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[9]The actual computation was done with the sum in Eq. (8) stopped at $\nu = 7$. This makes the result lower than the exact value, but the effect is in general very small, becoming slightly more important for very high multiplicities.

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Optical activity for neutrons

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When a neutron passes through an optically active medium, the transverse component of its polarization should precess around the direction of propagation while the longitudinal component should increase (or decrease) monotonically. For a representative medium, in a typical case, the rotatory power for neutron polarization is expected to be of order 10^{-5} cm^{-1}. The rate of acquisition of longitudinal polarization is also expected to be of similar magnitude, and the γ rays associated with thermal neutron capture should have circular polarization of order 10^{-7}.

It has been known for a long time that plane-polarized light, while traversing certain media which are said to be optically active, suffers a rotation of its plane of polarization by an amount and in a sense (right or left) which is characteristic of the medium. This seemingly arbitrary preference of a medium for one handedness over another is due not to any asymmetry of the laws of physics but, as shown by Pasteur, to the handed structure of the constituents of the medium. Since there is a one-to-one correspondence between the states of polarization of a light beam and those of spin-$\frac{1}{2}$ particles, one expects that similar effects should occur when polarized spin-$\frac{1}{2}$ particles[1] traverse an optically active medium.[2] For definiteness, we shall treat the case of slow neutrons transmitted through an optically active fluid,[3] i.e., one made up of handed molecules, and find that the detection of the phenomenon should be within reach of present experimental techniques.

The analog of optical activity for neutrons is the rotation of the neutron polarization, by an amount proportional to the distance traversed, about the direction of propagation $\hat{k}$. Such an effect requires that the mean forward scattering amplitudes f_R and f_L of longitudinally polarized neutrons be unequal for the two opposite helicities $(\vec{\sigma}\cdot\hat{k}) = \pm 1$. The refractive index for neutrons of wavelength $\lambda = 2\pi/k$ in a medium comprising a number density N of scatterers with a mean forward-scattering amplitude f is

$$n = 1 + \frac{2\pi N}{k^2} f \tag{1}$$

for $Nf \ll k^2$, a condition which is well satisfied for thermal neutrons in matter of normal densities. The rotatory power, viz., the amount by which the transverse component of the neutron spin precesses while traversing unit distance, is then

$$\Phi = \lambda N \mathrm{Re}(f_L - f_R) = \lambda N \,\mathrm{Re} f_S , \tag{2}$$

where f_S is the spin-dependent part, proportional to $-\frac{1}{2}(\vec{\sigma}\cdot\hat{k})$, of the mean forward-scattering amplitude for neutrons from the molecules of the medium. In addition, the neutrons will acquire a degree of longitudinal polarization in traversing a distance x, amounting to

$$P_L(x) = \tanh(\lambda N x \,\mathrm{Im} f_S) . \tag{3}$$

For small x, this is proportional to x, i.e., the acquired polarization is

$$\Psi = \lambda N \,\mathrm{Im} f_S \tag{4}$$

per unit distance, for $x \ll \Psi^{-1}$. Conversely, there will be differential absorption of longitudinally polarized neutrons with opposite helicities. This is the analog of the Cotton effect.

With parity-conserving interactions,[4] a difference f_S between the forward scattering amplitudes for left-handed and right-handed neutrons can arise only if the medium contains molecules having a

handed structure (i.e., which do not possess a center of symmetry) and an interaction is present which depends both on the neutron spin and on the molecular handedness. As a model of a handed molecule, we shall consider a spinless nucleus bound in a twisted potential well chosen by Condon[5] to have the form of an anisotropic harmonic-oscillator potential perturbed by a twisted anharmonicity xyz. To first order in the perturbation, the ground state $|A\rangle$ of this molecule is then a linear superposition of the ground state $|n_x=n_y=n_z=0\rangle$ of the anisotropic harmonic oscillator and an admixture ρ of the opposite-parity state $|n_x=n_y=n_z=1\rangle$:

$$|A\rangle = |000\rangle + \rho|111\rangle . \tag{5}$$

Condon's model should provide a reasonably accurate description for scattering of slow neutrons by a nucleus occupying an optically active site in an actual molecule. The two mechanisms which we have considered for inducing a spin-dependent contribution to the forward scattering amplitude of neutrons are (i) the spin-orbit interaction arising from the motion of the neutron magnetic moment through the Coulomb field of the nucleus, which is bound in a twisted potential well, and (ii) the corresponding interaction arising from the motion of the compound nucleus (of spin $\frac{1}{2}$), formed by resonant absorption of a slow neutron, in the twisted molecular potential before re-emitting the neutron.

