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Angular momentum composition of the proton in the quark-parton model*

L. M. Sehgal
III. Physikalisches Institut, Technische Hochschule Aachen, Aachen, West Germany†
and CERN, Geneva, Switzerland
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Assuming the angular momentum of a polarized proton ($J_Z=\frac{1}{2}$) to be the resultant of the total spin S_Z and total orbital momentum L_Z of its parton constituents (quarks and antiquarks), we find $S_Z=\frac{1}{2}(3F-D)$, $L_Z=\frac{1}{2}(1-3F+D)$. The approximation employed is the same as that leading to the Ellis-Jaffe sum rules for polarized electron-nucleon scattering. The result for L_Z, interpreted geometrically, implies that a polarized proton possesses a significant amount of rotation. On the assumption that S_Z and L_Z are dominated by $\mathcal{P}$ and $\mathfrak{N}$ quarks, an estimate is made of the separate contribution of these constituents.

This note concerns certain consequences arising out of the application of the quark-parton model to the angular momentum structure of a polarized proton.

Consider a positive-helicity proton moving with infinite momentum in the Z direction ($J_Z=\frac{1}{2}$). The angular momentum of such a proton is envisaged as being the resultant of the spin and orbital momenta of its parton constituents, which we take to be quarks and antiquarks. Each parton species ($\mathcal{P}, \mathfrak{N}, \lambda, \overline{\mathcal{P}}, \overline{\mathfrak{N}}, \overline{\lambda}$) contributes a net spin angular momentum proportional to the polarization of that species in the Z direction. In addition, each species contributes orbital angular momentum by virtue of the fact that partons may carry nonvanishing momenta transverse to the direction of motion of the proton. Denoting the total spin and total orbital momentum by S_Z and L_Z, respectively,

$$J_Z=S_Z+L_Z=\tfrac{1}{2}. \tag{1}$$

Let us define distribution functions[1] $u_\pm(x)$, $d_\pm(x)$, $s_\pm(x)$, $\bar{u}_\pm(x)$, $\bar{d}_\pm(x)$, and $\bar{s}_\pm(x)$ representing the average number of quarks of the types $\mathcal{P}$, $\mathfrak{N}$, λ, $\overline{\mathcal{P}}$, $\overline{\mathfrak{N}}$, and $\overline{\lambda}$ present in the proton with a fraction x of the proton's longitudinal momentum, the subscript + (−) denoting a quark of positive (negative) helicity. The total spin S_Z is then given by

$$S_Z=\tfrac{1}{2}\int[(u_+-u_-)+(\bar{u}_+-\bar{u}_-)+(d_+-d_-)+(\bar{d}_+-\bar{d}_-)+(s_+-s_-)+(\bar{s}_+-\bar{s}_-)]\,dx. \tag{2}$$

We now recall the sum rule of Bjorken[2] which relates the ratio G_A/G_V of coupling constants in $n\to pe^-\bar{\nu}$ to the structure functions g_{1p} and g_{1n} occurring in polarized electron-nucleon scattering. The sum rule reads

$$\left(\frac{G_A}{G_V}\right)_{n\to p}=6\int(g_{1p}-g_{1n})\,dx$$
$$=\int[(u_+-u_-)+(\bar{u}_+-\bar{u}_-)-(d_+-d_-)-(\bar{d}_+-\bar{d}_-)]\,dx, \tag{3}$$

where the second equality is the parton-model result.[3] A relation analogous to (3) can also be written for the coupling-constant ratio G_A/G_V in the decay $\Xi^-\to\Xi^0e^-\bar{\nu}$. Assuming SU(3) symmetry, the structure of Ξ^0 can be related to that of the proton,[4] giving the result

$$\left(\frac{G_A}{G_V}\right)_{\Xi^-\to\Xi^0}=\int[(d_+-d_-)+(\bar{d}_+-\bar{d}_-)-(s_+-s_-)-(\bar{s}_+-\bar{s}_-)]\,dx. \tag{4}$$

