strictions and additional counterterms.

(v) The Adler-Baker-Johnson function changes to some other function which now depends on the strong coupling as well. This new function is again the coefficient of a term with a single power of $\ln(q^2)$ (for $q^2 \to \infty$) in $\alpha\pi_c^{[1]}(q^2)$. (This together with the assumption of the vanishing of β does not necessarily overdetermine the parameters of the theory.) The whole attractive idea of determining α *within* pure electrodynamics,[1] according to the present dynamics with such an alternative solution, may be destroyed.

Based on the above (i)–(v) points, the author feels that the solution given in the bulk of the present paper is far more *physically* and *technically* attractive than the alternative type of solution presented in this appendix.

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Stability of the eigenvalue condition for the fine-structure constant α and short-distance behavior in strong interaction. II

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In the preceding paper, we have made a preliminary study of the short-distance behavior of quantum electrodynamics in the presence of neutral-meson theory with pseudoscalar-pseudoscalar coupling. We have learned in particular, by an elementary summation procedure, that in the single-fermion-loop contribution to the renormalized photon self-energy part, the Adler-Baker-Johnson eigenvalue condition for the fine-structure constant α, $F^{[1]}(x)|_{x=\alpha} = 0$, remains unaltered. We extend our results, in the single-fermion-loop context, to the pion self-energy part. As a consequence of the vanishing of the effective strong coupling at high energies, π^0-π^0 scattering graphs are finite and no ϕ^4 counterterm is required. We finally infer from the above work that the photon self-energy part in the *multi*loop contribution is asymptotically finite at the eigenvalue $[F^{[1]}(x)|_{x=\alpha} = 0]$ independently of the value of the strong coupling. The point $x = \alpha$ is the assumed (infinite order) zero of the single-loop electromagnetic-current-correlation functions in mass-zero pure electrodynamics.

In the preceding paper[1] we have studied the short-distance behavior of quantum electrodynamics in the presence of neutral-meson theory with pseudoscalar-pseudoscalar (ps-ps) coupling without closed fermion loops. The scaling equations for the various components (propagators, vertices, etc.) of the theory have been solved. It was then shown that in the single-closed-fermion-loop contribution $\pi_c^{[1]}$ to the (renormalized) photon self-energy part, the Adler-Baker-Johnson eigenvalue condition[2,3] for the (renormalized) fine-structure constant[2] α, $F^{[1]}(\alpha) = 0$, remains *unaltered*. This means that a possible zero of $F^{[1]}(x)$ does *not* "move" in the presence of the strong coupling. This leads to the beautiful idea that the value of α may be possibly determined within *pure* electrodynamics[2] (i.e., electrodynamics in isolation from the rest of the world). The mechanism which is responsible for the stability of the eigenvalue condition is that the *effective* strong coupling vanishes at high energies.[1] The approach we have used for the investigation of the above problem was through a study of the Callan-Symanzik scaling equations.[4] The method we assumed to study these scaling equations was to make an expansion in powers of the strong coupling and treat the electromagnetic coupling intact (i.e., to all orders) and then solve these equations by resumming back in the strong coupling in an elementary fashion. The effective strong coupling was then easily identified. This is a particular way of summation as clearly stated in Ref. 1 and it allowed us to treat the explicit derivative, appearing in the scaling equations, with respect to the strong coupling in a straightforward manner. [Roughly speaking, our way of summation corresponds to first summing virtual-photon corrections with all virtual-pion "variables" held fixed and then carrying out the virtual-pion integrations. This is suggestive from the scaling equations because of the presence of the derivative with respect to the strong coupling, as mentioned above (even in the absence of closed fermion loops) in the just-mentioned equations. We shall come back to this point later.] It is interesting to point out that the effective strong coupling (in this *Abelian* gauge field theory) vanishes even *faster* than in *non*-Abelian gauge field theories[5] and hence is of great practical interest. Thus we suggest that the definition of so-called asymptotic freedom be general enough to embrace such a situation. We have discussed in Ref. 1 how to check the stability of the eigenvalue condition for α, in the above sense, in other field theories as well. For the convenience of the reader, a brief summary of the study of the stability of the eigenvalue condition $F^{[1]}(\alpha)=0$ is given in the Appendix.

In the present work we wish to extend our results to the single-fermion-loop contribution to the pion self-energy part (and the scattering of the pion-photon system). π^0-π^0 scattering graphs vanish quite rapidly in the ultraviolet region and hence *no* ϕ^4 counterterm is needed in the theory. We finally discuss, with no rigor, the multiloop contribution to the photon self-energy part and the stability of the eigenvalue condition for α in the above sense in the full theory.

