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†Alfred P. Sloan Foundation Fellow.

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[4]The anomalous dimensions corresponding to a_n have been computed explicitly for integer $n \geq 0$. The expressions involve the sum $\sum_{j=2}^{n} j^{-1}$ characteristic of gauge theories. This sum may be analytically continued by rewriting it as $\psi(n+1)+\gamma-1$, where $\psi(z)$ is the digamma function and γ is Euler's constant.

[5]G. Miller *et al.*, Phys. Rev. D 6, 3011 (1972).

[6]Our results clearly depend on the parameters K and q_0^2/μ^2 only in the combination $K/\ln(q_0^2/\mu^2)$. However, we have in mind that q_0^2 is the smallest momentum transfer for which the asymptotic analysis applies (neglect of subdominant terms justified). Then if $K<1$ we would have that $F_2 \sim \cos[2(z \ln\omega)^{1/2}]$ as $\omega \to \infty$, an unacceptable behavior since F_2 is strictly positive.

[7]After completion of this work we received a report discussing the inversion of the nonsinglet moments: G. Parisi, Phys. Lett. 50B, 367 (1974).

[8]For a general group the parameters are

$$a=\frac{4C_2(G)}{11C_2(G)-4T(R)},$$

$$b=\frac{11C_2(G)+4T(R)-8C_2(R)T(R)/C_2(G)}{33C_2(G)-12T(R)},$$

$$c=3.2\,\frac{3C_2(R)}{11C_2(G)-4T(R)},$$

in the notation of Gross and Wilczek, Ref. 1. The numbers are not very sensitive to the quark content of the theory.

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π^+/π^- ratio in inclusive production and the triple-Regge formula

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Recently it has been reported in a photoproduction experiment that the ratio of π^+/π^- is high in the target fragmentation region, yielding a value of 10 near $x=-1$. The presence of this backward peak has been confirmed in πp and pp experiments. We explain this result by applying the triple-Regge formalism for π^+ and π^- production.

Single-particle inclusive distributions have been extensively studied with different beam and target particles. Recently, a study has been published on π^- inclusive production in a photoproduction experiment on deuterium.[1] From a comparison with the results of the SLAC-Berkeley-Tufts collaboration,[2] the ratio of π^+/π^- inclusive production was calculated as a function of the Feynman variable x (as described in Ref. 1). It is experimentally observed that this ratio is high in the target-fragmentation region, reaching a value of 10 for $-1.0<x<-0.8$; it drops off sharply at $x=-0.5$, reaching a value of unity in the pionization region ($x\sim 0$). The data available for π^+p (Ref. 3) and pp (Ref. 4) experiments seem to support the presence of this backward peak.

In this note we show that the observed high π^+/π^- ratio in the target-fragmentation region can be understood within the framework of the triple-Regge formalism. Consider the reactions

$$\gamma p \to \pi^{\pm} + \text{anything}.$$

For the fragmentation of the targets we can describe these reactions according to the triple-Regge diagrams shown in Figs. 1(a), 1(b), where P stands for Pomeron and the $\alpha(t)$ are the exchanged Regge trajectories. For π^+ production we can exchange either the neutron (N) or the Δ^0 trajectory. For π^- production only the Δ^{++} trajectory is allowed.

If we write now the formulas[5] corresponding to these diagrams, we obtain for the different exchanged trajectories

$$\left.\frac{d\sigma}{dt\,dx}\right|_N = \gamma_{P\gamma\gamma}(0)\gamma_{PNN}(t)|\beta_N(t)|^2(1-|x|)^{1-2\alpha_N(t)},$$

$$\left.\frac{d\sigma}{dt\,dx}\right|_\Delta = \gamma_{P\gamma\gamma}(0)\gamma_{P\Delta\Delta}(t)|\beta_\Delta(t)|^2(1-|x|)^{1-2\alpha_\Delta(t)},$$

where the γ's are the corresponding couplings and $\beta(t)$ the Regge residues. From kinematics

$$t = m^2(1-|x|)+\mu^2-(P_T^2+\mu^2)/|x|,$$

where m = mass of proton, μ = mass of pion, and P_T = transverse momentum. We have also used the relation

$$\frac{M^2}{s} \cong 1-|x|.$$

The first conclusion we can reach from the observation of these expressions is that the ratio π^+/π^- will be independent of the beam as $\gamma_{P\gamma\gamma}$ cancels out. γ_{PNN} and $\gamma_{P\Delta\Delta}$ can be related using SU(6) and the corresponding coefficients:

$$\gamma_{PNN} = 1.2\gamma_{P\Delta\Delta}.$$

For the calculation of the ratio it remains to introduce the trajectories $\alpha(t)$ and corresponding Regge residues $\beta(t)$. We will use the values of the effective trajectories obtained by Risk,[6] who uses an analogous formalism in an analysis of pp inclusive collisions. These trajectories should be related to the experimental backward πp collisions, and this relation between the inclusive amplitudes and backward elastic scattering has also been done in the same paper.[6] It is important to notice that the baryon trajectories obtained from inclusive stud-

TABLE I. Parameters obtained in Risk's analysis (Ref. 6).

