${ }^{5}$ S. L. Adler, Phys. Rev. 139, B1638 (1965).
${ }^{6}$ This approximation is good for the $V P$ intermediate state, and is plausible for the corrections to the VP contribution from higher-mass intermediates con-
sidered in Sec. III.
${ }^{7}$ The contribution of high-mass mesons at $q^{2}=0$ may be very small, however.

# Gauge-invariant models for two-body scattering with the Bethe-Salpeter ladder approximation* 

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#### Abstract

A natural and self-consistent method is given in the context of the ladder approximation of the Bethe-Salpeter equation for the construction of gauge-invariant models for two-body scattering involving photons. One vertex in these models is assumed to have structure and to be described by a Bethe-Salpeter equation. As examples, models for the following reactions are given: $\gamma N \rightarrow \gamma N ; \gamma N \rightarrow \pi^{ \pm, 0} N, \rho^{ \pm,}{ }^{0} N ; \gamma \pi \rightarrow \gamma \pi ; \gamma \pi \rightarrow \rho^{ \pm .0} \pi$.


## I. INTRODUCTION

In order to explain particular experimentally observed properties of scattering amplitudes, such as the existence of fixed $j$-plane poles, scaling, and forward peaks, it is useful to have several possible models that describe the process, so that the observed property can be traced back to features inherent in a limited class of such models. For example, the peaking of differential cross sections near the forward direction is associated with models in which cross-channel exchanges are important. For reactions involving the photon, the constraint that the models be gauge invariant makes it difficult to propose test models. Although it is usually possible to propose a set of Born diagrams whose sum is gauge invariant, any attempt to introduce particle structure by the insertion of form factors in these diagrams invariably destroys the gauge invariance. In order, then, to restore gauge invariance, it becomes necessary to add contact terms or contributions unmotivated by basic diagrams. ${ }^{1}$ The prescription for finding such gauge-invariance-restoring terms is often nonunique.

Within the framework of the Bethe-Salpeter equation in the ladder approximation it is possible to propose gauge-invariant models in which all contributions are motivated by scattering diagrams. In this framework, structure for the particles is introduced by assuming that vertex functions satisfy Bethe-Salpeter equations. The covariant poten-
tial responsible for the structure of the particles then naturally leads to a consideration of appropriate diagrams whose inclusion in the model ensures gauge invariance.

In the following we discuss several models, beginning with the standard reaction $\gamma N \rightarrow \pi N$ which illustrates this technique. Models of the type discussed here are most appropriate for considering the existence of fixed $j$-plane poles in Compton scattering ${ }^{2}$ and scaling of structure functions in deep-inelastic scattering. In fact one of the models we discuss is that used by Drell and Lee. ${ }^{3}$

## II. GAUGE-INVARIANT MODELS

To illustrate the technique for constructing gauge-invariant models with the framework of the Bethe-Salpeter equation in the ladder approximation we consider first pion photoproduction of nucleons, $\gamma_{q} N_{p} \rightarrow \pi_{q} N_{p^{\prime}}$, where the indices represent the respective four-momenta. The normal gaugeinvariant model for structureless particles is given by the sum of the Born diagrams in Fig. 1. If we denote the charge of the incoming and outgoing nucleons by $Q$ and $Q^{\prime}$, respectively, the gauge-invariant amplitude is just

$$
\begin{align*}
& T_{\mu}=\bar{u}\left(p^{\prime}\right)\left[Q \gamma_{5} P(p+q) \gamma_{\mu}+Q^{\prime} \gamma_{\mu} P\left(p^{\prime}-q\right) \gamma_{5}\right. \\
&\left.-\left(Q-Q^{\prime}\right) \gamma_{5} \pi\left(q-q^{\prime}\right)\left(-2 q^{\prime}+q\right)_{\mu}\right] u(p), \tag{1}
\end{align*}
$$

where $P(x)$ and $\pi(x)$ are the nucleon and pion propagators given by

(a)

(b)

(c)

FIG. 1. Primary diagrams for $\gamma N \rightarrow \pi N$ (structureless particles).

$$
\begin{aligned}
& P^{-1}(x)=-(i x \cdot \gamma+m-i \epsilon) \\
& \pi^{-1}(x)=x^{2}+p^{2}-i \epsilon
\end{aligned}
$$

