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Electron, muon, proton, and strong gravity

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Regarding the strong interaction as a manifestation of a strong gravity, the electron and the proton are identified as black holes in the strong gravitational field. The mass and classical radius of the electron are predicted to be $\frac{1}{2}\alpha m_\pi = 0.51$ MeV and $r_e = e^2/2m_p c^2 = 0.74 \times 10^{-16}$ cm, respectively. It is shown that the electron can be strongly interacting at energies above 240 GeV. The strong gravitational constant is predicted to be 3.9×10^{31} dyn cm^2 g^{-2}. The neutron-proton mass difference is partly explained. A neutral-pion–electron system bound by strong gravity is interpreted as the muon, and the muon-electron mass ratio is predicted to be $m_\mu/m_e = \frac{3}{2}(1/\alpha) + (2/\alpha)m_e/m_\pi = 206.6$. It is suggested that the weak interaction might also be a manifestation of strong gravity and the intermediate-boson mass is of order 240 GeV.

I. INTRODUCTION

The finite electron self-energy is a puzzling problem in both quantum theory and classical theory. Quantum electrodynamics, with its remarkable predictive power, fails to explain the origin of the finite electron mass, and none of the proposed regularization schemes have succeeded in predicting the observed mass. An electron of finite radius first proposed by Abraham and Lorentz makes the electrostatic energy of the electron finite.[1] Nevertheless, it is well known that an extended charge distribution interacting with itself cannot hold together and nonelectromagnetic forces are needed to prevent the electron from exploding. Recently, Isham, Salam, and Strathdee[2] proposed a theory in which gravitation was supposed to regularize lepton electrodynamics, whereas a strong gravity plays the same role in hadron dynamics. Motivated by their work, we have constructed a classical gravitational theory of the electron, the muon, and the proton. However, according to our theory the existence of the electron, the muon, and the proton depends on a strong interaction regarded as a form of strong gravity; even in the immediate neighborhood of the electron the spacetime structure is determined by strong gravity. The electron and the proton are identified as black holes in the strong gravitational field. Our theory predicts the electron mass and classical radius as 0.51 MeV and 0.74×10^{-16} cm, respectively. The constant of strong gravitation comes out as a by-product of the theory; its magnitude is 3.9×10^{31} dyn cm^2 g^{-2}. The theory also partly accounts for the neutron-proton mass difference. It is shown that the electron can be strongly interacting at energies above 240 GeV. A system of the electron and the neutral pion bound by strong gravity is interpreted as the muon. The muon-electron mass ratio is predicted to be 206.6. It is suggested that the weak interaction might also be a manifestation of strong gravity, and that the mass of the intermediate boson is of order 240 GeV.

II. SINGULARITIES IN THE STRONG GRAVITATIONAL FIELD

Black holes, which are singularities in the gravitational field, being characterized by a few observable parameters such as mass, angular momentum, and charge, resemble elementary particles more than composite objects. However, because of the extreme weakness of the gravitational coupling constant, the singularities in the gravitational field of a charged particle will have mass-to-charge ratios far greater than those for any of the known elementary particles. The Nordström-Reissner solution of the Einstein equation for a particle of mass m and charge e is given by the metric[3]

$$ds^2 = -\gamma^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + \gamma dt^2, \quad (1)$$

where

$$\gamma = 1 - \frac{2G_n m}{c^2 r} + \frac{G_n e^2}{c^4 r^2}, \quad (2)$$

with G_n the Newtonian gravitational constant. When $e=0$, we obtain the usual Schwarzschild form of the metric, which has a singularity at $r = 2G_n m/c^2$. The metric (1) will have two singularities if the quadratic equation

$$r^2 - \frac{2G_n m}{c^2} r + \frac{G_n e^2}{c^4} = 0 \quad (3)$$

has real roots, the condition for which is

$$\frac{G_n m^2}{e^2} \geq 1 \quad \text{or} \quad \frac{G_n m^2}{\hbar c} \geq \alpha\,, \tag{4}$$

where α is the fine-structure constant. For the proton $G_n m_p{}^2/\hbar c \simeq 10^{-40}$. Hence the gravitational field of the proton (or any other known elementary particle) will not have singularities corresponding to $\gamma = 0$.

However, if we assume that the structure of space-time in the immediate neighborhood of hadronic matter is determined by strong gravity, then the constant G_n in (2) should be replaced by the strong gravitational constant G_s. The dimensionless strong gravitational constant should have the same order of magnitude as the dimensionless strong-interaction coupling constant, i.e.,

$$\frac{G_s m_p{}^2}{\hbar c} \sim 1\,. \tag{5}$$

(Since we are interested in the strong gravitational field in the immediate neighborhood of hadronic matter, any exponential factors of the form $e^{-\mu r}$ occurring in γ due to the finite range of the strong gravity can be neglected.) Now we examine the singularities in the strong gravitational field of the charged nucleon given by the metric (1) with

