Quark "bags" and local field theory*

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We show how the "bag" model of Chodos, Jaffe, Johnson, Thorn, and Weisskopf for confining quarks within extended hadrons can be realized as a limit of a conventional local field theory.

A recently proposed picture for hadrons[1] explains the success of the predictions of the quark model without requiring the existence of free quark states. In this model the strongly interacting particles are constructed from quarks held inside a relativistic "bag." The quarks are nearly free in the interior of the bag, while they are forbidden from penetrating the surface. The energy of this system is the quark energy plus a term proportional to the volume of the bag. This latter term acts to compress the bag against the outward pressure of the quark "gas" serving to keep the bag inflated.

The interesting feature of this model is a nonlocal description for a relativistic system. From the outset hadrons are extended objects. This is to be contrasted with conventional relativistic field theories where the point of departure is a set of local fields. Indeed, bag theories might be expected to possess quite different properties from local theories. For example, such a commonly accepted principle as analyticity of scattering amplitudes could be questioned.

In this note we show that the bag model of Ref. 1 is obtainable as a limit of a conventional local field theory. We utilize a particular example of a type of state recently discussed by Lee and Wick.[2] The philosophy behind our discussion is similar to that presented by Vinciarelli.[3]

Beginning with a local theory, we construct states of finite spatial extent. These states become the states of the bag model in a certain limit of the parameters in the theory. The local theory we consider contains a field which assumes the value zero in the lowest energy state, or vacuum. However, the theory allows a metastable state in which this field assumes a constant nonvanishing value. Beginning with the vacuum, we produce a bag state by forming the metastable state in a finite region of space. Surrounding this bag is a "skin" or transition region in which the field adjusts from zero to the metastable value in a manner that minimizes the energy per unit area of skin.

Such a state by itself is unstable because it can lose energy by returning to the ground state. To stabilize the bag we fill it with other fields, the quarks, carrying conserved quantum numbers. We introduce a coupling between the quark fields and the bag in such a manner that the quarks have a large effective mass outside the bag, but they are massless in the interior. As a consequence, it takes less energy to create a quark inside the bag than to create it outside, and so the quarks are held inside. However, the quark kinetic energy increases as the bag becomes smaller; this energy balances the forces tending to make the bag collapse.

The main objective of this paper is to show that the parameters of this theory have a limit in which the energy and thickness of the bag skin go to zero, at the same time as all masses except the interior quark mass go to infinity. This limit gives the bag model of Ref. 1.

To have physical hadrons with normal charges in theories with fractionally charged quarks, the authors of Ref. 1 couple the quarks to a massless vector-gluon field which is also confined to the bag. We keep this mechanism in our local theory, although before we take the limit giving the theory of Ref. 1 the gluon field has a large but finite mass outside the bag, and can thus penetrate a short distance through the skin. In our model calculation a single charged scalar field represents the quarks; thus we ignore the quark spin and multiplicity, as well as the gluon fields.

Our discussion involves classical unquantized systems. We conjecture that the quantized bag theory is the limit of the quantized version of the local field theory discussed classically here. We shall restrict the interaction terms in our theory to be renormalizable in perturbation theory, although it is not clear that this is necessary since perturbation theory does not yield the bag states.

We begin with the Lagrangian density

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \phi(x)\partial_\mu \phi(x) + \partial_\mu \psi^*(x)\partial_\mu \psi(x) - V(\phi(x)) - \lambda\psi^*(x)\psi(x)[\phi(x)-\beta]^2 , \quad (1)$$

where

$$V(\phi) = \alpha[\tfrac{1}{4}\phi^4 - (\beta+\gamma)\tfrac{1}{3}\phi^3 + \beta\gamma\tfrac{1}{2}\phi^2] . \quad (2)$$

The quantities α, β, γ, and λ are positive parameters. Here $\phi(x)$ is the real scalar field which will produce the bag, and $\psi(x)$ is the complex quark field.[4] The potential $V(\phi)$ is plotted in Fig. 1. We constrain

$$\frac{1}{2} < \frac{\gamma}{\beta} < 1 \tag{3}$$

so that $V(\phi)$ has two minima, the lowest at $\phi = 0$ and another at $\phi = \beta$. The Hamiltonian density corresponding to this Lagrangian is

$$\mathcal{H}(x) = \tfrac{1}{2}[\partial_0\phi(x)\partial_0\phi(x) + \vec{\nabla}\phi(x)\cdot\vec{\nabla}\phi(x)] + \partial_0\psi^*(x)\partial_0\psi(x) + \vec{\nabla}\psi^*(x)\cdot\vec{\nabla}\psi(x) + V(\phi(x)) + \lambda\psi^*(x)\psi(x)[\phi(x)-\beta]^2. \tag{4}$$

