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Unitarity and threshold effects of diffractive production*

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Within the context of a general unitarization scheme of Baker and Blankenbecler and others, we reconsider the necessary ingredients needed to define a class of Feynman-diagram models at high energies. We then discuss the relationship between the threshold behavior in the total cross section of the diffractive production and the choice of the kernels that defines a class of diagrams.

I. INTRODUCTION

Recently considerable attention has been given to the rising total cross section observed in proton-proton interactions at the CERN ISR.[1] Of the papers on this subject, the more recent letter of Blankenbecler[2] is uniquely interesting in the sense that a different interpretation of the threshold effects of diffractive production is made. Namely, within a class of Feynman diagram models it is shown that the diffractively produced inelastic states lead to a decrease in the total cross section at high energies. While the result is in agreement with that of the high-energy eikonal approach, it is contrary to what is expected from triple-Pomeron effects which[3] regard the rise of the total cross section as due to the appearance of large-mass diffractive dissociation.

In this paper we would like to reexamine Blankenbecler's analysis by tracing the standard unitarization scheme.[4,5] We find the formalism introduced by Neff[6] in another context very convenient for our purpose. Particular attention is given to different ways of choosing the kernels that can be useful in the event that experiments do indeed prove the rising total cross section to be due to diffractive one-gap production. Certainly, the analysis of Ref. 2 does not include the nonplanar diagrams completely and a different choice of the kernels can define a different class of Feynman diagrams and take account of additional nonplanar contributions. Otherwise, our work may be regarded as giving corrections to the analysis of Ref. 2. To see how the kernels are chosen, it will prove useful to review the derivation of the basic unitary equations for the scattering amplitudes once the "Born" approximation is given.

We will see that the basic equations are obtained under rather general constraints of unitarity and they do not give the scattering amplitude uniquely. In actual calculations, however, one makes approximations directly on the equations by choosing the kernels semiphenomenologically and thus one ends up with a certain class of Feynman diagrams. The choice of the kernels is generally based on the existence of various rapidity gap states, the identification of the "bare"-Pomeron term, and the interpretation of the inelastic zero-gap overlap functions. While it is perfectly true that[7] in the multiperipheral scheme the inelastic overlap functions coming from the zero-gap intermediate states generate the Regge behavior, it is also possible that this bare Pomeron is a different object from the physical Pomeron. In fact, no multiperipheral bootstrap calculation has successfully given yet the Pomeron trajectory with the correct intercept and/or slope. On the other hand, there are several works which[8] distinguish between the two types of Pomeron on phenomenological grounds. Such a distinction is not made in Ref. 2. By taking into account the Pomeron contribution to production amplitudes, further multi-Pomeron inelastic sums can be introduced. Such situations can approximately be dealt with by keeping the diagonal kernels in the equations that are used. In view of the two types of Pomeron, we define throughout our paper the diffraction with reference to the bare-Pomeron exchange, i.e., the final state is connected only by the zero-gap propagators.

In Sec. II, derivation of the generalized Lippmann-Schwinger equations is traced more or less following the lines of Baker and Blankenbecler and others. The choice of the kernels made in Ref. 2 as well as a possible modification is discussed. In Sec. III, the threshold properties of diffractive and nondiffractive one-gap production are discussed for our choice of the kernels.

II. FORMALISM

We shall assume that the full scattering operator can be labeled by the number of large-rapidity gaps in each state as in Ref. 6. The transition amplitude for n particles with i large gaps scattering into m particles with j large gaps will be denoted as $T_{ni;mj}$. Although one can include the higher numbers of gaps trivially, we shall limit up to one-gap states only as we intend to review the derivation of the basic equations used in Ref. 2.

Then we can write the many-body unitarity relation

$$\operatorname{Im} T_{ni;mj} = \sum_{n'} \sum_{k} T^{*}_{ni;n'k}\,\rho_{n'k}\,T_{n'k;mj}\ , \qquad (2.1)$$

where $\rho_{n'k}$ is the appropriate projection of the phase space. As has been pointed out by other authors,[4-6] the only limitation on our formalism is that the amplitudes do not contain disconnected parts and anomalous thresholds. The aim is to write the general solutions for $T_{ni;mj}$ for given $B_{ni;mj}$ appropriate to the kinematic regions in such a way that the unitarity equation (2.1) is satisfied. It is obvious that the formalism based on the unitarity condition alone does not determine the S matrix uniquely, and one has to introduce additional dynamical assumptions. In any case the Born terms are assumed to be real and symmetric. Then following the generalized unitarization scheme of Baker and Blankenbecler[4] and others,[5,6] we relate the exact $T_{ni;mj}$ to $B_{ni;mj}$ by

