

Lepton pair production in hadron-hadron collisions

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Lepton production in hadron-hadron collisions is studied in a class of models. In the conventional quark-antiquark annihilation mechanism, the parton distributions incorporate the latest experimental information. The numerical estimates bring out unique signatures and represent realistic upper bounds. In studying the dependence of the results on different quark schemes we find that in models with charm the results remain practically unchanged, or are reduced by a multiplicative factor. We conclude that should the rates be considerably larger than the estimates, then they must be attributed to another origin.

I. INTRODUCTION

Ever since the original BNL-Columbia experiment¹ and the Drell and Yan suggestion² that the production of heavy-lepton pairs can be described within the framework of the parton model there have been numerous efforts trying to observe such processes. The interest is justified on several accounts. From the experimental point of view³ the electromagnetic production of lepton-antilepton pairs is important in normalizing other even more interesting pairs like $e^+\nu_e$ and $\mu^+\nu_\mu$. From the theoretical point of view,^{2, 4-11} the original suggestion gives results similar to those obtained if the product of the currents is dominated by free-field-theory singularities,¹² but the justification of the light-cone dominance in this process has never been complete. Even within the parton model the numerical estimates depend so critically on the assumptions governing the antiquark distributions that a conclusive test of the original idea has not been performed and must wait further experimental information. Alternatively, the experiments will determine antiquark distributions, which must then be compared with the constraints imposed by other reactions.

In the past year experimental results from neutrino and antineutrino experiments^{13, 14} indicate that the mean momentum carried by the antiquarks (nonstrange) is small. This implies that the production of lepton-pairs is greatly suppressed at large values of Q^2 and it provides a unique signature for the process. The need for an updated calculation is further enhanced by the observation that most of the calculations are concerned with the cross section $d\sigma/dQ^2$, which is not the quantity measured directly in the experiments. What are measured instead are double and triple differential

cross sections, subject to experimental efficiency limitations. These reasons compelled us to undertake this investigation of updating the calculation and studying its sensitivity to the underlying assumptions.

In Sec. II we present general formulas which can easily be adapted to diverse experimental situations. Parton distributions which incorporate the latest experimental information are also incorporated in the analysis. Section III gives a wide class of numerical estimates, pointing out signatures unique to this process. We have made an effort to present the expectations of the parton model in detail, so that a direct test with experiment is possible. If the experimental measurements are in the vicinity of the estimates, then the pursuit of further tests and correlations is desirable. If, on the other hand, the measurements are considerably larger than the estimates, then we must seek an alternative explanation.¹⁵ The parton contribution may still be there and a two-arm spectrometer could search for it. Section IV discusses briefly the effects of nuclear corrections, integrally charged quarks, charmed quarks, and the direct production of charmed particles.

II. GENERAL FORMULAS

Consider the reactions

$$p(p_+) + p(p_-) \rightarrow l(q_+) + \bar{l}(q_-) + \Gamma, \quad (2.1)$$

where $l\bar{l}$ is a lepton-antilepton pair like e^+e^- , $\mu^+\mu^-$, and Γ is any combination of hadronic states. The original Drell-Yan model² visualizes the scattering as proceeding through quark-antiquark annihilation into leptons. The kinematics are defined as follows:

$$S = (p_+ + p_-)^2, \quad (2.2)$$

$$Q^2 = (q_+ + q_-)^2 \approx xx'S, \quad (2.3)$$

while some of the other variables are defined in Fig. 1. The variables x and x' are given in terms of invariants

$$x = \frac{2}{S}(p_- \cdot q_+ + p_+ \cdot q_-), \quad (2.4)$$

$$x' = \frac{2}{S}(p_+ \cdot q_+ + p_- \cdot q_-). \quad (2.5)$$

Cross sections for such processes have been derived following standard techniques. It is useful to write a cross section, which is invariant under Lorentz transformations along the beam direction,

$$q_+^0 \frac{d\sigma}{dq_+^3} = \frac{8\alpha^2}{S^2 Q^4} [(p_- \cdot q_+)(p_- \cdot q_-) + (p_+ \cdot q_-)(p_+ \cdot q_+)] \times \frac{d\cos\theta_-}{\sin^2\theta_-} \Phi(x, x'), \quad (2.6)$$

where α is the fine structure constant; $\Phi(x, x')$ is a function of the parton distributions to be defined explicitly in the latter part of this section. The volume element $d\cos\theta_-/\sin^2\theta_-$ is invariant under boosts along the beam direction. Triple differential cross sections are obtained readily either in the laboratory frame,

$$\frac{d^3\sigma}{dq^0 dq^3 d\cos\theta_+ d\cos\theta_-} = \frac{8\pi\alpha^2}{SQ^4} q_+^3 \frac{\sin\theta_+}{\sin^3\theta_-} \times (2 - \cos\theta_+ + \cos\theta_-) \Phi(x, x'), \quad (2.7)$$

or the center-of-mass frame,

$$\frac{d^3\sigma}{dq^0 dq^3 d\cos\theta_+ d\cos\theta_-} = \frac{8\pi\alpha^2}{SQ^4} q_+^3 \frac{\sin\theta_+}{\sin^3\theta_-} \times (1 + \cos\theta_+ \cos\theta_-) \Phi(x, x'). \quad (2.8)$$