Combining (2), (3), and (4),

$$S_Z=\frac{1}{2}\left[\left(\frac{G_A}{G_V}\right)_{n\to p}+2\left(\frac{G_A}{G_V}\right)_{\Xi^-\to\Xi^0}\right]+\tfrac{3}{2}\int[(s_+-s_-)+(\bar{s}_+-\bar{s}_-)]\,dx. \tag{5}$$

We now argue that, on dynamical grounds, the term $[(s_+-s_-)+(\bar{s}_+-\bar{s}_-)]$ occurring in the above equation is likely to be small and may be neglected. The argument rests on the belief that strange partons present in a proton are part of a "sea" of quark-antiquark pairs to which some symmetry properties may reasonably be ascribed. One possible assumption is that the sea is unpolarized,[5] in which case $s_+=s_-$, $\bar{s}_+=\bar{s}_-$, and the above term vanishes. An alternative hypothesis also suffices, namely that the sea is CP-symmetric. This implies $s_+=\bar{s}_-$, $s_-=\bar{s}_+$ and again gives zero for the term in question. If we allow for a small degree of polarization of the sea as well as a small amount of CP asymmetry, the term $[(s_+-s_-)$

$+(\bar{s}_+-\bar{s}_-)]$ will be a quantity of the second order of smallness. Consequently its neglect should be a reasonable approximation.

If the validity of the above argument is accepted, we obtain for the spin angular momentum of the proton

$$S_Z \approx \frac{1}{2}\left[\left(\frac{G_A}{G_V}\right)_{n\to p} + 2\left(\frac{G_A}{G_V}\right)_{\Xi^-\to\Xi^0}\right]$$

$$= \tfrac{1}{2}(3F-D), \tag{6}$$

where we have used the octet-model results

$$(G_A/G_V)_{n\to p} = F+D, \quad (G_A/G_V)_{\Xi^-\to\Xi^0} = F-D.$$

Equation (1) then implies

$$L_Z = \tfrac{1}{2} - S_Z = \tfrac{1}{2}(1-3F+D). \tag{7}$$

Using the experimental values $D+F=1.25\pm0.01$, $F/D=0.58\pm0.03$,[6]

$$S_Z = 0.30\pm0.05, \quad L_Z = 0.20\pm0.05. \tag{8}$$

The above result shows that nearly 40% of the angular momentum of a polarized proton arises from the orbital motion of its constituents. In the geometrical picture of hadron structure, this implies that *a polarized proton possesses a significant amount of rotation*. (Interestingly, the concept of a rotating proton has been proposed by Yang[7] as a possible explanation for the recently discovered increase in total hadron cross sections.) It is interesting to contrast the result (8) with the angular momentum composition of the proton in the [SU(6)-symmetric] 3-quark model. In this model, the quarks are assumed to be in S-wave orbits, and consequently $L_Z=0$, $S_Z=\frac{1}{2}$. It may be recalled that SU(6) symmetry gives the results $(G_A/G_V)_{n\to p}=F+D=5/3$, $F/D=2/3$. When these values of F and D are substituted in (6) and (7) we indeed obtain $L_Z=0$, $S_Z=\frac{1}{2}$. The contrast between the parton-model results and those of the 3-quark model is an illustration of the difference between "current quarks" (or partons) and "constituent quarks."[8]

We now show that the assumptions underlying the derivation of the result (6) [namely, SU(3) symmetry and the neglect of the strange-parton contribution in Eq. (5)] also lead to sum rules for the structure functions g_{1p} and g_{1n}, first obtained by Ellis and Jaffe.[9] The parton-model expressions for these quantities are

$$2g_{1p} = \tfrac{4}{9}[(u_+-u_-)+(\bar{u}_+-\bar{u}_-)] + \tfrac{1}{9}[(d_+-d_-)+(\bar{d}_+-\bar{d}_-)] + \tfrac{1}{9}[(s_+-s_-)+(\bar{s}_+-\bar{s}_-)],$$
$$2g_{1n} = \tfrac{1}{9}[(u_+-u_-)+(\bar{u}_+-\bar{u}_-)] + \tfrac{4}{9}[(d_+-d_-)+(\bar{d}_+-\bar{d}_-)] + \tfrac{1}{9}[(s_+-s_-)+(\bar{s}_+-\bar{s}_-)]. \tag{9}$$