$$T_{ni;mj} = B_{ni;mj} + \sum_{n'} \sum_{k} T_{ni;n'k}\,G_{n'k}\,B_{n'k;mj}\ , \qquad (2.2)$$

where the $G_{n'k}$'s are collections of the k-gaps projection of the n'-particle Feynman propagators in the s channel whose imaginary part is $\rho_{n'k}$ in the appropriate physical region. It turns out that the representation (2.2) can be written either in the impact-parameter formalism or for the generalized partial-wave amplitudes.

To the extent that we limit ourselves to states with up to one gap, we have to solve the three equations for $T_{21;ni}$ (≥ 2, $i=0,1$) from (2.2) simultaneously, which involve the three Green's functions G_{ni} ($n\geq 2$, $i=0,1$). Following the notations of Ref. 2, the labels (2 1), (n 0), and (n 1) will be denoted simply as e, 0, and 1, respectively, hereafter. Accordingly, the equations for the scattering operators read

$$T_{ee} = K_{ee} + T_{e0}G_0K_{0e} + T_{e1}G_1K_{1e}\ , \qquad (2.3)$$

$$T_{e0} = (1 + T_{ee}\,G_e)K_{e0} + T_{e1}G_1K_{10}\ , \qquad (2.4)$$

$$T_{e1} = (1 + T_{ee}\,G_e)K_{e1} + T_{e0}G_0K_{01}\ , \qquad (2.5)$$

where

$$K_{ij} = B_{ij}(1 - G_jB_{jj})^{-1} \quad (i,j = e,0,1)\,. \qquad (2.6)$$

We note from (2.6) that the transition kernels are not necessarily restricted to the Hermitian form, but in general they can contain intermediate states of the G_e, G_1, and G_0 type. Nevertheless, the unitarity equations (2.1) are satisfied by the generalized Lippmann-Schwinger equations (2.3)–(2.5).

On the other hand, if we *choose* the transition kernel to be completely off-diagonal, i.e., $K_{ii}=0$, then the remaining elements K_{ij} ($i\neq j$) are necessarily real (or in general Hermitian) for the choice of the Born amplitudes specified above. With this choice for the kernel, we arrive at the basic equations for the scattering amplitudes that are used to define the class of diagrams considered in Ref. 2. In this case, the lowest-order contribution to the transition amplitude T_{e0} is K_{e0} so that the elastic amplitude T_{ee} contains the term $K_{e0}G_0K_{0e}$ which is the familiar ladder graphs representing the inelastic overlap functions when K_{e0} is chosen to contain the multiperipheral production graph. Although this sort of approximation does not allow us to treat the nonplanar diagram completely, it is shown in Ref. 2 that the result obtained under such approximation agrees with the eikonal at high energies. The term $K_{e0}G_0K_{0e}$ is often identified as the "bare-Pomeron" term and the main result of Ref. 2 is that the contribution of the bare Pomeron to the total cross section is larger than the actual total cross section, after taking into account the contributions of the elastic and one-gap intermediate states.

We devote the remainder of this paper to the examination of this result under a slightly different situation. Following the line of Refs. 4, 5, and 6, we can construct a high-energy model by appealing to phenomenology and choosing K_{ij} directly. In this way, we can avoid the step of choosing the Born amplitudes B_{ij}. In what follows, we shall put only

$$K_{00} = K_{11} = 0 \qquad (2.7)$$

in the basic equations and choose the off-diagonal terms to be real.[9] But we shall allow K_{ee} to be in general complex having the s-channel singularities of the G_e type.

There are several reasons for making this approximation. There have been numerous attempts[7] to build up the Pomeron through the inelastic overlap functions of ladder diagrams, i.e., the multiperipheral mechanism, but none of them has succeeded to obtain the bare-Pomeron intercept to be close to unity. In fact, there seems to be mounting phenomenological evidence[8] to choose the bare-Pomeron intercept less than one. In order to boost the physical (or renormalized) Pomeron intercepts to unity, we may need the elastic element of the kernels. Moreover, by keeping the elastic kernel K_{ee}, we may *effectively* take account of contributions of the nonplanar graphs that are not treated completely in Ref. 2. In addition, it is then possible to see to what extent the result obtained in Ref. 2 is dependent on the particular choice of the kernels.