Using the relations

$$
\begin{align*}
& P(p+q) q \cdot \gamma u(p)=i u(p) \\
& \bar{u}\left(p^{\prime}\right) q \cdot \gamma P\left(p^{\prime}-q\right)=-i \bar{u}\left(p^{\prime}\right), \tag{2}
\end{align*}
$$

and the fact that the external particles are on their mass shells, we obtain

$$
q^{\mu} T_{\mu}=i \bar{u}\left(p^{\prime}\right) \gamma_{5} u(p)\left[Q-Q^{\prime}-\left(Q-Q^{\prime}\right)\right]=0
$$

Structure can be introduced by assuming that the vertex functions describing the coupling of vector particles and pions to nucleons satisfy Bethe-Salpeter equations as illustrated in Fig. 2. The algebraic form of these equations is seen to be

$$
\left.\begin{array}{rl}
\Gamma_{\mu}^{N V N}(p, q, p+q) \equiv & \Gamma_{\mu}^{N V N}(p) \\
= & Z^{V} \gamma_{\mu}+
\end{array}\right) d^{4} x W_{N}(x) P(p+q+x) .
$$

and

$$
\begin{align*}
\Gamma^{\pi}(p, q, p+q) & \equiv \Gamma^{\pi}(p) \\
& =\int d^{4} x W_{N}(x) P(p+x) \Gamma^{\pi}(p+x) P(p+q+x) \tag{4}
\end{align*}
$$

where $W_{N}(x)$ is a potential and $Z^{\gamma}$ the $N \gamma N$ coupling

(a)

(b)

FIG. 2. Bethe-Salpeter equations for photon and pion vertices assuming the pion to be bound state of two nucleons.
constant. We assume that the potential is sufficiently well behaved so that the integrals (3) and (4) exist. In this model it is clear that the potential $W_{N}$ coupling the nucleons should be the same for photons as for pions. We have made the tacit assumption that this potential describes the exchange of scalar mesons. This is, of course, not necessary, for Dirac matrices, i.e., $\gamma_{5}, \gamma_{\mu}$, etc., could be inserted before the first propagator and after the last propagator to enable, the potential to describe the exchange of other types of mesons, i.e., pseudoscalar, vector, etc. We have also made the physically reasonable assumption that the pion, $\pi_{R}$, is a bound state of two nucleons (i.e., of an $N \bar{N}$ pair) or two quarks and thus that it lies on a Regge trajectory. Since it is impossible to conceive of a Regge trajectory for photons, we have written an inhomogeneous Bethe-Salpeter equation for its vertex function.

Clearly the potential $W_{N}$, which couples nucleons to give structure to the vertices, should also cause exchanges between the initial and final nucleons. This leads us to consider the model shown in Fig. 3. Denoting the contributions of the individual di-rect-channel diagrams of Fig. 3(a) by $D_{\mu}^{p}, D_{\mu}^{n}(n$ $=1,2,3, \ldots$ ) and those of the crossed channel diagrams of Fig. 3 (b) by $C_{\mu}^{p}, C_{\mu}^{n}(n=1,2,3, \ldots)$, we write

$$
\begin{align*}
& D_{\mu}=D_{\mu}^{p}+\sum_{n=1}^{\infty} D_{\mu}^{n} \\
& C_{\mu}=C_{\mu}^{p}+\sum_{u=1}^{\infty} C_{\mu}^{n} \tag{5}
\end{align*}
$$

The photoproduction amplitude $T_{\mu}$ is then given by

$$
\begin{equation*}
T_{\mu}=Q D_{\mu}+Q^{\prime} C_{\mu} \tag{6}
\end{equation*}
$$



(a)


(b)

FIG. 3. Diagrams necessary to give a gauge-invariant model for $\gamma N \rightarrow \pi_{R} N$.