$$\gamma = \gamma_s = 1 - \frac{2G_s m_n}{c^2 r} + \frac{G_s e^2}{c^4 r^2}\,, \tag{6}$$

where m_n is the uncharged nucleon (neutron) mass. Since $G_s m_n{}^2/e^2 \gg 1$, the equation $\gamma_s = 0$ has two real roots:

$$r_1 = \frac{G_s m_n}{c^2} - \frac{G_s m_n}{c^2}\left(1 - \frac{e^2}{G_s m_n{}^2}\right)^{1/2} \simeq \frac{e^2}{2m_n c^2}\,, \tag{7}$$

$$r_2 = \frac{G_s m_n}{c^2} + \frac{G_s m_n}{c^2}\left(1 - \frac{e^2}{G_s m_n{}^2}\right)^{1/2} \simeq \frac{2G_s m_n}{c^2} - \frac{e^2}{2m_n c^2}\,. \tag{8}$$

If m_{eff} is the effective mass[4] of the charged nucleon, then one should also be able to write the metric describing the strong gravitational field of the charged nucleon as (1) with

$$\gamma = \gamma_s = 1 - \frac{2G_s m_{\text{eff}}}{c^2 r}\,, \tag{9}$$

for $r \geq$ radius of the particle. Equating (6) and (9) we get

$$\frac{2G_s m_{\text{eff}}}{c^2 r} = \frac{2G_s m_n}{c^2 r} - \frac{G_s e^2}{c^4 r^2}\,. \tag{10}$$

But the quantity which we call the radius of the particle can also be taken as the range of strong interaction associated with the particle. Hence the condition (10) can hold only for those values of r which can be taken as the classical radii of the particles. We take r_1 and r_2 (roots of $\gamma_s = 0$) as these allowed values of r. When $r = r_1$ or r_2 the right-hand side of Eq. (10) becomes unity. Thus if m_1 and m_2 are the effective masses of the singularities in the strong gravitational field of the charged nucleon, we have

$$\frac{2G_s m_1}{c^2 r_1} = 1\,, \tag{11}$$

$$\frac{2G_s m_2}{c^2 r_2} = 1\,. \tag{12}$$

From (8) and (12) we get

$$m_2 = m_n - \frac{e^2}{4G_s m_n} \tag{13}$$

When $e = 0$ in (13), $m_2 = m_n$, we interpret m_2 as the proton mass. The most reasonable quantity that corresponds to the classical radius of the proton is the Compton wavelength of the pion; therefore, we set

$$r_2 = \frac{\hbar}{m_\pi c}\,, \tag{14}$$

where m_π = pion mass.

III. ELECTRON MASS

Equations (7), (8), (11), (12), and (14) are sufficient for us to determine m_1 and G_s. From (11) and (12)

$$m_1/m_2 = r_1/r_2\,. \tag{15}$$

Using (7) and (14) in the above we get

$$m_1 = \tfrac{1}{2}(e^2/\hbar c)\,\frac{m_\pi m_p}{m_n} \simeq \tfrac{1}{2}(e^2/\hbar c) m_\pi = 0.51\ \text{MeV}\,. \tag{16}$$

The mass m_1 is almost exactly the same as that of the electron. We identify the singularity corresponding to $r = r_1$ as the electron. In the following it will be shown that this singularity can have properties of the electron.

The masses m_1, m_2 and radii r_1, r_2 of the two singularities in the strong gravitational field of the charged nucleon, which we have interpreted as the electron and the proton, are connected by the same relations [Eqs. (11) and (12)] as those of the mass and radius of a black hole, suggesting that they are black holes in the strong gravitational field. The classical radius of the proton, which we have interpreted as the Schwarzschild radius of the singularity corresponding to the proton, is $r_2 = 1.4 \times 10^{-13}$ cm. The range of the proton's strong interaction, r_2, determines the mass of the strong-interacting quanta, that is, the pions associated with it. In the same way, since the classical radius of the electron according to our theory is $r_1 = e^2/2m_n c^2 = 0.74\times 10^{-16}$ cm, the strong-interacting quanta associated with it should have a mass of approximately 240 GeV. Present experiments do not rule out such a possibility. For the same reason any deviations of electron interaction from quantum electrodynamics can be expected to occur at energies above 240 GeV. We have not taken electron spin into account in the above treatment. It is well known that a charged black hole with angular momentum has a g factor of 2, the same value as for the electron.[5] This might be another justification for our identification of the electron as a black hole.

The strong gravitational constant comes out as a by-product of the theory. From (7) and (11) we have

$$G_s = e^2/4m_e m_n = 3.87\times 10^{31}\ \mathrm{dyn\,cm^2\,g^{-2}}, \tag{17}$$

where $m_1 = m_e$, or

$$G_s m_n{}^2/\hbar c = 3.42 .$$

IV. NEUTRON-PROTON MASS DIFFERENCE

With our identification of m_2 as the proton mass, it follows from (13) and (17) that

$$m_n - m_p = m_e . \tag{18}$$

The above value is smaller than the observed neutron-proton mass difference, roughly by a factor of 2. However, the result is encouraging because the quantities that we have denoted by m_p $(= m_2)$ and m_n are not exactly the physical masses of the proton and the neutron. The proton and the neutron also have electromagnetic self-energies associated with their magnetic moments which are not included in (18).