The basic assumption of parton-antiparton annihilation has several consequences. In the limit where the transverse momenta of the constituents are neglected, (i) the plane formed by the dilepton

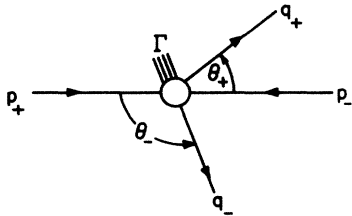


FIG. 1. Kinematics for the process.

pair contains the beam direction, and (ii) the transverse momentum of the dilepton pair is zero, i.e.,

$$q_- \sin\theta_- = q_+ \sin\theta_+. \quad (2.9)$$

Violations of this relation, arising from a perpendicular momentum dependence in the parton distribution, should be limited to a few hundred MeV. Most of the interesting physics is hidden in the function $\Phi(x, x')$, which is discussed next.

The overlap function $\Phi(x, x')$ is defined by

$$\Phi(x, x') = \sum_i f_i(x) f_{\bar{i}}(x') Q_i^2 + \sum_{\bar{i}} f_i(x') f_{\bar{i}}(x) Q_{\bar{i}}^2, \quad (2.10)$$

while the electroproduction structure function is

$$F_2(x) = x \sum_i Q_i^2 f_i(x) + x \sum_{\bar{i}} Q_{\bar{i}}^2 f_{\bar{i}}(x), \quad (2.11)$$

where \sum_i and $\sum_{\bar{i}}$ imply summations over quarks and antiquarks, respectively. There are, however, several measurements which indicate that the momentum carried by the antipartons and the strange quarks is much smaller than the momentum carried by the nonstrange partons. The observed ratio of the antineutrino to neutrino total cross sections on matter satisfies

$$\frac{\sigma^{\bar{\nu}}}{\sigma^{\nu}} = \frac{1}{3}(1 + \epsilon), \quad (2.12)$$

where

$$\epsilon = \begin{cases} 0.132 & \text{for } 1 \leq E \leq 10 \text{ GeV,} \\ 0.120 & \text{for } E \leq 80 \text{ GeV.} \end{cases}$$

For $\sin^2\theta_c = 0$ this implies¹⁶

$$\sum_{\bar{i}}' \int x f_{\bar{i}}(x) dx \leq \frac{3}{8} \epsilon \sum_i' \int x f_i(x) dx + O(\epsilon^2), \quad (2.13)$$

where the \sum' indicates summations over quarks or antiquarks which couple to the $\Delta S = 0$ part of the weak current. If in addition $\sigma_s/\sigma_T = 0$ in neutrino-induced reactions, then (2.13) becomes an equality. We do not make this additional assumption, because the corresponding ratio determined in electroproduction is different from zero. The main result is that the contribution of the antiquarks (nonstrange) is limited to small x . It is supposed that x is small enough so that the diffraction formula⁷ holds:

$$f_{\bar{i}}(x) = f_{\bar{i}}(x) \approx \frac{a}{x} G(x), \quad (2.14)$$

where $G(x)$ is a function with $G(0) = 1$ and decreasing rapidly with x .

To determine the significance of the strange quarks one must compare the electroproduction to neutrino results. The ratios^{16, 17}

$$\frac{\int (F_2^{\gamma n} + F_2^{\gamma p}) dx}{\int (F_2^{\nu n} + F_2^{\nu p}) dx} = 0.30 \pm 0.06 \leq \frac{5}{18} + \sigma \quad (2.15)$$

and

$$\frac{\int (F_2^{\gamma n} + F_2^{\gamma p}) x dx}{\int (F_2^{\nu n} + F_2^{\nu p}) x dx} \leq 0.32 \quad (2.16)$$

bound the strange-quark contribution

$$\frac{\int x(f_\lambda + f_{\bar{\lambda}}) dx}{\int x(f_p + f_{\bar{p}} + f_n + f_{\bar{n}}) dx} \leq 9\sigma = 0.7, \quad (2.17)$$

$$\frac{\int x^2(f_\lambda + f_{\bar{\lambda}}) dx}{\int x^2(f_p + f_{\bar{p}} + f_n + f_{\bar{n}}) dx} \leq 0.4. \quad (2.18)$$