It is interesting to notice that the assumption that the pion is a bound state of an $N \bar{N}$ pair means that the pion will appear as a bound state in the scattering process $N \bar{N} \rightarrow N \bar{N}$, and thus that the ladder diagrams shown in Fig. 3 simulate the exchange of a Reggeized pion as indicated in Fig. 4. This is not an unexpected feature since the exchange of an elementary pion was necessary to ensure gauge invariance for structureless scattering as depicted in Fig. 1. With the help of the generalized Ward identity for $\Gamma^{N \gamma N}$,

$$
\begin{align*}
& i q^{\mu} \Gamma_{\mu}^{N \gamma N}(p, p+q, q)=P^{-1}(p+q)-P^{-1}(p)+O(\Sigma) \quad \text { lation we consider } D^{p} \text { and } D^{1}: \\
& \simeq i q \cdot \gamma,(7)  \tag{7}\\
& q^{\mu} D_{\mu}^{p}= \bar{u}\left(p^{\prime}\right) \Gamma^{\pi}\left(p^{\prime}\right) P(p+q) q^{\mu} \Gamma^{\gamma}(p) u(p)=-i \bar{u}\left(p^{\prime}\right) \Gamma^{\pi}\left(p^{\prime}\right) P(p+q)\left[P^{-1}(p+q)-P^{-1}(p)\right] u(p) \\
&=i \bar{u}\left(p^{\prime}\right) \Gamma^{\pi}\left(p^{\prime}\right) P(p+q) P^{-1}(p) u(p)-i \bar{u}\left(p^{\prime}\right) \int d^{4} x W_{N}(x) P\left(p^{\prime}+x\right) \Gamma^{\pi}\left(p^{\prime}+x\right) P\left(p^{\prime}+q^{\prime}+x\right) u(p) \\
&=0-i \bar{u}\left(p^{\prime}\right) \int d^{4} x W_{N}(x) P\left(p^{\prime}+x\right) \Gamma^{\pi}\left(p^{\prime}+x\right) P\left(p^{\prime}+q^{\prime}+x\right) u(p) .
\end{align*}
$$

and the Bethe-Salpeter equation for $\Gamma^{\pi}$, it can be shown that the model given in Fig. 3 is gauge invariant. In particular, the generalized Ward identity results in each diagram giving two terms, one containing no propagators involving $q$ and the other with just one propagator containing $q$. If the BetheSalpeter equation (4) for $\Gamma^{\pi}$ is used to reexpress $\Gamma^{\pi}$ in the term whose propagators are independent of $q$, the resulting expression will just cancel the term having one propagator containing $q$ in the next higher diagram. As an illustration of this cancel-

Similarly,

$$
\begin{align*}
q^{\mu} D_{\mu}^{1} & =\bar{u}\left(p^{\prime}\right) \int d^{4} x_{1} W_{N}\left(x_{1}\right) P\left(p^{\prime}+x_{1}\right) \Gamma^{\pi}\left(p^{\prime}+x_{1}\right) P\left(p+q+x_{1}\right) q^{\mu} \Gamma_{\mu}^{\gamma}\left(p+x_{1}\right) P\left(p+x_{1}\right) u(p) \\
& =i \bar{u}\left(p^{\prime}\right) \int d^{4} x W_{N}(x) P\left(p^{\prime}+x\right) \Gamma^{\pi}\left(p^{\prime}+x\right) P(p+q+x) u(p)-i \bar{u}\left(p^{\prime}\right) \int d^{4} x W_{N}(x) P\left(p^{\prime}+x\right) \Gamma^{\pi}\left(p^{\prime}+x\right) P(p+q) u(p) \tag{9}
\end{align*}
$$

Clearly, the term left over from $q^{\mu} D_{\mu}^{p}$ just cancels the first term in $q^{\mu} D_{\mu}^{1}$, since $q+p=q^{\prime}+p^{\prime}$. Similarly the second term in $q^{\mu} D_{\mu}^{1}$ cancels the first term in $q^{\mu} D_{\mu}^{2}$-again with the help of the Bethe-Salpeter equation for $\Gamma^{\pi}$. This cancellation scheme is shown diagrammatically in Fig. 5. Since $D_{\mu}$ and $C_{\mu}$ are separately gauge-invariant, the model described by Fig. 3 can be used to describe chargedpion as well as neutral-pion photoproduction. In the proof of gauge invariance the fact that $\Gamma^{\pi}$ described the coupling of a bound-state pion to the nucleons was not essential. In fact, $\Gamma^{\pi}$ could be replaced by any vertex function describing the coupling of some particle to the nucleons. For example, if $\Gamma^{\pi}$ is replaced by $\Gamma_{\nu}^{V}$, where $\Gamma_{\nu}^{V}$ describes the coupling of a vector meson to the nucleons as illustrated in Fig. 2(a), the model is then a gaugeinvariant description for reactions of the type

$$
\gamma N \rightarrow \gamma N ; \quad \gamma N^{+} \rightarrow \rho^{+} N^{0}, \rho^{0} N^{+} ; \quad \text { etc. }
$$

