is, however, quite clear that one cannot calculate the *RR* cuts in this dual-SU(3) scheme by including all possible terms arising by convoluting the single exchange amplitude Eq. (4) with itself, in the fashion of the old absorption model.[17] In such a case, one finds a Ξ cross section much *smaller* than either Σ cross section, in clear contradiction to the data.

We conclude that the model presented is useful both phenomenologically and as a means for studying the duality and SU(3) properties of *RR* cuts. We have shown how to calculate *RR* cuts in terms of previously determined amplitudes in a manner that satisfies both theoretical and experimental constraints.

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Scaling in inelastic electron-photon scattering*

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We study moments of the cross section for inelastic electron-photon scattering. Simple scaling laws would result if the real photon were a particle which behaves predominantly like a light vector meson. We discuss possible modifications of these scaling laws originating in anomalies of the structure functions of the photon; they occur in quark models as well as in generalized vector-meson-dominance models.

I. INTRODUCTION

Deep inelastic electroproduction experiments have shown a local scaling law to hold for the electromagnetic structure functions of the nucleon.[1] For sufficiently large mass squared of the virtual photon, Q^2, and sufficiently high c.m. energy W of the photon-nucleon system, the structure functions depend only on the dimensionless ratio,[2] $2\nu/Q^2 = 1 + (W^2 - m_{\text{nucl}}{}^2)/Q^2$. The same types of inclusive experiments with (anti-) neutrino beams have proved the existence of a global scaling law: The total (anti-) neutrino-nucleon cross sections rise linearly with neutrino energy,[3] even for very low beam energies. The early onset of this type of scaling can be explained by Bloom-Gilman duality[4]: The W^2-dependent bumps in the structure functions are smoothed out by integrating them over a set of resonances. This way, a smoothly behaved total cross section results which exhibits its asymptotic properties already in the resonance region.

One might ask if similar simple scaling laws show up in the hadron production channels of inelastic electron-photon scattering, which will soon be accessible in electron-positron (electron-electron) colliding-beam machines. These experiments, proposed some time ago by Brodsky, Kinoshita and Terazawa, Walsh, and Carlson and Tung,[5] reveal the internal structure of the photon itself. Considering the real photon merely as a light vector meson, one would expect the structure functions of the photon to scale at least as rapidly as those of the nucleon. However, parton models[6-8] predict the existence of anomalous contributions to the photon structure functions which are no longer scale invariant. Furthermore, some of the diagrams which describe photon-photon annihilation in this model closely resemble the diagrams occurring in electron-positron annihilation into hadrons. This could cause additional problems for scaling.[9] On the other hand, we expect a breakdown of scaling to occur in generalized vector-meson-dominance models as well. Because of the quantum mechanical time-energy uncertainty the real photon can (for short times) transform into vector-meson states $V^{(n)}$ of very high mass. If $m_{V^{(n)}}{}^2 \gg \nu, Q^2$, we certainly can no longer apply the impulse approximation to the scattering of the electron on the constituents of the real photon because the interaction time is too long compared with the lifetime of this state. Only if the transitions of the photon into high-mass vector-meson states are very rare, an approximate scaling law should hold. It does not hold in generalized vector-meson-dominance models, which predict an s^{-1} scaling law (or slower falloff) for the cross section $\sigma(e^+e^- \rightarrow \text{all hadrons})$.[10,11]

The number of experimental $e\gamma$ events available in the near future is not large enough to allow a local analysis of the photon structure functions. Hence, it is expedient, as in neutrino-nucleon scattering, to investigate global quantities which can be predicted using theoretically transparent assumptions. In order to achieve as smooth a behavior of these quantities as possible, they should be defined in terms of the neutrinolike "cross section"

$$\frac{d^2\hat{\sigma}}{d\nu\, dQ^2} = Q^4 \frac{d^2\sigma}{d\nu\, dQ^2},$$

obtained from the actually measured cross section by dividing out the photon propagator. Besides the "total cross section" $\hat{\sigma}$,

$$\hat{\sigma} = \int\int d^2\hat{\sigma},$$

these (relativistically invariant) global quantities include the average energy and scattering angle of the outgoing electron,[12,13] defined as

$$\langle E' \cos^2(\tfrac{1}{2}\theta')[\sin^2(\tfrac{1}{2}\theta')]\rangle = \frac{\int\int E' \cos^2(\frac{1}{2}\theta')[\sin^2(\frac{1}{2}\theta')]\, d^2\hat{\sigma}}{\int\int d^2\hat{\sigma}}$$

in the laboratory system.

