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Finite quantum electrodynamics

T. Pradhan
Saha Institute of Nuclear Physics, Calcutta, India 700009

A. Lahiri
Vidyasagar College, Calcutta, India 700009
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It is shown that the ultraviolet divergences encountered in the lowest-order perturbation calculations of quantum electrodynamics no longer appear if the theory is expanded so as to include the μ meson, a triplet of heavy axial-vector bosons, and two heavy polar-vector bosons, in addition to the electron and photon, and suitably chosen couplings between them are introduced.

I. INTRODUCTION

Although the divergences occurring in the perturbation-theoretic calculation of physical processes in the present framework of quantum electrodynamics can be separated by the process of renormalization and meaningful results can be obtained, [1] the theory is not entirely satisfactory. The relations between the bare and renormalized quantities are formal, involving divergent expressions, and can lead to various paradoxes.[2] These problems cannot be bypassed by formulating the theory in terms of renormalized field operators because then the Lagrangian contains the renormalization constants, which are, strictly speaking, meaningless insofar as they are divergent quantities.

As far as the divergence in the selfenergy of the electron is concerned, it can be traced back to the classical theory, where the repulsive Coulomb energy diverges in the limit of a point electron. A cohesive force[3] would keep the electron stable by compensating for the repulsive Coulomb energy. Such cohesive forces can arise if, in addition to the photon, axial-vector bosons are coupled to the electron. As shown in the body of the paper, a coupling of the type $ig\bar{\psi}\gamma_\mu\gamma_5\psi a_\mu$, where a_μ is an axial-vector boson, gives a contribution to the self-energy part $\Sigma(p)$, whose divergent part is opposite in sign to the corresponding part arising from the minimal electromagnetic interaction, and for a proper choice of the coupling constant the two can be made to cancel.

Apart from the divergence in the self-energy of the electron, one encounters the divergence associated with the self-energy of the photon which has no classical analog. Ordinarily one would not expect that this divergence could be canceled by contributions arising from additional interactions as in the electron self-energy problem. This is because, for the minimal interaction, $Z_3 = \lim_{k^2\to 0} k^2 D_F(k^2)$ is of the form

$$[1-\alpha(\text{divergent part}+\text{finite parts})]$$

in perturbation theory, so that, for a cancellation of the divergence, the contribution of the additional interaction must be of the form

$$[1+\alpha(\text{divergent part}+\text{finite parts})],$$

which would ordinarily be a violation of the Lehmann bound $Z_3 \leq 1$ arising from the positivity condition on the corresponding spectral function. However, it is well known that on account of the negative metric associated with the fourth component of the photon field, the spectral function for the electron propagator is not positive-definite and the electron wave-function renormalization constant Z_2, calculated using the Landau gauge for the photon propagator, does not respect the Lehmann bound $Z_2 \leq 1$.[4] This suggests that in the case of Z_3, the constraint imposed by the Lehmann representation may also be avoided if there occur vacuum polarization diagrams with internal vector (or axial-vector) boson lines for which the propagator carries the projection operator $(\delta_{\mu\nu}-k_\mu k_\nu/k^2)$ similar to that of the photon propagator in the Landau gauge. In order that the effects of these bosons on the experimentally observed quantities be negligible they must be sufficiently heavy, in which case we would have a factor of $(\delta_{\mu\nu}+k_\mu k_\nu/M^2)$ in place of the desired $(\delta_{\mu\nu}-k_\mu k_\nu/k^2)$. However, if the massive vector bosons are suitably coupled to massless scalar particles through a mixing interaction, then the effective propagator resulting from the mixing does carry the desired factor $(\delta_{\mu\nu}-k_\mu k_\nu/k^2)$. It transpires that if the photon interacts with such vector particles $A_\mu^\pm$ carrying some internal charge (for which the natural choice is muon number) with a "Pauli-type" coupling

$$\mathcal{L}_{\rm int} = i\lambda A_\mu \partial_\nu (A_\mu^+ A_\nu^- - A_\mu^- A_\nu^+),$$

the spectral function appropriate for the photon self-energy resulting from this is not positive-definite. As a result of this, the divergence in Z_3 is opposite in sign to that arising from the electron-positron loop, thus making it possible to bring about a cancellation of the divergence by an appropriate choice of the coupling parameter λ.

The coupling of the photon to $A_\mu^\pm$ introduces a new divergence which occurs in the self-energy of the muonically charged bosons resulting from the emission and reabsorption of photons by these particles. This divergence can be compensated by contributions resulting from additional interactions where these bosons are coupled to muonic currents of the type $\bar{\mu}\gamma_\lambda e$ and $\bar{e}\gamma_\lambda\mu$. For the compensation of divergence in the selfenergy of the neutral axial-vector boson a_μ introduced earlier we need to have muonically charged axial-vector bosons $a_\mu^\pm$ which in turn will have their own divergent self-energy. The problem then reduces to that of removing the divergences from the self-energies of all the particles in the theory in a self-consistent manner. We have been able to achieve this. In the final form there are, all together, eight particles in the expanded quantum electrodynamics we are considering. In addition to the known particles (electron, muon, and photon), we have a doublet of massive electrically neutral but muonically "charged" vector bosons, a similar doublet of axial-vector bosons, and a single massive electrically and muonically neutral axial-vector boson. The coupling scheme as formulated in Sec. II requires, apart from the electric charge, several other coupling constants which are arbitrary to start with but are then fixed from requirements of cancellation of divergences in the selfenergies of all the particles. This is exhibited in Sec. III and IV. It turns out that no additional constraints on the coupling constants are needed to make the vertex parts finite in the second order.

The theory considered in this paper can be extended so as to encompass pure leptonic weak interactions by introducing two-component neutrinos associated with the electron and the muon. The neutrino currents, when coupled to massive bosons mentioned above, can account for the muon decay and other pure leptonic processes. In fact, in the limit of infinite mass of the vector bosons, the leptonic weak interactions in the theory will be identical to a Fierz-shuffled Fermi interaction.