The unrenormalized single-fermion-loop contribution to the pion self-energy part $\Sigma_\pi(q^2)$ is given by

$$\Sigma_\pi^{[1]}(q^2)=ig^2(Z_5/Z_2)^2 \times\int\frac{(dp)}{(2\pi)^4}\,\mathrm{Tr}\,[\gamma_5 S(p+q)\,\Gamma_5(p+q,p)\,S(p)]\,, \tag{1}$$

where $Z_4^{[0]}$ (the pion wave-function renormalization constant) is equal to one, Z_5 is the strong-vertex (Γ_5) renormalization constant,[1] and g^2 is the renormalized strong coupling (squared). In perturbation theory, $\Sigma_\pi^{[1]}$ has both quadratic and logarithmic divergences. We may formally introduce the renormalized (inverse) pion propagator in the single-fermion-loop context:

$$\tilde{D}_\pi^{-1}(q^2)=[Z_4(q^2+\mu_0^2+\Sigma_\pi(q^2))]^{[1]} \equiv q^2+\mu^2+\tilde{\Sigma}_\pi^{[1]}(q^2)\ . \tag{2}$$

The parameters μ and μ_0 denote the renormalized and the unrenormalized masses of the pion. Now we rely on the gauge invariance of the pion self-energy part and work in the gauge in which Z_5 is *finite*. This gauge has been explicitly written down in Ref. 1. We first consider the operation of the object

$$[m(\partial/\partial m)+g^2\beta(\partial/\partial g^2)]\equiv L$$

on $\Sigma_\pi^{[1]}(q^2)$ (with a fixed ratio kept for μ/m—see the Appendix), where β has been defined in the Appendix. We note that

$$Z_2\,LS^{-1}=(L+\gamma_2)\,\tilde{S}^{-1}\equiv L_2\tilde{S}^{-1}$$

and

$$(Z_5/Z_2)\,L(S\Gamma_5)=[L+(\beta/2)]\,(\tilde{S}\tilde{\Gamma}_5)\equiv L_\beta\tilde{S}\tilde{\Gamma}_5\ ,$$

where $(1/Z_2)\,LZ_2=-\gamma_2$, Z_2 is the proton wave-function renormalization constant, and the "tilde" sign denotes, as usual, renormalized quantities. We may define

$$L\Sigma_\pi^{[1]}(q^2)=I_\pi^{[1]}(q^2)\ , \tag{3}$$

and note that $I_\pi^{[1]}(q^2)$ is (symbolically) given by

$$I_\pi^{[1]}=-ig^2Z_5\int\mathrm{Tr}\,[\gamma_5\tilde{S}\tilde{\Gamma}_5\tilde{S}\,(L_2\tilde{S}^{-1})\tilde{S}-\gamma_5(L_\beta\tilde{S}\tilde{\Gamma}_5)\tilde{S}]\,. \tag{4}$$

By an elementary application of the scaling equations in Ref. 1 [see in particular Eqs. (27) and (31) in Ref. 1] we formally see that in the gauge in which Z_5 is finite, the integral in (4) behaves like

$$\int^\infty\frac{dp^2}{p^2}\,(p^2)^{-\beta_0(\alpha)/2}$$

for $p^2\to\infty$ ($e^2\neq0$) [see the definition of $\beta_0(\alpha)$ in the Appendix (which is determined in pure electrodynamics) which lies in the self-consistency range $2>\beta_0(\alpha)/2>0$] upon taking into consideration that the trace of an odd number of γ matrices is zero. Hence $I_\pi^{[1]}(q^2)$ is a finite object. From Eq. (2) we then have, in the single-fermion-loop context, the following equation:

$$\mu^2+L\tilde{\Sigma}_\pi^{[1]}(q^2)=\rho^{[1]}(q^2+\mu^2)+2Z_4^{[1]}\mu^2 +\mu^2\Delta^{[1]}+I_\pi^{[1]}(q^2)\ , \tag{5}$$

where we have arbitrarily defined $L\mu_0^2=(2+\Delta^{[1]})\mu^2$ and $LZ_4^{[1]}=\rho^{[1]}$. We note that $Z_4^{[1]}$ is finite (see also below). The reason is that to investigate its finiteness, we have to extract two powers of q in (1) and by working in the gauge in which Z_5 is finite we again see that the resulting integral behaves like

$$\int^{\infty} (dp^2/p^2)(p^2)^{-\beta_0(\alpha)/2}$$

for $p^2 \to \infty$ ($e^2 \neq 0$). The self-mass of the neutral pion, however, is not necessarily finite since we cannot extract two powers of q in (1) to investigate its finiteness and is quadratically divergent in perturbation theory. From Eq. (5) we may then infer that both $\rho^{[1]}$ and $\Delta^{[1]}$ (choose $q^2 = -\mu^2$) are also finite (with $e^2 \neq 0$). The scaling equation for $Z_4^{[1]}$ is given by