Summing up all Weizsäcker-Williams photons in an electron-positron collision, a simple-minded dimensional analysis for these functions yields the following dependence on the beam energy E in the laboratory system:

$$\hat{\sigma}(E) \propto E^2$$

and

$$\langle E' \cos^2(\tfrac{1}{2}\theta')[\sin^2(\tfrac{1}{2}\theta')]\rangle \propto E.$$

This behavior would obviously result if the (real) photon was a particle with predominantly hadronlike properties. Adopting the simple vector-meson-dominance (VMD) model for this case, we are able to predict the coefficients in these relations or at least to calculate their upper bounds. This is possible because $\hat{\sigma}$ can be interpreted in terms of the mean square charge per constituent of the vector meson. Thus, this model is a useful means for defining a standard measure for scaling quantities in electron-photon scattering. In electron-positron annihilation, this measure is set by the ratio of the cross sections $\sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$.

However, parton models and generalized VMD models as well predict logarithmic deviations from these scaling rules. In order to estimate their possible size, we apply a simplified version of the covariant parton model[14] to $e\gamma$ scattering. In this version, partons are treated as quarks with minimal electromagnetic coupling and small effective mass.[8] Even though the model generates only logarithmic scale-breaking terms, it nevertheless strongly affects the coefficients in the standard scaling relations. The most popular class of generalized VMD models[10,11] suggests the same results. This leads to the interesting possibility that dual relationships between resonance models and quark-parton models exist in photon-photon annihilation, too.

The outline of this article is as follows: In Sec. II we shall define the notation and discuss the kinematics of electron-photon scattering in colliding-beam machines. Section III is devoted to the derivation of the scaling rules for the various moments of the $e\gamma$ cross section. To get a firm basis, we first investigate those moments in the naive vector-meson-dominance model. (In an appendix we examine as well the consequences due to experimental restrictions of the electron scattering angle.) Then we estimate the size of anomalous contributions and show how they affect the scaling behavior of the moments. Some final remarks and a critical summary are the contents of the last section.

II. KINEMATICS OF ELECTRON-PHOTON SCATTERING

The large number of almost real photons which accompany electrons and positrons in colliding-beam machines, such as SPEAR and DORIS, allows us to study the inelastic scattering of electrons off photons.[5] To lowest order in QED, this corresponds to the annihilation of a real (γ) and a virtual (γ^*) photon into hadrons.

As shown in Fig. 1, we denote the electron momenta before and after collision by k and k', respectively; the momentum of the real photon, supposed to be radiated off the positron, should be p; denoting the momentum of the virtual photon by $q = k - k'$, we further introduce the invariant variables $Q^2 = -q^2 \geq 0$ and $\nu = p \cdot q$. Since the almost real photons are radiated off the positron with very small angles, we get the following expressions for all these variables in the laboratory frame.

ingoing electron: $k = E(1, 0, 0, 1)$

outgoing electron: $k' = E'(1, \sin\theta', 0, \cos\theta')$

real photon: $p = E_\gamma(1, 0, 0, -1)$

invariant variables: $Q^2 = 4EE' \sin^2(\frac{1}{2}\theta')$,

$$\nu = 2E_\gamma[E - E' \cos^2(\tfrac{1}{2}\theta')]$$

It should be noticed that the quantities Q^2 and $\nu/2E_\gamma E$ can be determined without measuring the positron momenta.

The expressions for the cross section become most transparent if one introduces the following (Lorentz invariant) scaling variables:

$x = Q^2/2\nu$ with $0 \leq x \leq 1$ (Bjorken variable),

$y = \nu/(kp)$ with $0 \leq y \leq 1$,

$\epsilon = (kp)/2E^2$ with $0 \leq \epsilon \leq 1$.

If the electron scattering angle has an experimental upper limit θ_0, the variable y is bounded from above by $y \leq 1/[1 + \epsilon x \cot^2(\frac{1}{2}\theta_0)]$.

As in inelastic electron-nucleon scattering, one describes the (real) photon (spin averaged) by two structure functions depending on the invariant variables ν and Q^2 or x and $y(kp)$:

$$\frac{1}{8\pi} \sum_{\text{spins}} \int d^4z\, e^{iqz} \langle \gamma(p) | [j_\mu(\tfrac{1}{2}z), j_\nu(-\tfrac{1}{2}z)] | \gamma(p) \rangle = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1 + \left(\frac{p_\mu}{\nu} - \frac{q_\mu}{q^2}\right)\left(\frac{p_\nu}{\nu} - \frac{q_\nu}{q^2}\right) \nu F_2 . \tag{1}$$

Figure 2 displays the triangle in the (ν, Q^2) plane where the structure functions are defined for the scattering process. For the sake of simplicity we have neglected the pion mass on the hadronic mass scale, and we are doing so, as well, in the following calculations. In terms of the structure functions, the cross section for the process $e + \gamma \to e'$ + hadrons reads

$$Q^4 \frac{d^2\sigma}{dx\,dy} \equiv \frac{d^2\hat{\sigma}}{dx\,dy} = 16\pi\alpha^2 E^2 \epsilon [(1 - y) F_2(x, y(kp)) + y^2 x F_1(x, y(kp))] . \tag{2}$$