II. FORMULATION OF THE THEORY

As mentioned in the Introduction, the proposed theory includes the quantized fields corresponding to the electron (e), the muon (μ), the photon (A_μ), a doublet of massive vector bosons ($A_\mu^\pm$), a doublet of massive axial-vector bosons ($a_\mu^\pm$), and a singlet massive axial-vector boson (a_μ). All the boson fields are electrically neutral, but the members of the doublets carry muonic number. The field a_μ is both electrically and muonically neutral. As pointed out earlier, the cancellation of the divergences appearing in the fermionic self-energy parts due to the minimal coupling

$$\mathcal{L}_{\rm int}^{(1)} = ie(\bar{e}\gamma_\lambda e + \bar{\mu}\gamma_\lambda\mu)A_\lambda$$

requires the additional coupling

$$\mathcal{L}_{\rm int}^{(2)} = ig(\bar{e}\gamma_\lambda\gamma_5 e + \bar{\mu}\gamma_\lambda\gamma_5\mu)a_\lambda,$$

where g is an as yet arbitrary coupling constant. Similarly, for the cancellation of the divergence in the charge renormalization constant arising from the minimal coupling, we need additional interactions between the photon and the doublets of the massive vector and axial-vector bosons. These interactions are to be of the Pauli type since otherwise these bosons would acquire a charge through these couplings. We therefore take these interactions to be of the form

$$\mathcal{L}_{\rm int}^{(3)} = i\lambda_V A_\mu \partial_\nu (A_\mu^+ A_\nu^- - A_\mu^- A_\nu^+) + i\lambda_A A_\mu \partial_\nu (a_\mu^+ a_\nu^- - a_\mu^- a_\nu^+).$$

Now, these interactions, while giving new contributions to the photon propagator, will also give similar contributions to the propagators $\langle 0|T(A_\mu^+(x)A_\mu^-(0))|0\rangle$ and $\langle 0|T(a_\mu^+(x)a_\nu^-(0))|0\rangle$. In order to cancel the divergences in these, we have to introduce off-diagonal couplings of these bosons to the fermions e and μ. Considerations such as C and P invariance lead us to adopt

$$\mathcal{L}_{\rm int}^{(4)} = g_V(e\gamma_\lambda\mu A_\lambda^- - \bar{\mu}\gamma_\lambda e A_\lambda^+) + ig_A(\bar{e}\gamma_\lambda\gamma_5\mu a_\lambda^- + \bar{\mu}\gamma_\lambda\gamma_5 e a_\lambda^+).$$

Finally, the interaction (2) will give a divergent contribution to the propagator $\langle 0|T(a_\mu(x)a_\nu(0))|0\rangle$. In order to cancel this, we introduce an interaction

$$\mathcal{L}_{\rm i\text{-}t}^{(5)} = \lambda(\partial_\mu a_\nu - \partial_\nu a_\mu)(a_\mu^+ A_\nu^- + a_\mu^- A_\nu^+).$$

Taking all the above interactions together we get the coupling scheme represented by the Lagrangian

$$\begin{aligned}\mathcal{L} = \mathcal{L}_0 &+ ie(\bar{e}\gamma_\lambda e + \bar{\mu}\gamma_\lambda\mu)A_\lambda + ig(\bar{e}\gamma_\lambda\gamma_5 e + \bar{\mu}\gamma_\lambda\gamma_5\mu)a_\lambda \\ &+ g_V(\bar{e}\gamma_\lambda\mu A_\lambda^- - \bar{\mu}\gamma_\lambda e A_\lambda^+) + ig_A(\bar{e}\gamma_\lambda\gamma_5\mu a_\lambda^- + \bar{\mu}\gamma_\lambda\gamma_5 e a_\lambda^+) \\ &+ i\lambda_\nu A_\mu \partial_\nu (A_\mu^+ A_\nu^- - A_\mu^- A_\nu^+) + i\lambda_A A_\mu \partial_\nu (a_\mu^+ a_\nu^- - a_\mu^- a_\nu^+) \\ &+ \lambda(\partial_\mu a_\nu - \partial_\nu a_\mu)(a_\mu^+ A_\nu^- + a_\mu^- A_\nu^+), \qquad (1)\end{aligned}$$

where $\mathcal{L}_0$ is the free part of the Lagrangian including the mass terms.

As pointed out earlier, the bosons $A^{\pm}_{\mu}$, $a^{\pm}_{\mu}$, and a_{μ} are massive. Ordinarily, a theory with such massive vector bosons would be nonrenormalizable. There has been considerable discussion in contemporary literature[5] of the ways to make such theories renormalizable. We shall adopt here a procedure which is straightforward and has the merit that once the divergences from the self-energies are removed, all the vertex parts are automatically finite (in second-order perturbation calculation). This method essentially consists of taking a certain limit of a local theory and arriving at an effective nonlocal theory where the massive bosons are coupled to nonlocal but conserved currents. The local theory contains couplings of the massive vector (and axial-vector) bosons with massless scalar (and pseudoscalar) bosons. The limiting procedure is so chosen as to effectively decouple the massless scalar bosons while at the same time rendering the theory renormalizable. We illustrate this procedure for the coupling of one of the axial-vector fields, say a_{μ}, coupled to the axial-vector current of the electron, for which the Lagrangian is (with $\hat{\partial}=\gamma\cdot\partial$)

$$\mathcal{L}=-\bar{e}(\hat{\partial}+m)e-\tfrac{1}{4}(\partial_{\mu}a_{\nu}-\partial_{\nu}a_{\mu})^2 -\tfrac{1}{4}M^2a_{\mu}a_{\mu}+ig\bar{e}\gamma_{\lambda}\gamma_5 e a_{\lambda}\,, \qquad (2)$$

M being the mass of a_{μ}. We now introduce a massless pseudoscalar boson field φ with the coupling

$$\mathcal{L}_{\rm int}=ig\epsilon\bar{e}\gamma_{\lambda}\gamma_5 e\partial_{\lambda}\varphi-\epsilon^{-1}a_{\lambda}\partial_{\lambda}\varphi\,. \qquad (3)$$

For finite ϵ, this describes a local interaction. The modifications of the vertices and propagators arising out of the mixing interaction $-\epsilon^{-1}a_{\lambda}\partial_{\lambda}\varphi$ are deduced in Appendix A. From the results obtained it will be clear that in the limit $\epsilon\to 0$, φ is effectively decoupled, leading to an effective interaction of a_{μ} with the conserved but nonlocal current $ig(\delta_{\lambda\sigma}-\partial_{\lambda}\partial_{\sigma}/\Box)\bar{e}\gamma_{\sigma}\gamma_5 e$ and an effective propagator $-i[(\delta_{\mu\nu}-k_{\nu}k_{\nu}/k^2)/(k^2+M^2)]$ for a_{μ}. It is thus clear that renormalizability is achieved by adopting the limiting procedure. In the following it will be understood that this procedure has been performed for all the massive vector and axial-vector fields and their interactions. If we assume this limiting procedure to have been performed, the relevant Feynman rules for the effective nonlocal theory are listed in Appendix B. This amounts to a modification of the vertices and propagators whereby any vector or axial-vector boson vertex with momentum k_{μ} will acquire a factor $G_{\mu\nu}(k)\equiv(\delta_{\mu\nu}-k_{\mu}k_{\nu}/k^2)$ and the correspongding propagator will be $-iG_{\mu\nu}(k)[1/(k^2+M^2)]$, where M is the mass of this boson. As mentioned in the Introduction, it is this form of the propagator that is responsible for the cancellation of the divergence of the charge-renormalization constant Z_3. The results obtained in Appendix A also contain the novel feature that for sufficiently small ϵ the effective φ propagator exhibits an indefinite metric, though the original Lagrangian has a positive-definite metric. The indefinite-metric approach to achieve finiteness of field theories has been considered by several authors.[6] In such theories an indefinite metric is introduced in the Lagrangian from the outset. The main problem in such theories is to prevent the appearance of particles with negative norm in physical processes. One way of achieving this is through the artifice of shadow states.[7] It is not clear how closely these approaches are related to ours.

