C. Barrett, in *Muonic Physics*, edited by C. S. Wu and V. Hughes (Academic, to be published).

[17]C. S. Wu and L. Wilets, Ref. 6.

[18]J. Shapiro and G. Breit, Phys. Rev. 113, 179 (1959).

[19]M. A. Ruderman, Phys. Rev. 118, 1632 (1960).

[20]H. Georgi and A. Pais, Phys. Rev. D (to be published).

[21]A more accurate approximation, which does not neglect higher powers of $Z\alpha$, is to multiply (A1) by the factor

$$[\tfrac{1}{3}(2\gamma_1+1)]^{1/2}\left(\frac{2}{1+\gamma_1}\right)^{5/2}\frac{2}{\Gamma(2\gamma_1+1)} \times \exp\left[2(\gamma_1-1)\ln\left(\frac{\sqrt{2}ZR_n}{(1+\gamma_1)^{1/2}a_0}\right)\right],$$

where

$$\gamma_1 = [1-(Z\alpha)^2]^{1/2}$$

and R_n is the nuclear radius. This factor would increase $\delta^{\text{el.}}_{2S_{1/2},2P_{1/2}}$ by about a factor of 2 for $Z\sim50$, and have a similar effect in the circular polarization.

[22]G. W. Erickson, Phys. Rev. Lett. 27, 780 (1971).