PHYSICAL REVIEW D VOLUME 10, NUMBER 5 1 SEPTEMBER 1974

Double-charge-exchange reactions and Regge-Regge cuts in a dual Regge model

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(Received 29 April 1974)

A model which previously achieved a good quantitative description of 19 different $0^- \frac{1}{2}^+ \to 0^- \frac{1}{2}^+$ reactions is extended to double-charge-exchange reactions. The underlying mechanism is double Regge exchange, calculated in an SU(3)-symmetric dual absorptive model. The results depend on whether a nonet or an octet of mesons is assumed in the intermediate state; strong theoretical reasons for favoring a nonet are given. The amplitudes obtained in the previous model produce good quantitative agreement with recent data on $\pi^- p \to K^+\Sigma^-$, $K^- p \to \pi^+\Sigma^-$, and $K^- p \to K^+\Xi^-$ forward differential cross sections.

I. INTRODUCTION

Recently a model[1] was described that quantitatively agrees with all elastic, charge-exchange (CEX), and hypercharge-exchange (HCEX) reactions of the type $0^-\frac{1}{2}^+ \to 0^-\frac{1}{2}^+$. This model consists basically of Regge-pole exchange amplitudes, strongly constrained by SU(3) symmetry and duality, and modified by absorptive Regge-Pomeron (*RP*) cuts in the manner suggested by Ringland *et al.*[2] With these results in hand, we shall extend the calculation to include Regge-Regge (*RR*) cuts. These *RR* cuts are expected to dominate double-charge-exchange (DCEX) reactions[3] such as $\pi^- p \to K^+\Sigma^-$, $K^- p \to \pi^+\Sigma^-$, and $K^- p \to K^+\Xi^-$. Recent data[4] on these reactions provide a test for our calculations. The salient feature of the new data is that the Ξ cross section is at least as large as (and probably larger than) either Σ cross section. Our model reproduces this relative magnitude relation very well, in contradiction with previous *RR*-cut models.[5,6] Also, the absolute magnitude of each reaction is in good agreement with the results of our model for a very reasonable value of the *RR*-cut strength.

II. A DUAL-REGGE MODEL FOR REGGE-REGGE CUTS

In lieu of a fundamentally justifiable procedure for calculating absorptive-cut corrections, we must use the generally accepted convolution prescription[7] for Regge-pole amplitudes (perhaps modified slightly[2]) in actual calculations. For this reason the absolute strength of *RR* cuts is not well determined and is best estimated phenomenologically by direct comparison with data (see Sec. III). However, the constraints of SU(3) and duality are sufficiently precise to allow a very reliable estimate of the *relative* amount of *RR* cut in related but different particle reactions. The constraints which SU(3) and duality imply for single Reggeon exchange amplitudes are, of course, well known,[8] and we shall assume, as input, amplitudes which do satisfy these constraints.

In both examples that we will consider below (meson-meson and meson-baryon scattering) the s and u channels are formally identical, hence the amplitudes must be s-u crossing (or line-reversal) invariant. This applies to *RR* cuts as well as to poles.[5]

The crucial constraint that we must apply is the result (provable in field theory,[9] the Reggeon calculus,[10] and in dual field theory[11]) that the third double-spectral function must exist for both particle-Reggeon scattering processes shown in Fig. 1. In the duality diagram language, this amounts to the existence of a planar s-u duality diagram for both particle-Reggeon scattering amplitudes M and N (Finkelstein's selection rule[12]). As we shall soon show, this constraint is intimately connected to the s-u crossing invariance constraint mentioned above.