III. FERMION SELF-ENERGIES

Let us first consider the electron self-energy. The diagrams shown in Figs. 1(a)–1(d) contribute to this self-energy. The Feynman matrix element for Fig. 1(a) is (we use the Landau gauge for the photon propagator)

$$\Sigma^{(a)}(p)=\frac{e^2}{(2\pi)^4}\int\frac{d^4k}{k^2}G_{\mu\nu}(k)\gamma_{\mu}\frac{i(\hat{p}-\hat{k})-m_e}{(p-k)^2+m_e{}^2}\gamma_{\nu}\,. \qquad (4a)$$

By applying the rules discussed in Sec. II we get the following contributions from the other three diagrams:

$$\Sigma^{(b)}(p)=\frac{g_V{}^2}{(2\pi)^4}\int\frac{d^4k}{k^2+M_V{}^2}G_{\mu\lambda}(k)\gamma_{\lambda}\frac{i(\hat{p}-\hat{k})-m_{\mu}}{(p-k)^2+m_{\mu}{}^2}\times G_{\nu\sigma}(k)\gamma_{\sigma}G_{\mu\nu}(k)\,,$$

$$\Sigma^{(c)}(p)=\frac{g_A{}^2}{(2\pi)^4}\int\frac{d^4k}{k^2+M_A{}^2}G_{\mu\lambda}(k)\gamma_{\lambda}\gamma_5\frac{i(\hat{p}-\hat{k})-m_{\mu}}{(p-k)^2+m_{\mu}{}^2}\times G_{\nu\sigma}(k)\gamma_{\sigma}\gamma_5 G_{\mu\nu}(k)\,,$$

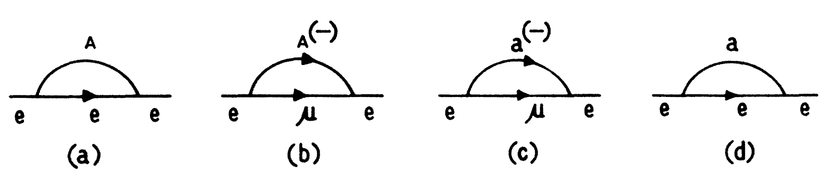

FIG. 1. Feynman diagrams for electron self-energy.

$$\Sigma^{(a)}(p)=\frac{g^2}{(2\pi)^4}\int\frac{d^4k}{k^2+M^2}G_{\mu\lambda}(k)\gamma_\lambda\gamma_5\frac{i(\hat{p}-\hat{k})-m_e}{(p-k)^2+m_e^2}\times G_{\nu\sigma}(k)\gamma_\sigma\gamma_5 G_{\mu\nu}(k)\,,$$

where M_V, M_A, and M are the masses of the relevant bosons and m_μ, m_e the masses of the fermions. After some simplifications we get

$$\Sigma^{(b)}(p)=\frac{g_V{}^2}{(2\pi)^4}\int\frac{d^4k}{k^2+M_V{}^2}\gamma_\lambda\frac{i(\hat{p}-\hat{k})-m_\mu}{(p-k)^2+m_\mu{}^2}\gamma_\sigma G_{\lambda\sigma}(k)\,, \tag{4b}$$

$$\Sigma^{(c)}(p)=\frac{g_A{}^2}{(2\pi)^4}\int\frac{d^4k}{k^2+M_A{}^2}\gamma_\lambda\frac{i(\hat{p}-\hat{k})+m_\mu}{(p-k)^2+m_\mu{}^2}\gamma_\sigma G_{\lambda\sigma}(k)\,, \tag{4c}$$

$$\Sigma^{(d)}(p)=\frac{g^2}{(2\pi)^4}\int\frac{d^4k}{k^2+M^2}\gamma_\lambda\frac{i(\hat{p}-\hat{k})+m_e}{(p-k)^2+m_e{}^2}\gamma_\sigma G_{\lambda\sigma}(k)\,. \tag{4d}$$

It is to be noted that the sign of the mass term in the numerator of $\Sigma^{(c)}(p)$ and $\Sigma^{(d)}(p)$, which is crucial for the removal of divergence, has its origin in the occurrence of the two γ_5 matrices. By using standard techniques of Feynman parameters and symmetric integration, we see that

$$\int\frac{d^4k}{k^2+M^2}G_{\lambda\sigma}(k)\gamma_\lambda\frac{i(\hat{p}-\hat{k})}{(p-k)^2+m^2}\gamma_\sigma$$

is free of ultraviolet divergence, while

$$\int\frac{d^4k}{(k^2+M^2)[(p-k)^2+m^2]}G_{\lambda\sigma}(k)\gamma_\lambda\gamma_\sigma$$

is not. Thus, $\Sigma(p)\equiv\Sigma^{(a)}(p)+\Sigma^{(b)}(p)+\Sigma^{(c)}(p)+\Sigma^{(d)}(p)$ will be free of ultraviolet divergence if

$$m_e e^2+m_\mu g_V{}^2=m_e g^2+m_\mu g_A{}^2\,. \tag{5}$$

We can treat the muon self-energy in the same manner. The diagrams shown in Figs. 2(a)–2(d) contribute in this case and we get the condition of finiteness as

$$m_\mu e^2+m_e g_V{}^2=m_\mu g^2+m_e g_A{}^2\,. \tag{6}$$

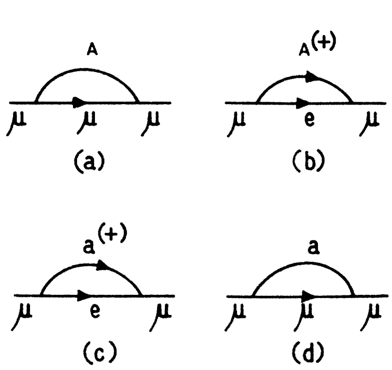

FIG. 2. Muon self-energy diagrams.