Actually, one measures electron-positron collisions in which the photons have a continuous energy distribution. If we allow the positron to be scattered into a cone with aperture $\theta_{\min} \leq \theta \leq \theta_{\max} \ll 1$, the leading term of the Weizsäcker-Williams spectrum of the photons is given[15] by

$$N(E_\gamma) dE_\gamma = \frac{\alpha}{\pi} \left\{ [1 + (1 - \epsilon)^2] \ln \frac{\theta_{\max}}{\theta_{\min}} - (1 - \tfrac{1}{2}\epsilon)^2 \ln \frac{\epsilon^2 + (1 - \epsilon)\theta_{\max}^2}{\epsilon^2 + (1 - \epsilon)\theta_{\min}^2} \right\} \frac{d\epsilon}{\epsilon} = H(\epsilon) \frac{d\epsilon}{\epsilon} . \tag{3a}$$

The spectrum is independent of the energy of the ingoing positron; it depends explicitly only on the fraction $\epsilon = E_\gamma/E$. It will be sufficient for our considerations to approximate the spectrum by the simplified expression

$$H(\epsilon) \frac{d\epsilon}{\epsilon} \to \frac{\alpha}{\pi} \ln \frac{\theta_{\max}}{\theta_{\min}} \frac{d\epsilon}{\epsilon} , \tag{3b}$$

leaving us at most with an error of the order of 20% for absolute predictions; some of our results will even be independent of the detailed form of the spectrum.

Let us now define the neutrinolike "cross section" $\hat{\sigma}$ as

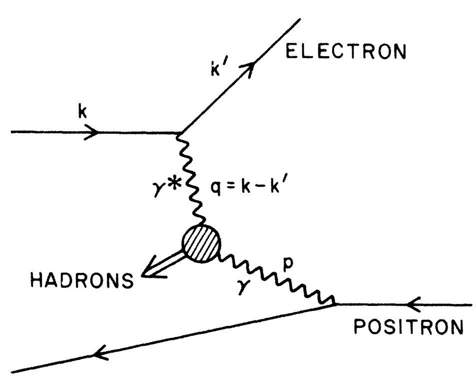

FIG. 1. Inelastic scattering of electrons off almost real photons in electron-positron collisions.

$$\hat{\sigma} = \int \frac{d\epsilon}{\epsilon} H(\epsilon) \int\int dx\,dy\,Q^4 \frac{d^2\sigma}{dx\,dy}$$
$$\equiv \left\langle \int\int d^2\hat{\sigma} \right\rangle_\epsilon, \tag{4}$$

where the integration is supposed to be taken over an arbitrary region in the cube $0 \leq x, y, \epsilon \leq 1$ (yet not varying with beam energy). Considering the moment $\hat{\sigma}$ instead of the cross section σ itself has the advantage of deemphasizing the low-x, y, ϵ region by dividing out the virtual photon propagator $1/Q^4 \propto 1/x^2y^2\epsilon^2$. Hence, we expect $\hat{\sigma}$ to reach its asymptotic value faster than σ itself. Other global quantities which are easily accessible are the average values of

$$f_1 = E' \cos^2(\tfrac{1}{2}\theta') \tag{5a}$$

and

$$f_2 = E' \sin^2(\tfrac{1}{2}\theta'). \tag{5b}$$

They should be defined as

$$\langle f_{1,2}\rangle = \frac{\langle \int\int f_{1,2}\, d^2\hat{\sigma}\rangle_\epsilon}{\langle \int\int d^2\hat{\sigma}\rangle_\epsilon}. \tag{6}$$

Investigating those quantities is particularly reasonable because they are simple, relativistically invariant functions of the scaling variables:

$$f_1 = E(1-y) \tag{5a$'$}$$

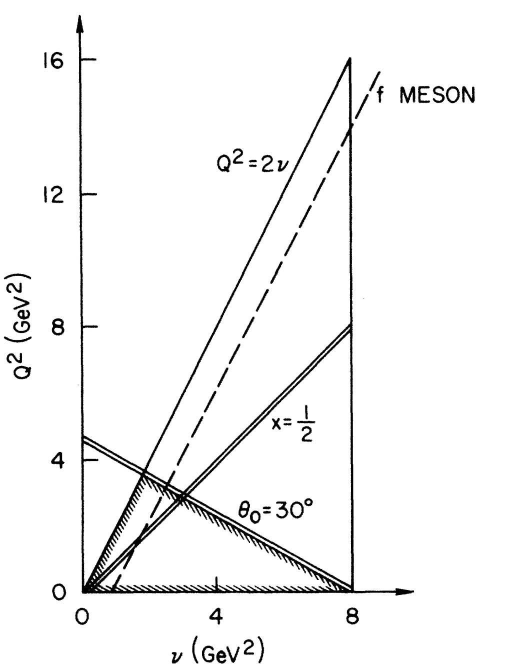

FIG. 2. (ν, Q^2) plane for $E = 4$ GeV and $E_\gamma = 1$ GeV. Events with scattering angle $\theta' \leq \theta_0$ fall into the hatched triangle formed by the lines $Q^2 = 0$, $\theta' = \theta_0$, and $Q^2 = 2\nu$ (the pion mass is neglected on the hadronic scale).

and

$$f_2 = Exy\epsilon. \tag{5b$'$}$$

From the results of inelastic neutrino scattering experiments we can conclude that measuring the moments $\hat{\sigma}$, $\langle f_1\rangle$, and $\langle f_2\rangle$ provides us with a useful tool for studying scaling phenomena in electron-photon scattering, even at low energies.