In the absence of $\psi(x)$ the state of lowest energy has $\phi(x) = 0$ for all x, but there exists a metastable state with $\phi(x) = \beta$. The equations of motion for the fields are

$$\begin{aligned} \Box\phi(x) &= -\alpha\phi(x)[\phi(x)-\gamma][\phi(x)-\beta] - 2\lambda\psi^*(x)\psi(x)[\phi(x)-\beta], \\ \Box\psi(x) &= -\lambda\psi(x)[\phi(x)-\beta]^2 . \end{aligned} \tag{5}$$

Since these equations involve second derivatives with respect to time, complete specification of the system at any one time requires knowledge of both the fields and their first time derivatives. The field $\psi(x)$ carries a conserved current

$$\begin{aligned} j_\mu(x) &= \psi^*(x)\partial_\mu\psi(x) - [\partial_\mu\psi^*(x)]\psi(x), \\ \partial\cdot j &= 0, \end{aligned} \tag{6}$$

and the associated charge

$$\begin{aligned} Q &= \int d^3x\, j_0(x), \\ \frac{dQ}{dt} &= 0 . \end{aligned} \tag{7}$$

We shall vary the parameters α, β, γ, and λ to obtain the bag theory of Ref. 1. The dynamics of the latter theory is determined by the Lagrangian

$$L_{\rm bag}(t) = \int_{R(t)} d^3x\,[\partial_\mu\chi^*(x)\partial_\mu\chi(x) - \beta] + \int_{\bar{R}(t)} d^3x\,[\partial_\mu\chi^*(x)\partial_\mu\chi(x) - M^2\chi^*(x)\chi(x)] \tag{8}$$

in the limit $M\to\infty$. Here $R(t)$ is the region of space referred to as the "bag" and $\bar{R}(t)$ is its complement.[5] The field $\chi(x)$ represents the quarks and corresponds to our field $\psi(x)$. The resulting bag Hamiltonian is

$$H_{\rm bag} = \int_{R(t)} d^3x[\partial_0\chi^*(x)\partial_0\chi(x) + \vec{\nabla}\chi^*(x)\cdot\vec{\nabla}\chi(x) + B] + \int_{\bar{R}(t)} d^3x[\partial_0\chi^*(x)\partial_0\chi(x) + \vec{\nabla}\chi^*(x)\cdot\vec{\nabla}\chi(x) + M^2\chi^*\chi]. \tag{9}$$

To obtain this theory from our local theory, we construct bag states characterized by $\phi(x)$ having the value β inside the bag. The field $\phi(x)$ adjusts itself outside the bag so as to minimize the energy of the system. We must now show the existence of a limit of the parameters in the theory such that these spatial variations in $\phi(x)$ occur only in a vanishingly small region about the surface of the bag. To avoid particles corresponding to fluctuations of $\phi(x)$ about its average value either inside or outside the bag, the above limit must give a large effective mass to such particles. The mass associated with excitations of $\psi(x)$ is also to become large outside the bag, but inside the bag ψ is massless and becomes effectively free.

We now give the required conditions more precisely. The effective mass of ψ excitation outside the bag is

$$m_{\psi,E}^2 = \lambda\beta^2 . \tag{10}$$

Thus, to obtain the bag model we require

$$\lambda\beta^2 \to \infty . \tag{11}$$

The exterior ϕ mass yields the condition

$$m_{\phi,E}^2 = \alpha\beta\gamma \to \infty , \tag{12}$$

while the interior ϕ mass gives

$$m_{\phi,I}^2 = \left.\frac{d^2V(\phi)}{d\phi^2}\right|_{\phi=\beta} = \alpha\beta(\beta-\gamma) \to \infty . \tag{13}$$

Because of Eq. (3), both Eqs. (12) and (13) are equivalent to

$$\alpha\beta^2 \to \infty . \tag{14}$$

To keep a term in the Hamiltonian equal to B times the bag volume, we require

$$V(\beta) = \frac{1}{6}\alpha\beta^4\left(\frac{\gamma}{\beta} - \frac{1}{2}\right) = B . \tag{15}$$