The bounds indicate that the strange-quark contribution is also peaked at small values of x . They suggest that f_λ and $f_{\bar{\lambda}}$ may be limited in the diffractive region, but they are not stringent enough to imply this conclusion. We shall also assume that

$$f_{\bar{\lambda}}(x) = \frac{b}{x} G'(x),$$

where b is a constant and $G'(x)$ is again a rapidly decreasing function of x , with $G'(0) = 1$. Point by point comparisons between $F_2^{\gamma}(x)$ and $F_2^{\nu}(x)$ will determine the importance of the strange and non-strange structure functions. In the absence of such detailed information we shall take $a = b$ and $G(x) = G'(x)$, leading to

$$f_{\bar{p}}(x) = f_{\bar{n}}(x) = f_{\bar{\lambda}}(x) = \frac{aG(x)}{x}. \quad (2.19)$$

We believe that the ambiguities arising from the specific form of $G(x)$ are far greater than those arising from equating all the antiquark structure functions. In any case, evidence of the limited antiquark distributions should be present in the numerical estimates. From (2.10), (2.11), and (2.19) we obtain

$$xx' \Phi(x, x') = \frac{F_2(0)}{2 \sum_i Q_i^2} [F_2(x)G(x') + F_2(x')G(x) - F_2(0)G(x)G(x')] , \quad (2.20)$$

where $a = F_2(0)/2 \sum_i Q_i^2$ has also been used. Quark models where the antiquark distributions satisfy (2.19) will lead to a formula of this form. The functional form is similar to the one suggested by Gronau.¹⁸ The main difference stems from the fact that we do not have to restrict the experiments to kinematic regions where x and x' are small, since nature automatically provides such a restriction for the antiquark distributions.

There are two other formulas which occur frequently in articles. The double differential cross section¹⁹ in the center-of-mass system

$$\frac{d\sigma}{dQ^2 dQ_{\parallel}} = \frac{8\pi\alpha^2}{3Q^4} \frac{Q_{\parallel} + (Q_{\parallel}^2 + Q^2)^{1/2}}{[Q_{\parallel} + (Q_{\parallel}^2 + Q^2)^{1/2}]^2 + Q^2} xx' \Phi(x, x') \quad (2.21)$$

and the Drell-Yan formula²

$$\frac{d\sigma}{dQ^2} = \frac{4}{3} \frac{\pi\alpha^2}{Q^2 S} \int \Phi\left(x, \frac{Q^2}{Sx}\right) \frac{dx}{x}. \quad (2.22)$$

We shall use these formulas in Sec. III in order to obtain estimates for a variety of experimental situations.

III. NUMERICAL ESTIMATES

In estimating the cross sections a choice must be made for the quantities $\sum_i Q_i^2$ and $G(x)$. Asymptotically the sum of the squares of the quark charges is related to the electron-positron annihilation²⁰ as follows:

$$\sum_i Q_i^2 = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}. \quad (3.1)$$

Present data do not seem to indicate a constant ratio associated with the asymptotic region. However, in production experiments the dilepton masses can be chosen to be so much larger that an asymptotic region could still make sense. Quark models give a wide range of values. For the estimates we shall select the value of $\sum_i Q_i^2 = \frac{2}{3}$, corresponding to Gell-Mann-Zweig quarks.

To accentuate the cutoff in momentum distributions we chose

$$G(x) = \theta(\xi - x), \quad (3.2)$$

with $\xi = 0.10$ and 0.20 . We have also chosen a $G(x)$ obtained in explicit parametrizations of electron production and neutrino-induced production data. Parametrizations²¹ satisfying the sum rules and threshold behavior give

$$G(x) = (1-x)^\eta, \quad (3.3)$$

with $\eta = 9$. In order to study the sensitivity of the results to the functional forms of $G(x)$ we varied ξ and the exponent η . Additional quantum numbers like color or charm will further reduce the cross sections. The effect of m such multiplets is to scale down the results by an over-all factor $1/m$.

In the BNL-Columbia experiment¹ μ pairs were observed with a longitudinal momentum ≈ 12 GeV/c. Theoretical curves,²² which account for this experimental constraint, are shown in Fig. 2. For $\xi = 0.20$ and $Q \lesssim 2.5$ GeV the theoretical curve could be compatible with experimental points. Significant deviations occur for larger values of Q .