III. SCALING AND ANOMALOUS CONTRIBUTIONS TO ELECTRON-PHOTON SCATTERING

In most photon-hadron reactions the real photon appears predominantly as a hadronic particle, being a superposition of a few virtual vector mesons. The only exceptions to this rule are processes in which two photons are involved, as in Compton scattering.[7] The deviation of the Compton cross section from the simple VMD prediction can be attributed, in parton models, to contributions from diagrams where one parton directly connects both photons without interacting with the remaining hadronic flux. Similar phenomena are expected to occur in photon-photon annihilation.[6,8] The quark-parton diagrams which contribute to this process are shown in Fig. 3. Dual relationships[10] could exist between various sets of diagrams. We shall comment on this possibility later. Figure 3(a) is the only diagram which has a parallel in inelastic electron-nucleon scattering. In the naive VMD model it can be approximated by replacing the quark-antiquark pair attached to the real photon by the lowest-lying U-singlet vector meson. Obviously, if this diagram represented the only contribution to electron-photon scattering, the absolute value of the moment $\langle f_1\rangle$ could be predicted, while $\langle f_2\rangle$ and $\hat{\sigma}$ could be bounded from above. A comparison with the nucleon case reveals that

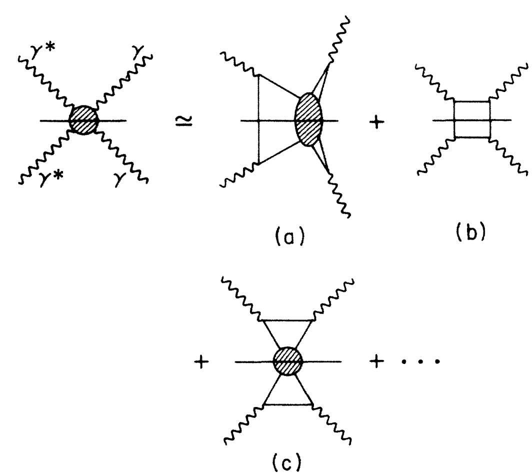

FIG. 3. Imaginary part of the $\gamma^*\gamma$ forward scattering amplitude in the covariant quark-parton model.

these bounds would not be far from the actual values. Therefore, deviations from those predictions would indicate how much the photon deviates from being a light vector meson. The remaining part of the diagrams of Fig. 3 [(b), (c), . . .] represents those disconnected pieces of the photon-photon scattering amplitude in which a parton coming from the real photon is directly connected to a parton in the virtual photon. These diagrams cannot be evaluated without knowing the quark propagators and the off-shell quark scattering amplitudes. Simplifying assumptions, however, should enable us to get an idea of the possible size of these contributions. Before turning to this speculative part of our considerations, we first derive the standard scaling rules in the simple VMD model, thereby ignoring nonscaling contributions for the moment.

A. Scaling contributions

We begin by imposing the hypothesis of rapid scaling on the structure functions of the photon in the spirit of Bloom-Gilman duality[4]:

$$F_{1,2}(x, y(kp)) \to F_{1,2}(x,\infty) \equiv F_{1,2}(x) . \quad (7)$$

As an immediate consequence we get the following global scaling rules:

$$\hat{\sigma}(E) \propto E^2 , \quad (8a)$$

$$\langle E' \cos^2(\tfrac{1}{2}\theta') \rangle \propto E , \quad (8b)$$

$$\langle E' \sin^2(\tfrac{1}{2}\theta') \rangle \propto E . \quad (8c)$$

For different beam energies it is understood that the range of integration [Eqs. (4), (6)] within the cube $0 \leq x, y, \epsilon \leq 1$ is chosen independent of the beam energy if experimental conditions do not allow one to exhaust the entire cube. The results agree with what one obtains from naive power counting if no mass parameters are involved. From neutrino-nucleon scattering, one has learned that global quantities display their asymptotic scaling properties even at very low beam energies. No lower bound for the variables ν and Q^2 has to be introduced so as to be definitely in the scaling region. Therefore, in the simple VMD model one could safely expect the same early onset of scaling in electron-photon scattering.

The proportionality coefficients in Eq. (8) can be calculated under weak additional assumptions.