Define P to represent generically the 0^- mesons π, η, K, η', and let a, b, c, d be specific members of this set. Then one can show very generally, assuming SU(3) and duality for the reaction $PP \to PP$, that the single Regge-pole exchange amplitude for the process $a+b \to c+d$ is

$$\langle cd|T^{(s)}|ab\rangle = [\langle a\tilde{c}b\tilde{d}\rangle + \langle c\tilde{a}\tilde{d}b\rangle + (\langle \tilde{c}ab\tilde{d}\rangle + \langle a\tilde{c}\tilde{d}b\rangle)e^{-i\pi\alpha}] \times \beta(t)s^{\alpha(t)}\,, \tag{1}$$

where $a, b, \tilde{c}, \tilde{d}$ are the 3×3 matrices defined in Ref. 1 ($\tilde{c}$ is the transpose of c), the brackets imply taking a trace,[13] $\beta(t)$ is the residue function, and $\alpha(t)$ is the exchange-degenerate trajectory for either ρ-A_2-ω-f or K^*-K^{**}. The traces may be represented by duality diagrams[14] as shown in Fig. 2.

The *RR* cuts contribute to PP scattering via "double-scattering" duality diagrams such as those shown in Fig. 3. To evaluate these diagrams it is necessary to sum over a complete set of

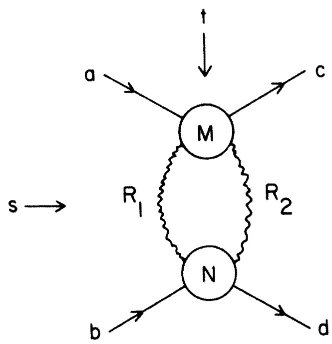

FIG. 1. The RR-cut contribution to the process $a+b \to c+d$. M and N represent particle-Reggeon scattering amplitudes.

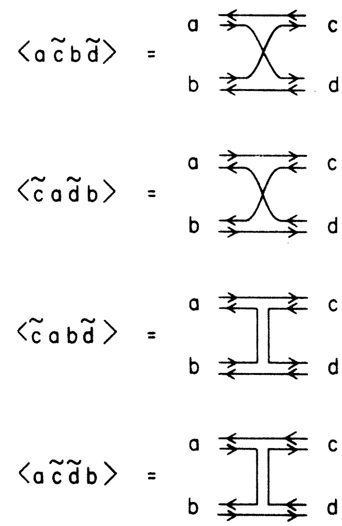

FIG. 2. The duality diagrams corresponding to the traces in Eq. (1).

intermediate states, labeled m, n in Fig. 3. The result depends crucially, of course, on what set of states is included—in particular, whether the sum is over an octet or a nonet. We account for the resulting uncertainty via a parameter δ multiplying the difference between the nonet and octet results in the following equations; for $\delta=0$ ($\delta=1$) the results correspond to summing over a nonet (octet) of intermediate states.

The SU(3) and duality properties of the RR cuts in $PP \to PP$ may now be calculated using Eq. (1) and the formulas

$$\sum_n \langle \tilde{n}a\rangle\langle nb\rangle = \langle ab\rangle - \tfrac{1}{3}\delta\langle a\rangle\langle b\rangle\,,$$
$$\sum_n \langle \tilde{n}anb\rangle = \langle a\rangle\langle b\rangle - \tfrac{1}{3}\delta\langle ab\rangle\,. \tag{2}$$

The results are

$$\sum_{n,m}\langle a\tilde{m}b\tilde{n}\rangle\langle \tilde{c}m\tilde{d}n\rangle = \langle a\tilde{d}\rangle\langle b\tilde{c}\rangle - \tfrac{1}{3}\delta\langle ab\tilde{c}\tilde{d}\rangle - \tfrac{1}{3}\delta\langle ba\tilde{d}\tilde{c}\rangle + \tfrac{1}{9}\delta\langle ab\rangle\langle\tilde{c}\tilde{d}\rangle\,,$$
$$\sum_{n,m}\langle \tilde{m}a\tilde{n}b\rangle\langle m\tilde{c}n\tilde{d}\rangle = \langle a\tilde{d}\rangle\langle b\tilde{c}\rangle - \tfrac{1}{3}\delta\langle ba\tilde{d}\tilde{c}\rangle - \tfrac{1}{3}\delta\langle ab\tilde{c}\tilde{d}\rangle + \tfrac{1}{9}\delta\langle ab\rangle\langle\tilde{c}\tilde{d}\rangle\,,$$
$$\sum_{n,m}\langle \tilde{m}ab\tilde{n}\rangle\langle m\tilde{c}\tilde{d}n\rangle = \langle ab\rangle\langle\tilde{c}\tilde{d}\rangle - \tfrac{2}{3}\delta\langle ab\tilde{c}\tilde{d}\rangle + \tfrac{1}{9}\delta\langle ab\rangle\langle\tilde{c}\tilde{d}\rangle\,,$$
$$\sum_{n,m}\langle a\tilde{m}\tilde{n}b\rangle\langle \tilde{c}mn\tilde{d}\rangle = \langle ab\rangle\langle\tilde{c}\tilde{d}\rangle - \tfrac{2}{3}\delta\langle ba\tilde{d}\tilde{c}\rangle + \tfrac{1}{9}\delta\langle ab\rangle\langle\tilde{c}\tilde{d}\rangle\,. \tag{3}$$