Since it is reasonable to consider the $\rho$ meson as a Regge pole and thus a bound state, $Z^{V}$ for the $\rho$ meson would be zero. The appropriate primary diagrams and the cancellation scheme are given in Fig. 5. The $q$-independent terms containing $Z^{V}$ which result from the replacement of $\Gamma_{\mu}^{V}$ by its Bethe-Salpeter equation cancel between $D^{n}$ and $C^{n}$. It is also easy to include models where the in-
coming and outgoing nucleons are considered as bound states. For example, we can consider the nucleon as satisfying a homogeneous Bethe-Salpeter equation. The diagrammatic form of this equation is given in Fig. 6. We then have the following equations:

$$
\begin{align*}
\phi(p+x) & \equiv \phi_{p}(x) \\
& =\int d^{4} x^{\prime} V\left(x^{\prime}\right) P\left(p+x+x^{\prime}\right) \phi_{p}\left(x+x^{\prime}\right) \pi\left(x+x^{\prime}\right) \\
\bar{\phi}(p+x) & \equiv \bar{\phi}_{p}(x)  \tag{10}\\
& =\int d^{4} x^{\prime} V\left(x^{\prime}\right) \pi\left(x+x^{\prime}\right) \bar{\phi}_{p}\left(x+x^{\prime}\right) P\left(p+x+x^{\prime}\right)
\end{align*}
$$

where $\pi$ is the appropriate propagator of the bare meson $x$ and $P$ is the propagator of the bare nucleon $N$. The potential $V$ describes the coupling of the

$\sim$

FIG. 4. Simulation of a Reggeized pion.


FIG. 5. (Continued on following page)
REACTION $\qquad$
$\qquad$
(5) $\quad \gamma N_{R} \rightarrow V_{Z} N_{R}$

REACTION $\qquad$
$\qquad$
$\qquad$
$D^{p} \quad C^{p} \quad S^{p}$

LADDER TERMS

$$
\square
$$




REACTION

(8) $\gamma \pi_{R} \rightarrow \pi_{R} \pi_{R}$
(AS UNDER 7) for $\mathrm{Z}=0$

FIG. 5. Gauge-invariance cancellation schemes.



FIG. 6. Bethe-Salpeter equation for the nucleon $N_{R}$ considered as a bound state of elementary particles $N$ and $X$.
nonidentical particles $X$ and $N$. We assume that the potential $V$ is sufficiently well behaved so that the integrals (10) exist. Previously we assumed the existence of a potential $W_{N}$ between two bare
nucleons $N$ which gives structure to the $\pi N N, V N N$, etc. vertices. Similarly there should exist a potential $W_{x}$ to describe structure at $\pi X X, V X X$, etc. vertices. The potentials $V$ and $W_{N}$ lead to basic or primary diagrams for the reactions

$$
\gamma N_{R} \rightarrow \pi_{R} N_{R}, \quad \gamma N_{R} \rightarrow V_{z} N_{R}, \quad \gamma \pi_{R} \rightarrow V_{z} \pi_{R}
$$

( $\pi_{R}$ is a bound state of an $N \bar{N}$ pair) as shown in Fig. 5. We also show in Fig. 5 the ladder diagrams which have to be added in order to make the model gauge invariant. The gauge-invariance cancellation scheme is again shown in the last column. Here $D^{n 0}$ is a diagram containing $n$ horizontal lines each denoting the potential $W_{N}$ between bare nucleons; $D^{0 n}$ is a corresponding diagram containing $n$ vertical lines with each denoting the potential $V$ between a bare nucleon $N$ and a bare meson $X$. As an example we consider the cancellation of $D^{00}$ by contributions from $D^{10}$ and $D^{01}$. The contribution of the primary direct diagram $D^{00}$ multiplied by $q^{\mu}$ is

$$
\int d^{4} x \bar{\phi}_{p^{\prime}}(x) P\left(p^{\prime}+x\right) \Gamma^{\pi}\left(p^{\prime}+x\right) P(p+q+x) q^{\mu} \Gamma_{\mu}^{N N}(p+x) P(p+x) \phi_{p}(x) \pi(x)
$$