(i) In the present model, we can take for granted the Callan-Gross relation $xF_1 = \frac{1}{2}F_2$ for the photon and carry out the integrations in Eqs. (4), (6) in such a way that the y integration can be factorized out.[16] Then, the slope in Eq. (8b) can be calculated:

$$\langle E' \cos^2(\tfrac{1}{2}\theta') \rangle = (1 - \langle y \rangle)E , \quad (9a)$$

where

$$\langle y \rangle = \frac{\int dy\, y(1 - y + \frac{1}{2}y^2)}{\int dy(1 - y + \frac{1}{2}y^2)} .$$

The coefficient in Eq. (9a) would be $\frac{9}{16}$ if the cross section was known over the entire y interval. Notice that this relation is an absolute prediction derived only from the assumptions of scaling and the Callan-Gross relation. The slope does not depend on the Weizsäcker-Williams spectrum.

(ii) Under the same assumptions, we can derive a model-independent upper bound for the coefficient in Eq. (8c), if, in addition, the x and ϵ integrals factorize. Defining

$$\langle \epsilon \rangle = \frac{\int d\epsilon\, \epsilon\, H(\epsilon)}{\int d\epsilon\, H(\epsilon)}$$

and

$$\langle x^k \rangle = \frac{\int dx\, x^k F_2(x)}{\int dx\, F_2(x)}$$

we get the inequality

$$\langle E' \sin^2(\tfrac{1}{2}\theta') \rangle = \langle \epsilon \rangle \langle y \rangle \langle x \rangle E \leq \langle \epsilon \rangle \langle y \rangle E . \quad (9b)$$

If we were able to cover experimentally the entire square $0 \leq y, \epsilon \leq 1$, we would obtain (for H = const)

$$\langle E' \sin^2(\tfrac{1}{2}\theta') \rangle = \tfrac{7}{32} \langle x \rangle E \leq \tfrac{7}{32} E . \quad (9b')$$

Comparing the results of Eqs. (9a) and (9b′) we easily recognize that small electron-scattering angles are dominant even after dividing out the photon propagator.

(iii) In order to determine $\hat{\sigma}(E)$ itself, we apply the VMD model to the real photon, as explained above. Since all functions in the expression

$$\frac{\hat{\sigma}(E)}{E^2} = 16\pi\alpha^2 \int d\epsilon\, H(\epsilon) \int dy(1 - y + \tfrac{1}{2}y^2) \int dx\, F_2(x) \quad (9c)$$

are positive-definite, we obtain an upper bound (which includes all experimental conditions) by extending the integration over the entire cube $0 \leq x, y, \epsilon \leq 1$. The integral over the structure function F_2 can be well estimated. Combining the VMD model with the quark model, we can write the state vector of the real photon (in the SU_3 symmetry limit) as

$$|\gamma\rangle = \frac{2e}{\sqrt{3}\, f_\rho} \frac{1}{\sqrt{6}} \{2|p\bar{p}\rangle - |n\bar{n}\rangle - |\lambda\bar{\lambda}\rangle\} . \quad (10)$$

em_ρ^2/f_ρ denotes the usual $\gamma\rho$ coupling constant. Since the integral over F_2 measures the mean square charge per constituent, the result for the integral in this approximation is

$$\int_0^1 dx\,F_2(x) = \frac{4}{9}\,\frac{\alpha}{f_\rho^2/4\pi} \simeq \frac{2}{9}\alpha\,. \tag{11}$$

However, this must be considered as an upper bound for the actual value of the integral. In parallel to the nucleon case, we do not expect a large quark-antiquark sea to be present in the hadronic part of the photon, yet there might be a substantial fraction of gluons present. They do not interact electromagnetically, but nevertheless reduce the average square charge per constituent. Thus, from simple VMD we derive the following upper bound for the "cross section" $\hat{\sigma}$:

$$\frac{\hat{\sigma}(E)}{E^2} \lesssim 2\alpha^4 \ln\frac{\theta_{\max}}{\theta_{\min}}\,. \tag{9c$''$}$$

For an actual experiment where the scattering angle of the electron has a maximum value θ_0, but where no other restrictions are imposed upon the variables, this estimate can be sharpened. We shall discuss this problem in the Appendix.

Hence, assuming the real photon to be a light vector particle with predominantly hadronic properties, one can well estimate the dependence of global quantities on the energy of the ingoing electron. They can serve as standard values with which experimental results can be confronted in order to extract the strength of the conventional hadronic component within the photon.

B. Anomalous contributions

To study, first, the possible effect of high-mass vector mesons, we adopt the following assumptions which are commonly used in generalized VMD models[10,11,17] [even though the model does not correctly describe $e^+e^- \to$ (all hadrons) for energies $s \gtrsim 12$ GeV^2, it might be applicable at smaller energies]:

(i) There is a Veneziano-type spectrum of the vector mesons, $m_{V(n)}{}^2 = m_\rho{}^2(1+2n)$.

(ii) The $V^{(n)}\gamma$ coupling constant falls off like the inverse mass, $f_{V(n)}{}^{-1} \propto m_{V(n)}{}^{-1}$.

(iii) For ν, $Q^2 \gtrsim m_{V(n)}{}^2$, the scattering on the constituents is incoherent.