With the choice of the kernels specified above, we obtain the elastic scattering operator

$$T_{ee} = (K_{ee} + W_{ee})\,(1 - G_e W_{ee})^{-1}, \tag{2.8}$$

where

$$\begin{aligned} W_{ee} &= K_{e0}G_0K_{0e} \\ &\quad + (K_{e1} + K_{e0}G_0K_{01}) \\ &\quad \times (1 - G_1K_{10}G_0K_{01})^{-1}G_1(K_{1e} + K_{10}G_0K_{0e}). \end{aligned} \tag{2.9}$$

At this point. we introduce the positive-definite operator used in Ref. 2,

$$\Sigma_T = (T_{ee}^* - T_{ee})/2i\,, \tag{2.10}$$

from which the total cross section can be calculated in the forward direction.

III. CONSEQUENCES OF DIFFRACTION

We reexamine in this section the analysis of Ref. 2 with (2.8). The elastic kernel kept in our equations serves to give additional elastic absorptions to the inelastic intermediate states, and thus the resulting total cross section of diffractive and nondiffractive production amplitudes will get additional corrections. To explore the consequences of diffraction, we assume as in Ref. 2 that G_e and G_0 are dominated by their imaginary parts at high energies,

$$G_e = -id_e,\ \ G_0 = -id_0\,, \tag{3.1}$$

where the real positive-definite d's contain the mass-shell δ functions of the particles in the rungs of the unitarity sums.

a. The case of $K_{01} = K_{10} = 0$. In this case the unitarity diagrams involving $K_{e0}G_0K_{01}$, $K_{10}G_0K_{01}$, and $K_{10}G_0K_{0e}$ are not contributing. Thus if we define the unitarity diagrams connected by the zero-gap propagator G_0 as the diffractive processes (which is equivalent to using the bare-Pomeron exchange in the t channel as criterion), then all of the inelastic diffractive productions are suppressed and we are left with nondiffractive and elastic (diffractive) productions only.

By introducing the x variable to control the strength of the one-gap states by $G_1 = -ixd_1$, we have

$$W_{ee} = -i(K_{e0}\,d_0K_{0e} + xK_{e1}\,d_1K_{1e}) \tag{3.2}$$

and

$$\begin{aligned} \Sigma_T(d_0, d_e, x d_1) &= (K_{e0}\,d_0K_{0e} + xK_{e1}\,d_1K_{1e} + \Delta K_{ee}) \\ &\quad \times [1 + d_e(K_{e0}d_0K_{0e} + xK_{e1}\,d_1K_{1e})]^{-1}, \end{aligned} \tag{3.3}$$

where

$$\Delta K_{ee} \equiv (1/2i)\,(K_{ee}^* - K_{ee}) \tag{3.4}$$

is positive-definite for any semiphenomenological choice of K_{ee}. As expected, (3.3) reduces to the situation considered in Ref. 2 when K_{ee} is taken to be real. But phenomenologically the Pomeron term is to a large extent imaginary. Moreover, for a given elastic Born term we see from (2.6) that

$$\begin{aligned} \Delta K_{ee} &= (1 - iB_{ee}\,d_e)^{-1}B_{ee}\,d_eB_{ee}(1 + id_eB_{ee})^{-1} \\ &\geq 0\,. \end{aligned} \tag{3.5}$$

The effect of varying x is given by

$$\begin{aligned} \frac{d\Sigma_T}{dx} &= [1 + (K_{e0}\,d_0K_{0e} + xK_{e1}\,d_1K_{1e})\,d_e]^{-1}K_{e1}\,d_1K_{1e}[1 + d_e(K_{e0}\,d_0K_{0e} + xK_{e1}\,d_1K_{1e})]^{-1} \\ &\quad - \Delta K_{ee}[1 + d_e(K_{e0}\,d_0K_{0e} + xK_{e1}\,d_1K_{1e})]^{-1}d_eK_{e1}\,d_1K_{1e}[1 + d_e(K_{e0}\,d_0K_{0e} + xK_{e1}\,d_1K_{1e})]^{-1}, \end{aligned} \tag{3.6}$$

where the second term is the additional contribution coming from the nonvanishing elastic kernel. Thus we may say that while the total cross section increases as nondiffractively produced one-gap states are introduced, the presence of the additional kernel can diminish the rate of increment. As we have mentioned before, this extra kernel may be regarded, for example, as the manifestation of the remaining nonplanar graphs that are not included by the inelastic overlap functions of the multiperipheral mechanism. Note that the presence of the diagonal kernel K_{ee} gives an absorptive correction to the inelastic production.