$$LZ_4^{[1]} = \rho^{[1]}, \tag{6}$$

and by following the procedure given in the Appendix we see that

$$Z_4^{[1]} \underset{\Lambda^2 \to \infty}{\sim} \text{finite} + O((m^2/\Lambda^2)^{\beta_0/2}) , \tag{7}$$

and as expected is finite in the limit of infinite cutoff $\Lambda^2 \to \infty$. The unrenormalized mass of the pion satisfies (in detail and note that $(\mu_0{}^2)^{[0]} = \mu^2$)

$$[m(\partial/\partial m) + 2 + g^2\beta(\partial/\partial g^2)]\, F = 2 + \Delta^{[1]},$$

with

$$\mu_0{}^2 = \mu^2 \times F . \tag{8}$$

The most general solution of (8) is given by

$$\mu_0{}^2 \underset{\Lambda^2 \to \infty}{\sim} \mu^2 \times \text{finite} + g^2\Lambda^2[c_1(m^2/\Lambda^2)^{\beta_0/2} + c_2(m^2/\Lambda^2)^{2(\beta_0/2)} + \cdots], \tag{9}$$

where the over-all Λ^2 factor appears because of the presence of the term 2 on the left-hand side of the first part of Eq. (8). For the internal consistency of the theory for the finiteness of $\mu_0{}^2$ (in the single-fermion-loop context), we must have at least one of the following three immediate conditions to be satisfied:

(i) All the coefficients $c_1, c_2, \ldots$ (which depend on α and g^2 as well) vanish simultaneously.[6] This possibility is not attractive and may possibly overdetermine the parameters in the theory.

(ii) Some of the just-mentioned coefficients vanish and $(\beta_0/2)$ is large enough to make the remaining terms vanish as $\Lambda^2 \to \infty$. An interesting situation, for example, would be $c_1 = 0$ and $\beta_0 > 1$. The coefficient c_1 is given by

$$c_1(\alpha, g^2) = c_1(\alpha, 0)\exp\left[-\int_0^{g^2} (dg'^2/g'^2)(\beta_0 + \beta)/\beta\right],$$

where the exponential is some finite expression. [The vanishing of $c_1(\alpha, 0)$ (which is determined in pure electrodynamics) would then be sufficient in this case. This condition with the vanishing of $F^{[1]}(\alpha)$ may overdetermine the value of α, however.]

(iii) The parameter $(\beta_0/2)$ lies in the range $2 > (\beta_0/2) > 1$, which is in the self-consistent range $2 > (\beta_0/2) > 0$ (see the Appendix and Ref. 1), and the above-mentioned coefficients remain arbitrary.

We shall not dwell upon these various possibilities. The third possibility seems, however, the most attractive one, and is quite suitable, since it allows one to extend the discussion of the finiteness result formally to the full theory by power-counting arguments. [To be honest, however, we should make the following remark concerning this latter possibility. Each time we apply the operator L to a certain amplitude, we introduce a new amplitude (the inhomogeneous part) which has at least one higher power (κ^{-1}, where the various external momenta are scaled by multiplying them by κ) of momentum decrease asymptotically ($\kappa \to \infty$) multiplied by arbitrary powers of $\ln(\kappa)$. Accordingly, the inhomogeneous part may be, generally speaking, neglected in comparison with the homogeneous part, asymptotically, if $(\beta_0/2) < 1$. If we assume, however, that the arbitrary powers of $\ln(\kappa)$ multiplying the inhomogeneous part do sum up to the form $(\kappa)^{-(\beta_0/2)}$, then the neglect of the inhomogeneous part, asymptotically, is certainly permissible. It is not difficult to check that such a solution is consistent with the scaling equations in Ref. 1 by repeated application of L to the inhomogeneous parts of the just-mentioned equations[1] in succession. We make no attempt, however, to actually prove the correctness of such a solution. In the work of Johnson, Baker, and Willey[7] such questions are automatically avoided.]