V. MUON MASS

If the muon mass and its classical radius also satisfy a relation of the form (11) or (12), that is, if

$$2G_s m_\mu/c^2 r_\mu = 1 , \tag{19}$$

then the muon radius r_μ turns out to be approximately 1.4×10^{-14} cm. According to our theory this would imply that the muon is strongly interacting at energies above 1.4 GeV, which is ruled out by experiment. Thus the black-hole interpretation is not possible for the muon. However, since the electron is strongly interacting at distances comparable to its classical radius r_e, a neutral pion could form a bound system with the electron if its mass were concentrated near the electron surface. We assume that the muon is such a system. The electron–neutral-pion system in the S state will have spin $\frac{1}{2}$ and the total energy of the bound system is given by

$$m_\mu = m_e + m_\pi + V + T , \tag{20}$$

where V is the strong gravitational potential energy of the system,

$$V = -\frac{G_s m_e m_\pi}{r_e c^2} ;$$

T is the kinetic energy of the bound particles. From the virial theorem for the inverse-square law

$$T = -\tfrac{1}{2}V ;$$

therefore, (20) becomes

$$m_\mu = m_\pi + m_e - \frac{G_s m_e m_\pi}{2r_e c^2}$$

and substitution for r_e from (11) yields

$$m_\mu = m_e + \tfrac{3}{4} m_\pi . \tag{21}$$

Hence from (16) and (21), taking m_π to be the neutral-pion mass, we get

$$m_\mu/m_e = \tfrac{3}{2}(1/\alpha) + (2/\alpha) m_e/m_\pi = 206.6 , \tag{22}$$

where α is the fine-structure constant. The above expression does not depend on G_s, and the predicted value for m_μ/m_e is remarkably close to the observed value $m_\mu/m_e = 206.765$. The theory also implies that the muon classical radius is of the same order as the electron classical radius. The mutual electrostatic repulsion prevents the formation of an electron–negative-pion bound system. Similarly, an electron–positive-pion system is impossible because the electrostatic binding energy of the system, $-e^2/r_e = -2m_n$, makes the total energy of the system negative. Thus doubly charged or neutral muons cannot exist. The fact that the electron and the muon have classical radii of the same order would account for the validity of electron-muon universality to a high degree of accuracy. However, as expected, the above classical theory has not explained why the electron and the muon carry different quantum numbers.

If the electron forms bound states with other mesons, it will give rise to heavy leptons. An interesting possibility is an η-e system bound by strong gravity. Such a system will have a mass of $\frac{3}{4} m_\eta + m_e \simeq 412$ MeV. Ramm[6] has reported evidence for a heavy charged lepton with a mass between 422 and 437 MeV, which is very close to the above value.

VI. WEAK INTERACTION

Since the radius of the electron is 0.74×10^{-16} cm, we have argued that strongly interacting quanta associated with it should have a mass of order 240 GeV. It is possible that these quanta are the same as the intermediate bosons which are supposed to mediate the weak interaction, and the weak interaction might also be a manifestation of the strong interaction (strong gravity). In such a case the dimensionless weak-interaction coupling constant $g^2/\hbar c$ should have the same order of magnitude as the dimensionless strong gravitational coupling constant given by (17). This in fact seems to be realized; for $m_W = 240$ GeV and $g^2 = G_s m_p^{\,2}$ the Fermi coupling constant $G_W = g^2 2^{1/2}/m_W$ turns out to be of order $10^{-5} m_p^{\,-2}$, which is just the observed order of magnitude of the Fermi constant. Even in the absence of an intermediate boson one would expect any nonlocality of the 4-fermion interaction to be exhibited at distances comparable to the electron radius; the unitarity requirement on the S-wave neutrino-electron scattering cross section[7] sets the above limit to be less than approximately 0.74×10^{-16} cm, which again is of the same order of magnitude as the classical radius of the electron predicted by us.

VII. CONCLUSION

A possible objection to our theory is that we have applied classical ideas to a highly relativistic quantum-mechanical problem. However, a more careful consideration reveals that the above argument is unjustified. The self-energy contribution to the electron mass deduced from quantum electrodynamics is of the order

$$\delta m_e \simeq \frac{3\alpha m_e}{2\pi}\ln(\Lambda/m_e)\,.$$

When the electron classical radius $r_e = 0.74\times10^{-16}$ cm the cutoff Λ takes the value 240 GeV, giving $\delta m_e \sim 0.01$ MeV. Thus a large percentage of the observed mass of the electron is its bare mass, and what the present theory has predicted is mostly this mass. Quantum electrodynamics holds up to distances comparable to r_e. The exact dynamics holding at distances less than r_e, approximately reducing to classical general relativity and even to Newtonian equations on a further approximation, cannot be ruled out.

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