Figure 3 shows the invariant and scaling quantity $Q^2 s d\sigma/dQ^2$ as function of $\tau = Q^2/s$ for different

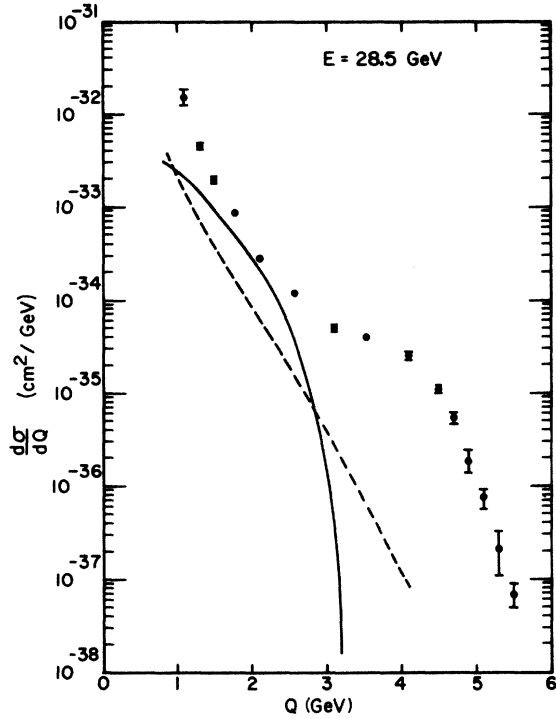


FIG. 2. Comparison of the BNL-Columbia experiment with parton-model expectations. Solid curve corresponds to $\theta(0.20-x)$ and dashed curve to $(1-x)^9$.

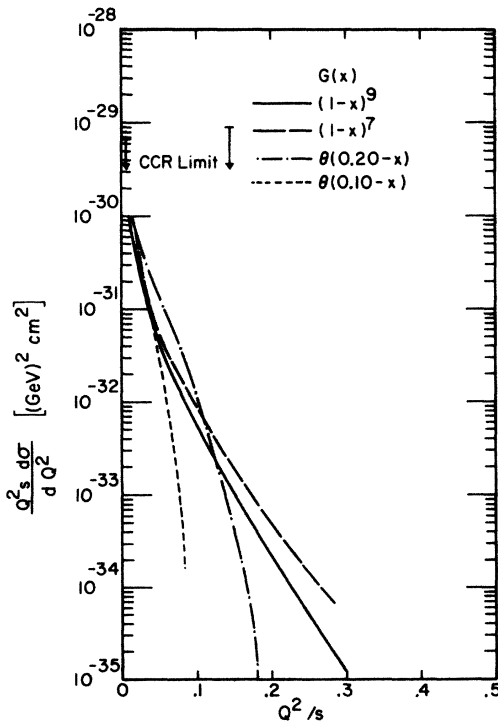


FIG. 3. Comparison with the CCR bound (Ref. 3) indicated by the arrows. Curves correspond to the parametrizations of $G(x)$ shown in the figure.

parametrizations of $G(x)$. We note that for small τ the shapes and normalizations of the curves are very similar. Substantial differences arise at larger values of τ . In the same figure is shown an upper bound from the CERN-Columbia-Rockefeller (CCR) experiment.³ All the estimates are consistent with the bound.

Estimates for the double differential cross section [e.g., (2.21)] in the center-of-mass system are shown in Fig. 4. An important signature, arising from the limitation of the antiquark momentum, is the substantial leveling (dashed curve) and perhaps decrease (solid curve) of the cross section at small $Q_{||}$.

In experiments of the NAL type, one-arm spectrometers seem to be favored. For such configurations we integrate over θ_- and present the results as functions of θ_+ and q_+ . Figures 5-7 show such curves²³ for different parametric forms of $G(x)$. We note that for small momentum of the observed lepton, the dependence on $G(x)$ is not critical, but it becomes more important as the momentum increases.²⁴ Figure 8 shows the dependence of the double differential cross section on the parameter η occurring in (3.3). In a two-arm spectrometer one would like to set q_+ and θ_+ .