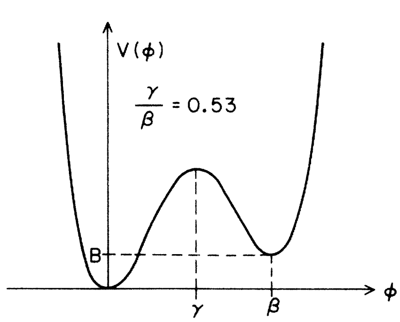

FIG. 1. The shape of the potential $V(\phi)$ of Eq. (2), plotted for $\gamma/\beta = 0.53$.

We must adjust the parameters so that both the thickness and energy of the bag skin go to zero. To estimate these quantities we use a variational calculation. We approximate the skin shape with a simple linear form, knowing that the true skin will have a lower energy content. At a particular time we consider a cross section of skin with $\phi(x)$ as shown in Fig. 2.[6] Here Δ parametrizes the skin thickness. We take $\partial_0\phi(x) = 0 = \psi(x) = \partial_0\psi(x)$. Later we shall argue that the presence of ψ will not significantly alter the skin properties. The energy per unit area of such a skin is

$$E_S(\Delta) = \int_0^\Delta dx \left[\frac{1}{2}\left(\frac{d}{dx}\phi(x)\right)^2 + V(\phi(x))\right]$$

$$= \frac{1}{2}\frac{\beta^2}{\Delta} + \frac{1}{12}\alpha\beta^4\Delta\left(\frac{\gamma}{\beta} - \frac{2}{5}\right). \tag{16}$$

Minimizing with respect to Δ gives

$$\Delta_{\min}{}^2 = \frac{6}{\alpha\beta^2}\left(\frac{\gamma}{\beta} - \frac{2}{5}\right)^{-1}, \tag{17}$$

$$E_{s,\min} = \left[\frac{\alpha\beta^6}{6}\left(\frac{\gamma}{\beta} - \frac{2}{5}\right)\right]^{1/2}. \tag{18}$$

Recalling that $\frac{1}{2} < \gamma/\beta < 1$, we find that to obtain $\Delta_{\min} \to 0$ and $E_{s,\min} \to 0$ requires

$$\alpha\beta^2 \to \infty, \tag{19}$$

$$\alpha\beta^6 \to 0. \tag{20}$$

Note that conditions (14) and (19) are equivalent.[7]

As another condition on our limit, we require the ψ particles inside the bag to become effectively free. Although the ϕ excitations in the bag have a large mass, they can produce an effective coupling among the ψ particles, just as massive intermediate vector bosons in theories of the weak interactions produce effective four-fermion couplings at low energy. Consequently, we require the term $\lambda\psi^*(x)\psi(x)[\phi(x) - \beta]^2$ to be unimportant inside the bag. Since variations of $\phi(x)$ about β are parametrized by $m_{\phi,I}$, we demand of our limit

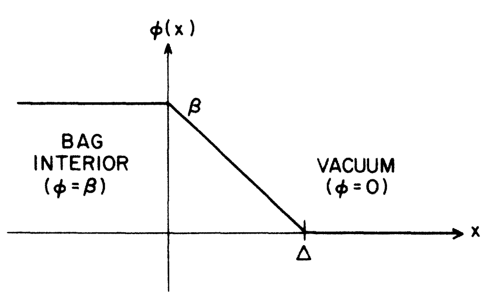

FIG. 2. The parametrization of $\phi(x)$ used to estimate the thickness and energy of the bag skin.

$$\frac{\lambda}{m_{\phi,I}{}^2} \sim \frac{\lambda}{\alpha\beta^2} \to 0. \tag{21}$$