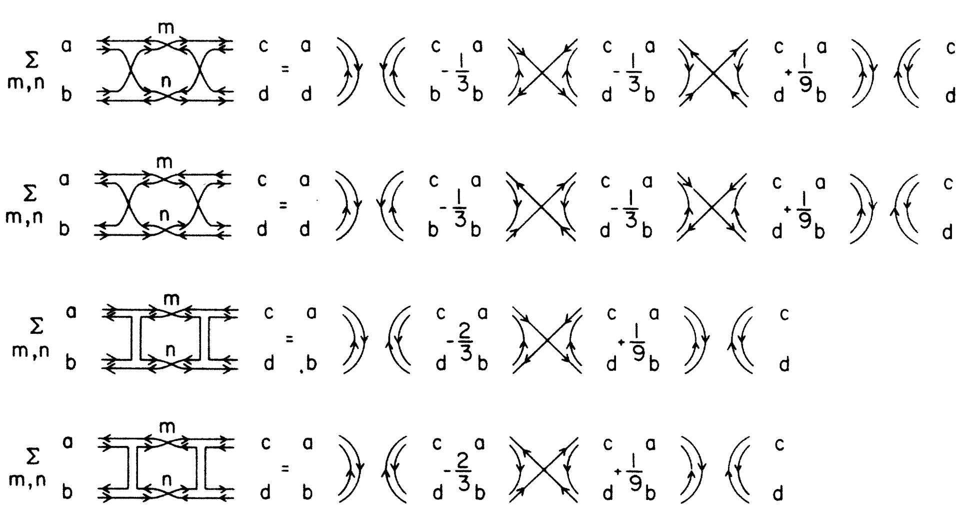

FIG. 3. The RR-cut contributions to the process $a+b \to c+d$ for PP scattering. The figures illustrate each equation in Eqs. (3), corresponding top to bottom with the text.

These four RR-cut terms are represented by the diagrams shown in Fig. 3. We see that the total RR-cut amplitude satisfies s-u crossing and Finkelstein's selection rule, but is probably not strictly dual to the Pomeron unless we sum over a *nonet* of 0^- mesons in the s-channel intermediate states [in which case $\delta = 0$, and only the leading terms survive in Eqs. (3)].

We may extend the calculation by defining B to represent the $\frac{1}{2}^+$ octet containing the proton and consider the reaction $PB \to PB$. Then the equation for the single Regge-pole exchange amplitude, analogous to Eq. (1), is

$$\langle cd|T^{(s)}|ab\rangle = \{[\langle a\tilde{c}b\tilde{d}\rangle(d+f) + (\langle a\tilde{c}\tilde{d}b\rangle - \langle a\tilde{c}\rangle\langle b\tilde{d}\rangle)(d-f)] + [\langle \tilde{c}ab\tilde{d}\rangle(d+f) + (\langle \tilde{c}a\tilde{d}b\rangle - \langle a\tilde{c}\rangle\langle b\tilde{d}\rangle)(d-f)]e^{-i\pi\alpha}\}\, s^{\alpha(t)}, \quad (4)$$

where $d(t)$ and $f(t)$ are independent residue functions. The traces may be represented by duality diagrams as shown in Fig. 4.