We may similarly study a general amplitude $\pi_{2n,r}^{[1]}(q_1, \ldots, q_{2n}; k_1, \ldots, k_r)$ with $2n$ external photon lines and r pion lines ($r \geq 1$) containing *one* closed fermion loop. Such a process is shown in Fig. 1. If we fix one of the integrations, say the one over loop p shown in Fig. 1, then all the remaining subintegrations are finite in perturbation theory. The final integration over p yields, however, generally a divergence of logarithmic type in perturbation theory. Clearly, $\pi_{2n,r}^{[1]}$ may be readily expressed in terms of renormalized components (note that $Z_3^{[0]} = 1$, $Z_4^{[0]} = 1$). Since such a process should be gauge-invariant, we may work in the gauge in which Z_2 is finite. In this gauge, $\tilde{\Gamma}_5(p, p) \sim \gamma_5 O((m^2/p^2)^{\beta_0/4})$ for $p^2 \to \infty$ ($e^2 \neq 0$, see Ref. 1). Accordingly, in a skeleton expansion of $\pi_{2n,r}^{[1]}$, with exact vertices, the integration over p (see Fig. 1) is actually finite. Now we scale the momenta $q_i \to \kappa q_i$,

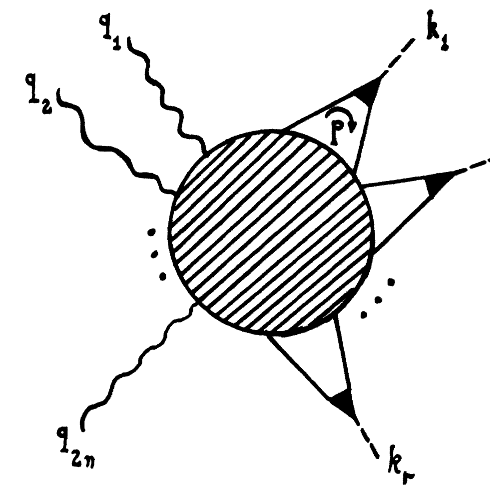

FIG. 1. Diagram representing the $(2n+r)$-point function $\pi^{[1]}_{2n,r}$ with $2n$ (r) external photons (pions). $\pi^{[1]}_{2n,r}$ contains one and only one closed fermion loop. The dark small triangles attaching two proton lines to a pion line (e.g., as shown in the loop integration over p) denotes the (full) strong vertex $\tilde{\Gamma}_5$. By definition the latter does not contain closed fermion loops.

$k_j \to \kappa k_j$ and consider the limit $\kappa\to\infty$ (with all the q_i's and k_j's being spacelike and nonexceptional). Clearly, the operation of L on $\pi^{[1]}_{2n,r}$ yields (even in perturbation theory) a finite expression behaving like[8] $\sim(\kappa)^{d-1}\times$[powers of $\ln(\kappa)$] for $\kappa\to\infty$ [more precisely like $(\kappa)^{d-1}(\kappa)^{-r(\beta_0/2)}$], where d is the "canonical" dimension of $\pi^{[1]}_{2n,r}$. Rewriting $\pi^{[1]} = (\kappa)^d \pi'^{[1]}$, we have in the usual manner[4] (note also $Z_3^{[0]} = Z_4^{[0]} = 1$)

$$[-\kappa(\partial/\partial\kappa) + g^2\beta(\partial/\partial g^2)] \times \pi'^{[1]}_{2n,r}(q_1,\ldots;k_1,\ldots;m/\kappa) \sim 0 \qquad (10)$$

[with a fixed ratio (μ/m) kept fixed—see the Appendix] or

$$\pi^{[1]}_{2n,r} \underset{\kappa\to\infty}{\sim} C(\kappa)^{d-r(\beta_0/2)} . \qquad (11)$$

We note that, to be more precise, we should have multiplied the integrand participating in the loop integration over p, say, as shown in Fig. 1, by a form factor $\Lambda^2/(p^2+\Lambda^2)$. When we scale the external momenta in the usual manner, we then have to scale the parameter $\Lambda^2 \to \Lambda^2/\kappa^2$ in the same way as we have to scale $m\to m/\kappa$. We may then argue that the limit $\Lambda^2\to\infty$ of the left-hand side of (10) exists (as discussed above) and then finally integrate (10) to yield (11). It is easy to see that the constant C in (11) is independent of any masses. From (11) we also see that π^0-π^0 scattering graphs (with $e^2\neq 0$) are finite and vanish quite rapidly in the limit $\kappa\to\infty$. The only diagram which seems troublesome in (11) corresponds to the case with $n=1$ and $r=1$. Such a process, however, is improved by at least one power of momentum decrease by the fact that the trace of an odd number of γ matrices is zero. Hence such an expression is reduced to a superficially logarithmic divergent one. (Actual perturbation-theory computations are well known to yield finite results for the latter process.) The damping of the strong vertex then guarantees the finiteness of such a process which is in turn proportional at worst to $(\kappa)^{-(\beta_0/2)}$ for $\kappa\to\infty$. The pion and the photon self-energy parts have been discussed above. [The corresponding renormalized propagators for the latter objects behave asymptotically $(q^2\to\infty)$ like $(1/q^2)$ [finite $+O((q^2)^{-(\beta_0/2)})$] in the single-fermion-loop context.]