Then Eq. (11) is to be replaced by

$$\int_0^1 dx\,F_2 \sim \frac{4\alpha}{9}\,\frac{1}{f_\rho^2/4\pi}\sum_{n=0}^{4E^2/2m_\rho^2}\frac{1}{1+2n} + \text{coherent part}$$

$$\sim \frac{2\alpha}{9}\,\frac{1}{f_\rho^2/4\pi}\ln\frac{4E^2}{2m_\rho^2} + \text{coherent part}\,. \tag{11$'$}$$

Numerically, the coefficient in front of the logarithmic terms is $\simeq \frac{1}{10}\alpha$. The interesting feature of this representation is the hint to a possible occurrence of scale-breaking terms by including higher vector mesons in the virtual energy fluctuations of the real photon. The (new) dimensional constant which governs the scale-breaking terms is the hadronic level spacing $2m_\rho{}^2$. However, it is not possible to estimate the coherent contribution in this model, and we turn to the quark-parton model, where more definite predictions can be obtained. This step might be justified by invoking a dual relationship between generalized VMD models and quark-parton models.

Parton contributions to $\gamma\gamma^*$ annihilation are expected to be of the same size as the simple vector-meson contributions.[6-8] This can easily be shown in a model where fractionally charged quarks with light effective mass are minimally coupled to photons. This model can certainly not be applied to e^+e^- annihilation for energies $s \gtrsim 12$ GeV^2. However, below this value it might have a chance to be, at least approximately, correct. Because we are interested in quantities which are not sensitive to energies $W^2 \gtrsim 10$ GeV^2 either, it is not unreasonable to apply this model to $\gamma\gamma^*$ annihilation. The corresponding diagrams are shown in Fig. 3. (Interference terms do not disturb the following discussion.[8]) Approximating the propagators of exchanged quarks by free fermion propagators with effective mass m_0 (~0.3 GeV) one finds three features which distinguish the set of diagrams (b), ... from the simple vector-meson-dominated diagram (a):

(i) In the transverse amplitudes, logarithmic scale-breaking terms are present;

(ii) the contributions from virtual scalar photons do not vanish anymore,

(iii) the piece F_2^B of the structure function vanishes linearly in x for $x \to 0$.

The detailed calculation gives the following result for the box diagram[8,18]:

$$F_2^B = \sum e_i{}^4 \cdot x\,\frac{\alpha}{\pi}\{[x^2+(1-x)^2][\ln(W^2/m_0{}^2)-1] + \tfrac{9}{2}x(1-x)\}\,, \tag{12a}$$

$$F_2^B - 2xF_1^B = \sum e_i{}^4\,\frac{4\alpha}{\pi}x^2(1-x)\,. \tag{12b}$$

The e_i denote the quark charge quantum numbers and W is the invariant energy of the $\gamma\gamma^*$ pair. Integrating F_2 over x, one recognizes a surprising numerical agreement between the leading logarithmic term in the generalized VMD model Eq. (11$'$) (which overestimates the integral because gluons are not taken into account) and the present quark model ($\sum e_i{}^4 = \frac{2}{3}$ for colored quarks):

$$\int_0^1 dx F_2^B = \alpha \frac{1}{3\pi}(\sum e_i^4)\ln(4E^2)+\cdots$$
$$\simeq \tfrac{1}{14}\alpha \ln(4E^2)+\cdots . \quad (13)$$

Even though this agreement could be accidental, it offers the interesting possibility of giving a dual relationship to generalized VMD models and quark models. Notice that the low-lying vector-meson contribution alone cannot be expected to be dual to the box diagram because the shapes of the corresponding pieces of the photon structure functions are different. In the present example, a dual relationship can exist only between the quark model and asymptotic vector-meson states.

The logarithmic scale breaking term in F_2^B is biggest for $x \gtrsim \frac{1}{2}$ and W^2 large. Unfortunately, this region is not easily accessible experimentally. Furthermore, the limited statistics in coming experiments will not permit detection of such weak deviations from scaling. Yet there might be some hope to investigate the presence of scalar contributions in the y distribution

$$\frac{d\hat{\sigma}}{dy} \propto (1-y+\tfrac{1}{2}y^2)+(1-y)R . \quad (14)$$

R is the ratio of the longitudinal to the transverse cross section. It becomes as large as 30% in this model. By contrast, for a simple hadronlike photon with spin-$\frac{1}{2}$ constituents, the second term would be absent. This might no longer be true if high-mass vector mesons give considerable contributions in the generalized VMD model.