We proceed under the same restrictions as above to calculate the four RR-cut contributions in PB scattering,

$$\sum_{m,n}\langle a\tilde{m}b\tilde{n}\rangle(\langle \tilde{c}m\tilde{d}n\rangle - \langle m\tilde{c}\rangle\langle n\tilde{d}\rangle) = \langle a\tilde{d}\rangle\langle b\tilde{c}\rangle - \langle a\tilde{c}b\tilde{d}\rangle - \tfrac{1}{3}\langle ba\tilde{d}\tilde{c}\rangle,$$

$$\sum_{m,n}(\langle \tilde{m}a\tilde{n}b\rangle - \langle a\tilde{m}\rangle\langle b\tilde{n}\rangle)\langle m\tilde{c}n\tilde{d}\rangle = \langle a\tilde{d}\rangle\langle b\tilde{c}\rangle - \langle a\tilde{c}b\tilde{d}\rangle - \tfrac{1}{3}\langle ab\tilde{c}\tilde{d}\rangle,$$

$$\sum_{m,n}(\langle a\tilde{m}\tilde{n}b\rangle - \langle a\tilde{m}\rangle\langle b\tilde{n}\rangle)\langle \tilde{c}mn\tilde{d}\rangle = \langle ab\rangle\langle \tilde{c}\tilde{d}\rangle - \langle \tilde{c}ab\tilde{d}\rangle - \tfrac{1}{3}\langle ba\tilde{d}\tilde{c}\rangle, \quad (5)$$

$$\sum_{m,n}\langle \tilde{m}ab\tilde{n}\rangle(\langle m\tilde{c}\tilde{d}n\rangle - \langle m\tilde{c}\rangle\langle n\tilde{d}\rangle) = \langle ab\rangle\langle \tilde{c}\tilde{d}\rangle - \langle \tilde{c}ab\tilde{d}\rangle - \tfrac{1}{3}\langle ab\tilde{c}\tilde{d}\rangle.$$

These four RR-cut terms are represented by the diagrams shown in Fig. 5. In calculating Eqs. (5) we have summed over a nonet m of 0^- mesons and an octet n of $\frac{1}{2}^+$ baryons. If both sums are over octets, then both s-u crossing and Finkelstein's selection rule are violated, i.e., a diagram contributes that is not a planar s-u diagram. We can see from Fig. 5 that the t-channel RR-cut amplitude is dual to s- and u-channel resonances, not to a Pomeron. This does not, however, rule out the possibility that the t-channel Pomeron is dual to other RR cuts (background) in other channels.[15] If needed, additional formulas can be derived for meson-decouplet and baryon-antibaryon reactions, as well as for backward meson-baryon scattering.

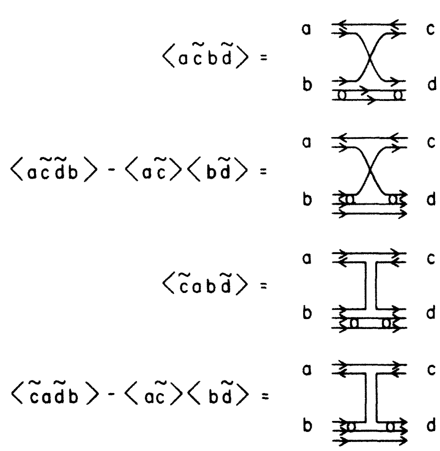

FIG. 4. The duality diagrams corresponding to the traces in Eq. (4).

III. PHENOMENOLOGICAL CONSEQUENCES OF THE RR-CUT MODEL

In phenomenological applications, the numbers resulting from Eqs. (3) or (5) are the coefficients of fundamentally unknown dynamical functions of s and t, which are generally approximated by the standard convolution formula[7] for Regge-pole exchange amplitudes. At the present time we wish