Combining Eq. (9) with Eqs. (3) and (4) and dropping the strange parton terms, we obtain the sum rules

$$2\int g_{1p}(x)\,dx = F - \tfrac{1}{9}D,$$
$$2\int g_{1n}(x)\,dx = \tfrac{2}{3}(F-\tfrac{2}{3}D). \tag{10}$$

These are exactly the results of Ellis and Jaffe, derived under identical assumptions but by using the technique of light-cone algebra. Verification of these sum rules would be indirect support for the conclusions (6) and (7) concerning the spin structure of the proton.

It is natural to inquire as to how the spin S_Z and the orbital momentum L_Z are shared among the various parton species within the nucleon. This question cannot be answered without making further hypotheses. We indicate here the results that are obtained if one *assumes* that the dominant contribution to S_Z and L_Z comes from the valence quarks $\mathcal{P}$ and $\mathcal{N}$. Then Eqs. (3) and (4) give

$$S_Z(\mathcal{P}) = \tfrac{1}{2}\int (u_+-u_-)\,dx \approx F,$$
$$S_Z(\mathcal{N}) = \tfrac{1}{2}\int (d_+-d_-)\,dx \approx \tfrac{1}{2}(F-D). \tag{11}$$

In the SU(6) limit ($F=\frac{2}{3}$, $D=1$) we recover the results of the 3-quark model, $S_Z(\mathcal{P})=\frac{2}{3}$, $S_Z(\mathcal{N})=-\frac{1}{6}$.[10] A similar decomposition of L_Z is possible by a naive application of the model to the proton and neutron magnetic moments. This gives

$$\mu_p\frac{e}{2M} = \frac{e}{2m}\left\{\tfrac{2}{3}\int(u_+-u_-)\,dx - \tfrac{1}{3}\int(d_+-d_-)\,dx + 2[\tfrac{2}{3}L_Z(\mathcal{P}) - \tfrac{1}{3}L_Z(\mathcal{N})]\right\},$$
$$\mu_n\frac{e}{2M} = \frac{e}{2m}\left\{\tfrac{2}{3}\int(d_+-d_-)\,dx - \tfrac{1}{3}\int(u_+-u_-)\,dx + 2[\tfrac{2}{3}L_Z(\mathcal{N}) - \tfrac{1}{3}L_Z(\mathcal{P})]\right\}, \tag{12}$$

where m (M) is the quark (nucleon) mass, and μ_p $(=2.79)$ and μ_n $(=-1.91)$ are the proton and neutron magnetic moments in units of μ_N. Combining Eq. (12) with the (approximate) angular momentum conservation equation

$$\tfrac{1}{2}\int[(u_+-u_-)+(d_+-d_-)]\,dx + L_Z(\mathcal{P}) + L_Z(\mathcal{N}) = \tfrac{1}{2}, \tag{13}$$

and using the result (9), we obtain

TABLE I. Angular momentum composition of the proton according to the parton model of this paper, compared with the results of the SU(6)-symmetric 3-quark model. (The numbers in the second column are based on $F \approx 0.46$, $D \approx 0.79$, $\mu_p = 2.79\mu_N$, $\mu_n = -1.91\mu_N$.)

	Parton model		3-quark model
S_z	$\frac{1}{2}(3F-D)$	($\approx$0.30)	$\frac{1}{2}$
L_z	$\frac{1}{2}(1-3F+D)$	($\approx$0.20)	0
$S_z(\mathcal{P})$	F	($\approx$0.46)	$\frac{2}{3}$
$S_z(\mathfrak{N})$	$\frac{1}{2}(F-D)$	(≈ -0.16)	$-\frac{1}{6}$
$L_z(\mathcal{P})$	$\frac{1}{4}\left[1-4F+\frac{1}{3}\frac{\mu_p-\mu_n}{\mu_p+\mu_n}\right]$	($\approx$0.18)	0
$L_z(\mathfrak{N})$	$\frac{1}{4}\left[1-2(F-D)-\frac{1}{3}\frac{\mu_p-\mu_n}{\mu_p+\mu_n}\right]$	($\approx$0.02)	0

$$L_z(\mathcal{P}) = \frac{1}{4}\left(1-4F+\frac{1}{3}\,\frac{\mu_p-\mu_n}{\mu_p+\mu_n}\right),$$
$$L_z(\mathfrak{N}) = \frac{1}{4}\left[1-2(F-D)-\frac{1}{3}\,\frac{\mu_p-\mu_n}{\mu_p+\mu_n}\right]. \tag{14}$$