Calculating the contribution of the box diagram to the moment $\hat{\sigma}$, one cannot expect to see a large deviation from the general scaling law $\hat{\sigma}(E)\propto E^2$. On the other hand, in the colored quark model, the proportionality coefficient changes by a factor of about 2 relative to the hadronic contribution:

$$\hat{\sigma}_{\text{box}} \simeq \frac{16}{\pi}\alpha^4 E^2 \ln\frac{\theta_{\max}}{\theta_{\min}}(\sum e_i^4)\left[\left(\frac{2}{9}\ln\frac{4E^2}{m_0^2}-1\right)+\frac{1}{6}\right],$$

where the term $\frac{2}{9}\ln(4E^2/m_0^2)-1$ inside the square bracket represents the transverse piece and the term $\frac{1}{6}$ represents the longitudinal piece. However, this happens only if one integrates over the entire x, y, ϵ cube. The influence of this diagram depends crucially on the maximum electron scattering angle, as one can see from Eq. (A2a) in the Appendix. If most of the "cross section" comes from the small-x region, the box diagram is negligible since F_2^B vanishes linearly in x. The same applies to the other expectation values $\langle E'\cos^2(\frac{1}{2}\theta')[\sin^2(\frac{1}{2}\theta')]\rangle$.

The last diagram [Fig. 3(c)] we have to consider is likely the most troublesome one. It could even destroy the general scaling law $\hat{\sigma}(E)\propto E^2$ (up to logarithmic terms), as it might do for $\sigma(e^+e^- \to \text{hadrons})$. Attempts have been made to estimate its contribution to e^+e^- annihilation by putting the quarks on their effective mass shell.[19] Applying the same approximation to diagram (c), one obtains a 25% correction to the box diagram if one describes the quark-antiquark interaction in a conventional Regge picture. In the spirit of the original covariantly formulated quark-parton model,[14] one could indeed assume the quarks to be kept near their effective mass shell. However, the extension of this assumption to large timelike parton momenta is not straightforward, but leaves us with a major uncertainty in the quark-parton analysis of photon-photon processes. This problem is intimately related to coherence effects in generalized VMD models.

Apart from the latter problem, anomalies in the photon-photon scattering amplitude result in a weak violation of the basic scaling rules (8) within the quark model. Yet collecting all anomalous contributions to $\hat{\sigma}$ changes the proportionality coefficient in $\hat{\sigma}(E)\propto E^2$ considerably when compared with the simple VMD prediction. The change amounts to a factor of about 3 and would clearly exceed the upper bound (9c″). However, if experiments are sensitive only to small x values, this effect is much smaller and harder to detect experimentally. In particular, the cross section σ itself cannot be expected to be influenced significantly by anomalous contributions in the present context, and its estimate by Brodsky *et al.* and Walsh[5] is not to be changed.

IV. CONCLUSION

Studying inelastic electron-photon scattering is primarily motivated by the desire to investigate the structure of the real (as well as the virtual) photon. The first stages of experimental analysis can include such global quantities as moments of the cross section, properly defined average energy losses, and average angles of the scattered leptons. Considering the real photon merely as a light vector particle with predominantly hadronic properties, as suggested by the success of the simple vector-meson-dominance model, we have obtained simple scaling rules for all those quantities: They grow with beam energy with the same power as their dimension; the slope of the growth can be calculated or at least estimated. These rules should be valid even if ν and Q^2 are not restricted to the scaling region. Therefore, they can serve as a useful standard measure for scaling effects in inelastic $e\gamma$ scattering.

However, the simple VMD piece of the photon-photon annihilation cross section might be superseded by anomalies attributable to a possible quark-antiquark substructure of both the real and virtual photon or, in a dual picture, to the excitation of high vector-meson states in the photon. Those anomalies are not accounted for by the simple vector-meson-dominance model. From the conventional assumption of the covariantly formulated parton model that only quark lines with finite mass couple to hadrons, we have derived two consequences: (i) The general scaling behavior of the moments is still valid (up to logarithmic corrections), yet the coefficients in these relations are changed. (ii) Experiments which are sensitive only to the small-x region should essentially reproduce the simple vector-meson-dominance predictions; this applies in particular to the cross section itself. However, we are not able to predict the rescattering correction from far off-shell quarks [diagram (c) in Fig. 3] and coherent scattering contributions of virtual high-mass excitations of the real photon. At high energies they might introduce scale-breaking pieces of the same size as the scaling VMD contributions. Thus, measurements of the energy dependence of such global quantities as average energy loss and average angle of the scattered electron are as interesting as the magnitude of the cross section itself (which is hard to predict more accurately than a factor of 2). The knowledge of their behavior is a useful correlate to the information obtained from electron-positron annihilation into hadrons.

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APPENDIX: LIMITED ELECTRON SCATTERING ANGLE

Two reasons require the limitation of the electron scattering angle in an $e\gamma$ experiment to a value below 30° in the present energy range: (i) The differential cross section $d\sigma/dQ^2$ is expected to fall off rapidly with Q^2, and high-Q^2 regions are sparsely populated compared with low-Q^2 regions. (ii) The 2γ process in $ee\to eeX$ is for large Q^2 superseded by radiative correction of the type $ee\to ee+\gamma^*$ ($\to$ hadrons with $C=-$).[20] Therefore, it is necessary to discuss briefly the consequences of a limited scattering angle for the moments of the cross section.