IV. BOSON SELF-ENERGIES

We start with the photon vacuum-polarization diagrams. The four diagrams shown in Figs. 3(a)–3(d) will contribute to this. The contribution of Fig. 3(a) to the vacuum-polarization tensor is given by

$$\Pi_{\alpha\beta}^{(a)}(k)=\frac{e^2}{(2\pi)^4}\int\frac{d^4p}{(p^2+m_e{}^2)[(p-k)^2+m_e{}^2]}\times\mathrm{Tr}[\gamma_\alpha(i\hat{p}-m_e)\gamma_\beta(i(\hat{p}-\hat{k})-m_e)]\,. \tag{7}$$

It is known that $\Pi_{\alpha\beta}^{(a)}(k)$ can be written in the form

$$\Pi_{\alpha\beta}^{(a)}(k)=k^2G_{\alpha\beta}(k)\Pi^{(a)}(k^2)+\delta_{\alpha\beta}D^{(a)}\,, \tag{8}$$

where the constant $D^{(a)}$ containing a quadratic divergence is of no physical significance since in all observable quantities it cancels with the so-called seagull terms. $\Pi^{(a)}(k^2)$, on the other hand, contributes to physical processes but is logarithmically divergent. By employing usual methods of calculation we get for its divergent part $\Pi_D^{(a)}(k^2)$

$$\Pi_D^{(a)}(k^2)=-\frac{4}{3}\frac{ie^2}{16\pi^2}\lim_{L\to\infty}\ln\frac{L^2}{m_e{}^2}\,. \tag{9}$$

In a similar manner, from Fig. 3(b) we get

$$\Pi_D^{(b)}(k^2)=-\frac{4}{3}\frac{ie^2}{16\pi^2}\lim_{L\to\infty}\ln\frac{L^2}{m_\mu{}^2}\,. \tag{10}$$

The contribution of Fig. 3(c) to the vacuum polarization tensor is given by

$$\Pi_{\alpha\beta}^{(c)}(k)=\frac{\lambda_V{}^2}{(2\pi)^4}2k_\nu k_\eta\times\int\frac{d^4q}{(q^2+M_V{}^2)[(k-q)^2+M_V{}^2]}\times[G_{\alpha\beta}(q)G_{\nu\eta}(k-q)-G_{\alpha\eta}(q)G_{\nu\beta}(k-q)]\,. \tag{11}$$

It can be easily seen that $\Pi_{\alpha\beta}^{(c)}$ is of the form

$$\Pi_{\alpha\beta}^{(c)}(k)=G_{\alpha\beta}(k)[k^2\Pi^{(c)}(k^2)+C]\,, \tag{12}$$

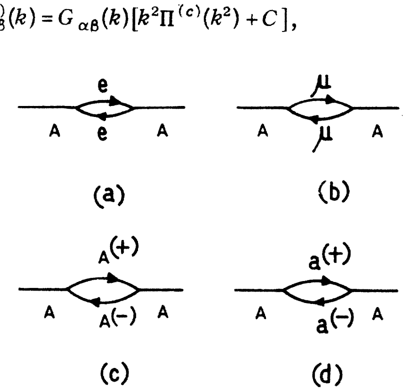

FIG. 3. Photon self-energy diagrams.

where C is a constant which, like D in Eq. (7), does not contribute to the physical processes or to charge renormalization and is compensated by seagull terms, and where

$$\Pi_D^{(c)}(k^2) = \frac{i\lambda_V^2}{16\pi^2}\lim_{L\to\infty}\ln\frac{L^2}{M_V^2}. \tag{13}$$

Similarly, from Fig. 3(d) we get

$$\Pi_D^{(d)}(k^2) = \frac{i\lambda_A^2}{16\pi^2}\lim_{L\to\infty}\ln\frac{L^2}{M^2}. \tag{14}$$

It is to be noted that the divergent parts of $\Pi^{(a)}$ and $\Pi^{(b)}$ are opposite in sign to those of $\Pi^{(c)}$ and $\Pi^{(d)}$. It is thus clear that the sum of all contributions,

$$\Pi(k^2) = \sum_{i=a,b,c,d}\Pi^{(i)}(k^2),$$

can be made divergence-free by suitably choosing the coupling constants. From Eqs. (9)–(14) we find that $\pi(k^2)$ is free of divergence if and only if

$$\tfrac{8}{3}e^2 = \lambda_V^2 + \lambda_A^2. \tag{15}$$

We now consider the self-energy of the $A^{(\pm)}$ vector bosons. Defining

$$\Pi_{\alpha\beta}(k) = \frac{1}{(2\pi)^4}\int d^4x\, e^{ik\cdot x}\langle 0|\,T(A_\alpha^+(x)A_\beta^-(0))\,|0\rangle, \tag{16}$$

we see that in the second order the three diagrams 4(a)–4(c) contribute to $\Pi_{\alpha\beta}(k)$. Defining $\Pi_{\alpha\beta}^{(a,b,c)}(k)$ as the respective contributions of these three diagrams to $\Pi_{\alpha\beta}(k)$ we can easily see that they are of the form

$$\Pi_{\alpha\beta}^{(a,b,c)}(k) = G_{\alpha\beta}(k)k^2\Pi^{(a,b,c)}(k^2), \tag{17}$$

and, dropping the quadratically divergent terms as discussed above, we get the logarighmically divergent parts as given below:

$$\Pi_D^{(a)}(k^2) = -\frac{ig_V^2}{16\pi^2}\frac{4}{3}\lim_{L\to\infty}\ln\frac{L^2}{m_e^2}, \tag{18a}$$

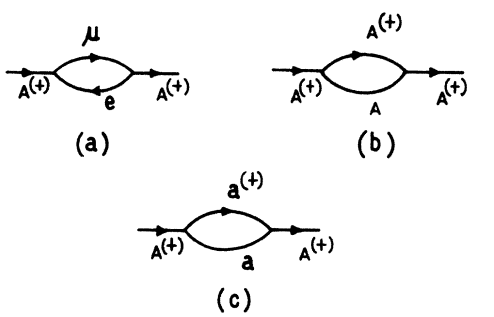

FIG. 4. Self-energy diagrams for the massive vector bosons.

$$\Pi_D^{(b)}(k^2) = \frac{i\lambda_V^2}{16\pi^2}\frac{1}{3}\lim_{L\to\infty}\ln\frac{L^2}{M_V^2}, \tag{18b}$$

$$\Pi_D^{(c)}(k^2) = \frac{i\lambda_A^2}{16\pi^2}\frac{1}{3}\lim_{L\to\infty}\ln\frac{L^2}{M_A^2}. \tag{18c}$$

Thus, $\Pi(k^2) = \sum_{i=a,b,c}\Pi^{(i)}(k^2)$ is free of divergence if

$$\tfrac{4}{3}g_V^2 = \tfrac{1}{3}\lambda_V^2 + \tfrac{1}{3}\lambda^2. \tag{19}$$

The condition of finiteness of the self-energy of the $a_\mu^\pm$ bosons can be worked out in a similar manner and is found out to be

$$\tfrac{4}{3}g_A^2 = \tfrac{1}{3}\lambda_A^2 + \tfrac{1}{3}\lambda^2. \tag{20}$$

The corresponding condition for the finiteness of the self-energy of the a_μ field is seen to be

$$\tfrac{8}{3}g^2 = \lambda^2. \tag{21}$$

Combining the conditions expressed by Eqs. (5), (6), (15), (19), (20), and (21) we see that we can choose the coupling constants in such a manner that all fermion and boson self-energies are finite. The most symmetric choice is

$$\begin{aligned} g^2 &= e^2, \\ g_V^2 &= g_A^2 = e^2, \\ \lambda_V^2 &= \lambda_A^2 = \tfrac{4}{3}e^2, \\ \lambda^2 &= \tfrac{8}{3}e^2. \end{aligned} \tag{22}$$

V. VERTEX DIAGRAMS

From the Lagrangian in Eq. (1) we see that we have to consider nine vertex functions. We shall see below that all of them are finite. Let us first consider the second-order eeA vertex given by the diagrams shown in Figs. 5(a)–5(e). The matrix element of Fig. 5(a) is finite in the gauge (Landau gauge) we have chosen. The matrix elements for the rest of the diagrams, i.e., Figs.