The first mechanism is present for neutrons of any energy and leads in general[6] to a value of f_S, for fixed molecular orientation, which is of third order in the neutron-nucleus scattering amplitude f:

$$f_S^{(3)} \sim (ka)(f/a)^2 \rho \xi_S f , \tag{6}$$

where $\xi_S f$ is the spin-dependent part of the neutron-nucleus scattering amplitude and a is a length characteristic of the size of the region within which the scattering nucleus is confined; it has been tacitly assumed that ρ, the parameter which represents the degree to which the molecular wave function (5) describes a twisted configuration, is of order unity. Substituting representative values of the other parameters, we find that $f_S^{(3)}$ is at most of order 10^{-21} cm for thermal neutrons, yielding according to Eq. (2) rotatory power of no more than 10^{-7} cm^{-1} in order of magnitude. A further difficulty is that $f_S^{(3)}$, and the corresponding rotatory power, disappear when we average over all molecular orientations as we must do in the case of usual fluids. For an assembly of randomly oriented molecules, the rotatory power is of fourth order and presumably smaller by a further factor of $(f/a) \approx 10^{-3}$.

The second mechanism requires the energy of the incident neutron to be close to that of a resonance of the compound nucleus. For an s-wave resonance, the polarization of the neutron is transferred *in toto* to the compound nucleus. The motion of the compound nucleus in the twisted molecular potential can turn this polarization because of the coupling $H_{SO} = h(r)\vec{L}\cdot\vec{S}$ between the spin $\vec{S}$ of the compound nucleus and its orbital angular momentum $\vec{L}$. Consequently, when the neutron is re-emitted, its spin will also have been turned. For this mechanism to be effective, the compound nucleus must move sufficiently to be aware that it is subject to a handed constraint. This requires that the nuclear lifetime be at least comparable to the period of molecular vibrations. As explained earlier, the effects of interest are proportional to the μ-dependent part t_μ of the forward scattering amplitude $f_\mu = \langle \alpha; k\mu | T | \alpha; k\mu \rangle$ for neutrons of momentum $\hbar k$ and helicity μ on a target α. We suppose that the nuclear state (except for its spin orientation) depends so little on its molecular coordinates that the state vector $|\alpha\rangle$ can be taken as the product of a nuclear part $|i\rangle$ which depends only on intrinsic nuclear coordinates and a molecular part $|A\rangle$ which depends only on molecular coordinates. The effect we wish to calculate is given, near a resonance, by

$$t_\mu = \sum_C \sum_B \frac{\langle A|g^* e^{-i\vec{k}\cdot\vec{r}}|C\rangle\langle C\mu|H_{SO}|B\mu\rangle\langle B|g e^{i\vec{k}\cdot\vec{r}}|A\rangle}{[E-(E_R - i\Gamma/2)-E_C][E-(E_R-i\Gamma/2)-E_B]}, \tag{7}$$

where E is the neutron energy, E_R and Γ are the position and width of the resonance corresponding to the nuclear state $|j\rangle$ formed by neutron absorption, and E_B is the excitation energy of the molecule in the state $|B\rangle$. Each of the summations on B and C extends over all the eigenstates of the twisted Condon oscillator. Since H_{SO} does not act on internal nuclear coordinates, it does not affect the intrinsic nuclear state $|j\rangle$; we have also made use of the fact that, for an s-wave resonance, the spin of the absorbing (emitting) nuclear state must coincide with that of the incident (emitted) neutron. g is the amplitude for neutron absorption to the resonant nuclear state $|j\rangle$. $f_S = t_L - t_R$ is then obtained by simply replacing H_{SO} in Eq. (7) by $h(r)$ times L_k, the component of $\vec{L}$ along the direction of neutron propagation. Expanding the exponentials, we find to lowest order in k

$$f_S = \frac{|g|^2}{E-(E_R-i\Gamma/2)} \sum_B \frac{2\,\mathrm{Im}[\langle A|h(r)L_k|B\rangle\langle B|\vec{k}\cdot\vec{r}|A\rangle]}{E-(E_R-i\Gamma/2)-E_B} . \tag{8}$$