Now we discuss the *multi*loop contribution to the theory. We follow Ref. 2 and make the two basic assumptions of Adler made in pure electrodynamics:

(1) The photon self-energy part may be correctly summed up loopwise as clearly discussed in Ref. 2.

(2) The $2n$-point electromagnetic-current-correlation functions (in the single-fermion-loop context), with the scale parameter m set equal to zero, and $F^{[1]}(x)$ *vanish* simultaneously at $x=\alpha$ in *pure* electrodynamics (see also below).

It is easy to check that a general amplitude with only external photon lines with a single-closed-fermion loop (containing both electromagnetic and strong corrections) goes over to its corresponding expression in pure electrodynamics in the limit $m\to 0$. The corrections for the latter object will go like $O((m)^{\beta_0})$ in the just-mentioned limit as a consequence of the damping of the strong interaction. At the eigenvalue, $F^{[1]}(x)|_{x=\alpha}=0$, the net expression for the just-mentioned amplitude for photon-photon scattering (determined in pure electrodynamics) vanishes in turn like $O(m^2)$.[2] We have also seen above that a general amplitude (in the single-fermion-loop context) with an arbitrary number of external photon lines and at least one pion line vanishes quite rapidly as $m\to 0$ (with a fixed ratio μ/m kept fixed). Accordingly, we see that the weaker assumption given in (2) is sufficient in the sense that all scattering amplitudes (in the single-fermion-loop context) with an arbitrary number of external photon and pion lines vanish [needless to say *including the independent vanishing* of (pure) π^0-π^0 scattering graphs] in the presence of the strong interaction in the limit $m\to 0$ at $x=\alpha$. Thus we clearly state that the two assumptions made above are the ones made in pure electrodynamics.[2] In Fig. 2 we show a two-loop contribution to the photon self-energy part with each blob shown containing one and only one closed

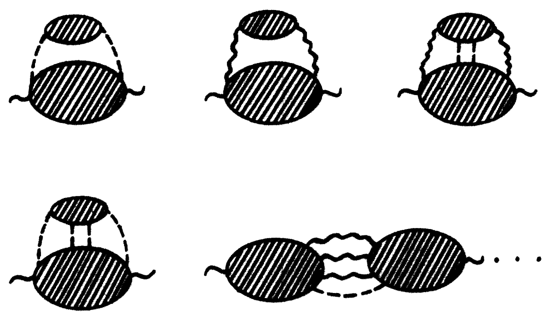

FIG. 2. Some diagrams contributing to $\pi_c^{[2]}$. Each blob shown contains one and only one closed fermion loop.

fermion loop. By simple power counting in the limit $m\to 0$, we easily see that the just-mentioned expression is asymptotically finite as we let all the integration variables become large simultaneously at $x=\alpha$. One may then argue, together with the two assumptions made above,[2] that at the eigenvalue $x=\alpha$ $[F^{[1]}(x)|_{x=\alpha}=0]$ the photon self-energy part containing n closed fermion loops $(n=1,2,\dots)$ is asymptotically finite. This study may be also carried out for the proton propagator and the strong vertex (again assuming the validity of a loopwise summation) with similar results as in Ref. 1 at $x=\alpha$. This is also carried out easily and simultaneously for the pion self-energy part. We shall briefly discuss this study, with no rigor, for the photon self-energy part—the main object of interest in this work.

In the loopwise summation,[2] the scaling equations for the photon self-energy part containing $n=1,2,\dots$ closed fermion loops, $\pi_c^{[n]}$, are formally given by

$$[m(\partial/\partial m)+\mu^2(\partial/\partial\mu^2)+g^2\chi^{[0]}(\partial/\partial g^2)]\,\alpha\pi_c^{[1]}(q^2)=\tilde{\chi}^{[1]}+I^{[1]}(q^2)\,, \tag{12a}$$

and for $n>1$,

$$[m(\partial/\partial m)+\mu^2(\partial/\partial\mu^2)+g^2\chi^{[0]}(\partial/\partial g^2)]\,\alpha\pi_c^{[n]}(q^2)$$
$$=\tilde{\chi}^{[n]}-\sum_{r=1}^{n-1}[\tilde{\chi}^{[r]}(\alpha(\partial/\partial\alpha)-1)+g^2\chi^{[r]}(\partial/\partial g^2)]\,\alpha\pi_c^{[n-r]}(q^2)+I^{[n]}(q^2)\,, \tag{12b}$$