Replacing $q^{\mu} \Gamma_{\mu}^{N \gamma N}(p+x)$ by $i P^{-1}(p+x)-i P^{-1}(p+q+x)$ and using (4) and (10), we may write this expression as

$$
\begin{align*}
& i \int d^{4} x \bar{\phi}_{p^{\prime}}(x) P\left(p^{\prime}+x\right) \Gamma^{\pi}\left(p^{\prime}+x\right) P(p+q+x)\left[\int d^{4} x_{1} V\left(x_{1}\right) P\left(p+x+x_{1}\right) \phi_{p}\left(x+x_{1}\right) \pi\left(x+x_{1}\right)\right] \pi(x) \\
& \quad-i \int d^{4} x \bar{\phi}_{p^{\prime}}(x) P\left(p^{\prime}+x\right)\left[\int d^{4} x_{1} W_{N}\left(x_{1}\right) P\left(p^{\prime}+x+x_{1}\right) \Gamma^{\pi}\left(p^{\prime}+x+x_{1}\right) P\left(p^{\prime}+q^{\prime}+x+x_{1}\right)\right] P(p+x) \phi_{p}(x) \pi(x) \tag{11}
\end{align*}
$$

The first of these two terms is canceled by a contribution from $D^{01}$. This diagram multiplied by $q^{\mu}$ is seen to give, on using (7),

$$
\begin{aligned}
& i \int d^{4} x d^{4} x_{1} \bar{\phi}_{p^{\prime}}(x) P\left(p^{\prime}+x\right) \Gamma^{\pi}\left(p^{\prime}+x\right) P(p+q+x) P\left(p+q+x+x_{1}\right) \phi\left(x+x_{1}\right) \pi\left(x+x_{1}\right) \pi(x) V\left(x_{1}\right) \\
&-i \int d^{4} x d^{4} x_{1} \bar{\phi}_{p^{\prime}}(x) P\left(p^{\prime}+x\right) \Gamma^{\pi}\left(p^{\prime}+x\right) P(p+q+x) P\left(p+x+x_{1}\right) \phi\left(x+x_{1}\right) \pi\left(x+x_{1}\right) \pi(x) V\left(x_{1}\right)
\end{aligned}
$$

Similarly $D^{10}$ yields, on using (7),

$$
\begin{aligned}
& i \int d^{4} x d^{4} x_{1} \bar{\phi}_{p}(x) P\left(p^{\prime}+x\right) P\left(p^{\prime}+x+x_{1}\right) \Gamma^{\pi}\left(p^{\prime}+x+x_{1}\right) P\left(p+q+x+x_{1}\right) P(p+x) \phi_{p}(x) \pi(x) W_{N}\left(x_{1}\right) \\
& \quad-i \int d^{4} x d^{4} x_{1} \bar{\phi}_{p^{\prime}}(x) P\left(p^{\prime}+x\right) P\left(p^{\prime}+x+x_{1}\right) \Gamma^{\pi}\left(p^{\prime}+x+x_{1}\right) P\left(p+x+x_{1}\right) P(p+x) \phi_{p}(x) \pi(x) W_{N}\left(x_{1}\right)
\end{aligned}
$$

Clearly the first of these terms cancels the second term in (11). The cancellation of the remaining terms proceeds in the manner indicated in Fig. 5, row 3.