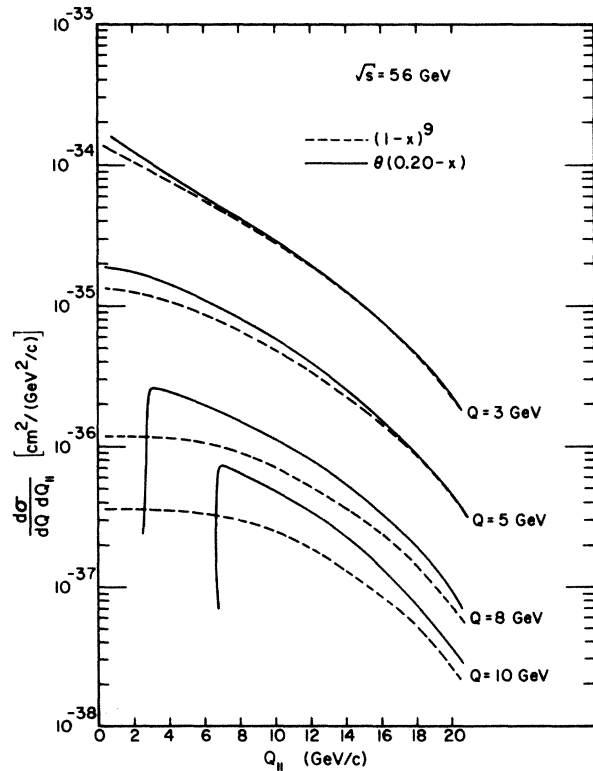


FIG. 4. Double differential cross section in the center-of-mass system.

at specific values and search for the other lepton at places where the cross section has a maximum. Several such curves are shown in Fig. 9. The main feature in this case is a narrow angular band of θ_+ , into which most of the events are concentrated.

IV. CONCLUSIONS AND REMARKS

So far we have presented a detailed discussion of the Drell-Yan model in view of the prevailing quark-parton ideas. We have gone into some detail in presenting numerical estimates so that a direct comparison would be possible. The numerical results should still be considered as estimates, because the antiquark contributions could be considerably smaller and diminish the total rate even further. The antiquark contributions cannot be larger than the cases we discussed. Thus an abnormally high rate will have to come from an alternative explanation. Several other effects could also be present and we shall elaborate on some of them.

Nuclear corrections. The experiments are done in nuclei and nuclear corrections could be important. The most important one seems to be the production of pions on the surface of the nucleus,

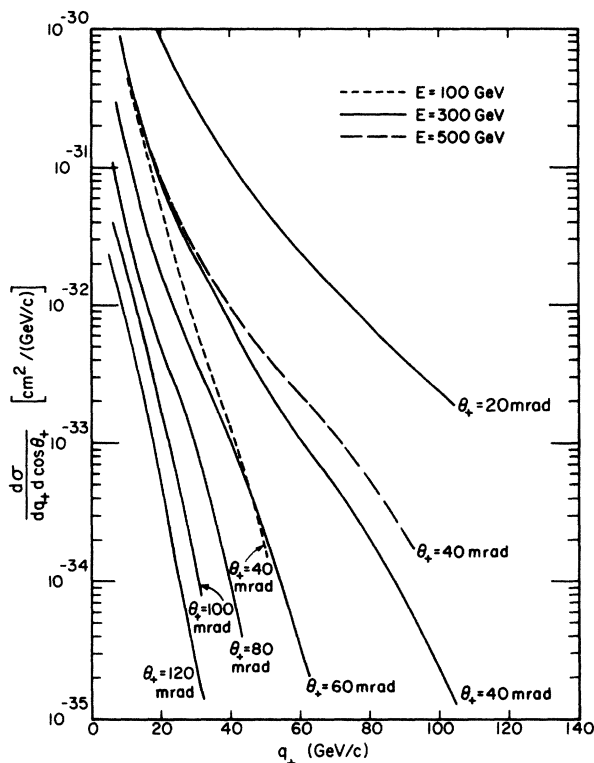


FIG. 6. Same as in Fig. 5, but for $G(x) = \theta(0.20 - x)$.

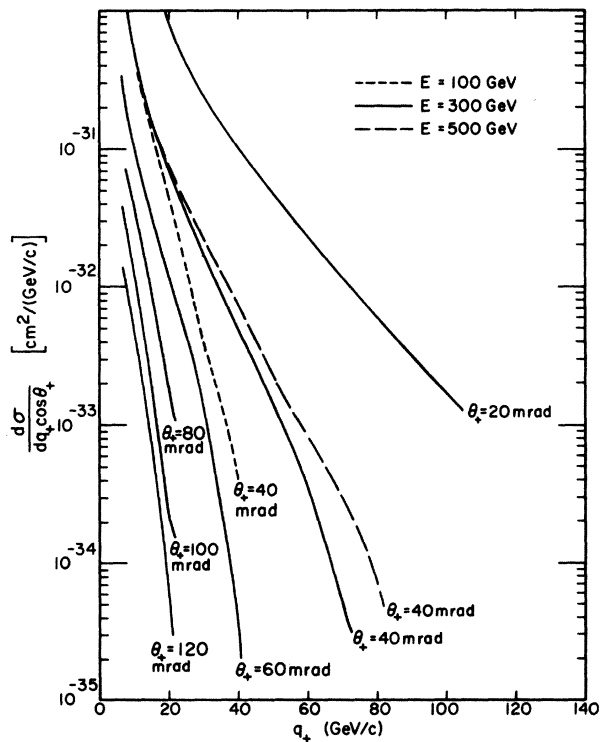


FIG. 5. Double differential cross section in the laboratory frame for different incident energies and angles θ_+ . For all the curves $G(x) = \theta(0.10 - x)$.