b. The case $K_{e1} = K_{1e} = 0$. In this case, the terms like $K_{e1}G_1K_{1e}$, $K_{e0}G_0K_{01}G_1K_{1e}$, and $K_{e1}G_1K_{10}G_0K_{0e}$ are missing from the unitarity sum and all the remaining terms are connected by the zero-gap propagator G_0 at the end of the graphs because we get from (2.9)

$$W_{ee} = K_{e0}(1 - G_0K_{01}G_1K_{10})^{-1}G_0K_{0e}\,. \tag{3.7}$$

Then, insofar as the zero-gap intermediate state can represent the bare-Pomeron exchange, the situation may be called diffractive. In fact, the one-gap states are produced basically through the zero-gap intermediate states. For this reason, one may group the contributions of the elastic and one-gap intermediate states to the zero-gap transition operator together by introducing

$$U = K_{0e}\,G_eK_{e0} + K_{01}G_1K_{10}\,. \tag{3.8}$$

Then the elastic amplitude can be written as

$$T_{ee} = \{1 + K_{ee}[K_{e0}(1 - G_0 K_{01} G_1 K_{10})^{-1} G_0 K_{0e}]^{-1}\} \times K_{e0}(1 - G_0 U)^{-1} G_0 K_{0e} . \quad (3.9)$$

As we are interested in the threshold properties of the diffractive one-gap states, we can no longer neglect the real part of G_1 which is negative-definite below threshold in our convention. For this reason, the real part of K_{ee} is also playing a role. We give here the results on Σ_T for the two interesting cases only, i.e., when K_{ee} is predominantly imaginary and when it is real.

(i) K_{ee} is imaginary. Then it must be of the form $K_{ee} = -i(\Delta K_{ee})$ where $\Delta K_{ee} > 0$ from (2.5). By introducing further variables y and z to control the strength of the real part of the one-gap propagators and of the elastic propagators, respectively, we obtain

$$\Sigma_T(d_0, zd_e, xd_1, y\,\mathrm{Re}G_1, \Delta K_{ee}) = (1 - z\Delta K_{ee} d_e) K_{e0} d_0^{1/2}(1+A)^{-1/2}(1+y^2L^2)^{-1}(1+A)^{-1/2} d_0^{1/2} K_{0e} + \Delta K_{ee} , \quad (3.10)$$

where we denote

$$L(z,x) = (1+A)^{-1/2} d_0^{1/2} K_{01}\,\mathrm{Re}G_1 K_{10} d_0^{1/2}(1+A)^{-1/2} \quad (3.11)$$

and

$$A(z,x) = d_0^{1/2}(zK_{0e} d_e K_{e0} + xK_{01} d_1 K_{10})\, d_0^{1/2} . \quad (3.12)$$

We can see from (3.10) that the positivity of Σ_T implies

$$1 - z\Delta K_{ee} d_e \geq 0 \quad (3.13)$$

unless ΔK_{ee} is actually the dominant term of the cross section. Provided that (3.13) holds, we obtain the inequality of Ref. 2:

$$\Sigma_T(d_0, d_e, d_1, \mathrm{Re}G_1, \Delta K_{ee}) \leq \Sigma_T(d_0, d_e, 0, 0, \Delta K_{ee}) \leq \Sigma_T(d_0, 0, 0, 0, \Delta K_{ee}) \quad (3.14)$$

so that Σ_T is a decreasing function of x, y, and z. In addition, we note that

$$\Sigma_T(d_0, zd_e, 0, 0, \Delta K_{ee}) = (K_{e0} d_0 K_{0e} + \Delta K_{ee}) \times (1 + zd_e K_{e0} d_0 K_{0e})^{-1} , \quad (3.15)$$

which is still true even when (3.13) is not valid. While it is a decreasing function of z, the cross section is an increasing function of ΔK_{ee} in the absence of diffractive one-gap production.