In the framework of the naive VMD model, we can completely exhaust the entire kinematically allowed region with $\theta'\leq\theta_0$ (hatched triangle in Fig. 2); we are not forced to introduce cuts which eliminate low-Q^2 and low-ν events. In this case, the scaling laws (8) do not change, but the coefficients do if one varies θ_0. In Eqs. (4) and (6) the y integration alone is restricted, in a scale-invariant fashion, to the interval $0\leq y\leq[1+\epsilon x\cot^2(\frac{1}{2}\theta_0)]^{-1}$ while x and ϵ vary independently between 0 and 1. These are the results:

(i) In the (unrealistic) case $\frac{1}{2}\pi\leq\theta_0\leq\pi$ the coefficients in Eq. (8) read

$$\frac{\hat{\sigma}(E)}{E^2}\simeq\frac{32}{3}\alpha^3\ln\frac{\theta_{max}}{\theta_{min}}\left[1-\frac{3}{8}\langle x\rangle\cot^2(\tfrac{1}{2}\theta_0)\right]\int_0^1 dx\,F_2(x)\,, \tag{A1a}$$

$$\langle E'\cos^2(\tfrac{1}{2}\theta')\rangle=\frac{9}{16}\left[1+\frac{1}{3}\langle x\rangle\cot^2(\tfrac{1}{2}\theta_0)\right]E\,, \tag{A1b}$$

$$\langle E'\sin^2(\tfrac{1}{2}\theta')\rangle=\frac{7}{32}\langle x\rangle\left[1-\frac{3}{4}\frac{\langle x^2\rangle}{\langle x\rangle}\cot^2(\tfrac{1}{2}\theta_0)\right]E\,. \tag{A1c}$$

In the quark-parton model, the average longitudinal parton momentum $\langle x\rangle$ is approximately $\frac{1}{2}$ and deviations of all quantities from their values at $\theta_0=\pi$ are to be small. In Fig. 4, the moments are shown for $\langle x\rangle=\frac{1}{2}$.

(ii) On the other hand, for small (but still finite) θ_0 we obtain quite a different behavior of the co-

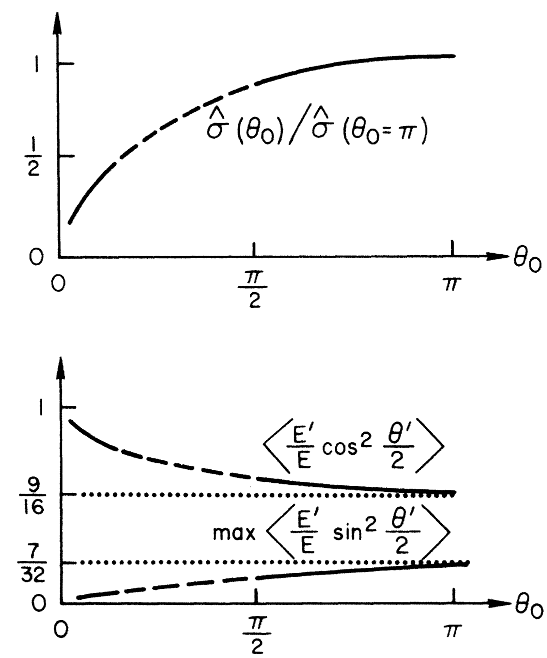

FIG. 4. Predictions for the moments of the inelastic electron-photon scattering cross section as functions of the maximum electron scattering angle θ_0. They are derived from the simple vector-meson-dominance model of the real photon, with spin-$\frac{1}{2}$ constituents carrying half of its momentum, $\langle x\rangle=\frac{1}{2}$ and $\langle x^2\rangle=\frac{1}{4}$.

efficients[21]:

$$\frac{\hat{\sigma}(E)}{E^2} \simeq 32\alpha^3 \ln \frac{\theta_{\max}}{\theta_{\min}} F_2(0)\left[\tfrac{1}{2}\theta_0 \ln(\tfrac{1}{2}\theta_0)\right]^2, \qquad \text{(A2a)}$$

$$\langle E' \cos^2(\tfrac{1}{2}\theta')\rangle \simeq \left[1 - \frac{\frac{3}{8}}{|\ln(\frac{1}{2}\theta_0)|}\right] E, \qquad \text{(A2b)}$$

$$\langle E' \sin^2(\tfrac{1}{2}\theta')\rangle \simeq \frac{1}{2}\left(\frac{\theta_0}{2}\right)^2 E. \qquad \text{(A2c)}$$

$F_2(0)$ can be estimated from the proton structure function[5] as

$$F_2(0) \simeq (\sigma_{\gamma\gamma}/\sigma_{\gamma p}) F_2^p(0) \simeq 0.3/300. \qquad \text{(A3)}$$

(iii) In the transition region, one cannot calculate the precise value of the moments. However, it can easily be shown that all moments are monotonic in θ_0. Thus, from the curves in Fig. 4 (the dashed part is interpolated by hand) we can read off the approximate size of the moments for intermediate scattering angles with a sufficient degree of accuracy.

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