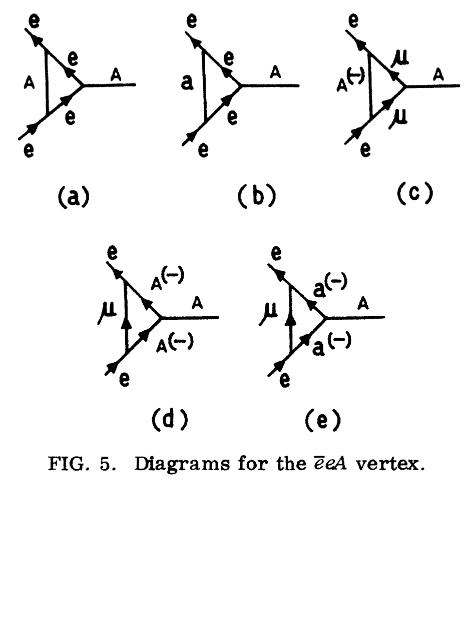

FIG. 5. Diagrams for the $\bar{e}eA$ vertex.

5(b)–5(e), can also be shown to be finite. In case of Figs. 5(d) and 5(e) this is evident from direct power counting of the integrand, while for 5(b) and 5(c) this can be seen by noting that the matrix elements are of the form

$$\Lambda_\mu(p_2, p_1) \propto \int \frac{d^4 l}{l^2+M^2}\gamma_\lambda \frac{i(\hat{p}_2 - \hat{l}) \pm m}{(p_2 - l)^2 + m^2} \times \gamma_\mu \frac{i(\hat{p}_1 - \hat{l}) \pm m}{(p_1 - l)^2 + m^2}\gamma_\sigma G_{\lambda\sigma}(l), \quad (23)$$

where M and m are some masses, and the $\pm$ signs are to be taken depending on whether we are considering axial-vector [Fig. 5(b)] or vector [Fig. 5(c)] coupling. By power counting we see that the entire divergence, if any, of the right-hand side is contained in

$$\int \frac{d^4 l}{(l^2+m^2)^2(l^2+M^2)} G_{\lambda\sigma}(l)\gamma_\lambda i\hat{l}\gamma_\mu i\hat{l}\gamma_\sigma,$$

which by symmetric integration is seen to be finite. Since the $\mu\mu A$ vertex diagrams are obtained by interchanging the roles of μ and e, this vertex will also be finite.

The diagrams for the eea vertex are shown in Fig. 6. Here the contributions of Figs. 6(e) and 6(f) are finite, as seen by direct power counting,

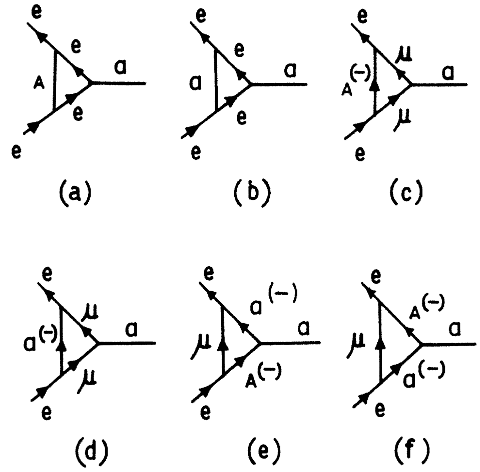

FIG. 6. Diagrams for the $\bar{e}ea$ vertex.

and the contributions of Figs. 6(a)–6(d) are all of the form given in Eq. (23) and hence are finite. The same applies to the corresponding diagrams of the $\mu\mu a$ vertex. We now consider the vertices off-diagonal in the fermion fields, i.e., the $e\mu A^\pm$ and $e\mu a^\pm$ vertices. The former is given by the diagrams in Fig. 7(a)–7(f). The contributions of Figs. 7(a) and 7(b) are of the form given in (23) and are thus finite. The contribution of Fig. 7(c) to the vertex function is

$$\Lambda_\mu^{(c)}(p_2, p_1) = -\frac{ie\lambda_V}{(2\pi)^4}\int \frac{d^4 p}{(p_1-p)^2[(p_2-p)^2+M_V{}^2](p^2+m_e{}^2)}\gamma_\alpha(i\hat{p} - m_e)\gamma_\beta G_{\alpha\eta}(p_2 - p)G_{\eta\tau}(p_2-p) \times G_{\mu\nu}(p_2-p_1)[i(p-p_1)_\nu G_{\beta\tau}(p-p_1) - i(p-p_1)_\tau G_{\beta\nu}(p-p_1)], \quad (24a)$$

whose divergent part is entirely contained in

$$[\Lambda_\mu^{(c)}(p_2, p_1)]_D = \frac{ie\lambda_V}{(2\pi)^4}\int \frac{d^4 p}{p^4(p^2+M_V{}^2)} p_\nu p_\sigma \gamma_\alpha \gamma_\sigma \gamma_\beta G_{\alpha\beta}(p) G_{\mu\nu}(p_2 - p_1). \quad (24b)$$

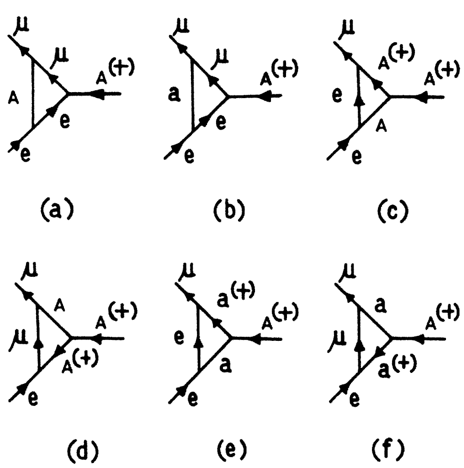

FIG. 7. Diagrams for the $\bar{\mu}eA^+$ vertex.

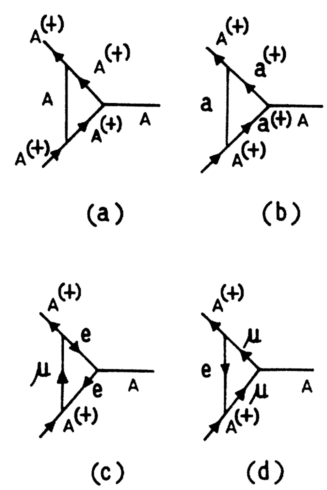

FIG. 8. Diagrams for the A^+A^-A vertex.