If we replace the pion vertex by a vector meson vertex we obtain-in an analogous manner-the gauge-invariance cancellation scheme shown in row 4 of Fig. 5. Unless the vector meson is composite, i.e., a $\rho$ meson, the sum of the $D$ (direct) diagrams and the sum of the $C$ (crossed) diagrams
are not individually gauge invariant. The primary diagrams for reactions of the type we have just been considering are box diagrams. The additional diagrams necessary for gauge invariance are all planar exchanges possible within the boxes, i.e., the sum of all diagrams with $n$ horizontal exchanges $W_{N}$ and all diagrams with $n$ vertical exchanges $V_{N}$ for $n=1$ to infinity. If we let $W \rightarrow 0$ in the case of Compton scattering, the resulting model is that of Drell and Lee. ${ }^{3}$

As another example we can consider the case when the meson $X$ carries an electric charge and couples to the photon. In this case the structure of vertices of the form $\pi X X, V X X$ is described by a potential $W_{x}$, and we can consider models for the reactions $\gamma N_{R} \rightarrow \pi_{R} N_{R}$ and $\gamma N_{R} \rightarrow V_{z} N_{R}$, where $\pi_{R}$ and $V_{R}(Z=0)$ would be considered as bound states of two $X$ particles. In the case of Compton scattering, in addition to the primary diagrams $D^{p}$ and $C^{p}$ a primary seagull diagram $S^{p}$ must be added, and gauge invariance requires the addition-to planar ladder diagrams obtained from $D^{p}$ and $C^{p}$ by inserting $W_{x}$ and $V$ rungs -of a corresponding ladder of seagull diagrams obtained by inserting any number of rungs $W_{x}$ between the meson lines. This model can describe the following reactions in a gauge-invariant form:

$$
\begin{align*}
& \gamma N_{R}^{+} \rightarrow\left(\rho_{R}^{0}, \gamma\right) N_{R}^{+}, \\
& \gamma N_{R}^{0} \rightarrow\left(\rho_{R}^{0}, \gamma\right) N_{R}^{0},  \tag{12}\\
& \gamma N_{R}^{+} \rightarrow \rho_{R}^{+} N_{R}^{0}, \\
& \gamma N_{R}^{0} \rightarrow \rho_{R}^{-} N_{R}^{+} .
\end{align*}
$$

Since the proof of the gauge invariance of these models is not at all trivial, we indicate the main steps. Consider the diagram for $D^{0 n}$ (see Fig. 5) multiplied by $q_{\lambda}$. Its contribution to the amplitude may be written

$$
\begin{align*}
D_{\mu}^{o n}= & -\bar{V}_{n}\left[\pi\left(\epsilon_{n}-p\right) q^{\mu} \Gamma_{\mu}^{X \gamma X}\left(q, p-\epsilon_{n}\right) \pi\left(p+q-\epsilon_{n}\right)\right] \\
& \times \phi_{p}\left(\epsilon_{n}-p\right) \\
= & -\bar{V}_{n} \pi\left(\epsilon_{n}-p\right) \phi_{p}\left(\epsilon_{n}-p\right) \\
+ & \bar{V}_{n+1} \pi\left(\epsilon_{n+1}-p\right) \phi_{p}\left(\epsilon_{n+1}-p\right) \tag{13}
\end{align*}
$$

where

$$
\epsilon_{n}=x+\sum_{i=1}^{n} x_{i}
$$

$$
\begin{align*}
& \Gamma_{\mu}\left(x-p^{\prime}\right)=\int d x^{\prime 4} \pi\left(x^{\prime}+x-p^{\prime}\right) \pi\left(x^{\prime}+x-p-q\right) \Gamma_{\mu}\left(x^{\prime}+x-p^{\prime}\right) W\left(x^{\prime}\right)+Z\left(2 x-2 p^{\prime}-q^{\prime}\right)  \tag{20}\\
& \Gamma_{\mu}\left(x-p+q^{\prime}\right)=\int d x^{\prime 4} \pi\left(x^{\prime}+x-p\right) \pi\left(x^{\prime}+x-p+q^{\prime}\right) \Gamma_{\mu}\left(x^{\prime}+x-p+q^{\prime}\right) W\left(x^{\prime}\right)+Z\left(2 x-2 p+q^{\prime}\right)
\end{align*}
$$

and

$$
\begin{align*}
& \bar{V}_{n}=\bar{V}_{n-1} \int d x_{n}^{4} V\left(x_{n}\right) P\left(\epsilon_{n}\right) \pi\left(\epsilon_{n-1}-p-q\right) \\
& \bar{V}_{0}=\int d x^{4} \Gamma_{\mu}\left(x-p^{\prime}\right) \pi\left(x-p^{\prime}\right) \bar{\phi}_{p^{\prime}}\left(x-p^{\prime}\right) P(x) \tag{14}
\end{align*}
$$