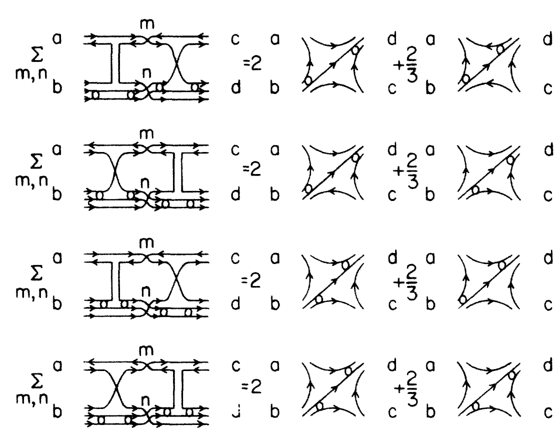

FIG. 5. The RR-cut contributions to the process $a + b \to c + d$ for PB scattering. The figures illustrate each equation in Eqs. (5), corresponding top to bottom with the text.

to consider only the helicity-nonflip amplitude for PB scattering, and in this circumstance the first and third equations in Eqs. (5) multiply equal functions and the second and fourth equations multiply equal functions. This fact allows for a cancellation whenever the first and third equations in Eqs. (5) are equal and opposite and the second and fourth equations are equal and opposite. As noticed first by Worden,[16] this cancellation actually occurs in $\pi^- p \to \pi^0 n$.

As noted by Finkelstein,[12] there are no RR cuts in PB scattering unless a planar s-u diagram exists for the reaction. This rule is made obvious by looking at Fig. 5, where only planar s-u diagrams contribute to the RR cuts. This rules out RR cuts in any KN elastic or CEX reaction, since the initial and final kaons must share a λ quark.

Line-reversed pairs, such as the HCEX reactions $\pi^+ p \to K^+\Sigma^+$, $K^- p \to \pi^-\Sigma^+$ and $\pi^- p \to K^0\Lambda$, $K^- n \to \pi^-\Lambda$, have equal RR cuts. These cuts have neither a purely real nor a rotating phase ($e^{-i\pi\alpha}$), so they can contribute to the line-reversal breaking observed in the above-mentioned reactions. Relative to the RP cuts, however, the RR cuts are probably too small to be of much influence for phenomenology, since the data on the relevant reactions are relatively imprecise.

On the other hand, DCEX reactions such as $\pi^- p \to K^+\Sigma^-$, $K^- p \to \pi^+\Sigma^-$, and $K^- p \to K^+\Xi^-$ presumably[3] do offer a chance to observe RR-cut amplitudes, since the reactions are forbidden to occur by single Regge-pole exchange. The best data[4] exist for 0° scattering over a range of energies. We have used Eqs. (5) and our PB amplitudes, which are known to agree with a large amount of PB data, to calculate the RR-cut contributions to each of the three exotic reactions mentioned above. The results are shown in Fig. 6.

The data for all three DCEX reactions show a precipitous s^{-10} behavior for incident lab momenta below about 3 GeV/c, and a gentler s dependence from 3 to 5 GeV/c, where unfortunately the data stop. No RR-cut model attempts to explain the s^{-10} behavior, which is presumably due to direct-channel resonance effects, but the high-energy behavior is expected[3] to be given approximately by

$$\frac{d\sigma}{dt} \sim s^{2\alpha_c - 2},$$

leading to the predictions $s^{-2.2}$ for the $\pi^- p \to K^+\Sigma^-$ and $K^- p \to \pi^+\Sigma^-$ forward cross sections and $s^{-2.6}$ for the $K^- p \to K^+\Xi^-$ forward cross section. As can be seen in Fig. 6, existing data are quite consistent with such predictions for the range 3–5 GeV/c.

We have used the amplitudes obtained in our previous PB work to generate the RR-cut amplitudes corresponding to the diagrams described by Eqs. (5). The results are in qualitative agreement with the data; by including a cut strength $\lambda_c \approx 0.7$, we can bring the model into good quantitative agreement with all three reactions, as shown in Fig. 6. We would like to emphasize that our PB amplitudes were originally determined by fitting data typically four orders of magnitude larger than the exotic cross section data, and therefore the agreement within a factor 0.7 achieved by the present calculation is quite remarkable.

The relative magnitudes of the three DCEX reactions are given primarily by the SU(3) factors calculated using Eqs. (5). The data agree well with the dual-SU(3) result that the cross sections for $\pi^- p \to K^+\Sigma^-$ and $K^- p \to \pi^+\Sigma^-$ are about equal, while the cross section for $K^- p \to K^+\Xi^-$ is somewhat larger than those cross sections.