The common factor is just the resonant neutron-nucleus scattering amplitude, f_R say. Since we are interested in thermal neutron resonances, we

have to do with neutrons which are in general not energetic enough to excite *molecular* resonances. It will then be seen that f_S will be largest when (a) the neutron energy coincides with a nuclear resonance *and* (b) the width of the resonance is less than or equal to the energy of the excited molecular states which contribute most to the sum in Eq. (8). We have thus confirmed the two conditions, deduced from physical arguments at the beginning of this paragraph, for nuclear resonant scattering to contribute significantly to neutron optical activity. The sum in Eq. (8) is formally quite similar to the corresponding expression[7,5] for ordinary optical activity and explicitly demonstrates that there can be no effect when the molecular states are eigenstates of parity. Since the two factors in the numerator of the summand are matrix elements of an even- and an odd-parity operator, respectively, both factors can be nonzero only if the eigenstates $|A\rangle$ or $|B\rangle$ have indefinite parity. Equation (8) also shows explicitly that f_S is proportional to k, a result analogous to the Born-Oseen-Gray theorem[5] which states that optical activity disappears in the limit $k \to 0$.

The choice (5) for the molecular ground state considerably simplifies the evaluation of Eq. (8) since r_k and L_k simultaneously have nonvanishing matrix elements between $|A\rangle$ and only a relatively small number of anisotropic oscillator states. For neutrons incident along the z axis of the Condon oscillator, Eq. (8) yields, at resonance,

$$(f_S)_Z = \tfrac{1}{4}\rho f_R\left(\frac{ke^2}{mc^2}\right) g_R Z F_E(\xi,\eta;\zeta), \tag{9}$$

where ξ, η, ζ are the angular frequencies for the principal axes of the anisotropic oscillator; Z and g_R are the atomic number and nuclear g factor of the compound nucleus, respectively, while m is the nucleon mass and

$$F_E(\xi,\eta;\zeta) = \frac{\hbar^2\zeta(\xi-\eta)\langle 110|\chi(r)|110\rangle}{[E-E_R+\hbar\zeta+i\Gamma/2][E-E_R+\hbar(\xi+\eta)+i\Gamma/2]} \tag{10}$$

is a dimensionless function of ξ, η, ζ obtained by setting

$$h(r) = \left(g_R\frac{e\hbar}{4mc}\right)\left(\frac{Ze\hbar}{2Mc}\right)\left(\frac{2M^3\xi\eta\zeta}{\hbar^3}\right)^{1/2}\chi(r),$$

where M is the mass of the compound nucleus. For an assembly of randomly oriented molecules, we must average f_S over all directions of incidence of the neutron relative to the anisotropic oscillator, obtaining

$$\bar{f}_S = \tfrac{1}{12}\rho f_R\left(\frac{ke^2}{mc^2}\right) g_R Z[F_E(\xi,\eta;\zeta)+F_E(\eta,\zeta;\xi) + F_E(\zeta,\xi;\eta)]. \tag{11}$$