where $-\chi=L\ln[(Z_5/Z_2)^2/Z_4)]$, $\tilde{\chi}=L\ln[Z_3]$, $L\equiv[m(\partial/\partial m)+\mu^2(\partial/\partial\mu^2)+g^2\chi(\partial/\partial g^2)+\alpha\tilde{\chi}(\partial/\partial\alpha)]$. In our previous notation $\chi^{[0]}\equiv\beta$. (To continue our discussion we omit the subscript c in $\pi_c^{[n]}$ to simplify the notation.) Quite generally, we write $\pi^{[n]}=\pi_0^{[n]}+\pi_1^{[n]}$ and $\tilde{\chi}^{[n]}=\tilde{\chi}_0^{[n]}+\tilde{\chi}_1^{[n]}$, where $\pi_0^{[n]}\equiv\pi^{[n]}|_{g^2=0}$, etc. For $n=2$, for example, Eq. (12b) becomes (in detail)

$$m(\partial/\partial m)\,\alpha\pi_0^{[2]}=\tilde{\chi}_0^{[2]}-\tilde{\chi}_0^{[1]}(\alpha(\partial/\partial\alpha)-1)\,\alpha\pi_0^{[1]}+I_0^{[2]}\,, \tag{13a}$$

$$[m(\partial/\partial m)+\mu^2(\partial/\partial\mu^2)+g^2\chi^{[0]}(\partial/\partial g^2)]\,\alpha\pi_1^{[2]}$$
$$=\tilde{\chi}_1^{[2]}-\tilde{\chi}_0^{[1]}(\alpha(\partial/\partial\alpha)-1)\,\alpha\pi_1^{[1]}-\tilde{\chi}_1^{[1]}(\alpha(\partial/\partial\alpha)-1)\,\alpha\pi^{[1]}-g^2\chi^{[1]}(\partial/\partial g^2)\,\alpha\pi^{[1]}+I_1^{[2]}\,. \tag{13b}$$

At the eigenvalue, the object $\tilde{\chi}_0^{[1]}(\alpha)$ $[\equiv -2\alpha F^{[1]}(\alpha)]$ vanishes by definition. According to the basic assumption (2),[2] we see that it implies that $\tilde{\chi}_0^{[2]}(x)$ and $\tilde{\chi}_1^{[1]}(x,g^2)(\partial/\partial x)\times F^{[1]}(x)$ vanish identically at $x=\alpha$ $[F^{[1]}(x)|_{x=\alpha}=0]$ independently of the value of g^2, and one obtains the asymptotic finiteness of $\pi_0^{[2]}$ *and* $\pi_1^{[2]}$ by following the procedure outlined in the Appendix. [The vanishing of $(\partial/\partial x)F^{[1]}(x)|_{x=\alpha}$ is well known[2].] The objects $I_{0,1}^{[2]}(q^2)$ are by definition constructed out of two closed fermion loops and vanish quite rapidly as $m\to 0$. Clearly the analysis may be carried out for any $n=1,2,\dots$. What we have learned from the above analysis alone is that for any $n=1,2,\dots,$ the photon self-energy part containing n closed fermion loops is asymptotically finite at $x=\alpha$, i.e., at the eigenvalue $F^{[1]}(x)|_{x=\alpha}=0$, and implies that the $\tilde{\chi}_0^{[n]}$'s vanish identically with (i) $\tilde{\chi}_1^{[n]}(\alpha,g^2)\equiv 0$ if $(\partial/\partial\alpha)\,\tilde{\chi}_0^{[n]}(\alpha)\neq 0$, or (ii) the $\tilde{\chi}_1^{[n]}(\alpha,g^2)$'s generally remain arbitrary if $(\partial/\partial\alpha)\tilde{\chi}_0^{[n]}(\alpha)=0$, with both cases independent of the value of g^2. The $\chi^{[n]}$'s obviously remain arbitrary. Since $\pi_0^{[n]}$ contains at least one single-fermion-loop current correlation function (attached to the rest of the graph solely by photon lines) and we know[2] that such a correlation function has an infinite-order zero when $m=0$ at $x=\alpha$, it follows that $(\partial/\partial\alpha)^s\,\tilde{\chi}_0^{[n]}(\alpha)\equiv 0$, $s=1,2,\dots$ [this is easily given by an elementary induction proof—see in particular Eqs. (121)–(127) in Ref. 2] and $(\partial/\partial x)^s\pi_0^{[n]}$ is also asymptotically finite at $x=\alpha$. The eigenvalue condition $F^{[1]}(x)|_{x=\alpha}=0$ (in the single-fermion-loop context), which we have shown remains *unaltered* in the presence of the strong interaction, implies [according to the two assumptions (1) and (2) above[2]] that the photon self-energy part containing arbitrarily n closed fermion loops is asymptotically finite at $x=\alpha$ independently of the value of g^2. [The actual computation of $\tilde{\chi}_1^{[n]}(\alpha, g^2)$ is not very hopeful (and not very interesting) since one is actually summing up an *infinite* set of diagrams coming from electrodynamics which makes the strong interaction damp out at high energies and render even π^0-π^0 scattering graphs finite.] Of course there are convergence problems related to the interchange of the limit $q^2 \to \infty$ with the summation over the number of loops (as n becomes large) which we make no attempt to discuss. (The functions $\tilde{\chi}_0^{[n]}$ are denoted by $\beta^{[n]}$ in Ref. 2; we shall not carry out the study of the Gell-Mann–Low function here.)