Entity	Free		Exact (to all orders in $\frac{1}{\epsilon}$)	
	Symbol	Value in momentum space	Symbol	Representation in momentum space
Propagator for a_μ	(a)	$-i\frac{\left(\delta_{\mu\nu}+\frac{k_\mu k_\nu}{M^2}\right)}{k^2+M^2}$	(a)	$-i\frac{\delta_{\mu\nu}-\alpha(k^2)\frac{k_\mu k_\nu}{k^2}}{k^2+M^2}$
Propagator for φ	(φ)	$-\frac{i}{k^2}$	(φ)	$-i\frac{\beta(k^2)}{k^2}$
eea vertex	(e) (a) (e)	$ig\gamma_\mu\gamma_5$	(e) (a) (e)	$ig\left(\delta_{\mu\nu}-\frac{a(k^2)}{k^2}k_\mu k_\nu\right)\gamma_\nu\gamma_5$
eeφ vertex	(e) (φ) (e)	$ig\epsilon\gamma_\mu\gamma_5(ik_\mu)$	(e) (φ) (e)	$ig\epsilon\gamma_\mu\gamma_5\left(\delta_{\mu\nu}-\frac{b(k^2)}{k^2}k_\mu k_\nu\right)ik_\nu$

FIG. 9. Momentum-space representations of free and exact (to all orders in $1/\epsilon$) propagators and vertices.

Similarly, the contribution of Fig. 7(d) is

$$\Lambda_\mu^{(a)}(p_2,p_1)=-\frac{ie\lambda_V}{(2\pi)^4}\int\frac{d^4p}{(p_2-p)^2[(p_1-p)^2+M_V{}^2](p^2+m_\mu{}^2)}\gamma_\alpha(i\hat{p}-m_\mu)\gamma_\beta G_{\beta\eta}(p_1-p)G_{\eta\tau}(p_1-p)$$
$$\times G_{\mu\nu}(p_2-p_1)[i(p_2-p)_\nu G_{\alpha\tau}(p_2-p)-i(p_2-p)_\tau G_{\alpha\nu}(p_2-p)]\,, \tag{24c}$$

whose divergent part is entirely contained in

$$[\Lambda_\mu^{(a)}(p_2,p_1)]_D=-\frac{ie\lambda_V}{(2\pi)^4}\int\frac{d^4p}{p^4(p^2+M_V{}^2)}p_\nu p_\sigma\gamma_\alpha\gamma_\sigma\gamma_\beta G_{\alpha\beta}^{(p)}G_{\alpha\beta}^{(p)}G_{\mu\nu}(p_2-p_1)\,. \tag{24d}$$

Though $\Lambda_\mu^{(c)}$ and $\Lambda_\mu^{(d)}$ are individually divergent, it is evident from Eqs. (24b) and (24d) that their sum is free of divergence. Similarly, $\Lambda_\mu^{(e)}+\Lambda_\mu^{(f)}$ may be seen to be finite. In an exactly similar manner we can show that the $e\mu a^\pm$ vertex is finite.

Next, we consider the $A^{(+)}$-$A^{(-)}$-A vertex for which the diagrams are given in Figs. 8(a)–8(d). The contribution of Fig. 8(a) to this vertex function is given by

$$\Lambda_{\alpha\beta\gamma}^{(a)}(p_1,p_2)=\frac{i\lambda_V{}^2}{(2\pi)^4}\int\frac{d^4l}{l^2[(l-p_1)^2+M_V{}^2][(l-p_2)^2+M_V{}^2]}G_{\alpha\alpha'}(p_1)G_{\beta\beta'}(p_2)i(p_2-p_1)_\nu$$
$$\times[l_{\alpha'}l_{\beta'}G_{\eta\eta'}(l)+l_\eta l_{\eta'}G_{\alpha'\beta'}(l)-l_{\alpha'}l_{\eta'}G_{\eta\beta'}(l)-l_{\beta'}l_\eta G_{\alpha'\eta'}(l)]$$
$$\times[G_{\gamma\eta}(p_1-l)G_{\nu\eta'}(p_2-l)-G_{\nu\eta}(p_1-l)G_{\gamma\eta'}(p_2-l)]\,, \tag{25}$$

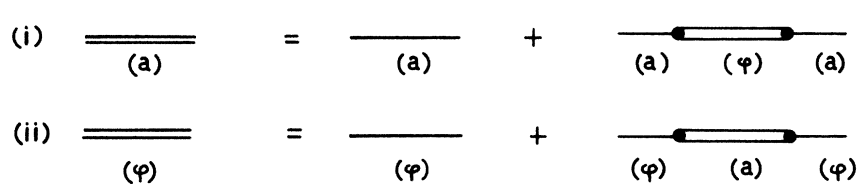

FIG. 10. Diagrammatic equations for propagators.

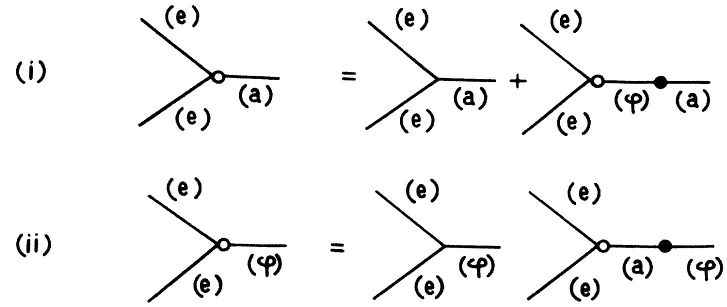

FIG. 11. Diagrammatic equations for vertices.

whose divergent part, if any, is entirely contained in

$$[\Lambda^{(a)}_{\alpha\beta\gamma}]_D=\frac{i\lambda_V^2}{(2\pi)^4}\int\frac{d^4l}{l^2(l^2+M_V^2)^2}\,l_{\alpha'}l_{\beta'}G_{\eta\eta'}(l)G_{\alpha\alpha'}(p_1)G_{\beta\beta'}(p_2)i(p_2-p_1)_\nu \times[G_{\gamma\eta}(l)G_{\nu\eta'}(l)-G_{\nu\eta}(l)G_{\gamma\eta'}(l)],$$

which is easily seen to be identically zero on account of the projection properties of $G_{\alpha\beta}(l)$. The finiteness of the contribution from Fig. 8(b) follows similarly. The contributions of Figs. 8(c) and 8(d) to the vertex function are respectively

$$\Lambda^{(c)}_{\alpha\beta\gamma}(p_1,p_2)=\frac{i}{(2\pi)^4}\frac{eg_V^2}{\lambda_V}G_{\alpha\alpha'}(p_1)G_{\beta\beta'}(p_2)\,\mathrm{Tr}\int d^4l\,\gamma_{\alpha'}\frac{i(\hat{l}-\hat{p}_1)-m_e}{(l-p_1)^2+m_e^2}\gamma_\gamma\frac{i(\hat{l}-\hat{p}_2)-m_e}{(l-p_2)^2+m_e^2}\gamma_{\beta'}\frac{i\hat{l}-m_\mu}{l^2+m_\mu^2}, \tag{26}$$

$$\Lambda^{(d)}_{\alpha\beta\gamma}(p_2,p_1)=\frac{i}{(2\pi)^4}\frac{eg_V^2}{\lambda_V}G_{\alpha\alpha'}(p_1)G_{\beta\beta'}(p_2)\,\mathrm{Tr}\int d^4l\,\gamma_{\beta'}\frac{i(\hat{l}+\hat{p}_2)-m_\mu}{(l+p_2)^2+m_\mu^2}\gamma_\gamma\frac{i(\hat{l}+\hat{p}_1)-m_\mu}{(l+p_1)^2+m_\mu^2}\gamma_{\alpha'}\frac{i\hat{l}-m_e}{l^2+m_e^2}. \tag{27}$$