Note that, since $F_E(\alpha,\beta;\gamma)$ is an antisymmetric function of α and β, $\bar{f}_S$ vanishes identically when any two of the frequencies ξ, η, ζ coincide, in agreement with the corresponding result for the optical case.[5] This is a requirement of symmetry because if $\xi = \eta$, for example, the potential (including the Condon perturbation) corresponding to (5) has $x = y$ as a plane of symmetry and therefore no twist, and the handedness of the molecule has disappeared. For a numerical *estimate* of the magnitude of the expected effect, we take $\chi(r) \propto r^{-3}$ and evaluate its expectation value for an isotropic harmonic oscillator of angular frequency ω since the corresponding value for an anisotropic oscillator cannot be expressed in a simple algebraic form. To lowest order in $\mathfrak{A}$, we find, near a resonance,

$$\bar{f}_S = \frac{1}{45}\left(\frac{2}{\pi}\right)^{1/2} f_R\left(\frac{ke^2}{mc^2}\right) g_R Z\mathfrak{A}, \tag{12}$$

where we have defined an anisotropy factor

$$\mathfrak{A} = \frac{(\xi-\eta)(\xi-\zeta)(\eta-\zeta)}{(\xi+\eta)(\xi+\zeta)(\eta+\zeta)}\rho$$

and we have neglected $(E - E_R)$ and Γ in comparison with molecular vibrational energies and used

$$\langle 110|r^{-3}|110\rangle = \frac{8}{15\sqrt{\pi}}\left(\frac{M\omega}{\hbar}\right)^{3/2},$$

setting $\omega = (\xi\eta\zeta)^{1/3}$. Combining Eqs. (12) and (2), we obtain

$$\Phi = \frac{2\sqrt{2\pi}}{45} N\left(\frac{e^2}{mc^2}\right) g_R Z\mathfrak{A}\,\mathrm{Re} f_R. \tag{13}$$

The absence of Planck's constant from this formula shows that the phenomenon of spin rotation does not depend on quantum effects, and illustrates the fact that spin polarization is amenable to a purely classical description.[8] Taking $g_R\mathfrak{A}$ to be of order unity for a case in which $f_R \approx 10^{-12}$ cm, $N \approx 10^{22}$ cm^{-3}, we find for a heavy nucleus that Φ, and similarly Ψ, is of order 10^{-5} cm^{-1}, i.e., the rotatory power is a few seconds of arc per cm.

We also note that, in addition to the analogs of optical activity described above, there will be another apparently parity-violating effect. We have seen that, in a handed medium, the absorption of left-handed and right-handed neutrons is not equally likely; consequently, even if the incident neu-

trons are unpolarized, the compound nucleus is expected to be longitudinally polarized to a degree f_S/f_R of order 10^{-7} for thermal neutrons in the case considered. Consequently, any secondary radiation emitted in the course of nuclear de-excitation will be polarized to a similar extent, in a direction determined by that of the incident neutrons. Since many compound nuclear resonances are highly inelastic, preferring to decay by γ emission, the circular polarization of the capture γ rays, which is of similar magnitude to that arising from parity-nonconserving nuclear forces, may also be worth measuring.[9]

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[1]For higher spins, similar *and* more complicated effects may occur.

[2]Professor J. A Janik kindly referred us to the work of V. G. Baryshevskii, Yad. Fiz. 4, 72 (1966) [Sov. J. Nucl. Phys. 4, 51 (1967)], of which we were ignorant. While Baryshevskii noted the existence of the effect, his estimate of its possible magnitude is much smaller than ours. It was based on an approximation which, if evaluated correctly, should yield no effect at all. See Ref. 6 below.

[3]Similar effects are expected in a crystalline medium with a handed structure, analogous to the optical activity of quartz. Polarized-neutron scattering from a lattice without inversion symmetry was discussed by F. Obermair [Z. Phys. 204, 215 (1967)], who did not, however, consider the effect discussed in this note.

[4]The neutron spin rotation due to parity-nonconserving nuclear forces has been considered by F. C. Michel [Phys. Rev. 133, B329 (1964)], who estimated the magnitude of the effect to be 10^{-8} cm^{-1}. We thank R. R. Lewis for this reference.

[5]E. U. Condon, Rev. Mod. Phys. 9, 432 (1937).

[6]Further details may be found in P. Kabir, G. Karl, and E. Obryk, Can. J. Phys. (to be published).

[7]L. Rosenfeld, Z. Phys. 52, 161 (1928).

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[9]The circular polarization of γ rays arising from this mechanism should reverse sign when the handedness of the medium (or its constituents) is reversed, whereas that arising from parity nonconservation has a fixed sign for a given γ transition.