Note that $L_z(\mathcal{P})$ and $L_z(\mathfrak{N})$ vanish separately in the SU(6) limit $F=\frac{2}{3}$, $D=1$, $\mu_n/\mu_p=-\frac{2}{3}$. We emphasize that the results (11) and (14) depend on the assumption that the angular momenta S_z and L_z as well as the magnetic moments μ_p and μ_n are dominated by the $\mathcal{P}$- and $\mathfrak{N}$-type quarks, which may well be an oversimplification. [Table I summarizes the results (8), (11), and (14).]

Finally, we note that our arguments concerning L_z and S_z make no mention of gluons, whose existence in the proton has been inferred on the basis of energy-momentum conservation.[11] Since gluons could, in principle, possess intrinsic spin as well as transverse momentum, they could contribute to both S_z and L_z. No fundamental justification for their neglect is available. In this connection, it is interesting to note that if one makes the extreme (and unrealistic) assumption that all partons (including gluons) move collinearly with the proton, so that $L_z=0$, one is driven to the conclusion that gluons carry spin, since the net spin of quarks and antiquarks adds up to $\frac{1}{2}(3F-D)$, which is short of the total angular momentum of the proton. Since the assumption of zero transverse momentum is clearly unwarranted, such a conclusion is unjustified.[12] The role of gluons in the angular momentum composition of the proton remains an unknown.

Note added in proof. It may be of some interest to note that Eqs. (12) and (13) together imply a quark mass $m=\frac{1}{3}M(\mu_p+\mu_n)^{-1}=356$ MeV.

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†Permanent address.

[1]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972). We follow Feynman's notation.

[2]J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

[3]C. H. Llewellyn Smith, talk presented at the Naples meeting on Phenomenology and Models of Electromagnetic and Strong Interactions at High Energy, 1970 (unpublished); CERN Report No. TH-1188, 1970 (unpublished); M. Gourdin, CERN Report No. TH-1360, 1971 (unpublished); J. F. Kuti and V. F. Weisskopf, Phys. Rev. D 4, 3418 (1971); Feynman, Ref. 1, p. 155.

[4]In the limit of SU(3) symmetry, the $(\mathcal{P}, \mathfrak{N}, \lambda)$ content of the Ξ^0 is the same as the $(\mathfrak{N}, \lambda, \mathcal{P})$ content of the proton. See, e.g., Feynman, Ref. 1, p. 214.

[5]Kuti and Weisskopf, Ref. 4; J. Kuti, in *Neutrino '72*, Proceedings of the Euro-Physics Conference, Balatonfüred, Hungary, 1972, edited by A. Frenkel and G. Marx (OMKDK-TECHNOINFORM, Budapest, 1972), Vol. II, p. 101.

[6]H. Ebenhoh *et al.*, Z. Phys. 241, 473 (1971).

[7]C. N. Yang, SUNY Stony Brook Report No. ITP SB 73-51, 1973 (unpublished).

[8]H. J. Melosh, thesis, Caltech, 1973 (unpublished). For a review, see J. Weyers, lectures at the Louvain Summer School on Particle Interactions at Very High Energies, 1973 [CERN Report No. TH-1743, 1973 (unpublished)]; F. J. Gilman, lectures at the 14th Scottish Universities Summer School, 1973 [SLAC Report No. SLAC-PUB-1320 (T/E) 1973 (unpublished)].

[9]J. Ellis and R. Jaffe, Phys. Rev. D 9, 1444 (1974).

[10]Gourdin, Ref. 3.

[11]C. H. Llewellyn Smith, Phys. Rev. D 4, 2392 (1971).

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