Phenomenologically, our model results do not provide a test to decide whether one should sum over a nonet or an octet of mesons in Eqs. (5) and Fig. 5. As we remarked in Sec. II, summing over an octet does provide some theoretical problems. Some ambiguity in the magnitudes at the 20% level should be expected anyway, since there is every reason to expect vacuum absorption to modify our amplitudes. Presently available data do not encourage such a detailed calculation. It

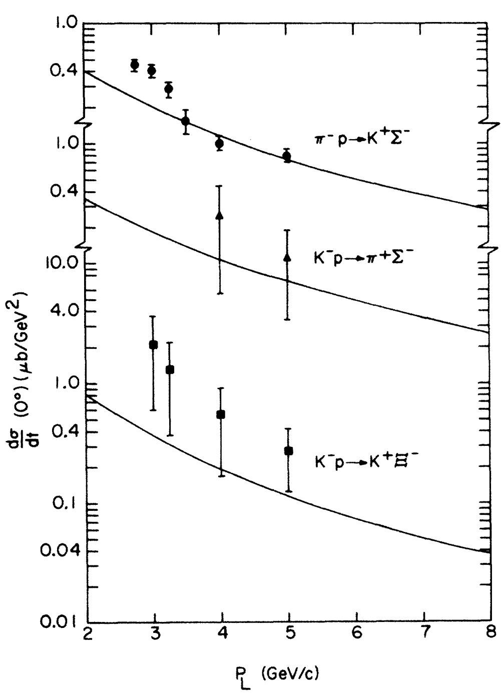

FIG. 6. Differential cross sections at zero degrees and model results for DCEX reactions.

is, however, quite clear that one cannot calculate the *RR* cuts in this dual-SU(3) scheme by including all possible terms arising by convoluting the single exchange amplitude Eq. (4) with itself, in the fashion of the old absorption model.[17] In such a case, one finds a Ξ cross section much *smaller* than either Σ cross section, in clear contradiction to the data.

We conclude that the model presented is useful both phenomenologically and as a means for studying the duality and SU(3) properties of *RR* cuts. We have shown how to calculate *RR* cuts in terms of previously determined amplitudes in a manner that satisfies both theoretical and experimental constraints.

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PHYSICAL REVIEW D VOLUME 10, NUMBER 5 1 SEPTEMBER 1974

Scaling in inelastic electron-photon scattering*

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(Received 20 May 1974)

We study moments of the cross section for inelastic electron-photon scattering. Simple scaling laws would result if the real photon were a particle which behaves predominantly like a light vector meson. We discuss possible modifications of these scaling laws originating in anomalies of the structure functions of the photon; they occur in quark models as well as in generalized vector-meson-dominance models.

I. INTRODUCTION

Deep inelastic electroproduction experiments have shown a local scaling law to hold for the electromagnetic structure functions of the nucleon.[1] For sufficiently large mass squared of the virtual photon, Q^2, and sufficiently high c.m. energy W of the photon-nucleon system, the structure functions depend only on the dimensionless ratio,[2] $2\nu/Q^2 = 1 + (W^2 - m_{\rm nucl}{}^2)/Q^2$. The same types of inclusive experiments with (anti-) neutrino beams have proved the existence of a global scaling law: The total (anti-) neutrino-nucleon cross sections rise linearly with neutrino energy,[3] even for very low beam energies. The early onset of this type of scaling can be explained by Bloom-Gilman duality[4]: The W^2-dependent bumps in the structure functions are smoothed out by integrating them over a set of resonances. This way, a smoothly behaved total cross section results which exhibits its asymptotic properties already in the resonance region.

One might ask if similar simple scaling laws show up in the hadron production channels of inelastic electron-photon scattering, which will soon be accessible in electron-positron (electron-electron) colliding-beam machines. These experiments, proposed some time ago by Brodsky, Kinoshita and Terazawa, Walsh, and Carlson and Tung,[5] reveal the internal structure of the photon itself. Considering the real photon merely as a light vec-