It is clear that the way of summation over the electromagnetic coupling first (to all orders) and then over g^2 is very suggestive from the initial Eq. (12a). The latter does *not* contain a derivative with respect to α (since $Z_1 = Z_2$, the Ward identity) and our method of summation seems most natural and perhaps the simplest. Also since the effective electromagnetic coupling can never be "switched off," however high the energies may be, according to positivity considerations ($0 \leq Z_3 \leq 1$; $e_0^2 = e^2/Z_3$) a high-energy analysis with a reversed way of summation compared with ours [i.e., sum over g^2 (to all orders) first and then over α] does not seem to be hopeful. See, for example, the damping of the terms coming from the strong interaction at high energies inside the square brackets in Eqs. (45)–(48) in Ref. 1 to further clarify this point. Ordinary perturbation theory results in the strong interaction, with an effective strong coupling $g^2 \ln(q^2/m^2)$ for $q^2 \gg m^2$, have reduced to expressions each being the *sum of terms that vanish quite rapidly at high energies* [$|q^2|^{1/2} \gg m(g^2)^{1/\beta_0}$]. In particular we have seen in our method of summation that no ϕ^4 counterterm is needed. We make no attempt to discuss here other possible ways of summation and other possible types of solutions to the problem (with respect to the latter see also Appendix B in Ref. 1).

We have studied the short-distance behavior of quantum electrodynamics in the presence of neutral-meson theory with ps-ps coupling. We have assumed an elementary way of summation of the theory, which was clearly stated throughout this work, leading to the physically very interesting results presented in this work. First we have considered the theory with no closed fermion loops and then demonstrated that in the single fermion-loop contribution to the (renormalized) photon self-energy part the eigenvalue condition for the (renormalized) fine-structure constant $F^{[1]}(\alpha)=0$ remains *unaltered* in the presence of the strong interaction. As we mentioned earlier this leads to the beautiful idea[2] that the value of α may be possibly determined within electrodynamics in isolation from the rest of the world. (The explicit evaluation of the function $F^{[1]}(x)$ seems to be becoming more urgent than before.) It was seen that the effective strong coupling vanishes quite rapidly at high energies (even *faster* than in *non*-Abelian gauge theories[5]). The finiteness problem of the self-mass of the neutral pion was also discussed without necessarily imposing additional constraints than the well-known self-consistency ones in pure electrodynamics. These self-consistency conditions may be roughly summarized by the positivity of the parameter $\beta_0(\alpha)$ $(2 > \beta_0/2 > 0)$ and the vanishing of $F^{[1]}(\alpha)$ (the eigenvalue condition for α). The parameter β_0 has also taken the important physical role as a measure of how fast the strong interaction is damped out at high energies. We have then discussed the extension of the above results to an asymptotically finite photon self-energy part in the *multi*loop contribution. We have seen that the photon self-energy part containing arbitrary n closed fermion loops is asymptotically finite at $x=\alpha$ (i.e., at the eigenvalue $F^{[1]}(x)|_{x=\alpha}=0$) independently of the value of g^2. The point $x=\alpha$ is the assumed (infinite order) zero of the single-loop electromagnetic-current-correlation functions in mass zero pure electrodynamics.[2]

Earlier we could discuss finite quantum electrodynamics in isolation from other dynamics; now we could also, formally, discuss it in a "larger" part of the world with (neutral) pions and protons. The photon has the remarkable property of "dissolving" the strong coupling at high energies. Finally, we remark that for an arbitrary and finite value of the renormalized (physical) strong coupling $(g^2/4\pi)$, the unrenormalized strong coupling vanishes in the limit of infinite cutoff (in the same way as the unrenormalized mass of the fermion vanishes in this limit for a fixed and finite value of its physical mass) as a consequence of the presence of the electromagnetic interaction. Should we take this as a hint that strong interaction is electromagnetic in origin?