The inequality (3.14) implies that the total cross section is decreasing when the diffractively produced one-gap intermediate states are being produced. On the other hand, *if* experiments prove that the increasing total cross section is indeed due to the threshold phenomenon of producing diffractive one-gap states, then it may mean within the present formalism that (3.13) is no longer valid and Σ_T must get the dominant contribution from ΔK_{ee}. Namely, the other nonplanar contributions are more important since we then get

$$\Sigma_T(d_0, d_e, d_1, \mathrm{Re}G_1, \Delta K_{ee}) \geq \Sigma_T(d_0, d_e, 0, 0, \Delta K_{ee}) . \quad (3.16)$$

(ii) K_{ee} is real. In this case, we arrive at

$$\Sigma_T(d_0, zd_e, xd_1, y\,\mathrm{Re}G_1, K_{ee}) = [K_{e0} d_0^{1/2}(1+A)^{-1/2} + zyK_{ee} d_e K_{e0} d_0^{1/2}(1+A)^{-1/2} L]\,(1+y^2L^2)^{-1}(1+A)^{-1/2} d_0^{1/2} K_{0e} . \quad (3.17)$$

However, as we do not have any reason to prefer a particular sign of K_{ee} at this level of sophistication, we cannot make any definite statements about the threshold behavior.

To sum up, the threshold properties concluded in Ref. 2 for the nondiffractive as well as the diffractive production are true so long as the nonplanar graphs missing there play a minor role in the total cross section. But if independent evidence supports the view that the rising total cross section is a local phenomenon due to the threshold effect of producing the diffractive one-gap states, then the formalism should be modified to include more nonplanar diagrams, for example, through nonvanishing diagonal kernels. In such cases, the nondiffractive case is expected also to give different behavior as we see from (2.5). Finally, we add that the most thorough analysis would be the one solving the 3×3 matrix equation including the elements T_{00} and T_{11}. The situations considered in Ref. 2 as well as in the present work will be only special cases of such an ambitious program.

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[9]Under the conditions of Eq. (2.7), K_{0e} and K_{1e} can in general have intermediate states of G_e type, but such elastic absorptions to the inelastic kernels are expected to give much smaller corrections to T_{ee}.

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Fragmentation and energy conservation in the eikonal-Regge model*

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We have studied the effect of fragmentation and energy conservation in the eikonal-Regge model. A generalized eikonal representation involving multi-impact parameters is given for the elastic and inelastic amplitudes when fragmentation takes place. The generalized eikonal function which describes a many-body potential depends on more than one impact parameter. In the strong-absorption model and at high energy, however, the elastic amplitude can be approximated by a single-impact-parameter representation with an effective eikonal function. As a result of the fragmentation, we find that although σ_E and σ_T still increase as $\ln^2 s$, their ratio is no longer $\frac{1}{2}$. Instead, it is the sum of the elastic and diffractive cross sections which remains to be one half of the total cross section. To enforce the energy conservation, we propose a thermodynamic approach by introducing an impact-parameter-dependent temperature. Using well-known thermodynamics relations, we obtain various corrections to the naive eikonal-Regge model predictions due to energy conservation. Experimental consequences are discussed.

I. INTRODUCTION

The eikonal model[1] for high-energy scattering of hadrons offers a semiclassical picture for a very complicated process. A most striking characteristic of high-energy hadron collisions is the fact that the number distributions in phase space are very different in transverse and longitudinal momentum axes; they are rather limited in the former but apparently not in the latter. The impact-parameter representation in the eikonal model is ideal for describing this disparate situation. It nicely separates the transverse degrees of freedom from the dynamics in the longitudinal space.

The main features of the eikonal approach are that, on the one hand, the s-channel unitarity is automatically enforced, and on the other, it can incorporate any energy dependence of the total cross sections consistent with unitarity by a proper choice of the eikonal function. This is in distinction with the conventional Regge approach in which the Pomeron is assumed to be a simple pole. The upper bound for the total cross sections in this case is a constant asymptotically. If the rise in the pp total cross section[2] observed in recent CERN ISR experiments continues to hold in the future, the simple Regge-pole approach must be abandoned. In that case the eikonal model may be a simple alternative to organize the data. Assuming this possibility to exist, we will reexamine and explore further certain aspects in an eikonal model with rising total cross sections.

The eikonal approximation has been studied most thoroughly for the elastic amplitude at high ener-