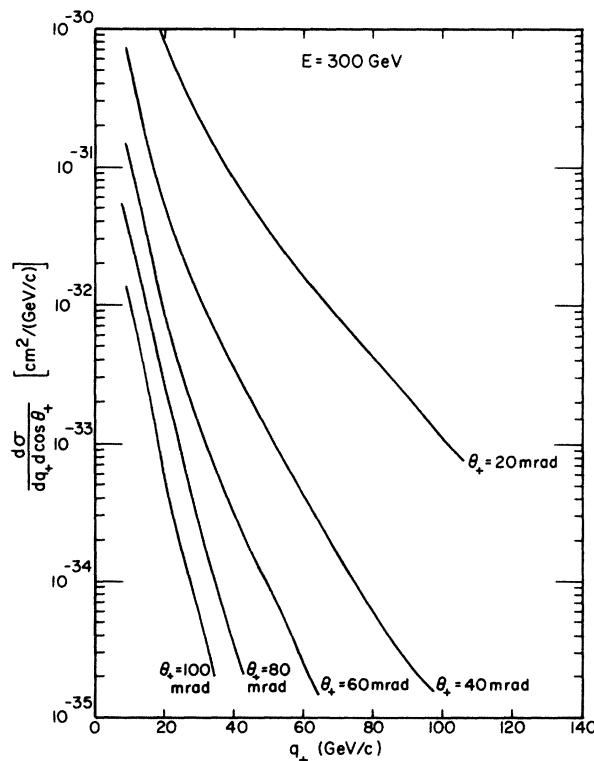


FIG. 7. Same as in Fig. 5, but for $G(x) = (1 - x)^9$.

which subsequently produce leptons through the reaction

$$\pi + \text{matter} \rightarrow l\bar{l} + \text{anything} . \quad (4.1)$$

This effect could be analyzed as a two step process. First the mesons are produced which then rescatter to produce the leptons. Since the antiquark distribution in mesons is not expected to be limited to small x , the p_{\perp} dependence of the leptons is expected to be considerably different.²⁵

Integrally charged quarks. Considering again the basic interaction to be

$$q + \bar{q} \rightarrow l + \bar{l} , \quad (4.1)$$

we can inquire whether different representations of quarks could lead to considerably different conclusions. A representative case is three integrally charged triplets of the Han-Nambu²⁶ type. Limitations on the nonstrange antiquark distributions again follow from a helicity argument and remain unchanged. Detail features in such models depend on the specific structure of the weak and electromagnetic currents. Assuming again that the λ -type quark distributions are limited, we arrive at similar cross sections, except for an over-all normalization factor. The cross section is reduced by a

factor of $\frac{1}{3}$ due to the three multiplets and in addition by the fact that $\sum Q_i^2 = 4$ in this case.

Charmed quarks. Charmed quarks are frequently introduced through the Glashow-Iliopoulos-Maiani scheme²⁷

$$\begin{pmatrix} \rho \\ \mathcal{N}_c \end{pmatrix}_L \text{ and } \begin{pmatrix} \rho' \\ \lambda_c \end{pmatrix}_L , \quad (4.2)$$

where $\mathcal{N}_c = \mathcal{N} \cos \theta_c + \lambda \sin \theta_c$ and $\lambda_c = -\mathcal{N} \sin \theta_c + \lambda \cos \theta_c$. The charged weak currents

$$J^W = \bar{\rho} \mathcal{N}_c + \bar{\rho}' \lambda_c \quad (4.3)$$

contain transitions into the charmed states. Most likely, low-energy neutrino experiments have not excited charmed states. Consequently, the effective form of the structure functions is the same as in the absence of charm. Couplings of the electromagnetic current, on the other hand, do not excite charmed states, so that available determinations of νW_2 must include contributions from charmed quarks.²⁸ Comparisons among the structure functions in the two processes determine the importance of charmed states. Omitting again the antiquark contributions, we arrive at

$$\frac{4f_{\rho'} + f_{\lambda}}{f_{\rho} + f_{\mathcal{N}}} \leq 9\sigma . \quad (4.4)$$

It is now evident that the presence of charmed quarks will not seriously modify the previous re-