Finally we discuss the behavior of $\psi(x)$ in the vicinity of the skin. Combining Eqs. (10) and (17) we see that

$$m_{\psi,E}{}^2\Delta_{\min}{}^2 \sim \lambda/\alpha. \tag{22}$$

The conditions (19), (20), and (21) imply that the right-hand side of Eq. (22) goes to zero in our limit. Unless the ψ field has important contributions with momenta large compared to the external ψ mass, ψ effectively cannot see the thickness of the skin. Neglecting such high-momentum components, we consider varying the field $\phi(x)$ so as to produce an infinitesimal coherent shift in the skin position. The action derived from the Lagrangian in Eq. (1) must, of course, be invariant under such a variation. This immediately yields boundary conditions for ψ identical with those found by variation of the bag boundary in the Lagrangian of Eq. (8).[5] For consistency, we still must show that the interaction between $\psi(x)$ and $\phi(x)$ does not appreciably alter the properties of the skin. Thus we ask that

$$\Delta_{\min}\langle\lambda\psi^*(x)\psi(x)[\phi(x) - \beta]^2\rangle_{\text{skin}} \ll E_s. \tag{23}$$

This will follow if

$$\langle\psi^*(x)\psi(x)\rangle_{\text{skin}} \ll \frac{\alpha\beta^2}{\lambda}. \tag{24}$$

On the other hand, condition (21) implies that the right-hand side of relation (24) goes to infinity. Since in the limit $\psi(x)$ vanishes on the skin, this condition is automatic.

We summarize all the conditions we wish to satisfy:

$$\lambda\beta^2 \to \infty, \tag{25a}$$

$$\alpha\beta^2 \to \infty, \tag{25b}$$

$$\frac{\gamma}{\beta} = \frac{1}{2} + \frac{6B}{\alpha\beta^4}, \tag{25c}$$

$$\alpha\beta^6 \to 0, \tag{25d}$$

$$\frac{\lambda}{\alpha\beta^2} \to 0. \tag{25e}$$

Conditions (25b) and (25d) immediately tell us that $\beta \to 0$. This means that the numerical shift in ϕ required to produce a bag is small. We now parametrize α and λ by

$$\alpha = R_1\beta^{-(p_1+4)}, \quad \lambda = R_2\beta^{-(p_2+2)}, \tag{26}$$

where R_1 and R_2 are positive constants. The conditions (25) now become

$$\beta \to 0\ , \tag{27}$$

$$0 < p_2 < p_1 < 2\ , \tag{28}$$

$$\gamma = \frac{1}{2}\beta\left(1 + \frac{12B}{R_1}\beta^{p_1}\right)\ . \tag{29}$$

These conditions are clearly easy to satisfy.

In conclusion, the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \phi(x)\partial_\mu \phi(x) + \partial_\mu \psi^*(x)\partial_\mu \psi(x) - V(\phi(x)) - R_2\beta^{-(p_2+2)}[\phi(x) - \beta]^2\ , \tag{30}$$

where

$$V(\phi) = R_1\beta^{-(p_1+4)}\left[\frac{\phi^4}{4} - \frac{1}{2}\beta\left(1 + \frac{4B\beta^{p_1}}{R_1}\right)\phi^3 + \frac{\beta^2}{4}\left(1 + \frac{12B\beta^{p_1}}{R_1}\right)\phi^2\right]\ , \tag{31}$$

describes, in the limit $\beta \to 0$, the bag theory of Ref. 1. The positive parameters R_1, R_2 are arbitrary, while p_1 and p_2 are constrained by the inequality (28). As the limit $\beta \to 0$ is taken, the two minima of $V(\phi)$ approach each other but are separated by a barrier of growing height. The depths of these two minima differ by the bag constant B.

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[1]A. Chodos, R. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974).

[2]T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974).

[3]P. Vinciarelli, Nuovo Cimento Lett. 4, 905 (1972).

[4]Our model calculation differs slightly from that in Ref. 1 in that we fill the bag with a charged field (ψ), while Ref. 1 considers a neutral one.

[5]We are discussing the system of Ref. 1 with the boundary conditions referred to as Dirichlet rather than the alternative theory with Neumann boundary conditions.

[6]Of course, in the true minimum-energy configuration, $\phi(x)$ has a small nonzero value even for large x. The skin thickness Δ parametrizes the distance beyond which $\phi(x)$ is in some sense negligible. Also note that our parametrization for $\phi(x)$ has a discontinuous first derivative. Since the equations of motion involve second derivatives with respect to x, to avoid infinities we should slightly round off the corners of our parametric form. This problem does not enter the calculation of the energy; so, we can ignore it here.

[7]We remark here that a possibly interesting variation on the bag model follows if $E_{s,\min}$ is allowed to remain finite. This generates an effective term in the bag Hamiltonian proportional to the surface area of the bag. Such a surface term also acts to hold the bag together, removing the necessity for nonzero B.