$\alpha_N(t) = -1.35+1.25t$	$t>0$
$-1.35+0.35t$	$t<0$
$\alpha_\Delta(t) = -1.9+1.25t$	$t>0$
$-1.9+0.75t$	$t<0$
$\beta_N{}^2(t) = 50e^{1.3t}$	$t>-1.2$
$10.5e^{0.345(t+1.2)}$	$-4<t<-1.2$
4	$t<-4$
$\beta_\Delta{}^2(t) = 35e^{1.03(t+0.2)}$	$t>-0.2$
$35e^{0.47(t+0.2)}$	$-1<t<-0.2$
$35e^{-0.38}$	$t<-1$

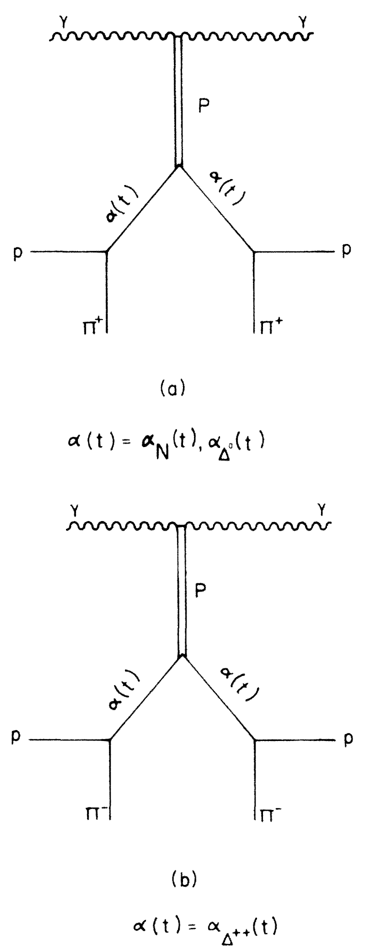

FIG. 1. Triple-Regge diagrams of the reactions (a) $\gamma p \to \pi^+ +$ anything, (b) $\gamma p \to \pi^- +$ anything.

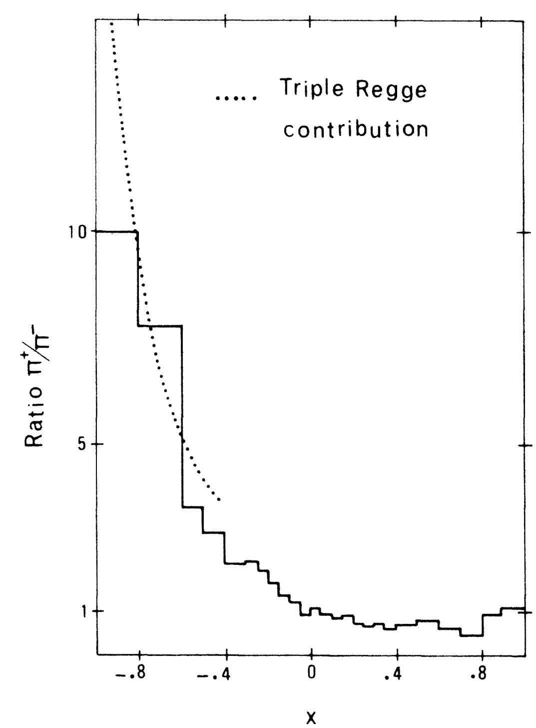

FIG. 2. Comparison of the triple-Regge predictions to the ratio π^+/π^- for γp reactions.

ies have intercepts one or two units lower than those obtained from two-body reactions.[7] This result has been confirmed in other analyses.[2,8]

Thus, using the values of Risk (reproduced in Table I) and integrating with respect to P_T^2, with the corresponding weight due to the experimental decay with a slope of ~6,[9] we calculate the triple-Regge contribution to the ratio as shown in Figs. 2 and 3. (We would like to stress the fact that our triple-Regge prediction is not a fit to the experimental data.) In Fig. 2 we have plotted also the experimental values for the ratio as obtained in the photoproduction experiment of Ref. 1. In Fig. 3 a comparison is made with the compilation of the ratio π^+/π^- for π^+p reactions as given by Morrison.[10] As we can see, the triple-Regge calculation contributes with a strong backward peak in agreement with the experimental results. The double-Regge model[11] predicts a value of one for the ratio in the pionization region.

For the photoproduction case, if we would apply the same formalism but now in the fragmentation region of the beam, using vector dominance, it is easy to see that the same trajectory would be exchanged for π^+ and π^- production. Therefore, we would predict a value of unity for the π^+/π^- ratio in the photon-fragmentation region, as is confirmed experimentally (see Fig. 2).

It is interesting to note that the comparison of triple-Regge predictions is being made with an experiment where there is not enough beam energy to be strictly in the triple-Regge limit (s large and s/M^2 large), but, as was pointed out by Lam *et al.*,[12] according to duality one can extend the validity of the triple-Regge formula to lower energies and a larger region in x and in that case it should describe the average behavior. In conclusion, we find that the triple-Regge formula[13] can be used to understand the strong backward peak observed for the π^+/π^- ratio in inclusive experiments.

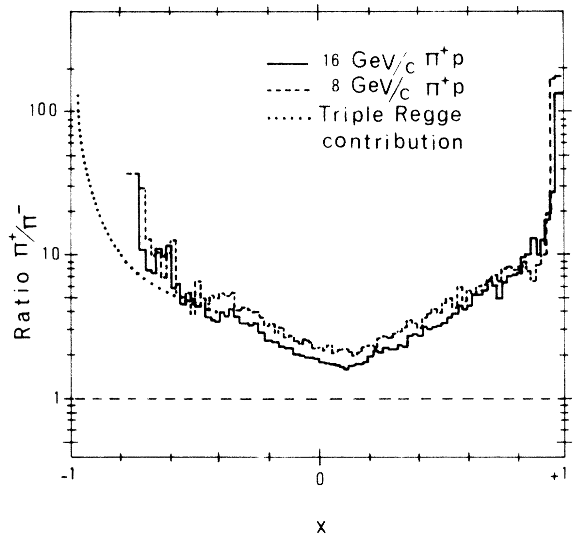

FIG. 3. Comparison of the triple-Regge predictions to the ratio π^+/π^- for π^+p reactions.

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