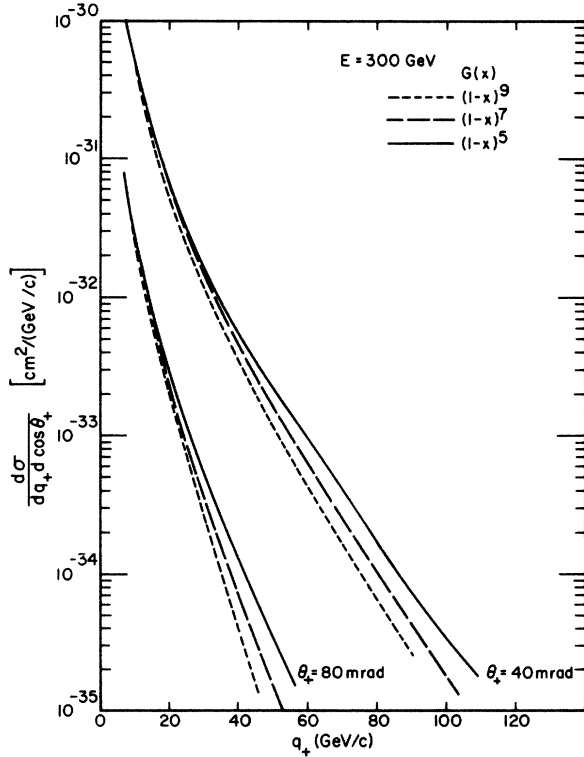


FIG. 8. Double differential cross section in the laboratory frame at the angles $\theta_+ = 40$ and 80 milliradians. Three different parametrizations of $G(x)$ are shown.

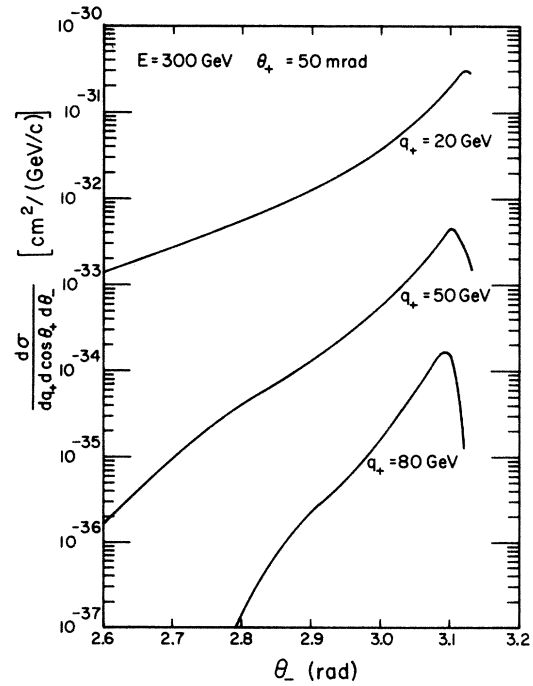


FIG. 9. Triple differential cross section in the laboratory frame for $G(x) = (1-x)^9$.

sults, because the combined λ and ϕ' distributions are limited.

So far we considered conventional quark-models where the quarks are left-handed. We could in general consider cases where besides the multiplets (4.2) there are also right-handed multiplets. In such cases the ratio of the cross sections being $\frac{1}{3}$ must follow from a detailed choice of the structure functions. The predictions for the production of heavy leptons in this class of models can be quite different.

Other mechanisms. If charm states exist, they should be produced directly in hadronic reactions²⁹

either singly or in pairs. They can be detected by their leptonic decays. Lepton-antilepton pairs could be produced in this manner, but the correlations and distinct signatures associated with the electromagnetic production of pairs should now be absent.

Other mechanisms like two-photon contributions,³⁰ direct W production,^{31, 32} and the effects of neutral currents³¹⁻³³ have also been studied and we refer to the available articles.

Note added in proof. Recently we received a paper on the same topic by S. Pakvasa, D. Parashar, and S. F. Tuan.

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¹J. Christenson *et al.*, Phys. Rev. Lett. **25**, 1523 (1970).

²S. D. Drell and T.-M. Yan, Phys. Rev. Lett. **25**, 316 (1970); and Ann. Phys. (N.Y.) **66**, 578 (1971).

³B. G. Pope, J. Phys. (Paris) Suppl. **34**, C1-409 (1973).

⁴J. Kuti and V. Weisskopf, Phys. Rev. D **4**, 3418 (1971).

⁵P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. **B33**, 221 (1971); **B36**, 642 (1972).

⁶S. M. Berman, J. D. Bjorken, and J. B. Kogut, Phys. Rev. D **4**, 3388 (1971). For a summary, see J. D. Bjorken, J. Phys. (Paris) Suppl. **34**, C1-385 (1973).