The divergent parts of $\Lambda^{(c)}_{\alpha\beta\gamma}(p_1,p_2)$ and $\Lambda^{(d)}_{\alpha\beta\gamma}(p_1,p_2)$ are contained respectively in

$$[\Lambda^{(c)}_{\alpha\beta\gamma}(p_1,p_2)]_D=\frac{eg_V^2}{(2\pi)^4}G_{\alpha\alpha'}(p_1)G_{\beta\beta'}(p_2)\,\mathrm{Tr}\int\frac{d^4l}{(l^2+m_\mu^2)[(l-p_1)^2+m_e^2][(l-p_2)^2+m_e^2]} \times\gamma_{\alpha'}\gamma_\lambda\gamma_\gamma\gamma_\sigma\gamma_{\beta'}\gamma_\eta\;(l_\lambda l_\sigma-l_\lambda p_{2\sigma}p_{1\lambda})l_\eta,$$

$$[\Lambda^{(d)}_{\alpha\beta\gamma}(p_1,p_2)]_D=\frac{1}{(2\pi)^4}\frac{eg_V^2}{\lambda_V}G_{\alpha\alpha'}(p_1)G_{\beta\beta'}(p_2)\,\mathrm{Tr}\int\frac{d^4l}{(l^2+m_e^2)[(l+p_1)^2+m_\mu^2][(l+p_2)^2+m_\mu^2]} \times\gamma_{\beta'}\gamma_\lambda\gamma_\gamma\gamma_\sigma\gamma_{\alpha'}\gamma_\eta\;(l_\lambda l_\sigma+l_\lambda p_{1\sigma}+l_\sigma p_{2\lambda})l_\eta .$$

By utilizing the trace properties of the γ matrices and the usual techniques of symmetric integration one can easily convince oneself that $[\Lambda^{(c)}_{\alpha\beta\gamma}(p_1,p_2)]_D+[\Lambda^{(d)}_{\alpha\beta\gamma}(p_1,p_2)]_D$ is finite.

In a similar manner we can see that the matrix elements of the a^+a^-A and a^+A^-a vertices are also finite. We therefore conclude that all the second-order vertices are finite, without having to impose any more constraints on the coupling constants than are necessary to make the self-energies finite. This finiteness of the vertex functions is a particularly satisfying feature because as a result of this there will be no overlap divergences in higher orders.

VI. OBSERVABLE QUANTITIES

As pointed out earlier, all the new vector bosons introduced in the theory must have large masses in order that they do not affect the excellent agreement of the existing theory with experiment. An estimate of the masses of these particles ob-

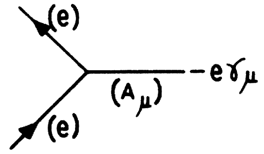

FIG. 12. $\bar{e}eA_\mu$ vertex.

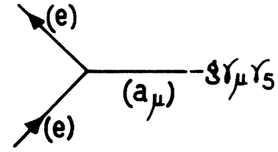

FIG. 13. $\bar{e}ea_\mu$ vertex.

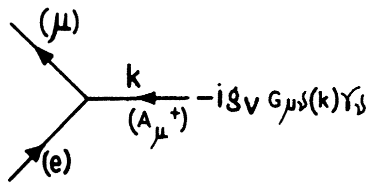

FIG. 14. $\bar{\mu}eA_\mu^+$ vertex.

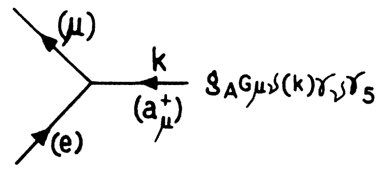

FIG. 15. $\bar{\mu}ea_\mu^+$ vertex.

tained from the extension of our theory to muon decay considered in Sec. VII yields a value of the order of 100 GeV. Hence these particles will be produced only at very high energies and will have very small magnetic moments. As an example of their effect on physical quantities we consider the modification of the electron anomalous magnetic moment coming from Fig. 5(b). We have

$$\Lambda_\mu^{(b)}(p_2,p_1)=\frac{ig^2}{(2\pi)^4}\int\frac{d^4k}{k^2+M^2}\gamma_\alpha\frac{i(\hat{p}-\hat{k})+m_e}{(p_2-k)^2+m_e{}^2}\times\gamma_\mu\frac{i(\hat{p}_1-\hat{k})+m_e}{(p_1-k)^2+m_e{}^2}\gamma_\beta G_{\alpha\beta}(k)\,. \tag{28}$$

Defining the form factors f_1 and f_2 by

$$\Lambda_\mu^{(b)}(p_2,p_1)=\gamma_\mu f_1(q^2)+i\sigma_{\mu\nu}q_\nu f_2(q^2)\,,$$

we find on calculation that $f_2(0)$ is of the order of $m_e{}^2/M^2$, which is 10^{-10} times the $\alpha/2\pi$ value. The contribution of Fig. 5(c) will be $O(m_\mu{}^2/M^2)$, which is $\sim 10^{-6}(\alpha/2\pi)$. The contributions of Figs. 5(d) and 5(f) are yet smaller, being at best $O([\ln(M^2/m^2)]/(M^4/m^4))$.

We need not go into a detailed discussion of other physical processes, such as Möller scattering and Lamb shift, because it is clear that the modifications due to the massive vector bosons will be negligible on account of their very large mass.

VII. EXTENSION TO LEPTONIC WEAK INTERACTIONS

The uncharged bosons introduced in the theory may be coupled to two-component neutrinos as well. Since the neutrinos are massless, such couplings will not alter the fermion self-energies; they will nevertheless alter the conditions of finiteness of the boson self-energies. However, the main point to note here is that the theory will be able to describe the leptonic weak interactions. The fact that the intermediate bosons here have muonic rather than ordinary electric charge means that the equivalent current-current theory would be related to that obtained from charged-intermediate-boson theory by a Fierz transformation.