Note added in proof. That the restriction $2 > \frac{1}{2}\beta_0 > 1$ is not necessarily ruled out, in our study in paper I, may be formally seen from a recent work of S. Weinberg [Phys. Rev. D 8, 3497 (1973)]. Accordingly the condition (iii) stated after Eq. (9) of the present article is not excluded.

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APPENDIX

We summarize the study of the stability of the eigenvalue condition $F^{[1]}(x)|_{x=\alpha}=0$ in the present theory in the single-fermion-loop context. Instead of considering three scale (mass) parameters m, μ, and λ as in Ref. 1 for the proton, pion, and photon, we may consider only one, say m, and introduce dimensionless and fixed parameters $a\equiv(\mu^2/m^2)$ and $b\equiv(\lambda^2/m^2)$. Accordingly, we may write for the inverse (free) pion and photon propagators (q^2+am^2) and (q^2+bm^2), respectively. The Callan-Symanzik scaling equation, in the single-fermion-loop contribution, for $\alpha\pi_c^{[1]}(q^2)$ is given by[1]

$$[m(\partial/\partial m)+g^2\beta(\partial/\partial g^2)]\,\alpha\pi_c^{[1]}(q^2)=\rho^{[1]}\ \text{for}\ q^2\to\infty, \tag{A1}$$

where

$$-\beta=(Z_2/Z_5)^2[m(\partial/\partial m)+g^2\beta(\partial/\partial g^2)](Z_5/Z_2)^2 \tag{A2}$$

and

$$\rho^{[1]}=[(1/Z_3)(m(\partial/\partial m)+g^2\beta(\partial/\partial g^2))Z_3]^{[1]}. \tag{A3}$$

We have seen in Ref. 1 that all the expansion coefficients in

$$\beta=-\beta_0+\beta_1 g^2+\cdots,$$
$$\rho^{[1]}=-2\alpha F^{[1]}(\alpha)+\rho_1 g^2+\cdots \tag{A4}$$

(which are treated to all orders in α) are finite. The self-consistency of the finiteness of the self-mass of the fermion in pure electrodynamics (which we assume) requires that $\beta_0(\alpha)>0$. This is because the object (m_0Z_2/Z_5) in pure electrodynamics is finite[9] (where m_0 is the unrenormalized mass of the fermion), and from the gauge invariance[1] of (Z_5/Z_2) we easily see, for example, from Ref. 2 that $\beta_0(\alpha)/2=\delta(\alpha)$, where[7] $\delta(\alpha)=3(\alpha/2\pi)+\frac{3}{4}(\alpha/2\pi)^2+\cdots$ [with the self-consistent range[7] for $\delta(\alpha)$: $0<\delta(\alpha)<2$]. Assuming an expansion of the form

$$\pi_c^{[1]}=(\pi_c^{[1]})_0+(\pi_c^{[1]})g^2+\cdots, \tag{A5}$$

we then immediately obtain the following differential equations:

$$m(\partial/\partial m)(\pi_c^{[1]})_0=-2\alpha F^{[1]}(\alpha), \tag{A6}$$

and for $n\geq 1$

$$[(m/n)(\partial/\partial m)-\beta_0(\alpha)](\alpha\pi_c^{[1]})_n+\sum_{r=1}^{n-1}[1-(r/n)]\beta_r(\alpha\pi_c^{[1]})_{n-r}=(\rho_n/n). \tag{A7}$$

The solution to the above equations is elementary and is given by

$$\alpha\pi_c^{[1]}(q^2)\underset{q^2\to\infty}{\sim}C^{[1]}+\alpha F^{[1]}(\alpha)\ln(q^2/m^2)+O((m^2/q^2)^{\beta_0(\alpha)/2}), \tag{A8}$$

and hence the eigenvalue condition $F^{[1]}(\alpha)=0$ remains unaltered in the presence of the strong interaction. The effective strong coupling is clearly given by $g^2(m^2/q^2)^{\beta_0/2}$ and vanishes at high energies. When we naively expand the correction $(m^2/q^2)^{\beta_0/2}$ in (A8) in powers of β_0 we generate not only a single power of $\ln(q^2/m^2)$, which modifies the eigenvalue condition, but arbitrary powers of the latter as well. Accordingly, one should be very careful when making contact with perturbation theory results. Questions of this sort originated the investigation carried out in this work. The limit $b\to 0$ exists[10] in the gauge-invariant object Z_3. Finally, we remind the reader of the simple fact that $\pi_c^{[1]}$, by definition, contains *one* over-all closed fermion loop. Accordingly, $\pi_c^{[1]}$ does *not* contain *pion-pion*, pion-photon, photon-photon scattering graphs and has no pion and photon self-energy parts as well.

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