⁷R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972).

⁸R. W. Fiddler, Phys. Lett. **46B**, 455 (1973).

⁹K. J. Evans, Nucl. Phys. **B65**, 540 (1973).

¹⁰H. B. Thacker, Phys. Rev. D **9**, 2567 (1974).

¹¹G. R. Farrar, Caltech Report No. 68-422, 1974 (unpublished).

¹²K. Wilson compares the two approaches in *Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies*, edited by N. B. Mistry (Laboratory of Nuclear Studies, Cornell University, Ithaca, N. Y., 1972). G. Altarelli, R. A. Brandt, and G. Preparata, Phys. Rev. Lett. **26**, 42 (1971).

¹³T. Eichten *et al.*, Phys. Lett. **46B**, 274 (1973).

¹⁴A. Benvenuti *et al.*, Phys. Rev. Lett. **30**, 1084 (1973); **32**, 125 (1974).

¹⁵A larger rate than expected was observed in the reactions (a) $\gamma p \rightarrow \mu^+ \mu^- + x$: J. F. Davis *et al.*, Phys. Rev. Lett. **29**, 1356 (1972); (b) $\gamma p \rightarrow \gamma + x$: D. O. Caldwell *et al.*, Univ. of Calif. at Santa Barbara Report No. 73-3649 (unpublished); (c) $\mu + p \rightarrow \mu + \gamma + x$: K. C. Königsmann, Univ. of Rochester Report No. 471, 1974 (unpublished); (d) $e^+ + e^- \rightarrow$ hadrons: A. Litke *et al.*, Phys. Rev. Lett. **30**, 1189 (1973); G. Tarnopolsky, Phys. Rev. Lett. **32**, 432 (1974); B. Richter, invited talk at the APS meeting, Chicago, 1974 (unpublished).

¹⁶E. A. Paschos and V. I. Zakharov, Phys. Rev. D **8**, 215 (1973); E. A. Paschos, NAL Report No. NAL-Conf-73/27-THY, 1973 (unpublished).

¹⁷H. J. Lipkin and E. A. Paschos, Phys. Rev. D **8**, 2325 (1973).

¹⁸M. Gronau, Phys. Lett. **39B**, 395 (1972).

¹⁹R. Jaffe and J. Primack (private communication to S.C.C. Ting); C. H. Llewellyn Smith, CERN Report No. TH 1710-CERN (unpublished).

²⁰J. D. Bjorken, SLAC Report No. SLAC-PUB-1318, 1973 (unpublished).

²¹V. Barger and R. J. N. Phillips, Nucl. Phys. **B73**, 269 (1974); R. McElhaney and S. F. Tuan, Phys. Rev. D **8**,

2269 (1973). The antiquark distributions considered are also consistent with the quark counting arguments of S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973).

²²We were informed that M. Einhorn and R. Savit are completing work related to this cross section [NAL Report No. NAL-Pub-74/35-THY (unpublished)].

²³We have ascertained that throughout the range of integration Q^2 remains large enough for the model to be valid.

²⁴The results in Figs. 5-8 are relevant to the NAL experiments. A comparison with the BBK paper (Ref. 6) shows that the two calculations agree (within the uncertainties of the models) for small q_+ (≤ 40 GeV), but they diverge for larger values of q_+ . This is expected because $\phi(x, x')$ for the two calculations coincide when both x and x' are small and differ when one of them is large.

²⁵Charge-exchange nuclear corrections have been found to be important at low energies in several reactions: M. M. Sternheim and R. R. Silbar, Phys. Rev. D **6**, 8117 (1972); S. L. Adler, S. Nussinov, and E. A. Paschos, Phys. Rev. D **9**, 2125 (1974).

²⁶M. Y. Han and Y. Nambu, Phys. Rev. **139**, B1006 (1965); Y. Nambu and M. Y. Han, Phys. Rev. D **10**, 674 (1974).

²⁷S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1972).

²⁸A. De Rújula, H. Georgi, S. L. Glashow, and H. R. Quinn, Harvard report, 1974 (unpublished).

²⁹G. Snow, Nucl. Phys. **B55**, 445 (1973).

³⁰M.-S. Chen, I. Z. Muzinich, H. Terazawa, and T. P. Cheng, Phys. Rev. D 7, 3485 (1973).

³¹R. L. Jaffe and J. R. Primack, Nucl. Phys. B61, 317 (1973).

³²L. M. Lederman and D. H. Saxon, Nucl. Phys. B63,

315 (1973).

³³R. W. Brown, K. O. Mikaelian, and M. K. Gallard, NAL Report No. NAL-Pub-74/14-THY, 1974 (unpublished).