In obtaining (13) we have used (10) and the generalized Ward identity

$$
\begin{equation*}
q^{\mu} \Gamma_{\mu}^{X \gamma X}(q, p)=\pi^{-1}(p+q)-\pi^{-1}(p) \tag{15}
\end{equation*}
$$

where in the case of the bare point interaction $q^{\mu} \Gamma_{\mu}^{X \gamma X}(q, p)=q \cdot(2 p+q)$. From (13) it follows that

$$
\begin{align*}
\sum_{n=0}^{\infty} D_{\mu}^{o n}=-\int & d x^{4} \Gamma_{\mu}\left(x-p^{\prime}\right) \pi\left(x-p^{\prime}\right) \\
& \times \bar{\phi}_{p^{\prime}}\left(x-p^{\prime}\right) P(x) \phi_{p}(x-p) \pi(x-p) \tag{16}
\end{align*}
$$

Proceeding in a similar manner we find for the contributions of the crossed terms

$$
\begin{align*}
\sum_{n=0}^{\infty} C_{\mu}^{0 n}=\int & d x^{4} \Gamma_{\mu}\left(x-p+q^{\prime}\right) \pi(x-p) \\
& \times \phi_{p}(x-p) P(x) \pi(x-p) \bar{\phi}_{p^{\prime}}\left(x-p^{\prime}\right) . \tag{17}
\end{align*}
$$

Next we consider the terms containing horizontal rungs $W_{x}$. The contribution of the direct-channel diagram containing $n$ rungs is

$$
\begin{align*}
D_{\mu}^{n 0}=- & \bar{W}_{n} \pi\left(\epsilon_{n}-p^{\prime}\right) \Gamma_{\mu}\left(\epsilon_{n}-p^{\prime}\right) \pi\left(\epsilon_{n}-p\right) \\
& \times q^{\mu} \Gamma_{\mu}^{X \gamma X}\left(q, p-\epsilon_{n}\right) \pi\left(\epsilon_{n}-p-q\right), \tag{18}
\end{align*}
$$

where

$$
\begin{align*}
& \bar{W}_{n}=\bar{W}_{n-1} \int d x_{n}^{4} W\left(x_{n}\right) \pi\left(\epsilon_{n-1}-p\right) \pi\left(\epsilon_{n-1}-p^{\prime}\right), \\
& \bar{W}_{0}=\int d x^{4} \phi_{p}(x-p) P(x) \bar{\phi}_{p^{\prime}}\left(x-p^{\prime}\right) . \tag{19}
\end{align*}
$$

Proceeding as before but using now the first of the vector vertex equations,
we find that (18) may be written

$$
\begin{align*}
D_{\mu}^{n 0}= & \bar{W}_{n} \pi\left(\epsilon_{n}-p^{\prime}\right) \boldsymbol{\Gamma}_{\mu}\left(\epsilon_{n}-p^{\prime}\right) \pi\left(\epsilon_{n}-p-q\right) \\
& -Z \pi\left(\epsilon_{n}-p^{\prime}\right) \bar{W}_{n}\left(2 \epsilon_{n}-2 p^{\prime}-q^{\prime}\right)_{\mu} \pi\left(\epsilon_{n}-p\right) \\
& -\Gamma_{\mu}\left(\epsilon_{n+1}-p^{\prime}\right) \bar{W}_{n+1} \pi\left(\epsilon_{n+1}-p^{\prime}\right) \pi\left(\epsilon_{n+1}-p-q\right) . \tag{22}
\end{align*}
$$