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APPENDIX A

In this appendix we show how the effective nonlocal interaction of the massive vector and axial-vector bosons can be obtained from a local theory by a limiting procedure. For this we start with the local Lagrangian

$$\begin{aligned}\mathcal{L}=&-\bar{e}(\hat{\partial}+m)e-\tfrac{1}{4}(\partial_\mu a_\nu-\partial_\nu a_\mu)^2-\tfrac{1}{2}M^2a_\mu a_\mu\\&+ig\bar{e}\gamma_\mu\gamma_5 ea_\mu-\tfrac{1}{2}\partial_\mu\varphi\partial_\mu\varphi\\&+ig\epsilon\bar{e}\gamma_\mu\gamma_5 e\partial_\mu\varphi-(1/\epsilon)a_\mu\partial_\mu\varphi\,.\end{aligned}\tag{A1}$$

We first study, for finite ϵ, the effect of the mixing term $-(1/\epsilon)a_\mu\partial_\mu\varphi$ by calculating to all orders in $1/\epsilon$, but to zeroth order in g, the effective propagators and vertices. The bare propagators and vertices along with their exact values (to all orders in $1/\epsilon$) are given below in momentum space (see Fig. 9).

The corrected propagators for a_μ and φ may be obtained from the coupled equations represented diagramatically in Fig. 10, where the dot denotes the insertion of the mixing term. In momentum space these equations for the a_μ and φ propagators take the form

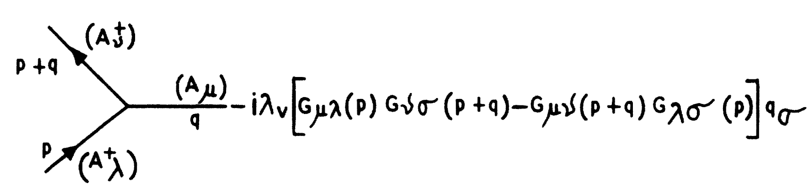
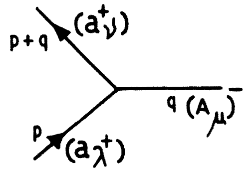


FIG. 16. $A_\mu A_\nu^- A_\lambda^+$ vertex.



$$-i\lambda_A\left[G_{\mu\lambda}(p)\,G_{\nu\sigma}(p+q)-G_{\mu\nu}(p+q)\,G_{\lambda\sigma}(p)\right]q_\sigma$$

FIG. 17. $A_\mu a_\nu^- a_\lambda^+$ vertex.

$$-i\frac{\delta_{\mu\nu}-\alpha(k^2)k_\mu k_\nu/k^2}{k^2+M^2}=-i\frac{\delta_{\mu\nu}+k_\mu k_\nu/M^2}{k^2+M^2}-\frac{i}{\epsilon^2}\frac{\beta(k^2)k_\lambda k_\sigma}{k^2(k^2+M^2)^2}\left(\delta_{\mu\lambda}+\frac{k_\mu k_\lambda}{M^2}\right)\left(\delta_{\nu\sigma}+\frac{k_\nu k_\sigma}{M^2}\right), \tag{A2}$$

and

$$-i\frac{\beta(k^2)}{k^2}=-\frac{i}{k^2}+\frac{1}{\epsilon}\frac{-i}{k^2+M^2}\frac{1}{k^4}k_\lambda k_\sigma\left(\delta_{\lambda\sigma}-\alpha(k^2)\frac{k_\lambda k_\sigma}{k^2}\right). \tag{A3}$$

These equations are to be solved to obtain $\alpha(k^2)$ and $\beta(k^2)$. On solving them, one finds

$$\alpha(k^2)=\frac{1+\epsilon^2k^2}{1-\epsilon^2M^2} \tag{A4}$$

and

$$\beta(k^2)=-\frac{\epsilon^2M^2}{1-\epsilon^2M^2}. \tag{A5}$$

The coupled equations for the vertices are shown diagrammatically in Fig. 11. In momentum space these correspond to

$$ig\left(\delta_{\mu\nu}-a(k^2)\frac{k_\mu k_\nu}{k^2}\right)=ig\delta_{\mu\nu}-ig\frac{k_\mu k_\lambda}{k^2}\left(\delta_{\nu\lambda}-k_\nu k_\lambda\frac{b(k^2)}{k^2}\right), \tag{A6}$$

and

$$ig\epsilon\left(\delta_{\mu\nu}-b(k^2)\frac{k_\mu k_\nu}{k^2}\right)ik_\nu=ig\epsilon\,\delta_{\mu\nu}ik_\nu-ig\frac{1}{k^2+M^2}\left(\delta_{\mu\lambda}-\frac{a(k^2)}{k^2}k_\mu k_\lambda\right)\times\left(\delta_{\nu\lambda}+\frac{k_\nu k_\lambda}{M^2}\right)ik_\nu, \tag{A7}$$

whose solution gives

$$\left.\begin{aligned}a(k^2)&=1\,,\\ b(k^2)&=0\,.\end{aligned}\right\} \tag{A8}$$

Using the values of $a(k^2)$, $b(k^2)$, $\alpha(k^2)$, and $\beta(k^2)$ in the vertices and propagators listed in the table we find the following:

propagator for a_μ to all orders in $1/\epsilon$:

$$-i\left(\delta_{\mu\nu}-\frac{1+\epsilon^2k^2}{1-\epsilon^2M^2}\frac{k_\mu k_\nu}{k^2}\right)\Big/(k^2+M^2),$$

propagator for φ to all orders in $1/\epsilon$:

$$-i\frac{-\epsilon^2M^2}{k^2(1-\epsilon^2M^2)},$$

eea vertex to all orders in $1/\epsilon$:

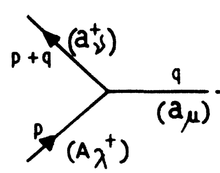
$$ig\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\gamma_\nu\gamma_5,$$

$ee\varphi$ vertex to all orders in $1/\epsilon$:

$$ig\epsilon\gamma_\mu\gamma_5(ik_\mu).$$

It should be noted that even for finite ϵ the effective eea vertex is purely transverse, which implies that a_μ is effectively coupled to a nonlocal conserved current.

However, since for finite ϵ the φ field is not decoupled, the diagrams containing the internal φ lines will make the theory nonrenormalizable on account of the derivative couplings. It is for this reason that we have to go to the limit $\epsilon\to 0$ under which $\beta(k^2)\to 0$. Therefore all the diagrams



$$-\lambda\,G_{\mu\sigma}(q)\left[G_{\sigma\lambda}(p)\,G_{\nu\eta}(p+q)-G_{\sigma\nu}(p+q)\,G_{\lambda\eta}(p)\right]q_\eta$$

FIG. 18. $a_\mu a_\nu^- A_\lambda^+$ vertex.

with internal φ lines drop out. The role of the φ field here is similar to that of a Goldstone boson.

Finally, we make the observation that for $\epsilon < 1/M$, $\beta(k^2) < 0$, and hence the φ propagator simulates a negative metric. This is rather unexpected since the original Lagrangian did not have any negative metric to start with.

APPENDIX B

The effective Feynman rules for the vertices corresponding to the interactions in the Lagrangian given by Eq. (1) are given in Figs. 12–18. These are obtained in the manner discussed in Appendix A.

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