The contribution of the corresponding crossed term is similarly found to be

$$
\begin{aligned}
C_{\mu}^{n 0}= & -\bar{W}_{n} \pi\left(\epsilon_{n}-p\right) \Gamma_{\mu}\left(\epsilon_{n}-p+q^{\prime}\right) \pi\left(\epsilon_{n}-p^{\prime}+q\right) \\
& +Z \pi\left(\epsilon_{n}-p\right) \bar{W}_{n}\left(2 \epsilon_{n}-2 p+q^{\prime}\right)_{\mu} \pi\left(\epsilon_{n}-p^{\prime}\right) \\
& +\Gamma_{\mu}\left(\epsilon_{n+1}-p+q^{\prime}\right) \bar{W}_{n+1} \pi\left(\epsilon_{n+1}-p^{\prime}\right) \pi\left(\epsilon_{n+1}-p+q^{\prime}\right) .
\end{aligned}
$$

The appropriate contribution of the seagull diagram containing $n$ horizontal rungs $W_{x}$ is

$$
\begin{equation*}
S_{\mu}^{n}=-2 q_{\mu} \bar{W}_{n} Z \pi\left(\epsilon_{n}-p\right) \pi\left(\epsilon_{n}-p^{\prime}\right) \tag{23}
\end{equation*}
$$

Using (19) and (20) one now finds that

$$
\sum_{n=1}^{\infty}\left(D_{\mu}^{n 0}+S_{\mu}^{n}+C_{\mu}^{n 0}\right)=-S_{\mu}^{0}-\sum_{n=0}^{\infty}\left(D_{\mu}^{0 n}+C_{\mu}^{0 n}\right),
$$

which proves our claim.
Finally it is also possible to replace the incoming and outgoing nucleons by pions. This results in gauge-invariant models for $\gamma \pi \rightarrow V_{z} \pi$ (Fig. 5, row 6) and $\gamma \pi_{R} \rightarrow V_{z} \pi_{R}$ (Fig. 5, row 7), where in the last reaction the vector particle if it is a $\rho$ and the pion are considered as bound states of two nucleons. The vertex functions for the two external pions may be written
$\Gamma(p+x)=\int d^{4} x_{1} W\left(x_{1}\right) P\left(p+x+x_{1}\right) \Gamma\left(p+x+x_{1}\right) P\left(x+x_{1}\right)$,
$\bar{\Gamma}(p+x)=\int d^{4} x_{1} W\left(x_{1}\right) P\left(x+x_{1}\right) \Gamma\left(p+x+x_{1}\right) P\left(p+x+x_{1}\right)$.
The vertex function for the external vector particle is, of course, given by (3). The model describes in a gauge-invariant manner the reactions

$$
\begin{equation*}
\gamma \pi_{R}^{ \pm} \rightarrow\left(\gamma, \rho_{R}^{0}\right) \pi_{R}^{ \pm}, \quad \gamma \pi_{R}^{ \pm} \rightarrow \rho_{R}^{ \pm} \pi_{R}^{0}, \quad \gamma \pi_{R}^{0} \rightarrow \rho_{R}^{ \pm} \pi_{R}^{\mp} . \tag{25}
\end{equation*}
$$

## III. CONCLUSION

It has been possible to construct within the framework of the Bethe-Salpeter equation in the ladder approximation gauge-invariant models containing structured vertices for reactions of the form $\gamma N \rightarrow \pi_{R} N, V_{z} N$ where the nucleon $N$ could be
considered either as elementary or as a bound state and $\gamma \pi \rightarrow V_{2} \pi$ where the pion $\pi$ could be considered either as elementary or as a bound state. Clearly a bound-state pion model will provide a gauge-invariant description of $\gamma \pi_{R} \rightarrow \pi_{R} \pi_{R}$. Thus we have presented a method of constructing gaugeinvariant models for an apparently unlimited variety of photonic two-body reactions that is natural and self-consistent. The models are useful for the investigation of numerous properties of photonic reactions such as, for instance, their behavior in the deep Regge region in analogy to the investigations of Blankenbecler et al. ${ }^{4}$ Of course, in the one- $Z$ models consider explicitly above, the photon is assumed to be bare so that the Ward identity is protected, and there is no contradiction between Eqs. (3) and (7). The case of structure in both photon vertices in our models requires simply replacement of the bare charged particle propagator by its appropriately dressed counterpart. This need not be an additional complication since in many applications the explicit form of the dressed propagator is not required. Two special cases of the models discussed here are implicit in the work of Brodsky et al. ${ }^{5}$ and Scott. ${ }^{6}$

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